CSC242: Introduction to Artificial Intelligence

Lecture 2.4

Please put away all electronic devices

Announcements

- Project 2 due Sunday 1159PM

- Unit 2 Exam: One week from Thursday (Fall Break next Mon/Tue)

Factored Representation

- Splits a state into variables (or attributes) that can have values

- Factored states can be more or less similar (unlike atomic states)

- Can also represent uncertainty (don't know value of some attribute)

Constraint Satisfaction Problem (CSP)

- X: Set of variables $\{ X_1, \ldots, X_n \}$
- D: Set of domains $\{ D_1, \ldots, D_n \}$

  - Each D_i : set of values $\{ v_1, \ldots, v_k \}$

- C: Set of constraints $\{ C_1, \ldots, C_m \}$
- Solution: Assign to each X_i a value from D_i such that all the C_i are satisfied

Propositional Logic

- Programming language for knowledge
- Factored representation (Boolean CSP)
  - Propositions, connectives, sentences
- Possible worlds, satisfiability, models
- Entailment: $\alpha \models \beta$
  - Every model of α is a model of β
  - Intractable! (co-NP-complete)

Propositional Theorem Proving

- Inference rules: Soundness, Completeness
- Proof: $\alpha \vdash \beta$
    - Searching for proofs is an alternative to enumerating models; "can be more efficient" (at least sometimes)
- Resolution: sound and complete inference rule
    - Works on clauses (CNF); requires refutation
- Forward and backward chaining

Forward Chaining

- Reasons forward from new facts
  - Data-driven
- Done by humans to some extent
  - When to stop?
- For KBs using only definite clauses
  - Sound, complete, linear time

AIMA 7.5.4

Backward Chaining

- Reasons backward from query
  - Goal-directed
- Useful for answering specific questions
- For KBs using only definite clauses
  - Sound, complete, linear time

AIMA 7.5.4

Propositional Inference

- Computing whether $\alpha \models \beta$
- Model Checking
  - Intractable (but see AIMA 7.6)
- Inference rules: Soundness, Completeness
- Derivation: $\alpha \vdash \beta$
  - Searching for proofs is an alternative to enumerating models
    - May be faster in practice
- Resolution is a sound and complete-ish inference rule
  - Works on clauses (CNF), requires refutation proof

PL 👍

- Declarative: based on a truth relation between sentences and possible worlds

PL 👍

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- Expressive: can represent partial information (e.g., disjunction, negation)

PL 👍

- Declarative: based on a truth relation between sentences and possible worlds
- Expressive: can represent partial information (e.g., disjunction, negation)
- Compositional: the meaning of a sentence is a function of the meanings of its parts

PL 👎

- Model checking takes exponential time
  - Theorem proving may help

PL 👎

- Model checking takes exponential time
  - Theorem proving may help
- Lacks the expressive power to describe concisely complex environments (many objects, relationships between them)

$$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$

$$B_{1,2} \Leftrightarrow (P_{1,3} \vee P_{2,2} \vee P_{1,1})$$

$$B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$$

$$B_{2,2} \Leftrightarrow (P_{2,1} \vee P_{3,2} \vee P_{2,3})$$

$$\cdots$$

$$B_{4,4} \Leftrightarrow (P_{3,4} \vee P_{4,3})$$

$$B_{1,1} \Leftrightarrow (P_{1,2} \vee P_{2,1})$$

$$B_{1,2} \Leftrightarrow (P_{1,3} \vee P_{2,2} \vee P_{1,1})$$

$$B_{2,1} \Leftrightarrow (P_{1,1} \vee P_{2,2} \vee P_{3,1})$$

$$B_{2,2} \Leftrightarrow (P_{2,1} \vee P_{3,2} \vee P_{2,3})$$

$$\cdots$$

$$B_{4,4} \Leftrightarrow (P_{3,4} \vee P_{4,3})$$

"Rooms adjacent to pits are breezy"

- Rooms adjacent to pits are breezy
- Socrates is a person
  All people are mortal
- Anybody's grandmother is either their mother's or their father's mother

Logic 2.0

- Define a language based on propositional logic that will allow us to say all these things
- Define entailment ("follows from")
- Figure out how to compute what follows from our knowledge

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Ontology

ontology |änˈtäləjē|
noun
1 the branch of metaphysics dealing with the nature of being.

2 a set of concepts and categories in a subject area or domain that shows their properties and the relations between them

ORIGIN early 18th cent.: from modern Latin ***ontologia***, from Greek ***ōn***, ***ont-*** ***'being'*** + **-logy**.

Ontology

- Objects: people, houses, numbers, theories, Socrates, colors, wars, ...
- Relations:

  - Unary (Properties): breezy, mortal, red, round, bogus, prime, ...
  - n-ary: brother of, bigger than, inside, part of, has color, occurred after, owns, above, between
- Functions: "single-valued" relations: mother of, father of, best friend, one more than, ...

Ontologies (Factored Representations)

- Logic 1.0 (Propositional Logic)
  - Identify Boolean features of the world
- Logic 2.0:
  - Identify objects, relations, and functions in the world

Ontology (Domain of Discourse, Conceptualization)

- Objects: people, houses, numbers, theories, Socrates, colors, wars, ...
- Relations:

  - Unary (Properties): breezy, mortal, red, round, bogus, prime, ...
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- Functions: "single-valued" relations: mother of, father of, best friend, one more than, ...

A Programming Language for Knowledge

- Syntax:
  - What counts as a well-formed statement, formula, sentence, or program
- Semantics:
  - What these statements, formulas, sentences, or programs mean

Propositional Logic: Proposition Symbols

- Symbols denoting propositions (things that can be true of false)
- $W_{1,1}$, $Raining$, $Hungry$, $Cranky$, ...

denote |diˈnōt|

verb [trans.]
be a sign of; indicate *: this mark denotes purity and quality.*
• (often **be denoted**) stand as a name or symbol for *: the level of output per firm, denoted by X.*

Constant Symbols

- Symbols denoting objects in the world
- *Socrates*, *George*, *Snoopy*, ...

denote |diˈnōt|

verb [trans.]
be a sign of; indicate *: this mark denotes purity and quality.*
• (often **be denoted**) stand as a name or symbol for *: the level of output per firm, denoted by X.*

Relation (Predicate) Symbols

- Symbols denoting relations
- $Mortal(\cdot)$, $Smelly(\cdot)$, $Breezy(\cdot)$, $On(\cdot,\cdot)$, $Above(\cdot,\cdot)$, $Equals(\cdot,\cdot)$ a.k.a. "$\cdot = \cdot$", …
- Arity: number of arguments

pred·i·cate
noun | ˈpredəkət |
• *Logic* something that is affirmed or denied concerning an argument of a proposition.
From Latin ***praedicatum*** 'something declared'

Function Symbols

- Symbols denoting functions
- $mother(\cdot)$, $father(\cdot)$, $oneMoreThan(\cdot)$, $hat(\cdot)$, $plus(\cdot,\cdot)$ a.k.a. "$\cdot + \cdot$", ...
- Arity: number of arguments

Symbols

- Constant symbols: $Socrates$, $George$
- Relation symbols: $Mortal(\cdot)$, $On(\cdot,\cdot)$
- Function symbols: $mother(\cdot)$, $plus(\cdot,\cdot)$

Term

- A logical expression that denotes (refers to) an object
- Constant symbol; or
- Function symbol and tuple of terms of appropriate arity

$Socrates$ $mother(George)$

$plus(1,2)$ a.k.a. "$1+2$" $mother(father(George))$

Atomic Sentence

- States a fact
- Predicate (relation) symbol and tuple of terms of appropriate arity

$$Mortal(Socrates)$$

Atomic Sentence

- States a fact
- Predicate (relation) symbol and tuple of terms of appropriate arity

$$Mortal(Socrates)$$

$$On(A, B)$$

$$Brother(Richard, John)$$

$$Married(father(Richard), mother(John))$$

Connectives

- Connect sentences into larger sentences that can also be true or false

- Negation (not): $\neg$
- Conjunction (and): $\wedge$
- Disjunction (or): $\vee$
- Implication (if-then): $\Rightarrow$
- Biconditional (iff): $\Leftrightarrow$

Connectives

$\neg On(A, B)$

$King(Richard) \vee King(John)$

$\neg King(Richard) \Rightarrow King(John)$

Logic 2.0 (Syntax)

- Constant symbols
- Predicate (relation) symbols & arity
- Function symbols & arity
- Terms
- Atomic sentences
- Complex sentences (using connectives)

Predicate Logic

- Constant symbols
- Predicate (relation) symbols & arity
- Function symbols & arity
- Terms
- Atomic sentences
- Complex sentences (using connectives)

First-Order Predicate Logic

- Constant symbols
- Predicate (relation) symbols & arity
- Function symbols & arity
- Terms
- Atomic sentences
- Complex sentences (using connectives)

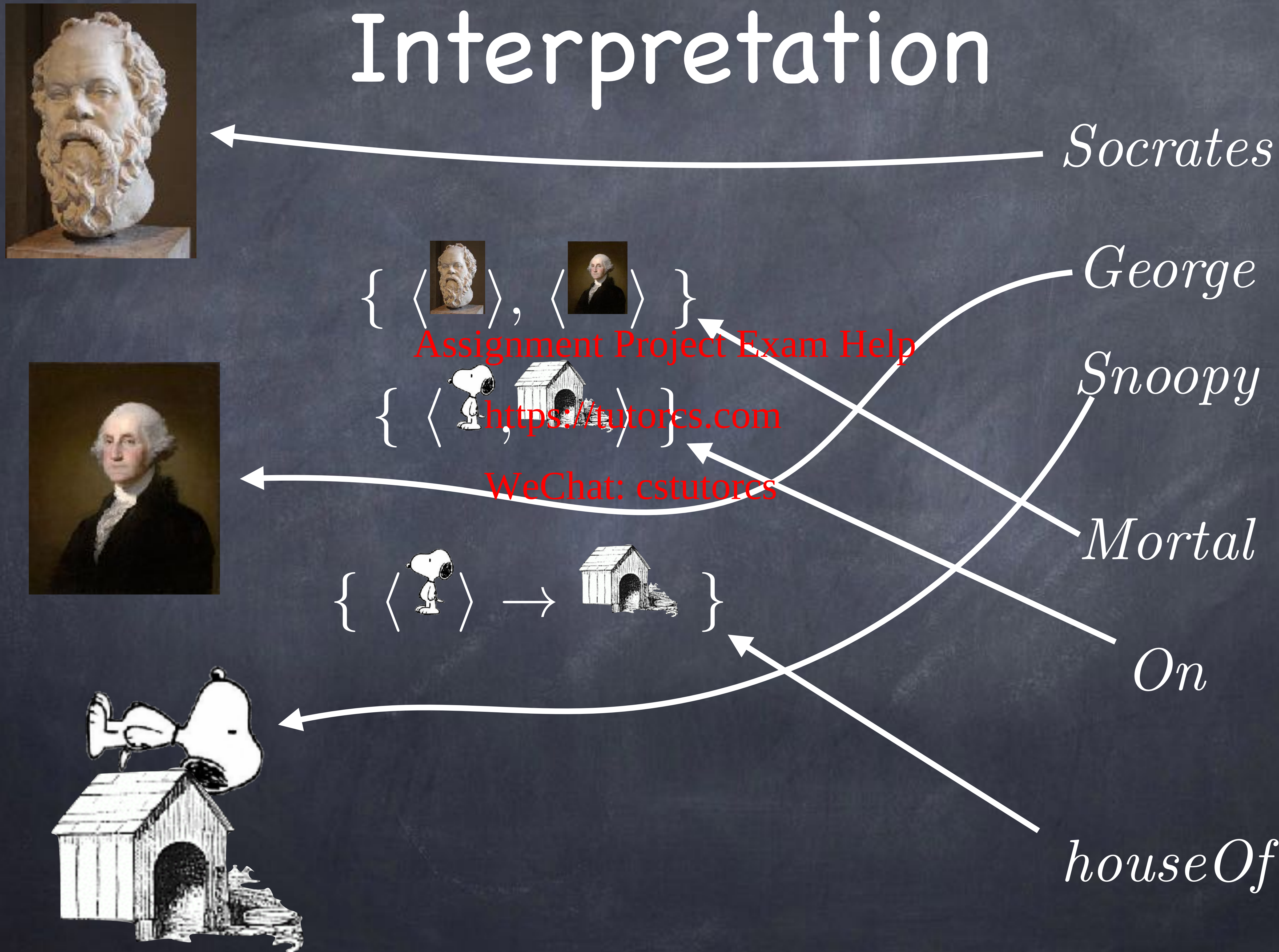
Propositional Logic
Possible World

- Assignment of true or false to all the atomic propositions
- A possible world satisfies a sentence if it makes the sentence true
  - "A model of the sentence"

Ontology (Domain of Discourse, Conceptualization)

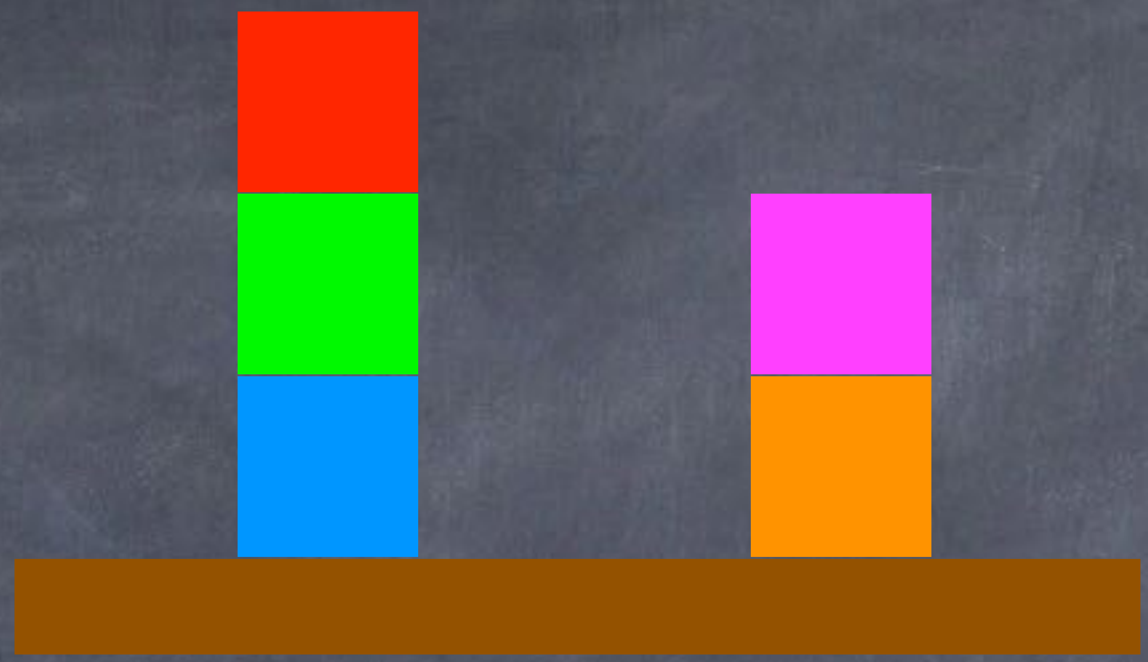
- **Objects**: people, houses, numbers, theories, Socrates, colors, wars, ...
- **Relations**:
  - Unary (Properties): breezy, mortal, red, round, bogus, prime, ...
  - n-ary: brother of, bigger than, inside, part of, has color, occurred after, owns, above, between
- **Functions**: "single-valued" relations: mother of, father of, best friend, one more than, ...

Interpretation



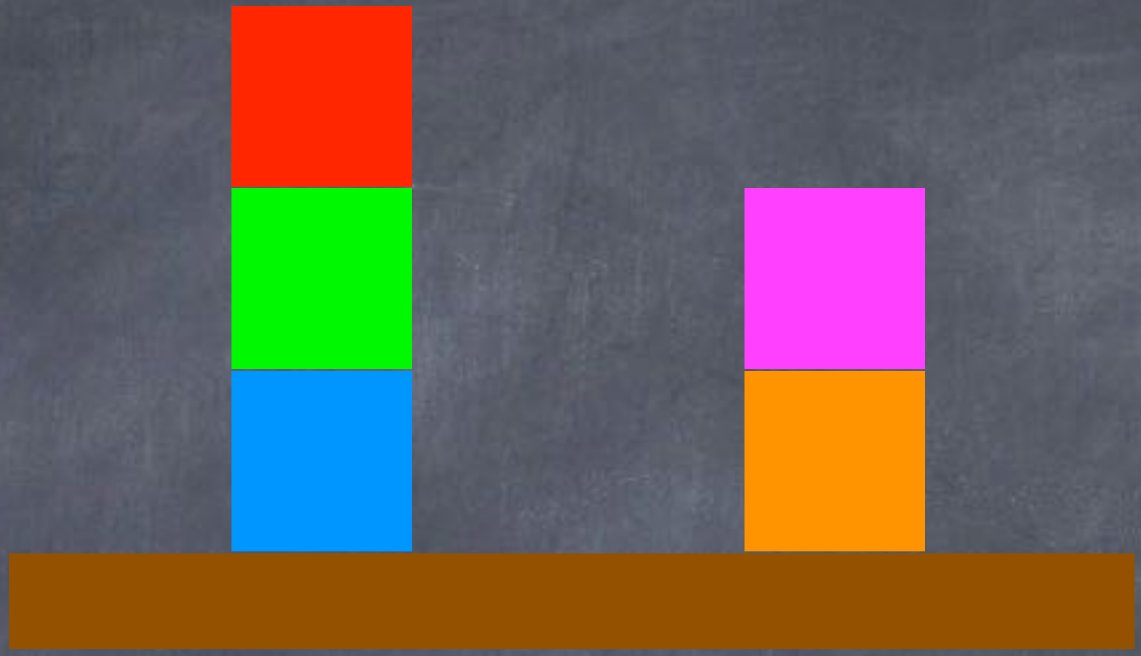
Interpretation

Language Elements		Interpretation I Objects: Ω_I
Constant Symbols	σ	$I(\sigma) \in \Omega_I$
Predicate Symbols	π_n	$I(\pi_n) \subseteq \Omega_I^n$
Function Symbols	ϕ_n	$I(\phi_n) : \Omega_I^n \to \Omega_I$



- Objects: , , , ,

- Relations: being on, being above, being clear, being on the table
- Functions: “the block on top of me”

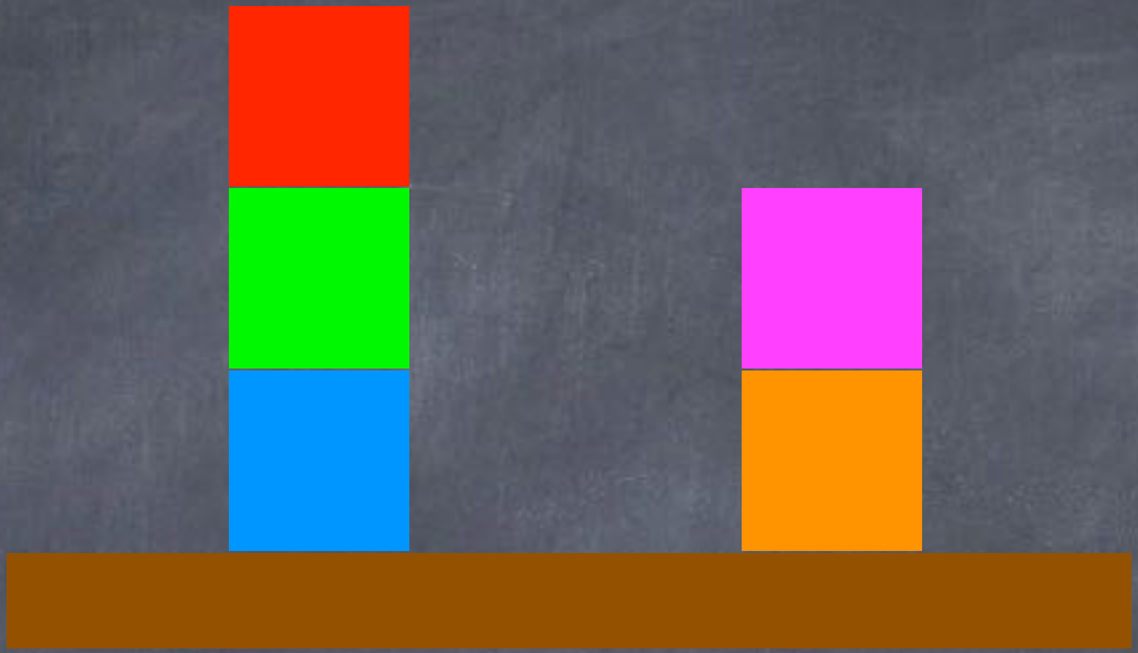


Constant Symbols: A, B, C, D, E

Predicate Symbols: On(·,·), Above(·,·), OnTable(·), Clear(·)

Function Symbols: Hat(·)

$$\Omega_I = \{\ \textcolor{red}{\blacksquare},\ \textcolor{green}{\blacksquare},\ \textcolor{blue}{\blacksquare},\ \textcolor{magenta}{\blacksquare},\ \textcolor{orange}{\blacksquare}\ \}$$



$$I(\texttt{A}) = \textcolor{red}{\blacksquare}$$

$$I(\texttt{B}) = \textcolor{green}{\blacksquare}$$

$$I(\texttt{C}) = \textcolor{blue}{\blacksquare}$$

$$I(\texttt{D}) = \textcolor{magenta}{\blacksquare}$$

$$I(\texttt{E}) = \textcolor{orange}{\blacksquare}$$

$$I(\texttt{On}) = \{\ \langle \textcolor{red}{\blacksquare}, \textcolor{green}{\blacksquare} \rangle,\ \langle \textcolor{green}{\blacksquare}, \textcolor{blue}{\blacksquare} \rangle,\ \langle \textcolor{magenta}{\blacksquare}, \textcolor{orange}{\blacksquare} \rangle\ \}$$

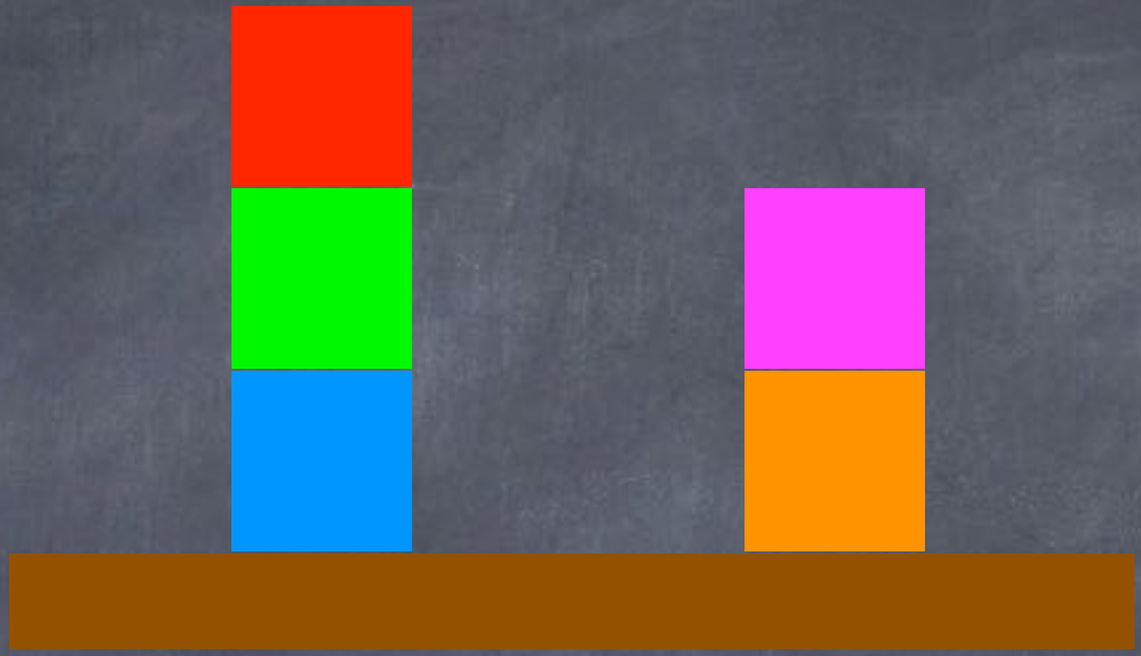
$$I(\texttt{Above}) = \{\ \langle \textcolor{red}{\blacksquare}, \textcolor{green}{\blacksquare} \rangle,\ \langle \textcolor{green}{\blacksquare}, \textcolor{blue}{\blacksquare} \rangle,\ \langle \textcolor{red}{\blacksquare}, \textcolor{blue}{\blacksquare} \rangle,\ \langle \textcolor{magenta}{\blacksquare}, \textcolor{orange}{\blacksquare} \rangle\ \}$$

$$I(\texttt{OnTable}) = \{\ \langle \textcolor{blue}{\blacksquare} \rangle,\ \langle \textcolor{orange}{\blacksquare} \rangle\ \}$$

$$I(\texttt{Clear}) = \{\ \langle \textcolor{red}{\blacksquare} \rangle,\ \langle \textcolor{magenta}{\blacksquare} \rangle\ \}$$

$$I(\texttt{Hat}) = \{\ \langle \textcolor{green}{\blacksquare} \rangle \to \textcolor{red}{\blacksquare},\ \langle \textcolor{blue}{\blacksquare} \rangle \to \textcolor{red}{\blacksquare},\ \langle \textcolor{orange}{\blacksquare} \rangle \to \textcolor{magenta}{\blacksquare}\ \}$$

$$\Omega_I = \{\ {\color{red}\blacksquare},\ {\color{lime}\blacksquare},\ {\color{cyan}\blacksquare},\ {\color{magenta}\blacksquare},\ {\color{orange}\blacksquare}\ \}$$



$$I(\texttt{A}) = {\color{red}\blacksquare}$$

$$I(\texttt{B}) = {\color{lime}\blacksquare}$$

$$I(\texttt{C}) = {\color{cyan}\blacksquare}$$

$$I(\texttt{D}) = {\color{magenta}\blacksquare}$$

$$I(\texttt{E}) = {\color{orange}\blacksquare}$$

$$I(\texttt{On}) = \{\ \langle {\color{lime}\blacksquare}, {\color{red}\blacksquare} \rangle,\ \langle {\color{cyan}\blacksquare}, {\color{lime}\blacksquare} \rangle,\ \langle {\color{orange}\blacksquare}, {\color{magenta}\blacksquare} \rangle\ \}$$

$$I(\texttt{Above}) = \{\ \langle {\color{lime}\blacksquare}, {\color{red}\blacksquare} \rangle,\ \langle {\color{cyan}\blacksquare}, {\color{lime}\blacksquare} \rangle,\ \langle {\color{cyan}\blacksquare}, {\color{red}\blacksquare} \rangle,\ \langle {\color{orange}\blacksquare}, {\color{magenta}\blacksquare} \rangle\ \}$$

$$I(\texttt{OnTable}) = \{\ \langle {\color{red}\blacksquare} \rangle,\ \langle {\color{magenta}\blacksquare} \rangle\ \}$$

$$I(\texttt{Clear}) = \{\ \langle {\color{cyan}\blacksquare} \rangle,\ \langle {\color{orange}\blacksquare} \rangle\ \}$$

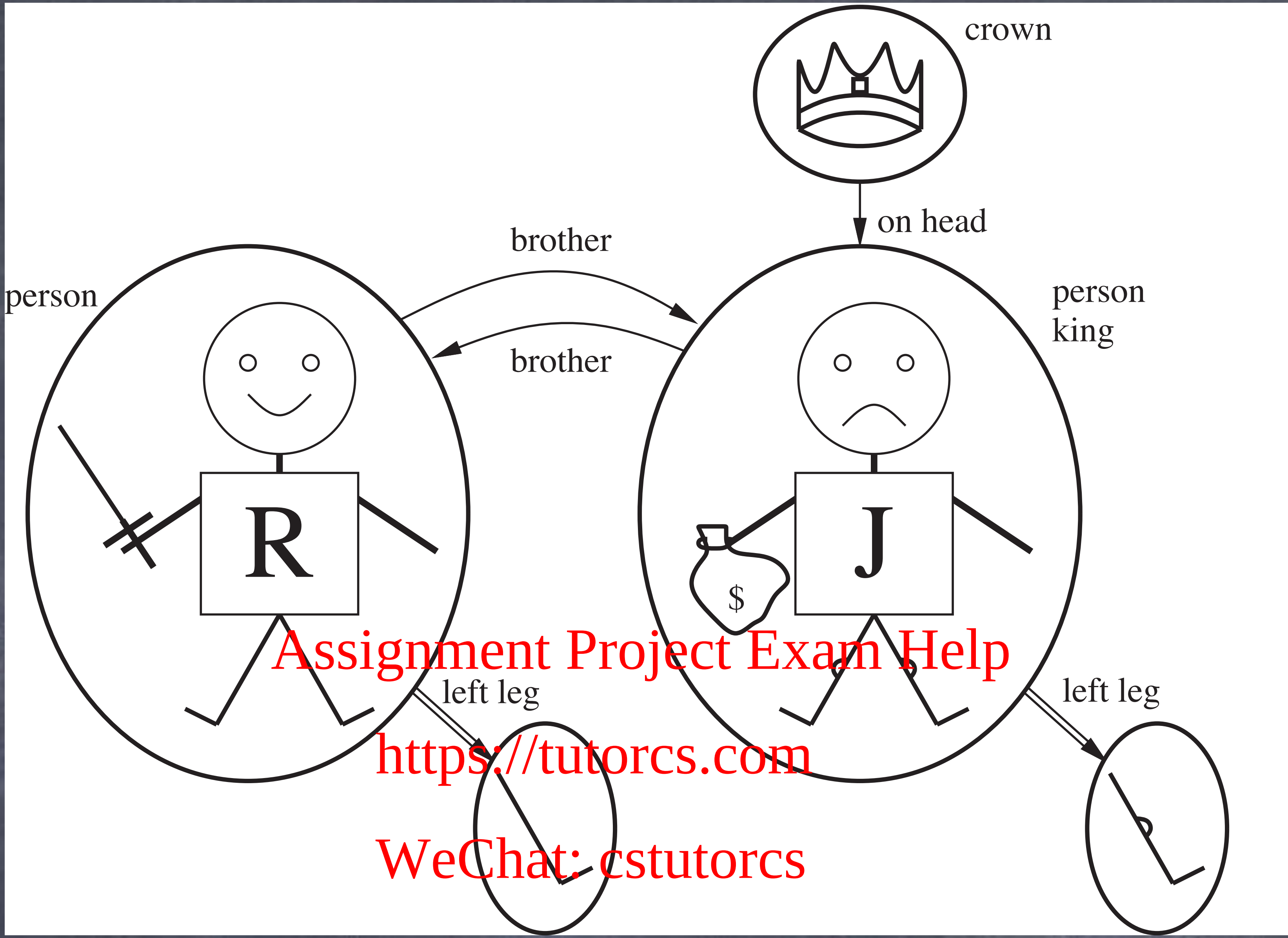
$$I(\texttt{Hat}) = \{\ \langle {\color{lime}\blacksquare} \rangle \rightarrow {\color{red}\blacksquare},\ \langle {\color{cyan}\blacksquare} \rangle \rightarrow {\color{red}\blacksquare},\ \langle {\color{orange}\blacksquare} \rangle \rightarrow {\color{magenta}\blacksquare}\ \}$$

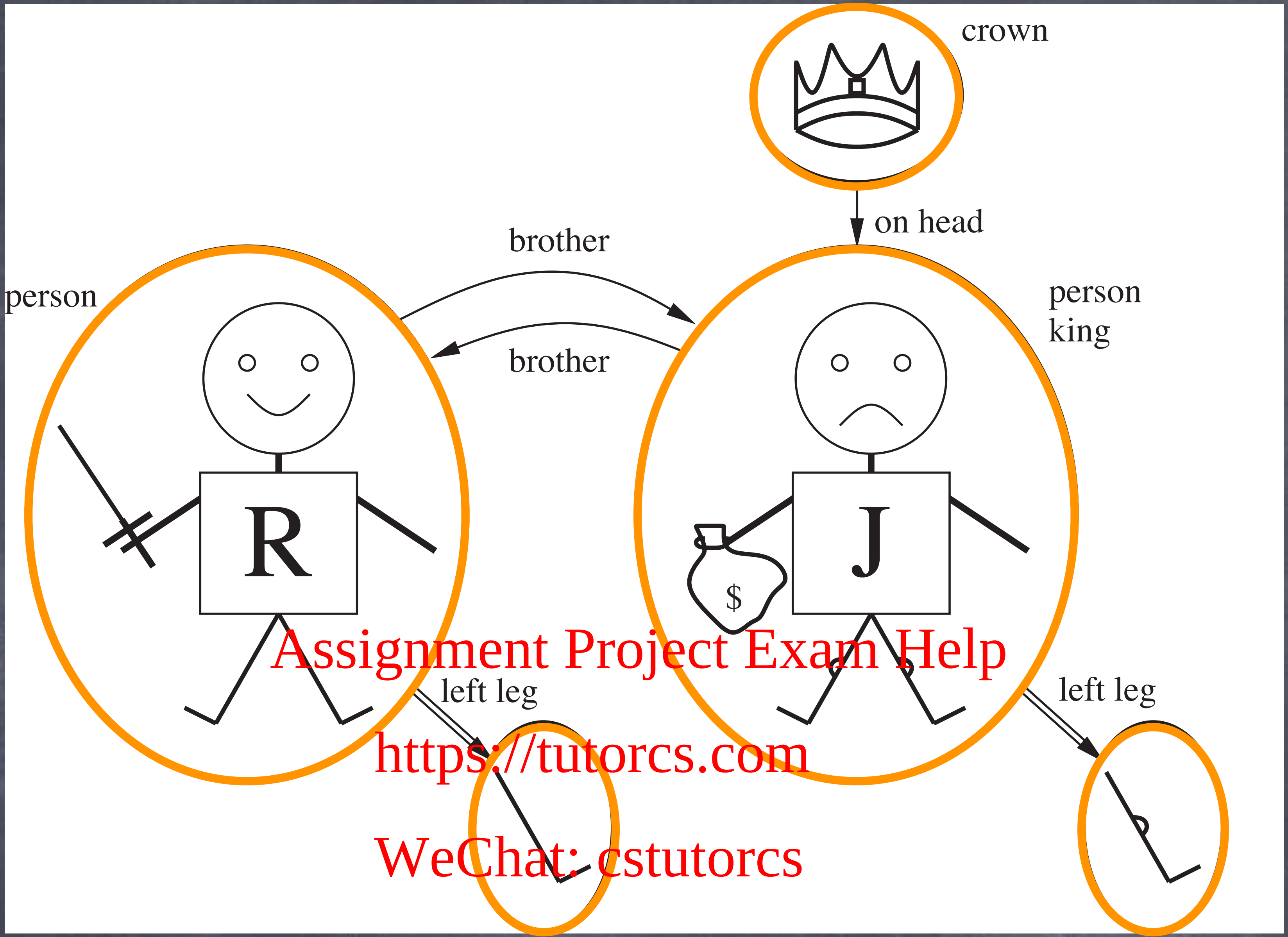
Richard

(1157-1199)

John

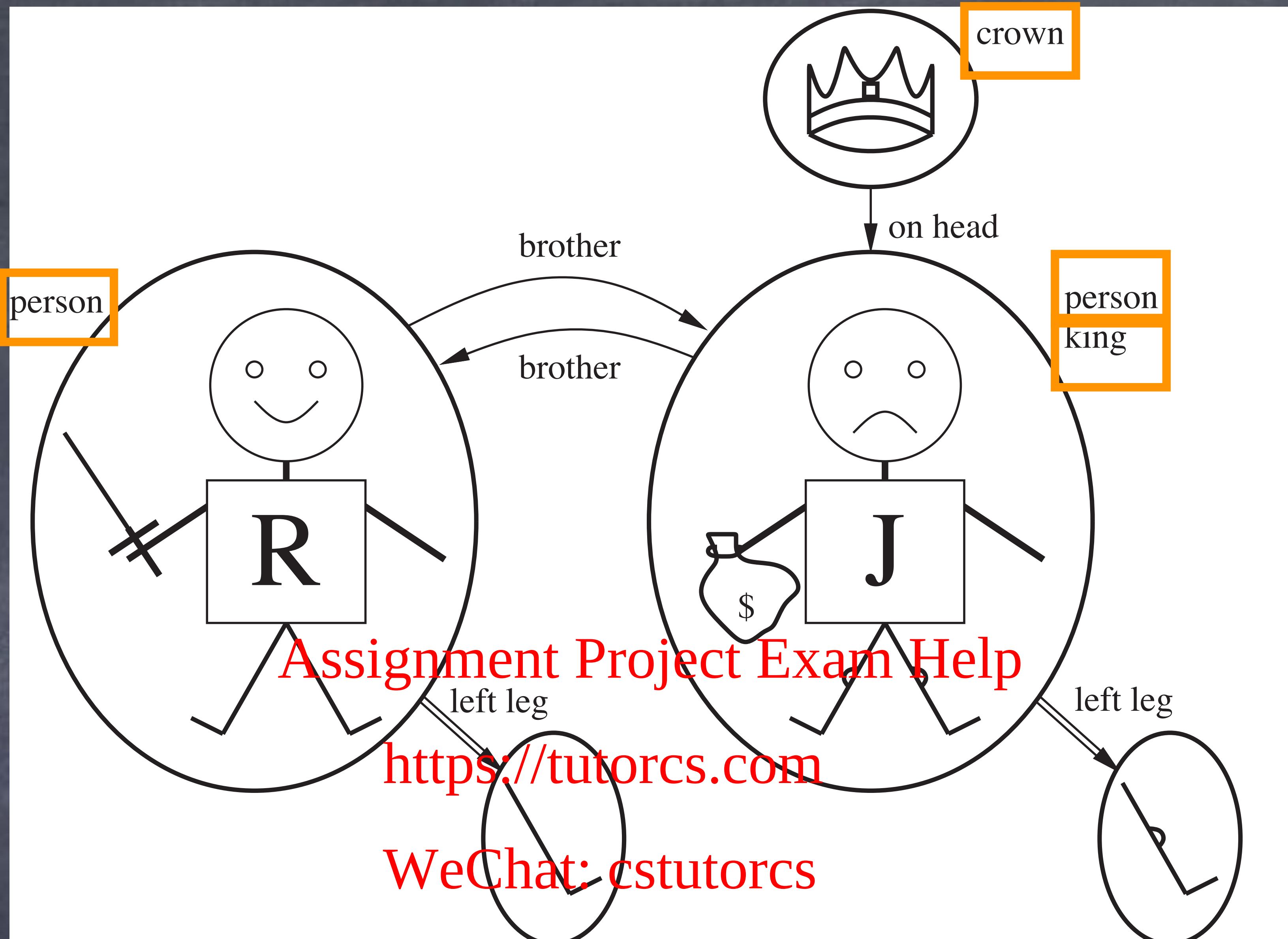
(1166-1216)


crown
on head
brother
brother
person
person
king
R
J
$
left leg
left leg



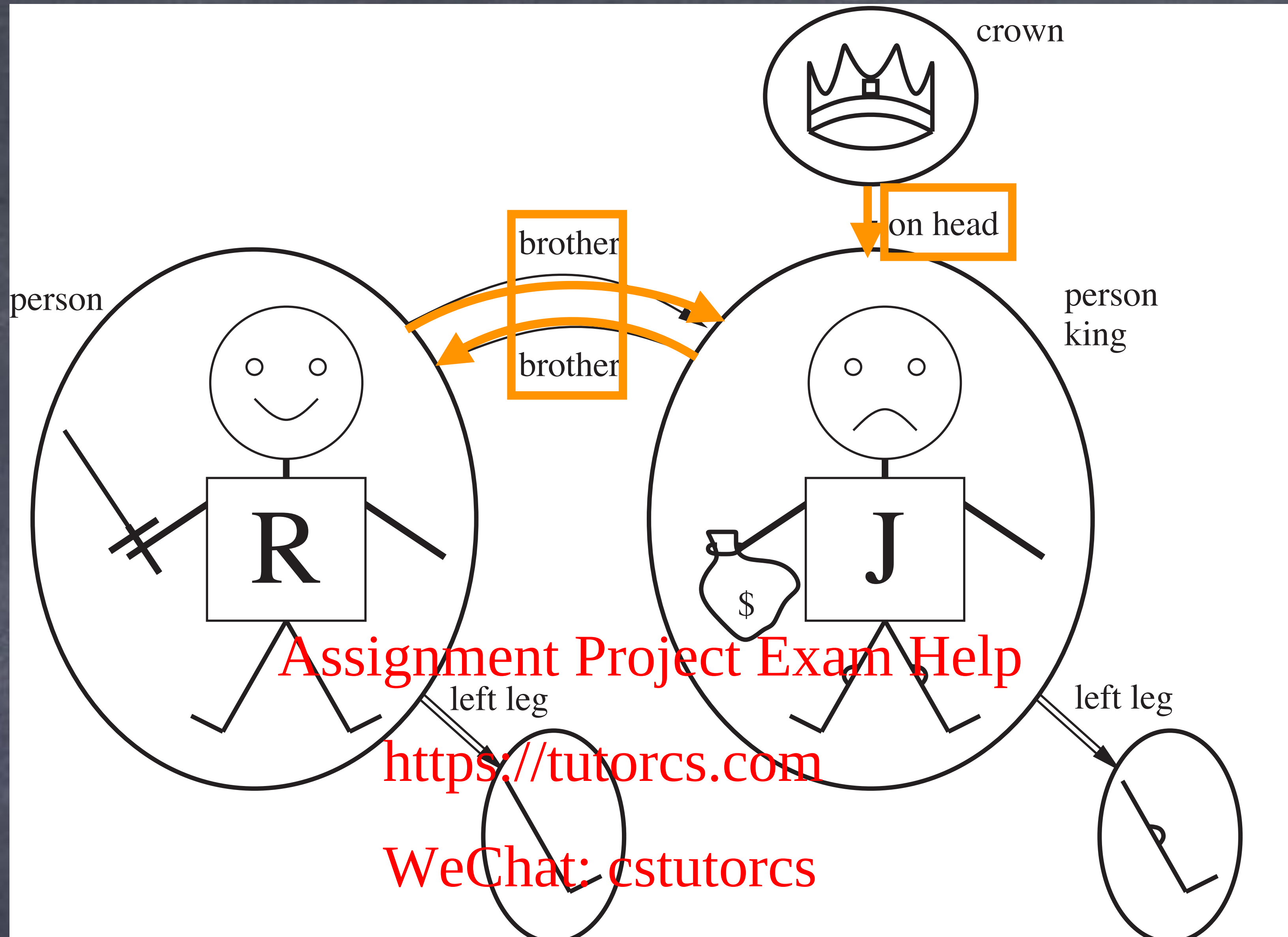
Objects (Ω_I):

Richard, John, left leg 1, left leg 2, the crown



Unary Relations (Properties):

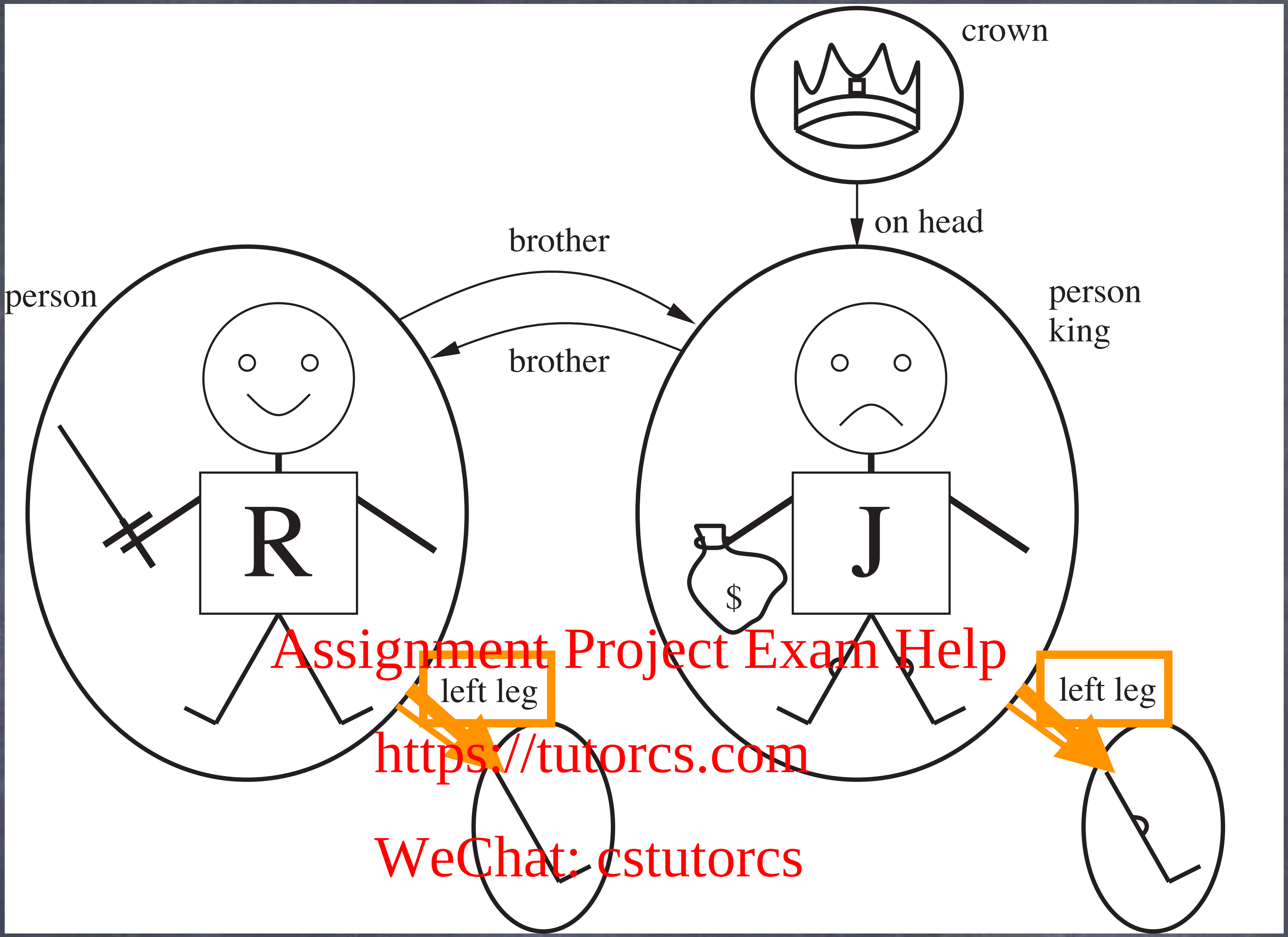
- being a person
- being a crown
- being a king



Binary Relations:

two things being brothers

one thing being on the head of another



Functions:

the left leg of something

$$\Omega_I = \{\ \boxed{\mathbf{R}},\ \boxed{\mathbf{J}},\ \text{♕},\ \llcorner,\ \lrcorner\ \}$$

$$I(\texttt{Richard}) = \boxed{\mathbf{R}}$$

$$I(\texttt{John}) = \boxed{\mathbf{J}}$$

$$I(\texttt{Person}) = \{\ \langle\boxed{\mathbf{R}}\rangle,\ \langle\boxed{\mathbf{J}}\rangle\ \}$$

$$I(\texttt{King}) = \{\ \langle\boxed{\mathbf{J}}\rangle\ \}$$

$$I(\texttt{Crown}) = \{\ \langle\text{♕}\rangle\ \}$$

$$I(\texttt{Brother}) = \{\ \langle\boxed{\mathbf{R}},\boxed{\mathbf{J}}\rangle,\ \langle\boxed{\mathbf{J}},\boxed{\mathbf{R}}\rangle\ \}$$

$$I(\texttt{OnHead}) = \{\ \langle\text{♕},\boxed{\mathbf{J}}\rangle\ \}$$

$$I(\texttt{leftLegOf}) = \{\ \langle\boxed{\mathbf{R}}\rangle \rightarrow \llcorner\ ,\ \langle\boxed{\mathbf{J}}\rangle \rightarrow \lrcorner\ \}$$

$$\Omega_I = \{ \boxed{\mathbf{R}}, \boxed{\mathbf{J}}, \quad , \quad , \quad \}$$

$$I(\texttt{Richard}) = \boxed{\mathbf{J}}$$

$$I(\texttt{John}) = $$

$$I(\texttt{Person}) = \{ \langle \boxed{\mathbf{R}} \rangle, \langle \boxed{\mathbf{J}} \rangle \}$$

$$I(\texttt{King}) = \{ \langle \boxed{\mathbf{J}} \rangle \}$$

$$I(\texttt{Crown}) = \{ \langle \quad \rangle \}$$

$$I(\texttt{Brother}) = \{ \langle \boxed{\mathbf{R}}, \boxed{\mathbf{J}} \rangle, \langle \boxed{\mathbf{J}}, \boxed{\mathbf{R}} \rangle \}$$

$$I(\texttt{OnHead}) = \{ \langle \quad , \boxed{\mathbf{J}} \rangle \}$$

$$I(\texttt{leftLegOf}) = \{ \langle \boxed{\mathbf{R}} \rangle \to \quad , \\ \langle \boxed{\mathbf{J}} \rangle \to \quad \}$$

$\Omega_I = \{$ $\boxed{\mathbf{R}}$, $\boxed{\mathbf{J}}$, , ,  $\}$

$I(\texttt{Richard}) =$ 

$I(\texttt{John}) =$ 

$I(\texttt{Person}) = \{ \langle \boxed{\mathbf{R}} \rangle, \langle \boxed{\mathbf{J}} \rangle \}$

$I(\texttt{King}) = \{ \langle \boxed{\mathbf{J}} \rangle \}$

$I(\texttt{Crown}) = \{ \langle$  $\rangle \}$

$I(\texttt{Brother}) = \{ \langle \boxed{\mathbf{R}}, \boxed{\mathbf{J}} \rangle, \langle \boxed{\mathbf{J}}, \boxed{\mathbf{R}} \rangle \}$

$I(\texttt{OnHead}) = \{ \langle$ , $\boxed{\mathbf{J}} \rangle \}$

$I(\texttt{leftLegOf}) = \{ \langle \boxed{\mathbf{R}} \rangle \rightarrow$ ,

$\langle \boxed{\mathbf{J}} \rangle \rightarrow$  $\}$

$$\Omega_I = \{\ \boxed{\mathbf{R}},\ \boxed{\mathbf{J}},\ \ ,\ \ ,\ \ \}$$

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$$I(\texttt{Crown}) = \{\ \langle\ \rangle\ \}$$

$$I(\texttt{Brother}) = \{\ \langle \boxed{\mathbf{R}}, \boxed{\mathbf{J}} \rangle,\ \langle \boxed{\mathbf{J}}, \boxed{\mathbf{R}} \rangle\ \}$$

$$I(\texttt{OnHead}) = \{\ \langle\ , \boxed{\mathbf{J}} \rangle\ \}$$

$$I(\texttt{leftLegOf}) = \{\ \langle \boxed{\mathbf{R}} \rangle \to\ ,\ \langle \boxed{\mathbf{J}} \rangle \to\ \}$$

First-Order Model (Possible World)

- Ontology (Domain of Discourse, Conceptualization)
  - Objects, relations, and functions

- Interpretation function I
  - Constant symbols $\rightarrow$ Objects
  - Predicate symbols $\rightarrow$ Relations (sets of tuples)
  - Function symbols $\rightarrow$ Functions (mappings)

AIMA p. 290

Satisfaction

- A model (possible world) satisfies a sentence if it makes the sentence true
  - "A model of the sentence"

Terms

- Constant term c
  - $I(c) \in \Omega_I$

- Function term $f(t_1, \ldots, t_n)$
  - $I(f)$ = some function F
  - $I(t_i)$ = some object $d_i \in \Omega_I$
  - $I(f(t_1, \ldots, t_n)) = F(d_1, \ldots, d_n)$

Terms

- Constant term c
  - $I(c) \in \Omega_I$

- Function term $f(t_1, \ldots, t_n)$
  - $I(f)$ = some function F
  - $I(t_i)$ = some object $d_i \in \Omega_I$
  - $I(f(t_1, \ldots, t_n)) = F(d_1, \ldots, d_n)$

"The interpretation fixes the referent (or denotation) of every term."

Atomic Sentences

- Atomic sentence $P(t_1, ..., t_n)$
  - $I(P)$ = some relation Φ
  - $I(t_i)$ = some object $d_i \in \Omega_I$
  - $P(t_1, ..., t_n)$ is true iff $\langle d_1, ..., d_n \rangle \in \Phi$

Atomic Sentences

- Atomic sentence $P(t_1, \ldots, t_n)$
  - $I(P)$ = some relation Φ
  - $I(t_i)$ = some object $d_i \in \Omega_I$
  - $P(t_1, \ldots, t_n)$ is true iff $\langle d_1, \ldots, d_n \rangle \in \Phi$

"An atomic sentence is true in a given model if the relation referred to by the predicate symbol holds among the objects referred to by the arguments."

Complex Sentences

α	β	$\neg\alpha$	$\alpha\wedge\beta$	$\alpha\vee\beta$	$\alpha\Rightarrow\beta$	$\alpha\Leftrightarrow\beta$
F	F		F	F	T	T
F	T	T	F	T	T	F
T	F		F	T	F	F
T	T	F	T	T	T	T

Semantics of First-Order Logic

- Set of objects, with relations & functions
- Interpretation function

  - Constant symbols → objects
  - Predicate symbols → relations (tuples)
  - Function symbols → functions (mappings)
- An interpretation satisfies a sentence if it makes the sentence true

- Rooms adjacent to pits are breezy
- Socrates is a person
  All people are mortal
- Anybody's grandmother is either their mother's or their father's mother

- Rooms adjacent to pits are breezy
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$$Person(Socrates)$$

- Rooms adjacent to pits are breezy
- Socrates is a person
  All people are mortal
- Anybody's grandmother is either their mother's or their father's mother

$$Person(Socrates)$$

True in I if $\langle I(Socrates)\rangle \in I(Person)$

- Rooms adjacent to pits are breezy
- Socrates is a person
  All people are mortal
- Anybody's grandmother is either their mother's or their father's mother

All people are mortal

Every object that is a person is also mortal

For every object x, if x is a person, then x is mortal

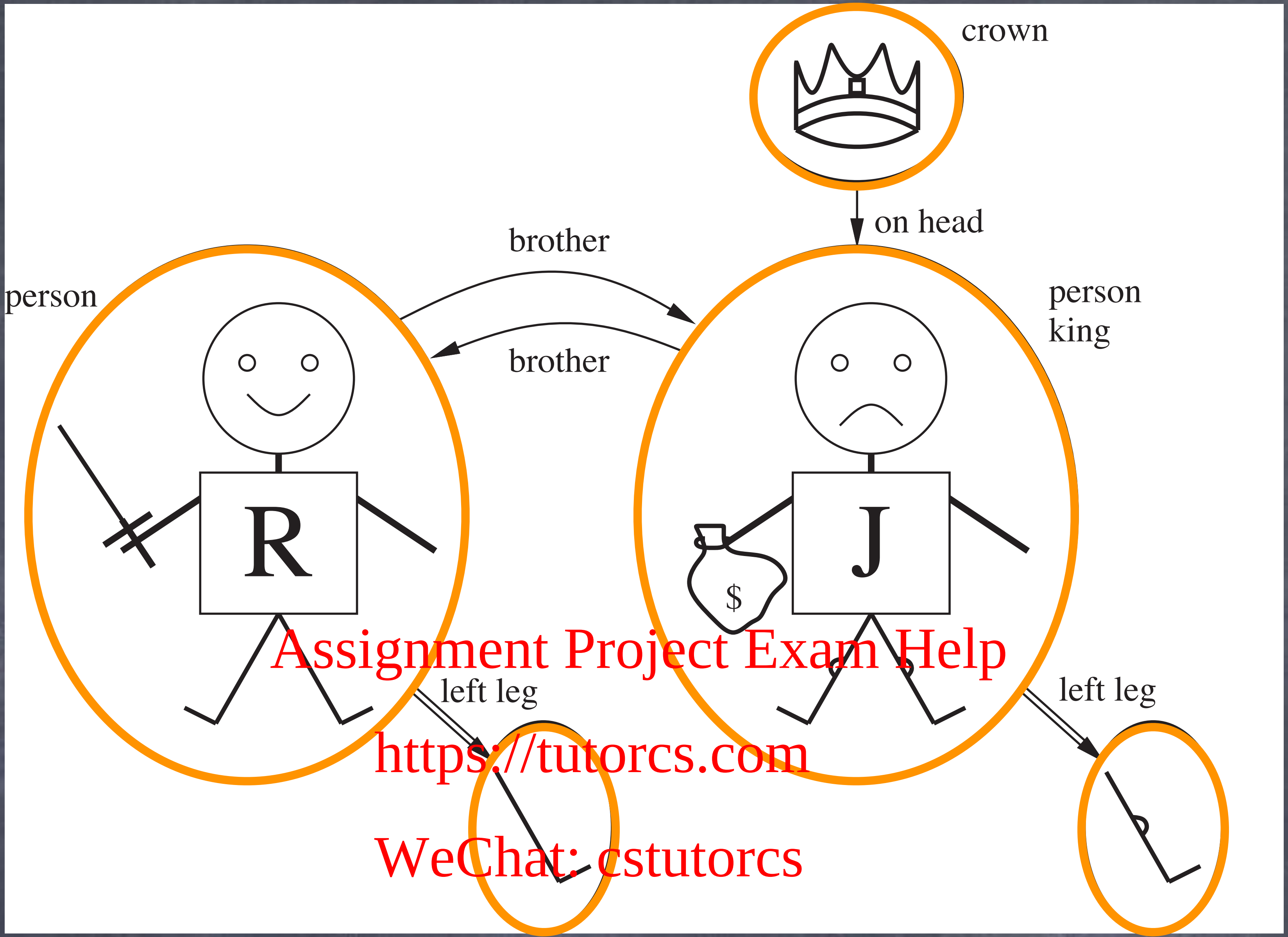
For every object x: $Person(x) \Rightarrow Mortal(x)$

Universal Quantification

- Syntax: $\forall x\, \varphi$
- Semantics: φ is true for every object x

  - Extended interpretation maps every variable to an object in the domain
  - $\forall x\, \varphi$ is true if φ is true in every extended interpretation

$$\forall x \; King(x) \Rightarrow Person(x)$$



Objects (Ω_I):

Richard, John, left leg 1, left leg 2, the crown

$$\forall x\ King(x) \Rightarrow Person(x)$$

$x \rightarrow$ Richard

$x \rightarrow$ John

$x \rightarrow$ Richard's left leg

$x \rightarrow$ John's left leg

$x \rightarrow$ the crown

$$\forall x\ King(x) \Rightarrow Person(x)$$

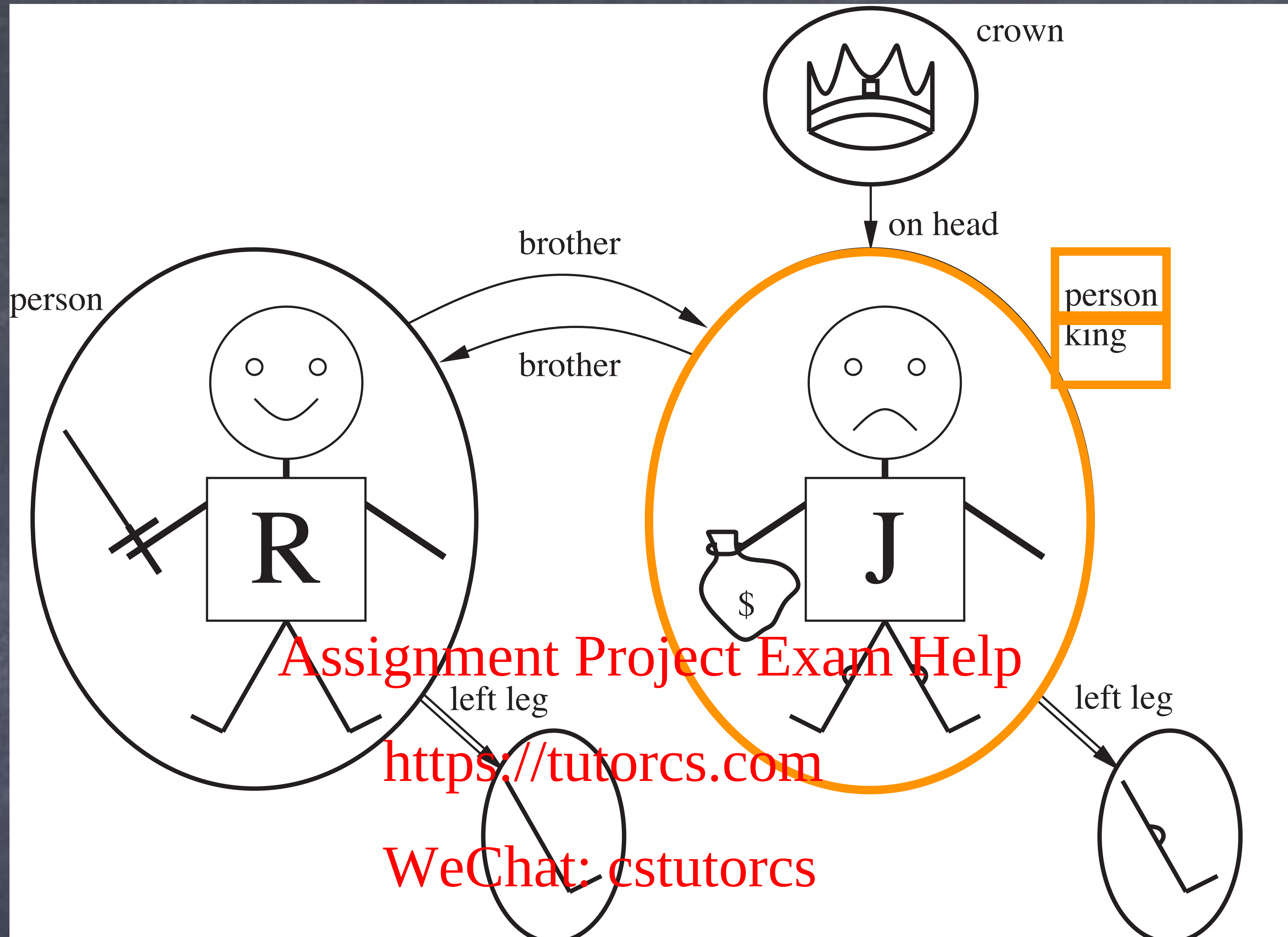
Richard is a king ⇒ Richard is a person

John is a king ⇒ John is a person

Richard's left leg is a king ⇒ Richard's left leg is a person

John's left leg is a king ⇒ John's left leg is a person

the crown is a king ⇒ the crown is a person



Objects (Ω_I):

Richard, John, left leg 1, left leg 2, the crown

$$\forall x\ King(x) \Rightarrow Person(x)$$

Richard is a king ⇒ Richard is a person

true true

John is a king ⇒ John is a person True

Richard's left leg is a king ⇒ Richard's left leg is a person

John's left leg is a king ⇒ John's left leg is a person

the crown is a king ⇒ the crown is a person

$$\forall x\ King(x) \Rightarrow Person(x)$$

false

Richard is a king ⇒ Richard is a person True

true true

John is a king ⇒ John is a person True

false

Richard's left leg is a king ⇒ Richard's left leg is a person True

false

John's left leg is a king ⇒ John's left leg is a person True

false

the crown is a king ⇒ the crown is a person True

$$\forall x\ King(x) \Rightarrow Person(x)$$ True!

false
Richard is a king $\Rightarrow$ Richard is a person True

true true
John is a king $\Rightarrow$ John is a person True

false
Richard's left leg is a king $\Rightarrow$ Richard's left leg is a person True

false
John's left leg is a king $\Rightarrow$ John's left leg is a person True

false
the crown is a king $\Rightarrow$ the crown is a person True

Universal Quantification

- Syntax: $\forall x\, \varphi$
- Semantics: φ is true for every object x
  - Extended interpretation maps every variable to an object in the domain
  - $\forall x\, \varphi$ is true if φ is true in every extended interpretation

All people are mortal.

$$\forall x \; Person(x) \Rightarrow Mortal(x)$$

Rooms adjacent to pits are breezy.

$$\forall x \forall y \; Room(x) \wedge Pit(y) \wedge Adjacent(x, y) \Rightarrow Breezy(x)$$

Anybody's grandmother is either their mother's or their father's mother

$$\forall x \forall y \; Grandmother(x, y) \Rightarrow$$
$$x = mother(mother(y)) \vee x = mother(father(y)))$$

$$\forall x \; King(x) \wedge Person(x)$$

$$\forall x \; King(x) \land Person(x)$$ False!

Richard is a king ∧ Richard is a person False

John is a king ∧ John is a person True

Richard's left leg is a king ∧ Richard's left leg is a person False

John's left leg is a king ∧ John's left leg is a person False

the crown is a king ∧ the crown is a person False

Rule: $\forall x \; King(x) \Rightarrow Person(x)$

Probably false statement:

$$\forall x \; King(x) \wedge Person(x)$$

Existential Quantification

- Syntax: $\exists x \, \varphi$
- Semantics: φ is true for some object x
  - Extended interpretation maps every variable to an object in the domain
  - $\exists x \, \varphi$ is true if φ is true in some extended interpretation

John has a crown on his head.

$$\exists x \; Crown(x) \land OnHead(x, John)$$

John has a crown on his head.

$$\exists x\ Crown(x) \land OnHead(x, John)$$

$x \rightarrow$ Richard

$x \rightarrow$ John

$x \rightarrow$ Richard's left leg

$x \rightarrow$ John's left leg

$x \rightarrow$ the crown

John has a crown on his head.

$\exists x \; Crown(x) \wedge OnHead(x, John)$ True!

$x \rightarrow$ Richard

$x \rightarrow$ John

$x \rightarrow$ Richard's left leg

$x \rightarrow$ John's left leg

$x \rightarrow$ the crown True

Existential Quantification

- Syntax: $\exists x\ \varphi$
- Semantics: φ is true for <u>some</u> object x
  - <u>Extended interpretation</u> maps every variable to an object in the domain
  - $\exists x\ \varphi$ is true if φ is true in <u>some</u> extended interpretation

Nested Quantifiers

Everyone (every person) loves someone

$$\forall x\ Person(x) \Rightarrow \exists y\ Person(y) \wedge Loves(x,y)$$

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Someone is loved by everyone

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Someone loves everyone

$$\exists x \; Person(x) \wedge \forall y \; Person(y) \Rightarrow Loves(x,y)$$

$$\exists x \; \forall y \; Person(x) \wedge Person(y) \Rightarrow Loves(x,y)$$

First-Order Predicate Logic

- Syntax:
  - Constant, predicate, and function symbols
  - Terms, atomic sentences, connectives
  - Quantifiers and variables
- Semantics:
  - Ontology of objects, relations, functions
  - First-order interpretation
  - Extended interpretation
  - Satisfaction (sentence true in a possible world)

Entailment

- α entails β : $\alpha \vDash \beta$
  - Every model of α is also a model of β
  - Whenever α is true, so is β
  - β is true in every world consistent with α
  - $Models(\alpha) \subseteq Models(\beta)$
  - β logically follows from α

Model Checking

- Given knowledge α and query β
  - For every possible model I
    - If α is satisfied by I
      - If β is not satisfied by I
        - Conclude that $\alpha \not\models \beta$
  - Conclude that $\alpha \models \beta$

All Possible Models

- # of objects in the world from 1 to ∞
- Some constants refer to the same object
- Some objects are not referred to by any constant ("unnamed")
- Relations and functions defined over sets of subsets of objects
- Variables range over all possible objects in extended interpretations

Constant symbols: $\{ R, J \}$

Relation symbol: $P(\cdot,\cdot)$

Constant symbols: $\{\ R,\ J\ \}$

Relation symbol: $P(\cdot,\cdot)$

$$\Omega_I = \{\ \bullet_1\ \}$$

$$I(R) = \bullet_1$$

$$I(J) = \bullet_1$$

$$I(P) = \{\ \}$$

$$\begin{tikzcd} R \arrow[dr] & & J \arrow[dl] \\ & \bullet_1 & \end{tikzcd}$$

Constant symbols: $\{ R, J \}$

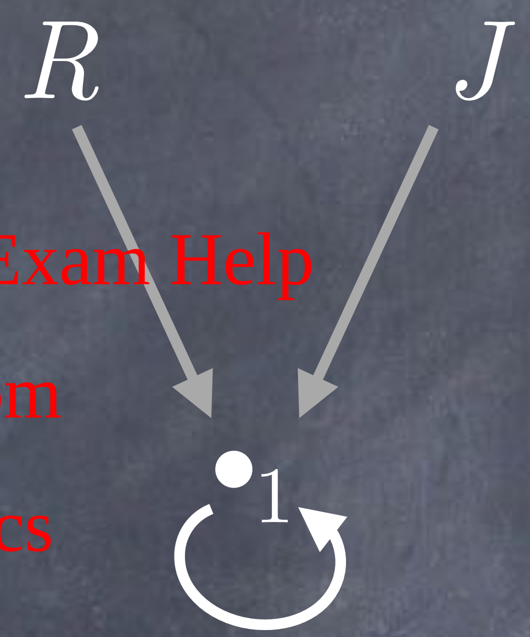
Relation symbol: $P(\cdot, \cdot)$

$$\Omega_I = \{ \bullet_1 \}$$

$$I(R) = \bullet_1$$

$$I(J) = \bullet_1$$

$$I(P) = \{ \langle \bullet_1, \bullet_1 \rangle \}$$



Constant symbols: $\{ R, J \}$
Relation symbol: $P(\cdot, \cdot)$

$$\Omega_I = \{ \bullet_1 \}$$

1 interpretation of R and J

2 interpretations of P

2 possible interpretations

Constant symbols: $\{ R, J \}$

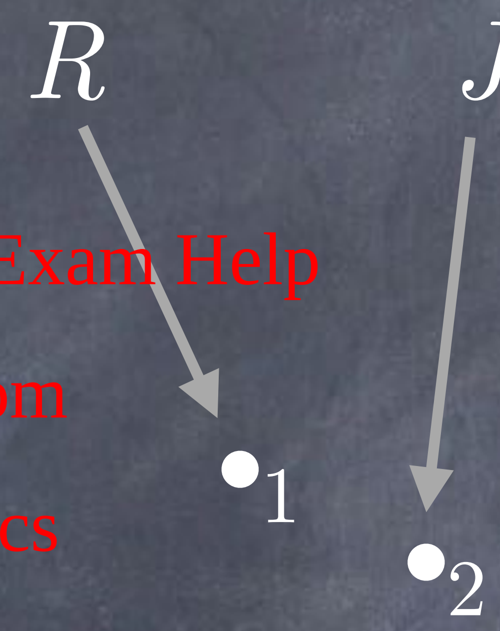
Relation symbol: $P(\cdot,\cdot)$

$$\Omega_I = \{ \bullet_1, \bullet_2 \}$$

$$I(R) = \bullet_1$$

$$I(J) = \bullet_2$$

$$I(P) = \ldots$$



2^2=4 interpretations of R and J

2^{2^2}=16 interpretations of P

64 possible interpretations

Constant symbols: $\{ R, J \}$

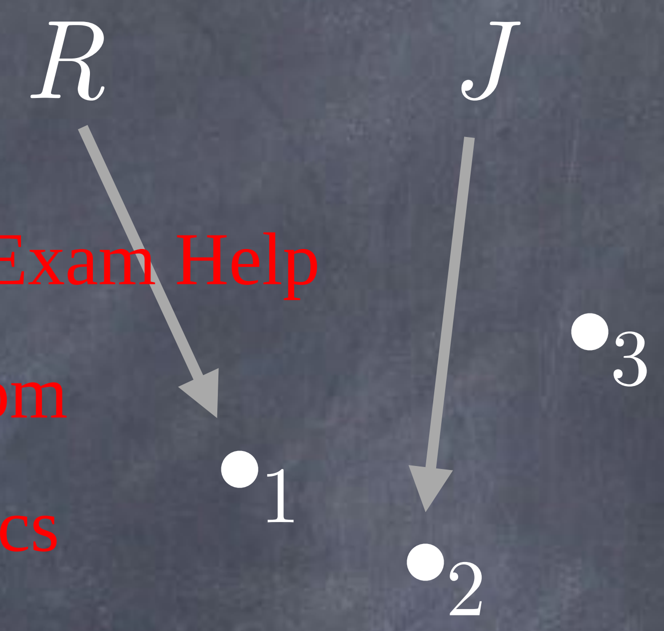
Relation symbol: $P(\cdot,\cdot)$

$$\Omega_I = \{ \bullet_1, \bullet_2, \bullet_3 \}$$

$$I(R) = \bullet_1$$

$$I(J) = \bullet_2$$

$$I(P) = \ldots$$



3^2=9 interpretations of R and J

2^{3^2}=2^9=512 interpretations of P

4608 possible interpretations

Constant symbols: $\{ R, J \}$

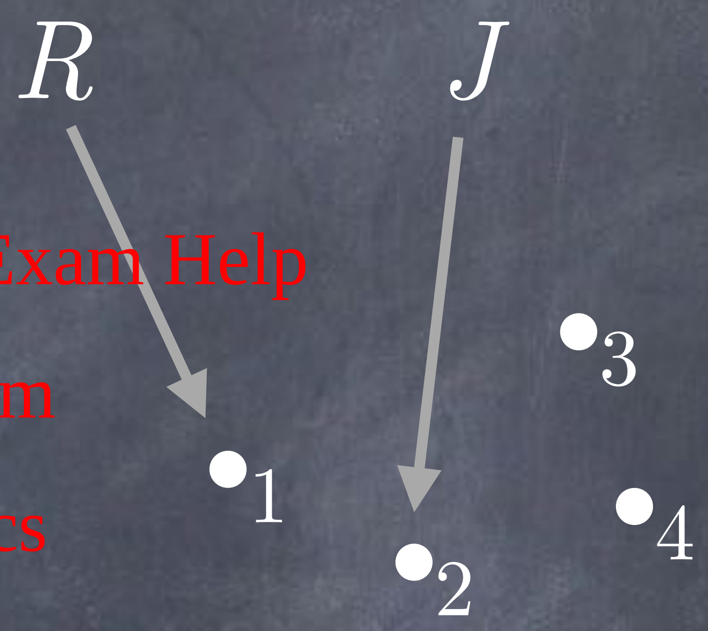
Relation symbol: $P(\cdot,\cdot)$

$$\Omega_I = \{ \bullet_1, \bullet_2, \bullet_3, \bullet_4 \}$$

$$I(R) = \bullet_1$$

$$I(J) = \bullet_2$$

$$I(P) = \ldots$$



1,048,576 possible interpretations

Computing Entailment

- Number of possible models HUGE
  - Possibly unbounded

Computing Entailment

- Number of possible models HUGE
  - Possibly unbounded
- Can't do model checking

Computing Entailment

- Number of possible models HUGE
  - Possibly unbounded
- Can't do model checking
- Look for inference rules, do theorem proving

For next time:

AIMA Ch 9