Q1. How many 1’s are there in the binary representation of the decimal numbers from 22 to 32, inclusive?

Do not rush to convert every number in the range from decimal to binary! Notice 32 is a power-of-2 number but the previous one is 16 which is a bit far less than 22. Let’s induce by converting from lower end 22 into binaries - the reason why we list them in 6-digit is b/c 32 has 6-digits in binary:

Decimal	Binary	
22	**010**110	
23	**010**111	
24	**011**000	011... leading pattern starts from here
25	**011**001	
...		
31	**011**111	power-of-2 minus 1, the pattern stops here
32	100000	power-of-2

We do not need to convert more after 24(10)! Notice that the first/left 3-digit starting from 24 is always 011...Therefore we wonder how many 1’s in 3-digit binaries on the right from 24 through 31 inclusive:

0 1’s - C(3, 0) = 1 count (2+0) x 1 = 2 1’s (The 2 is the count of 1’s in the leading 011...)
1 1’s - C(3, 1) = 3 count (2+1) x 3 = 9 1’s
2 1’s - C(3, 2) = 3 count (2+2) x 3 = 12 1’s
3 1’s - C(3, 3) = 1 count (2+3) x 1 = 5 1’s

Don’t forget to include the following “edge” numbers:

22(10) = 010110(2) has 3 1’s
23(10) = 010111(2) has 4 1’s
32(10) = 100000(2) has 1 1’s

Total: 2 + 9 + 12 + 5 + 3 + 4 + 1 = 36

Answer: C

Q2. Which number is the largest? 101100000(2) 535(8) 15F(16)

It is much easier to convert octal and hexadecimal into binary, and we only need to perform two conversions (octal to binary and hexadecimal to binary). Align the binaries vertically will make the comparison more obvious:

```
                      101100000(2)
535(8) = 101 011 101 = 101011101(2)
15F(16)= 1 0101 1111 = 101011111(2)
                         ^
                         Confirmed at this digit!                                    Answer: A
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Q3. Find f(7), given:

$$f(x) = \begin{cases} 1 & \text{if } x = 1 \\ x + f(x-1) & \text{if } x > 1 \end{cases}$$

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f(7) = 7 + f(6)       = 7 + 21 = 28
f(6) = 6 + f(5)       = 6 + 15 = 21
f(5) = 5 + f(4)       = 5 + 10 = 15
f(4) = 4 + f(3)       = 4 + 6 = 10
f(3) = 3 + f(2)       = 3 + 3 = 6
f(2) = 2 + f(1)       = 2 + 1 = 3
f(1) = 1                                                                         Answer: B. 28
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Q4. Find f(20), given:

$$f(x) = \begin{cases} f(x/2)+1 & \text{if } x \text{ is even} \\ f(x+3)-1 & \text{if } x \text{ is odd and } x > 3 \\ (x+1)/2 & \text{otherwise} \end{cases}$$

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f(20) = f(20/2=10) + 1     = 4 + 1 = 5
f(10) = f(10/2=5) + 1      = 3 + 1 = 4
f(5) = f(5+3=8) - 1        = 4 - 1 = 3
f(8) = f(8/2=4) + 1        = 3 + 1 = 4
f(4) = f(2) + 1            = 2 + 1 = 3
f(2) = f(1) + 1            = 1 + 1 = 2
f(1) = (1 + 1) / 2 = 1                                                           Answer: D. 5
```

Q5. What is the output when this program is executed?

Make a spreadsheet of variables: a, b, and c. Then track their values:

	a	b	c	
a = 24 : b = 2 : c = 3	24	2	3	
if a / c == b ^ c then				a/c == 8, b^c == 8, true
b = b * c	24	6	3	
end if				
if c + a / b > b then				c+a/b == 3+24/6 == 7 > 6, true
a = a - b * c	6	6	3	24 - 6 * 3 == 24 - 18 == 6
end if				
b = b + a / 3	6	8	3	6 + 6 / 3 == 6 + 2 == 8
if b > a + c then				b == 8, a + c == 6+3 == 9, 8 > 9, false
b = 2 * b				
Else				
c = a / c	6	8	2	6 / 3 == 2
end if				
output a + b / c ^ 2				6 + 8 / 2 ^ 2 == 6 + 8 / 4 == 6 + 2 == 8

Answer: 8