

Q1. Convert 2018(10) to octal.

Convert decimal to octal by repeatedly dividing quotients by base 8:

$$2018 / 8 = 252 \text{ R } 2$$

$$252 / 8 = 31 \text{ R } 4$$

$$31 / 8 = 3 \text{ R } 7$$

$$3 / 8 = 0 \text{ R } 3$$

Answer: 3742 (8)

Q2. How many decimal numbers from 1 to 32 have the same number of 1's and 0's in their binary representation?
Note: ignore leading zeroes.

32(10) in binary is 100000(2), which is a power-of-2 number. So in binary, it is a leading 1 with 5 succeeding 0's in 6-digit;

In order to have equal numbers of 0's and 1's, the numbers of digits must be even numbers: 2, 4, and 6. However, there is only 1 6-digit number which is 32(10) and it does not qualify;

For 2-digit binaries, we have only 1 number: 10(2); for 4-digit binaries, we have 3 numbers b/c with given 4 we can only have two 1's and two 0's, where 1 of the two 1's is the leading 1. Therefore the other 1 must be in each of the 3 non-leading digits respectively.

Answer: $1 + 3 = 4$

Q3. Find $f(18)$ given:

$f(x) = f(x-5)+1$	if $x > 5$
$= 7$	if $x = 5$
$= f(x+3)-2$	if $x < 5$

Entering (top-down)

$$\begin{aligned}f(18) &= f(18-5) + 1 = f(13) + 1 && \text{(top rule)} \\f(13) &= f(13-5) + 1 = f(8) + 1 && \text{(top rule)} \\f(8) &= f(8-5) + 1 = f(3) + 1 && \text{(top rule)} \\f(3) &= f(3+3) - 2 = f(6) - 2 && \text{(bottom rule)} \\f(6) &= f(6-5) + 1 = f(1) + 1 && \text{(top rule)} \\f(1) &= f(1+3) - 2 = f(4) - 2 && \text{(bottom rule)} \\f(4) &= f(4+3) - 2 = f(7) - 2 && \text{(bottom rule)} \\f(7) &= f(7-5) + 1 = f(2) + 1 && \text{(top rule)} \\f(2) &= f(2+3) - 2 = f(5) - 2 && \text{(bottom rule)} \\f(5) &= 7 && \text{(middle rule)}\end{aligned}$$

Exiting (bottom-up)

$$\begin{aligned}&= 3 + 1 = \mathbf{4} \\&= 2 + 1 = 3 \\&= 1 + 1 = 2 \\&= 3 - 2 = 1 \\&= 2 + 1 = 3 \\&= 4 - 2 = 2 \\&= 6 - 2 = 4 \\&= 5 + 1 = 6 \\&= 7 - 2 = 5\end{aligned}$$

Answer: 4

Q4. Find $f(f(f(f(24))))$ given: $f(x) = \begin{cases} \lfloor x/2 \rfloor + 1 & \text{if } x \text{ is even} \\ \lfloor x/3 \rfloor - 2 & \text{if } x \text{ is odd} \end{cases}$

Note: $\lfloor x \rfloor$ is the greatest integer $\leq x$

Work from inside-out for the embedded functions by solving the following top-down:

$$\begin{aligned}&f(x) \\&f(f(x)) \\&f(f(f(x))) \\&f(f(f(f(x))))\end{aligned}$$

$$\begin{aligned}f(24) &= \lfloor 24/2 \rfloor + 1 = 12 + 1 = 13 \\f(f(24)) &= f(13) = \lfloor 13/3 \rfloor - 2 = 4 - 2 = 2 \\f(f(f(24))) &= f(2) = \lfloor 2/2 \rfloor + 1 = 2 \\f(f(f(f(24)))) &= f(2) = 2\end{aligned}$$

Answer: 2

Q5. What is the output when this program is executed?

	a	b	c	d	e	f	
a = 2 : b = 1 : c = 0 : d = 3 : e = 4	2	1	0	3	4		
f = a + b + c + d + e						10	2+1+0+3+4==10
if f / 5 == a then							10/5==2, true
f = f / 5						2	
else							
f = a + 2							
end if							
if a + b < d * e / 2 then							2+1<3*4/2, 3<6, true
b = d	2	3	0	3	4	2	
else							
a = e							
end if	2	3	0	3	4	2	
if 2 * d ↑ c == e / a then							2*3^0==4/2, 2==2, true
d = e	2	3	0	4	4	2	
else							
c = a							
end if							
if (b < d) && (c < e) then							Pop-quiz: Are the parentheses useful in the following expression? (3<4) && (0<4), true
b = d	2	4	0	4	4	2	
else							
c = e							
end if							
if (c ↑ a > d * e) (f < d / e) then							Pop-quiz: Are the parentheses useful in the following expression? (0^2>4*4) (2<4/4), false
c = a							
else							
d = c	2	4	0	0	4	2	
end if							
output 2 * a + b * (c - d) + e / 2 * f							2*2+4*(0-0)+4/2*2
end							=4+0+4=8

Answer: 8