

	B	S
B	2, 1	0, 0
S	0, 2	1, 2

• APPLY FICTITIOUS PLAY ALGORITHM FROM THE UNIFORM START STRATEGIES

t	$G_1(B)$	$G_1(S)$	$BR_1$	$G_2(B)$	$G_2(S)$	$BR_2$
0	1/2	1/2	B	1/2	1/2	S
1	3/4	1/4	S	1/4	3/4	B
2	1/2	1/2	B	1/2	1/2	S
3	5/8	3/8	B	3/8	5/8	S
4	1/2	1/2	...	1/2	1/2	...
5	...	...	...	...	...	...

t = 0

$$BR_1(G_2^0) = B$$

$$BR_2(G_1^0) = S$$

INITIALIZATION

t = 1

$$G_1^1 = \frac{1}{2} \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3/4 \\ 1/4 \end{bmatrix}$$

$$G_2^1 = \frac{1}{2} \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/4 \\ 3/4 \end{bmatrix}$$

AVERAGE STRATEGY

$$BR_1(G_2^1) = S$$

$$BR_2(G_1^1) = B$$

BEST RESPONSE

t = 2

$$G_1^2 = \frac{1}{3} \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} + \frac{1}{3} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{1}{3} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$$

$$G_2^2 = \frac{1}{3} \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} + \frac{1}{3} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \frac{1}{3} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$$

AVERAGE STRATEGY

$$BR_1(G_2^2) = B$$

$$BR_2(G_1^2) = S$$

BEST RESPONSE

t = 3

$$G_1^3 = \frac{1}{4} \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} + \frac{1}{4} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{1}{4} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \frac{1}{4} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 5/8 \\ 3/8 \end{bmatrix}$$

$$G_2^3 = \frac{1}{4} \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} + \frac{1}{4} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \frac{1}{4} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{1}{4} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3/8 \\ 5/8 \end{bmatrix}$$

AVERAGE STRATEGY

$$BR_1(G_2^3) = B$$

$$BR_2(G_1^3) = S$$

BEST RESPONSE

...

	H	T
H	1, -1	-1, 1
T	-1, 1	1, -1

• APPLY THE FICTITIOUS PLAY FROM THE UNIFORM STRATEGIES

t	$G_1(H)$	$G_1(T)$	$BR_1$	$G_2(H)$	$G_2(T)$	i
0	1/2	1/2	H	1/2	1/2	1
1	3/4	1/4	H	3/4	1/4	2
2	5/6	1/6	H	1/2	1/2	3
3	7/8	1/8	T	3/8	5/8	4
4	...	...	...	...	...	...
5	...	...	...	...	...	...

t = 0

$$BR_1(G_2^0) \in \{H, T\} \rightarrow \text{WE CHOOSE H}$$

$$BR_2(G_1^0) \in \{H, T\} \rightarrow \text{WE CHOOSE H}$$

TIE BREAKING

t = 1

$$G_1^1 = \frac{1}{2} \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3/4 \\ 1/4 \end{bmatrix}$$

$$G_2^1 = \frac{1}{2} \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 3/4 \\ 1/4 \end{bmatrix}$$

$$BR_1(G_2^1) = H$$

$$BR_2(G_1^1) = T$$

t = 2

$$G_1^2 = \frac{1}{3} \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} + \frac{1}{3} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{1}{3} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 5/6 \\ 1/6 \end{bmatrix}$$

$$G_2^2 = \frac{1}{3} \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} + \frac{1}{3} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{1}{3} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$$

$$BR_1(G_2^2) \in \{H, T\} \rightarrow \text{WE CHOOSE H}$$

$$BR_2(G_1^2) = T$$

TIE BREAKING

t = 3

$$G_1^3 = \frac{1}{4} \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} + \frac{1}{4} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{1}{4} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{1}{4} \begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 7/8 \\ 1/8 \end{bmatrix}$$

$$G_2^3 = \frac{1}{4} \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} + \frac{1}{4} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + \frac{1}{4} \begin{bmatrix} 0 \\ 1 \end{bmatrix} + \frac{1}{4} \begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3/8 \\ 5/8 \end{bmatrix}$$

$$BR_1(G_2^3) = T$$

$$BR_2(G_1^3) = T$$

...

	H	T
H	2, -2	-3, 3
T	-1, 1	2, -2

• APPLY REGRET MATCHING

t	$G_1(H)$	$G_1(T)$	$r_1(H)$	$r_1(T)$	$G_2(H)$	$G_2(T)$	$r_2(H)$	$r_2(T)$
0	1/2	1/2	-0.5	0.5	1/2	1/2	-0.5	0.5
1	0	1	-5	0	0	1	3	0
2	0	1			5/6	1/6		
3								

t = 0

$$r_1^0(H) = -0.5 - 0 = -0.5$$

$$r_1^0(T) = 0.5 - 0 = 0.5$$

$$r_2^0(H) = -0.5 - 0 = -0.5$$

$$r_2^0(T) = 0.5 - 0 = 0.5$$

INSTANTANEOUS REGRET

$$R_1^0(H) = -0.5$$

$$R_1^0(T) = 0.5$$

$$R_2^0(H) = -0.5$$

$$R_2^0(T) = 0.5$$

CUMULATIVE REGRET

$$R_1^{0,+}(H) = 0$$

$$R_1^{0,+}(T) = 0.5$$

$$R_2^{0,+}(H) = 0$$

$$R_2^{0,+}(T) = 0.5$$

CUMULATIVE REGRET PLUS

t = 1

$$G_1^1(H) = \frac{R_1^{0,+}(H)}{R_1^{0,+}(H) + R_1^{0,+}(T)} = 0$$

$$G_1^1(T) = \frac{R_1^{0,+}(T)}{R_1^{0,+}(H) + R_1^{0,+}(T)} = 1$$

$$G_2^1(H) = 0$$

$$G_2^1(T) = 1$$

STRATEGY UPDATE

$$r_1^1(H) = -3 - 2 = -5$$

$$r_1^1(T) = 2 - 2 = 0$$

$$r_2^1(H) = 1 - (-2) = 3$$

$$r_2^1(T) = -2 - (-2) = 0$$

INSTANTANEOUS REGRET

$$R_1^1(H) = -5.5$$

$$R_1^1(T) = 0.5$$

$$R_2^1(H) = 2.5$$

$$R_2^1(T) = 0.5$$

CUMULATIVE REGRET

$$R_1^{1,+}(H) = 0$$

$$R_1^{1,+}(T) = 0.5$$

$$R_2^{1,+}(H) = 2.5$$

$$R_2^{1,+}(T) = 0.5$$

CUMULATIVE REGRET PLUS