19/05:

Intro: Apprentissage.

$\Sigma = \{0,1\}$

.1) n-state DFA A

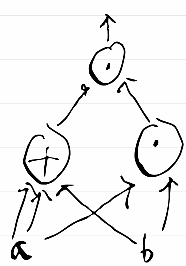
- Basis $\mathbb{B}$. (of DFA) → langage $\subseteq \Sigma^{\leq n}$.
- I know: $\forall\ B_i \in \mathbb{B}\ |L(B_i \cap A)| = k_i \in \mathbb{N}$.
- goal: $(k_1, \ldots, k_m)$ doit identifier A uniquement.

. $|\mathbb{B}| \in \mathcal{O}(P(n))$

$(n, (k_1, \ldots, k_m)) \to$ construct uniquely A.

I) Learning algo.
II) Non-comm circuits : equivalence.

↳ Low. bound PTIME.
Upp. bound ~~Z~~ PSPACE.



$= 2\,aab + bab.$

↳ to decide: intersect with weight. DFA

(not direct)

1) $|\Sigma^n| = 2^{n+1} - 1$
2) # (n-states DFA) with 1-final state : $\leq n^5$

3) $\#(\text{Languages of } \Sigma^n) = 2^{(2^{n+1})}$

4) $\#(\text{1-final, n-state DFA languages})$

Basis DFA's:

$$\mathbb{B} = \left\{ |W| \equiv i \pmod{p_j} \;\middle|\; W \in \Sigma^n,\ p_j \text{ the } j \text{ prime},\ j \leq n,\ 0 \leq i \leq p_j \right\}.$$

$$\underbrace{|\mathbb{B}|}_{=m} \leq n^2 \log n$$

$$\bar{k} = (k_1, \dots, k_m)$$

$$\#(\bar{k}) \leq n \binom{2^{n+1}}{n \log n} \approx O(2^n)$$

↳ not all vectors represent an automaton.

n-state DFA

$A \to A'$

$B \to B'$

$\to$ very different.

diff

↳ does there exist such a procedure?

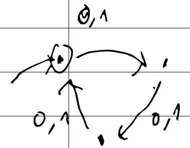
0m2, 1m2

0m3, 1m3, 2m3

0m5, 1m5 ... 4m5

10(0+1)* = 1000 1001 1010 1011 | 10 101 100 | 1101100 1111101 1110 1111

11(0+1)+



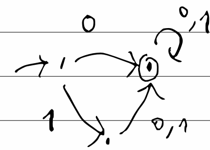
1

,2

,3

1, 1000 – 1111 | 10 – 11

1100 – 111



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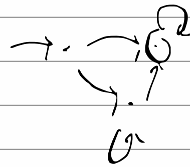

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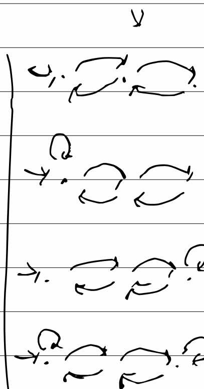
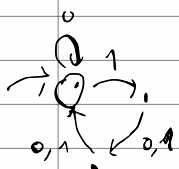


6

12

12

24

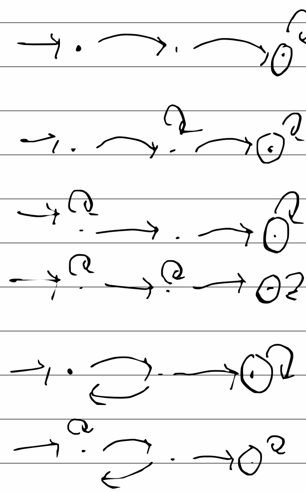


4 7

5 8

6 9

6



1

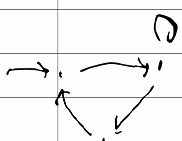
2

2

8

2

4

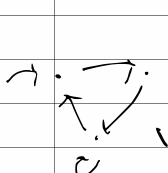


10 13

11 14

12 15

6

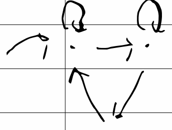


16 19

17 20

18 21

6



22 28

23 29

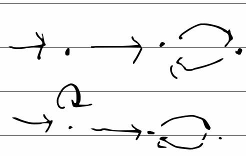
24 30

25 31

26 32

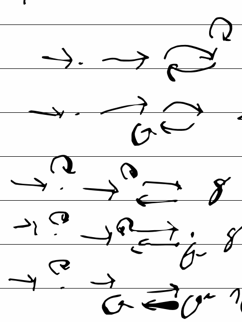
27 33

12



2

4



4

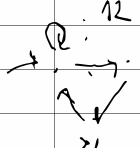
4

8

8

16

12



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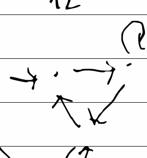
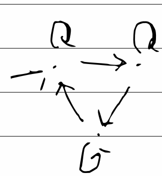


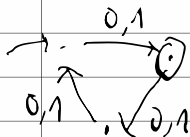


24

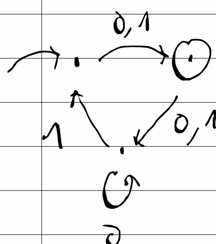
→, 45

57

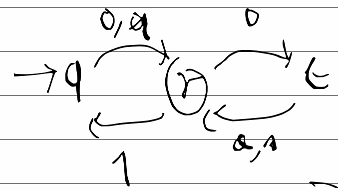
214



$\to L = 1(0+1)((0+1)^3)^*$

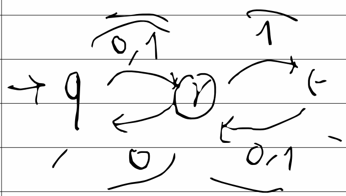


$\to L = 1(0+1)((0+1)0^*1(0+1))^*$



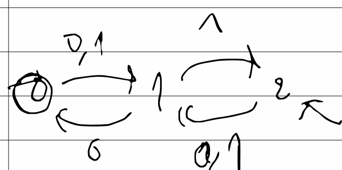
00110

$\equiv$ $\neq$



odd length words

→ 2-DFA ✓

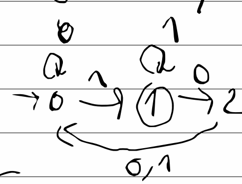


even, finish 0.

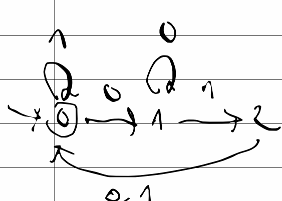
even, finish 1.

odd len even

odd long odd.



$(0^*1^*0(0+1))^*$



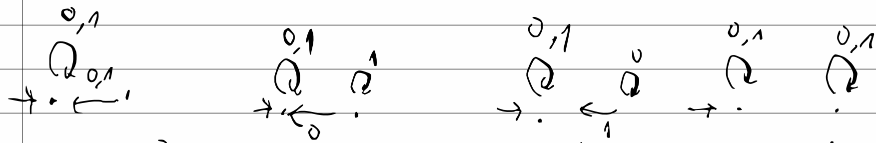
$(1^*0+1(0+1))^*$

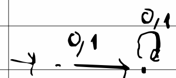
$L = \{0,\ 10,\ 11\}$ → encode as 1 string.

$\forall F$

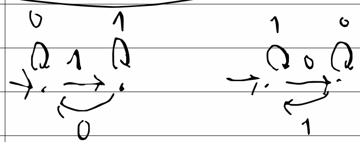
$F(1u,\ 1w) \to 1uw$

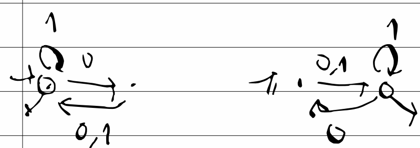
$= 1u \times 2^{|w|} + 1w - 2^{|w|}.$





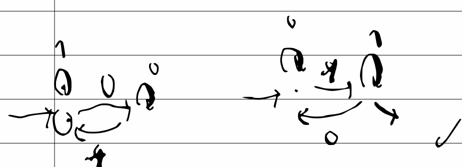
+ plein de langages vides.

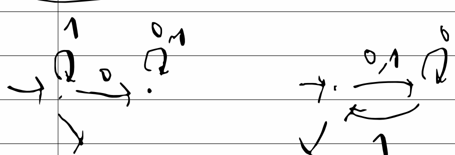




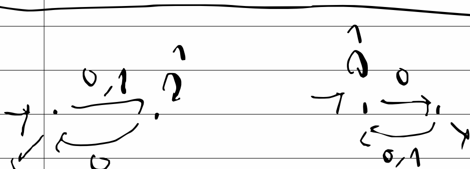
!

(1, 3, 2, 2, 0)

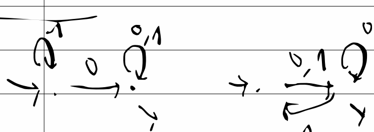




! (0, 2, 1, 1, 0)

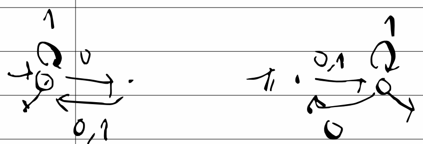


! (2, 0, 1, 0, 1)



! (3, 1, 2, 1, 1).

A B



(1, 1, 2, 2, 0)

$L_A = 1^*\left(1^*\,0(0+1)\right)^*$ $L_B = (0+1)\,1^*\left(0(0+1)1^*\right)^*$

(1:...)

ε	(1)	(0)	0	(2)	(0)
1	(3)	(1)	1	(3)	(1)
11	(7)	(3)	01	(5)	(1)
00	(4)	(0)	11	(7)	(3)
01	(5)	(1)	000	(8)	(0)
111	(15)	(7)	001	(9)	(1)
100	(12)	(4)	100	(12)	(4)
101	(13)	(5)	101	(13)	(5)
			111	(15)	(7)

(0, 1, 3, 4, 5, 7 → 0, 1, 3, 4, 5, 7)

1, 3, 4, 5, 7, 12, 13, 15 2, 3, 5, 7, 8, 9, 12, 13, 15

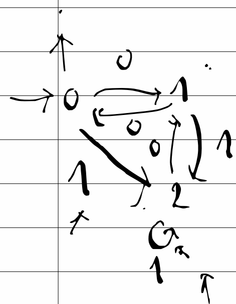
(1, 4, 1, 3, 1)

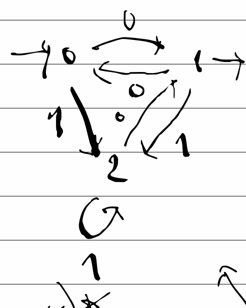
(2, 6, 3, 4, 1) (3, 6, 3, 2, 1)

?

238 - 41 answer-vekt suspicious.

$(5,0,3,2,0,20,1,0,2)$





$$\left(\left(0(10)^*0\right)^* + \left(1^+(01^*)^*00\right)^*\right)^*$$

ε	1	0	2
00	4	000	8
100	12	10	6
0000	16	1010	26
0100	20	010	10
1100	28	0110	22

(5,1

(6,0

Conjecture: $\rightarrow$ 1 final state.

Input n-DFA A, B.

$L(A) \neq L(B) \Rightarrow V_n(L(A)) \neq V_n(L(B))$

$V_n(L(A)) : (i,p) \rightarrow \#\{w \in L(A) \cap \Sigma^{2n-2} \mid 1w \equiv i(p)\}$

Finite DFA $C \rightarrow L(C) \subseteq \Sigma^n$?

$\rightarrow$ carry over multiple final states!

$A \rightarrow V_1$

$A' \rightarrow V_2$

$B \rightarrow V_3$

$B' \rightarrow V_4$

A, A' same but final state

B, B'

$x \sim 4 \leftarrow xw \rightarrow 13$

$x \sim 8 \leftarrow xu \rightarrow 24$

$\rightarrow V_1 + V_2 \overset{?}{=} V_3 + V_4$

$x = 1$

$w = 01$

$u = 000$

Conj $\rightarrow$ NO ?

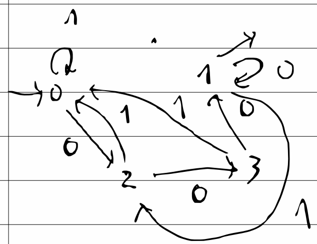
1DFA A, A' identical but not final state.

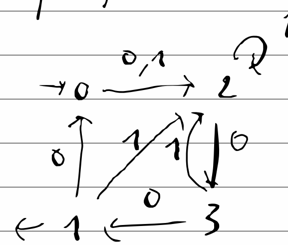
naturally similar proof $\leftarrow$ $L(A) \cap L(A') = \emptyset$ $\quad \exists w, x, \; xw \in L(A) \wedge w \in L(A')$

$\Rightarrow \forall u, \; xu \in (L(A)) \Rightarrow \; \in L(A')$

$n=4 \quad 2n-2=6$

→ only equal vector For $2n-2$!





$L_1 \cap \Sigma^{2n-2}$ $\quad$ $L_A \cap \Sigma^n$ $\cancel{L_B \cap \Sigma^2}$ $\quad$ $L_B \cap \Sigma^{2n-2}$

		mod3	m5
000	8	2	3
0000	16	1	1
1000	24	0	4
00000	32	2	2
01000	40	1	0
10000	48	0	3
11000	56	2	1
000000	64	1	4
000100	68	2	3
001000	72	0	2
010000	80	2	0
011000	88	1	3
100000	96	0	1
101000	104	2	4
110000	112	1	2
111000	120	0	0

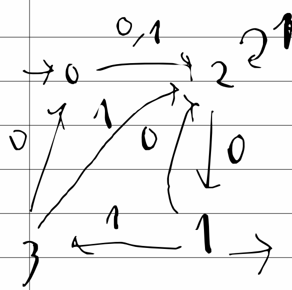
		mod3	m5
000	8	2	3
100	12	0	2
0100	20	2	0
1100	28	1	3
00100	36	0	1
01100	44	2	4
10100	52	1	2
11100	60	0	0
000100	68	2	3
001100	76	1	1
010100	84	0	4
011100	92	2	2
100100	100	1	0
101100	108	0	3
110100	116	2	1
111100	124	1	4

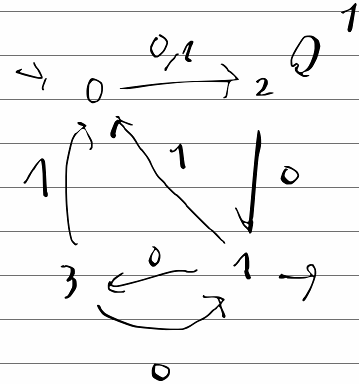
$L_A = \{8,68\} + L_1 + L_2$ $\qquad$ $L_B = \{8,68\} + L_1 - 4 + L_2 + 4$

$\|L_1\| = \|L_2\|$

~~Would fail for any prime!~~ → no!

But if we go further... i.e. 1 length more → solved!





0101000

Samevectors for n, $2n$, $3n$, $4n$

but not the other values

(though total sum stable)

with one more prime, only equal for n.