University of Toronto Mississauga
STA312 H5S: Computational Statistics - Winter 2020
Instructor: Dr. Luai Al Labadi

Assignment # 1

Due Date: Wed. January 29th, 2020 at beginning of lecture. **Late assignments will not be accepted.**
If you cannot make it to the lecture, you can submit through a colleague.
Electronic submissions will not be accepted.
Solve all questions. However, **not all questions on the assignments are marked**. The questions to be marked are not announced.

Instructions:

- Start all your R codes with the following information: Question Number, Assignment 1, Your Last Name, First Name, Student Number. For Example,
  # Question 3, Assignment 1
  # Last Name: ABCD, First Name: XYZW
  # St. #: 0123456789

- In this assignment, you should use R only when it is stated in the question "[**Computational Using R**]".

- You may not alter the output (by typing or handwriting anything). Output should be directly copied and pasted to the assignment where needed. If you do not follow these rules, your assignment will not be accepted.

- Format, spelling and grammar will be marked in addition to the statistical aspects.

- Assignments should presently neatly. Use the following format:

- Theoretical questions can be **hand-written** or **typed** using Times New Roman or Arial size 12 font, single spaced, printed double-sided on 8.5x11 inch paper and stapled together in the top left corner, without any other packaging.

- Attach the cover page (provided) at the front of your assignment. (Be sure to include an appropriate title for your assignment and fill in/circle all the information required).

- For marking purposes, include the question numbers/part letters (1a,b,c, etc.) in your answers.

- Do not simply hand in pages of R output without further explanation. Only include the relevant tables or plots that are asked in each question. Make sure to interpret the results in plain English in terms of the problem, quote relevant numbers from the output, and give justifications as a part of your solutions.

- Do not include unnecessary code or output in the body of the assignment. At the end, include an appendix with ALL your R code and output.

- The appendix will be checked to make sure the work was completed individually by the student and that the instructions were followed.

Answer All the following questions.
Follow the form of densities given in the formula sheet.

Question 1 Let F_X and f_X be, respectively, the cumulative distribution function and the probability density function of X. Let $Y = h(X)$, where h is a strictly monotone function. let G_Y, g_Y be a the cumulative distribution function and the probability density function of Y.

  (a) **[Ch. 2]** Assume that h is strictly increasing. Show that $G_Y(y) = F_X(h^{-1}(y))$.

  (b) **[Ch. 2]** Write g_Y in terms of f_X. Assume that h is differentiable.

Question 2 Let $X_1 \sim Gamma(\alpha, 1)$ and $X_2 \sim Gamma(\beta, 1)$, where X_1 and X_2 are independent. Let $Y_1 = \frac{X_1}{X_1 + X_2}$ and $Y_2 = X_1 + X_2$.

  (a) **[Ch. 2]** Show that $Y_2 \sim Gamma(\alpha + \beta, 1)$. **Hint:** Use moment generating function.

  (b) **[Ch. 2]** Derive the distribution of $5Y_2$. **Hint:** Use moment generating function.

  (c) **[Ch. 2]** Use the Jacobian method to find $f_{Y_1,Y_2}(y_1, y_2)$, the joint probability density function of Y_1 and Y_2. Don't forget to specify where the density is greater than zero.

  (d) **[Ch. 2]** Use Part (c) to show that $Y_1 \sim Beta(\alpha, \beta)$.

Assignment Project Exam Help

Question 3 [Ch. 2] Let $X \sim Gamma(n, \lambda)$, where n is an integer. Show that $X \overset{d}{=} Y_1 + \cdots + Y_n$, where $Y_1, \ldots, Y_n$ are independent $Exopnential(\lambda)$. Here $\overset{d}{=}$ means equals in distribution (i.e. have the same distribution).

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Question 4 Assume that U has a uniform distribution on $[0, 1]$.

  (a) **[Ch. 2]** Find the cumulative distribution function of U.

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  (b) **[Ch. 2]** Show that U^2 has $Beta(0.5, 1)$.

  (c) **[Ch. 3]** Use (b) to write an algorithm to generate a sample of size 1000 from $Beta(0.5, 1)$. Be as precise as possible.

  (d) **[Ch. 1; Computational Using R]** Now, based in the algorithm in (c), simulate a sample of size 1000 from $Beta(0.5, 1)$. For the generated sample, plot the relative frequency histogram and the corresponding density on the same picture. Make appropriate titles. **Hint:** Consult Ch. 1.

  (e) **[Ch. 1; Computational Using R]** Use the built-in R function "rbeta" to simulate a sample of size 1000 from $Beta(0.5, 1)$. For the generated sample, plot the relative frequency histogram and the corresponding density on the same picture. Make appropriate titles. **All plots need to be in one main plot**; use "par(mfrow=c(1,2))". **Hint:** Consult Ch. 1.

  (f) **[Ch. 2]** Now based on Question 2 and assuming that your computer can generate a sample from $Gamma(a, b)$, $a > 0, b > 0$, write an algorithm to generate a sample of size 1000 from $Beta(0.5, 1)$. Be as precise as possible.

Question 5 Let X be a continuous random variable with the following probability density function (pdf)

$$f_X(x) = \begin{cases} 0.5e^x & x < 0 \\ 0.5e^{-x} & x \geq 0 \end{cases}.$$

  (a) **[Ch. 2]** Show that $f_X(x)$ is a valid pdf.

(b) [**Ch. 2**] Find the cumulative distribution function $F_X(x)$.

(c) [**Ch. 3**] Find $F_X^{-1}(x)$.

(d) [**Ch. 3**] Write an algorithm to generate a sample of size 1000 from the distribution of X using the inverse-transform method. Be as precise as possible.

(e) [**Ch. 3; Computational Using R**] Now, based in the algorithm in (d), simulate a sample of size 1000 from the distribution of X. For the generated sample, plot the relative frequency histogram and the corresponding density on the same picture. Make appropriate titles. **Hint:** Consult Ch. 1.

Question 6 (Book Question 3.4) The Rayleigh density is

$$f_X(x) = \begin{cases} \frac{x}{\sigma^2} e^{-x^2/(2\sigma^2)} & x \geq 0, \sigma > 0 \\ 0 & \text{otherwise} \end{cases}.$$

(a) [**Ch. 2**] Show that $f_X(x)$ is a valid pdf.

(b) [**Ch. 2**] Show that $E(X) = \sigma\sqrt{\pi/2}$.

(c) [**Ch. 2**] Find the cumulative distribution function $F_X(x)$.

(d) [**Ch. 3**] Find $F_X^{-1}(x)$.

(e) [**Ch. 3**] Develop an algorithm to generate a sample of size 10^4 from $Rayleigh(\sigma)$ distribution using the inverse-transform method. Be as precise as possible.

(f) [**Ch. 3; Computational Using R**] Now, based in the algorithm in (e), simulate a sample of size 10^4 from $Rayleigh(\sigma)$ for the values of $\sigma = 1, 2, 3$ and 4. For the generated sample in each case, plot the relative frequency histogram and the corresponding density on the same picture. Make appropriate titles. [All plots need to be in one main plot; use "par(mfrow=c(2,2))". Check that the mean is close to the theoretical mean $\sigma\sqrt{\pi/2}$. **Hint:** Consult Ch. 1.

Good Luck