```
Q1. Find f(0) given:  f(x)  = 2 * f(x+2) - 3      if x <= 6
                            = 3 * x + 2           if x > 6
Entering (top-down)                                           Exiting (bottom-up)
f(0) = 2 * f(0 + 2) - 3 = 2 * f(2) - 3 (upper rule)          = 2 * 187 - 3 = 374 - 3 = 371
f(2) = 2 * f(2 + 2) - 3 = 2 * f(4) - 3 (upper rule)          = 2 *  95 - 3 = 190 - 3 = 187
f(4) = 2 * f(4 + 2) - 3 = 2 * f(6) - 3 (upper rule)          = 2 *  49 - 3 =  98 - 3 =  95
f(6) = 2 * f(6 + 2) - 3 = 2 * f(8) - 3 (upper rule)          = 2 *  26 - 3 =  52 - 3 =  49
f(8) = 3 * 8 + 2 = 24 + 2 = 26         (lower rule) Replace each f(x)'s above (turning point)

Answer: f(0) = 371
```

```
Q2. Find f(5) given: f(x)  = (2*x+1)*f(x-1)      if x > 1
                           = 1                   if x = 1
Entering (top-down)                           Exiting (bottom-up)
f(5) = (2*5+1)*f(4)   (upper rule)          = 11* 315= 3465
f(4) = (2*4+1)*f(3)   (upper rule)          = 9 * 35 = 315
f(3) = (2*3+1)*f(2)   (upper rule)          = 7 * 5  = 35
f(2) = (2*2+1)*f(1)   (upper rule)          = 5 * 1  = 5
f(1) = 1              (lower rule) Replace each f(x)'s above (turning point)

Answer: f(5) = 3465
```

Q3. Which has the most 1's in its binary representation?

414(8) 1B5(16) 178(10) 200(16) 600(8)

Notice the 2 numbers on the right-hand side 200(16) and 600(8) are a power-of-2 number and a simple combination of power-of-2 number:

2 (8, 10, 16) are 10 (2). Adding two 0's on the right if 2 won't add more 1's in its binary representation. Therefore, 200 (16) is also a power-of-2 number, whereas all power-of-2 numbers have only one 1;

6 (8, 10, 16) are 4 (8, 10, 16) + 2 (8, 10, 16) which is 110 (2). Adding two 0's on the right won't add more 1's in its binary representation. So 600(8) has only two 1's;

414(8) is a combination of 3 power-of-2 numbers: 400(8) + 10(8) + 4(8). So 414(8) has three 1's;

These numbers can all be evaluated mentally: 414(8), 200(16), and 600(8).

1B5 (16)= 1 11(10) 5 = 0001 1011 1001 (2) has 6 ones
178 (10)= 128 + 50 = 128 + 32 + 18 = 128 + 32 + 16 + 2 has 4 ones (b/c 4 power of 2 numbers!)

Answer: 1B5 (16)

Q4. Determine the number of 1's in the binary representation of the solution of the following expression:

(743(8) − AF(16) + 110100101000(2)) * 256(10)

256(10) is a power-of-2 number. Any number multiply a power-of-2 number the result in binary is like appending all the zeros from the power-of-2 operand on the left-hand side of this number. Therefore there will be no 1's added! So the expression can be simplified with only the left-hand side of the multiplication sign:

743(8) − AF(16) + 110100101000(2)

There are several approaches to continue:

1) Convert non-binary numbers into binary, perform one subtraction and one addition in binary, then count the 1's from the result;
2) Convert non-octal numbers into octal numbers, perform one subtraction and one addition in octal, then convert the result into binary and count the 1's.

I recommend the 2nd approach b/c the subtraction and addition in octal are simple with less writing. The final step to convert the arithmetic result in octal into binary can be done in very little time if you are fluent in power-of-2 numbers.

In my earlier solution, I used the 1st approach. I realized that binary subtraction and addition can be somewhat easier to make mistakes when writing lengthy strings 0’s and 1’s.

AF(16) = 10(10) 15(10) = 1010 1111 (2) = 10 101 111 (2) = 237 (8)

743(8) - 237(8) = 504(8)

110100101000(2) = 110 100 101 000 (2) = 6450(8)

504(8) + 6450(8) = 7254(8) = 111 010 101 100 (2)

Answer: 7 one’s

Q5. What is printed when this program is run?

You do not need to change the indents of the original program b/c the binary if-else branches induced with each line of code below has only one set of value for all variables: a, b, c, d.

Prepare a table of 4 columns each tracking 1 variable like below. Make sure in your worksheet, each step is evaluated precisely:

	a	b	c	d	
a = 4: b = 10: c = 1: d = 2	4	10	1	2	
if a <= b then a = a + b	14	10	1	2	4<=10,true
if c * d >= a then b = b + c else c = c - d	14	10	-1	2	1*2>=14,false,1-2=-1
if a / b <= 1 then a = c + a else b = d + b	14	12	-1	2	14/10<=1,1.4<=1,false
if d ^ c < c ^ d then c = d else d = c	14	12	2	2	2^(-1)<(-1)^2,true
if ((a > b) or (c < d)) then b = a - b else c = d + c	14	2	2	2	(14>12)||(2<2),true
print a / b + c * (a * b − c / d) / b					14/2+2*(14*2-2/2)/2
End					=7+2*(28-1)/2=7+27=34