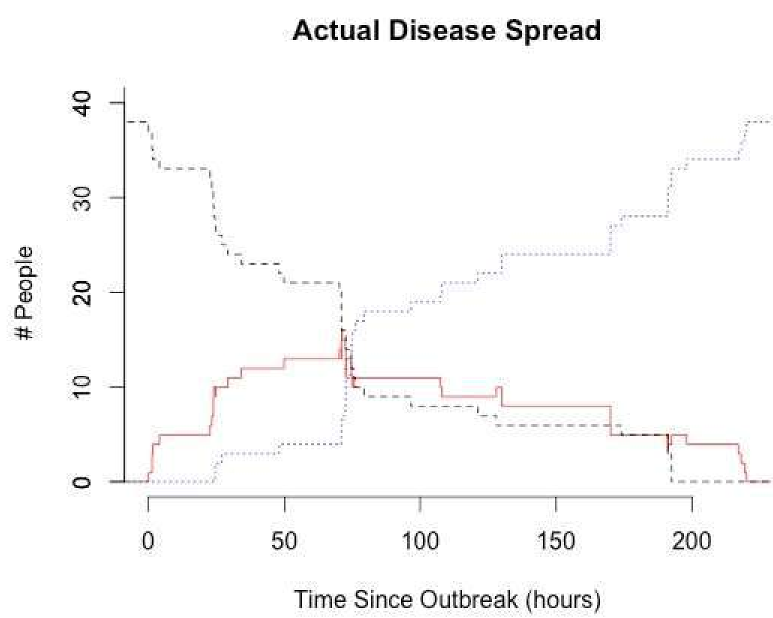
Problem Set 2 ANSWER KEY

1.a) S-I-R Model (Susceptible-Infectious-Recovered) [**4 pts]**
1.b) S-I-R is a basic compartmental model in which all individuals begin as susceptibles- they can be infected with the parasite at any time. One or more individuals are infected with the disease and then randomly mix through the population as infectious individuals- those who can spread the parasite. If these individuals infect another individual, these too become infectious immediately (no latent period). After a certain infectious period, the individual is recovered and assumes lifelong immunity to the parasite. (Deaths may be mentioned, but are not imperative to this description, since out model did not incorporate parasite-induced death). **[10 pts; -2 pts if no mention of compartmental model]**

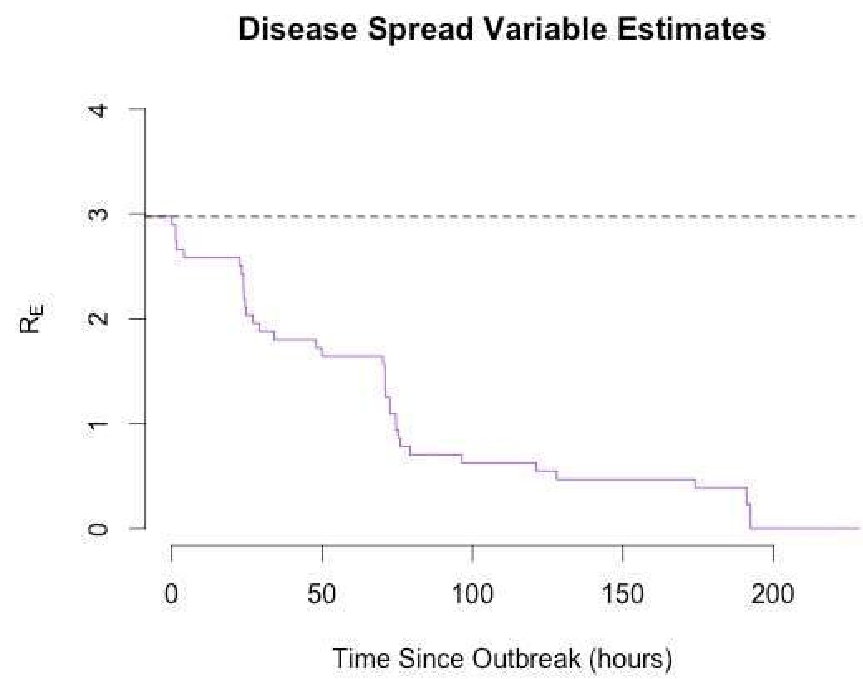
2.a) Graph pasted below: **[4 pts]**



2.b) Black line: susceptible individuals at a given time **[3 pts]**; red line: infected individuals at a given time **[3 pts]**; blue line: recovered individuals at a given time **[3 pts] [-2 pts overall if no mention of variable *at a certain time*]**
2.c) Since the S-I-R model is a compartmental model, all individuals must fit into a certain compartment (susceptible, infected, or recovered) at any given time step (unless they die, and deaths were not built into this real-world simulation), this means that at each time step, the values of S + I + R always equal the population size. As individuals begin to be infected, the number of susceptible individuals decreases as the number of infectious individuals increases; peak number of infectious individuals is reached at approximately 75 hours after the outbreak. As individuals begin to recover, the number of infectious individuals also begins to decrease, until all individuals in the population are recovered and immune at the end of outbreak.
[10 pts; -2 pts if no mention of relationships between decreases in one compartment and increases in another]

3.a) R_0 (Basic Reproduction Number). **[4 pts]**
3.b) Spreads when $R_0 > 1$ **[3 pts]**; Does not spread when $R_0 < 1$ **[3 pts]**
3.c) Because the parasite *did* spread through the population, we would predict that $R_0 > 1$. **[4 pts]**
3.d) $R_0 = 2.973684$; this result agrees with the prediction made in 3.c. **[3 pts]**

4.a) R or R_E (Effective Reproduction Number) **[4 pts]**
4.b) 55 values were estimated for R_E **[2 pts]**; the maximum value was 2.973684 and the minimum value was 0 **[2 pts].** There were 55 values for this number because it was recalculated depending on the proportion of susceptible individuals in the population at each time step (time steps included not only transitions from susceptible to infectious, but also from infectious to recovered, which explains why there were more R_E outputs than the number of individuals in the population; if we look at only the number of unique R_E values, then we can see that the number of unique values for R_E approximately equals the number of individuals in the population). **[4 pts]**
4.c) Graph pasted below (with y-axis label): **[4 pts, -2 pts if no y-axis label]**



4.d) Black line: R_0 (constant over time) **[3 pts]**; purple line: R or R_E over time **[3 pts; -1 pt if no mention of time]**
4.e) The two lines start at the same value because when all individuals are susceptible, $R_E = R_0 * 1$, or $R_E = R_0$ **[4 pts].** The purple-lined value (R_E) dips below the black-lined value (R_0) because R_E decreases as individuals become infected/the number of susceptibles diminishes; the purple-lined value (R_E) eventually reaches 0 when all of the susceptibles in the population have been depleted (infected). **[4 pts]**

5.a) The vaccination equation is $p_c = 1 - \frac{1}{R_0}$, where p_c is the proportion of individuals needing vaccination

By substituting our calculated R_0 into the equation, we get $p_c = 1 - \frac{1}{2.973684}$, or $p_c = 0.6637168$

Then, by multiplying this proportion by the number of susceptibles in Boston (4,500,000), we get $N_{vaccinated} = 0.6637168 \cdot 4500000 = 2986726$ **[5 pts]**

5.b) This vaccination strategy would most likely work for the population of Boston, because it would bring down R_E to 1, which is essentially the break-even point between spreading and not spreading. This does not mean that absolutely no one would become infected, but that the parasite would not be able to spread through the population. **[4 pts]** This number is, however, a minimum number of individuals that would need vaccinating, and if we were to vaccinate more than necessary, this would slow the spread of the parasite through the population even further. **[4 pts]**