POLITECNICO
MILANO 1863

Consistency and Replication Exercises

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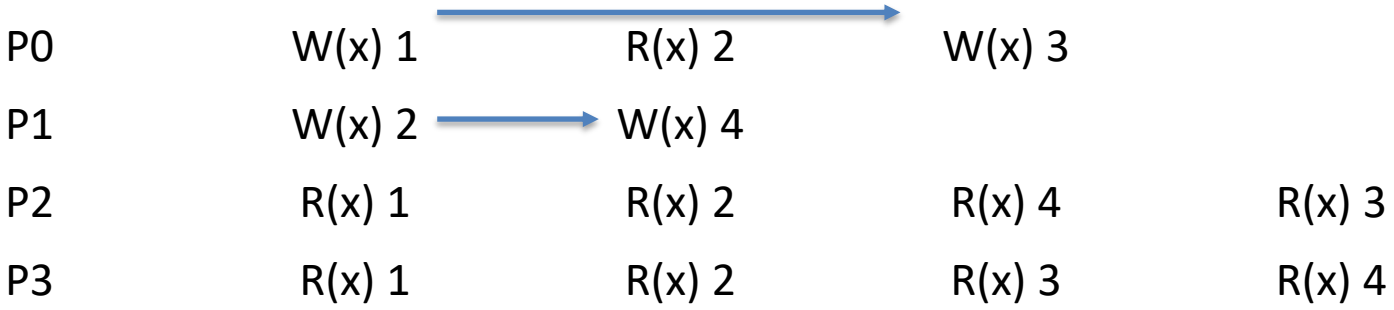
Exercise 1

P0	W(x) 1	R(x) 2	W(x) 3	
P1	W(x) 2	W(x) 4		
P2	R(x) 1	R(x) 2	R(x) 4	R(x) 3
P3	R(x) 1	R(x) 2	R(x) 3	R(x) 4

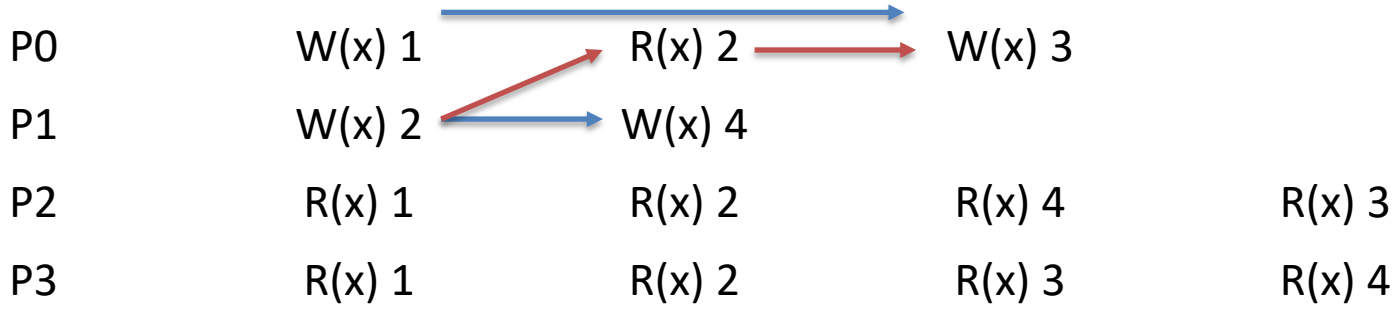
- FIFO? Causal? Sequential?
- If not sequential, can it become sequential by removing a single operation?

Exercise 1



- FIFO?
  - 1 → 3, 2 → 4
  - OK!

Exercise 1



- Causal?
  - 1 → 3, 2 → 4
  - 2 → 3
  - OK!

Exercise 1

P0	W(x) 1	R(x) 2	W(x) 3	
P1	W(x) 2	W(x) 4		
P2	R(x) 1	R(x) 2	R(x) 4	R(x) 3
P3	R(x) 1	R(x) 2	R(x) 3	R(x) 4

- Sequential?
  - 1 → 2 → 3
  - 2 → 4
  - 1 → 2 → 4 → 3
  - 1 → 2 → 3 → 4
  - NO!

Exercise 1

P0	W(x) 1	R(x) 2	W(x) 3	
P1	W(x) 2	W(x) 4		
P2	R(x) 1	R(x) 2	**R(x) 4**	**R(x) 3**
P3	R(x) 1	R(x) 2	**R(x) 3**	**R(x) 4**

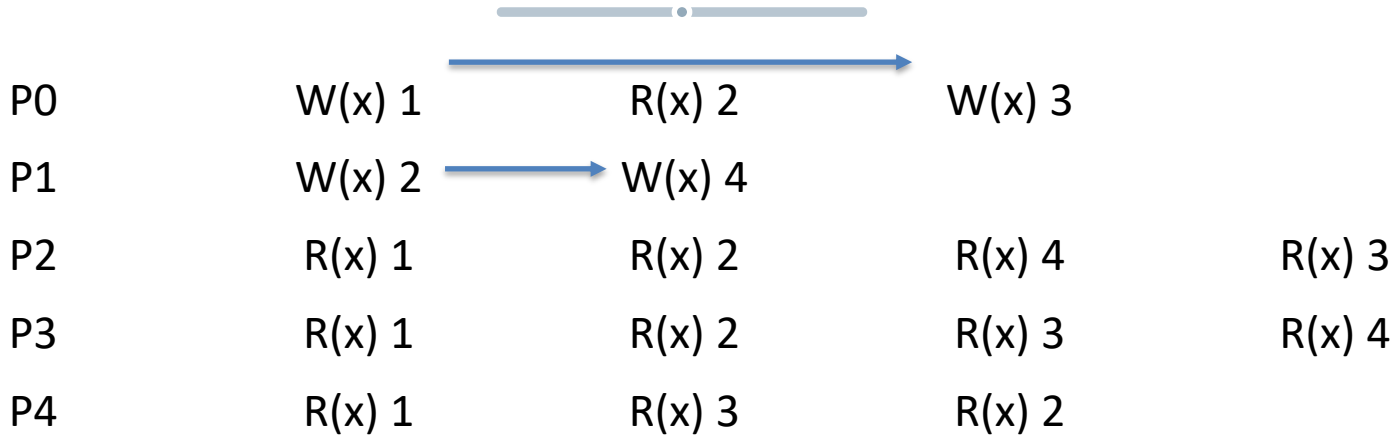
- Sequential?
  - 1 → 2 → 3
  - 2 → 4
  - 1 → 2 → **4 → 3**
  - 1 → 2 → **3 → 4**
  - It is sufficient to relax the order by removing <u>one</u> of the four red operations

Exercise 2

P0	W(x) 1	R(x) 2	W(x) 3	
P1	W(x) 2	W(x) 4		
P2	R(x) 1	R(x) 2	R(x) 4	R(x) 3
P3	R(x) 1	R(x) 2	R(x) 3	R(x) 4
P4	R(x) 1	R(x) 3	R(x) 2	

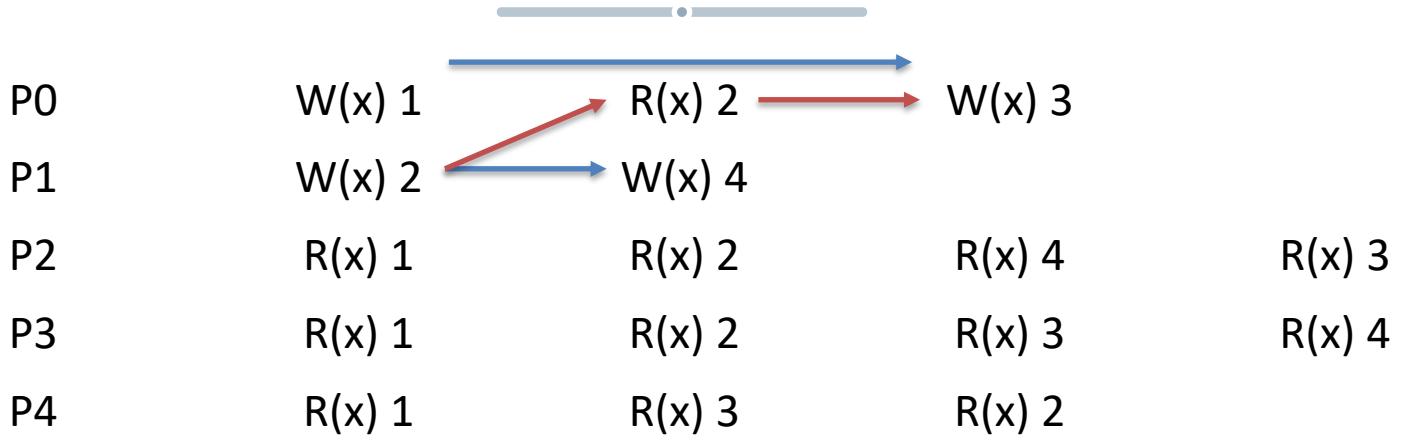
- FIFO? Causal? Sequential?
- If not sequential, can it become sequential by removing a single operation?

Exercise 2



- FIFO?
  - 1 → 3, 2 → 4
  - OK!

Exercise 2



- Causal?
  - 1 → 3, 2 → 4
  - 2 → 3
  - NO!

Exercise 2

P0	W(x) 1	R(x) 2	W(x) 3	
P1	W(x) 2	W(x) 4		
P2	R(x) 1	R(x) 2	R(x) 4	R(x) 3
P3	R(x) 1	R(x) 2	R(x) 3	R(x) 4
P4	R(x) 1	R(x) 3	R(x) 2	

- Sequential? No, it is not causal …

Exercise 2

P0	W(x) 1	R(x) 2	W(x) 3	
P1	W(x) 2	W(x) 4		
P2	R(x) 1	R(x) 2	R(x) 4	R(x) 3
P3	R(x) 1	R(x) 2	R(x) 3	R(x) 4
P4	R(x) 1	R(x) 3	R(x) 2	

- We cannot make it sequentially consistent by removing one operation, because the schedule has two conflicts
  - 3 → 4, 4 → 3
  - 2 → 3, 3 → 2

Exercise 3

P0	W(x) 1	R(x) 2	W(x) 3
P1	R(x) 1	W(x) 2	R(x) 3
P2	R(x) K	R(x) ?	

K	Consistency model	Set of allowed values?
1	FIFO	
1	Sequential	
2	FIFO	
2	Sequential	
3	FIFO	
3	Sequential	

Exercise 3

P0	W(x) 1	R(x) 2	W(x) 3
P1	R(x) 1	W(x) 2	R(x) 3
P2	R(x) **1**	R(x) ?	

K	Consistency model	Set of allowed values?
1	FIFO	{ 1, 2, 3 }
1	Sequential	{ 1, 2, 3 }
2	FIFO	
2	Sequential	
3	FIFO	
3	Sequential	

Exercise 3

P0	W(x) 1	R(x) 2	W(x) 3
P1	R(x) 1	W(x) 2	R(x) 3
P2	R(x) **2**	R(x) ?	

K	Consistency model	Set of allowed values?
1	FIFO	{ 1, 2, 3 }
1	Sequential	{ 1, 2, 3 }
2	FIFO	{ 1, 2, 3 }
2	Sequential	{ 2, 3 }
3	FIFO	
3	Sequential	

Exercise 3

P0	W(x) 1	R(x) 2	W(x) 3
P1	R(x) 1	W(x) 2	R(x) 3
P2	R(x) **3**	R(x) ?	

K	Consistency model	Set of allowed values?
1	FIFO	{ 1, 2, 3 }
1	Sequential	{ 1, 2, 3 }
2	FIFO	{ 1, 2, 3 }
2	Sequential	{ 2, 3 }
3	FIFO	{ 2, 3 }
3	Sequential	{ 3 }

Exercise 4

P0	R(x) 1	W(y) 2	W(x) 3	R(y) 2
P1	W(x) 2	R(y) 1	R(x) 3	R(y) 2
P2	R(x) 0	W(y) 1	W(x) 1	R(y) 2
P3	R(y) 0	R(x) 1	R(y) 1	W(x) 4

- FIFO?
- Causal?
- Sequential?

Exercise 4

P0	R(x) 1	W(y) 2	W(x) 3	R(y) 2
P1	W(x) 2	R(y) 1	R(x) 3	R(y) 2
P2	R(x) 0	W(y) 1	W(x) 1	R(y) 2
P3	R(y) 0	R(x) 1	R(y) 1	W(x) 4

- R(x)0 → R(y)0 → W(y)1 → W(x)2 → W(x)1 → R(x)1 → R(x)1 → R(y)1 → R(y)1 → W(y)2 → W(x)3 → R(x)3 → R(y)2 → R(y)2 → R(y)2 → W(x)4
- Sequentially consistent!
- And thus also causally consistent and FIFO consistent

Exercise 5

Consider a distributed data store with a numeric variable (initial value = 0).
Three processes, A, B and C, interact with the store, running the following instructions.

```
A:      while(true) {
                if (x<3) x++;
        }

B:      while(true) {
                if (x>0) x--;
        }

C:      while(true) {
                if (x>0) x--;
        }
```

The following requirement has to be satisfied: "Each process must always read a value of x between 0 and 3 (0 and 3 included)"

Exercise 5

Consider 4 implementations (1, 2, 3, 4) with the following properties.

1. The store presents a FIFO consistency model. Read and write operations are considered as separate instructions.
2. The store presents a sequential consistency model. Read and write operations are considered as separate instructions.
3. The store presents a FIFO consistency model. A read operation, the condition evaluation and the subsequent write operation are considered as a single atomic instruction.
4. The store presents a sequential consistency model. A read operation, the condition evaluation and the subsequent write operation are considered as an atomic instruction.

In all the cases, when a write operation is performed, an identifier of the operation (and not the new value of x) is propagated to all replicas.

Write, for each implementation, if it satisfies the requirement. If not, show an example in which it is violated.

Exercise 5

A:
```
while(true) {
        if (x<3) x++;
}
```

B:
```
while(true) {
        if (x>0) x--;
}
```

C:
```
while(true) {
        if (x>0) x--;
}
```

FIFO consistency model.
Read and write operations are considered as separate instructions.

A reads x = 0 → writes x = 1
B reads x = 1
C reads x = 1
B writes x-- → x = 0
C writes x-- → x = -1

Exercise 5

A:
```
while(true) {
        if (x<3) x++;
}
```

B:
```
while(true) {
        if (x>0) x--;
}
```

C:
```
while(true) {
        if (x>0) x--;
}
```

Sequential consistency model. Read and write operations are considered as separate instructions.

A reads x = 0 → writes x = 1

B reads x = 1

C reads x = 1

B writes x-- → x = 0

C writes x-- → x = -1

Exercise 5

A:
```
while(true) {
        if (x<3) x++;
}
```

B:
```
while(true) {
        if (x>0) x--;
}
```

C:
```
while(true) {
        if (x>0) x--;
}
```

FIFO consistency model. Read and write operations are considered as an atomic instruction.

A reads x = 0 → writes x = 1

B reads x = 1 and propagates x--

C reads the instruction from B before the instruction from A and (temporarily) observes x = -1

Exercise 5

A:
```
while(true) {
        if (x<3) x++;
}
```

B:
```
while(true) {
        if (x>0) x--;
}
```

C:
```
while(true) {
        if (x>0) x--;
}
```

Sequential consistency model. Read and write operations are considered as an atomic instruction.

This implementation satisfies the requirements. A, B, C need to see the same interleaving of operations, and so the previous counter-example is not possible anymore.

Exercise 6

P0	R(x) 2	W(y) 3	W(x) 4	
P1	W(x) 1	W(x) 2	W(x) 5	R(y) 3
P2	R(x) 2	R(x) 5	R(y) 3	R(x) 4
P3	R(x) K	R(y) 3	R(x) ?	

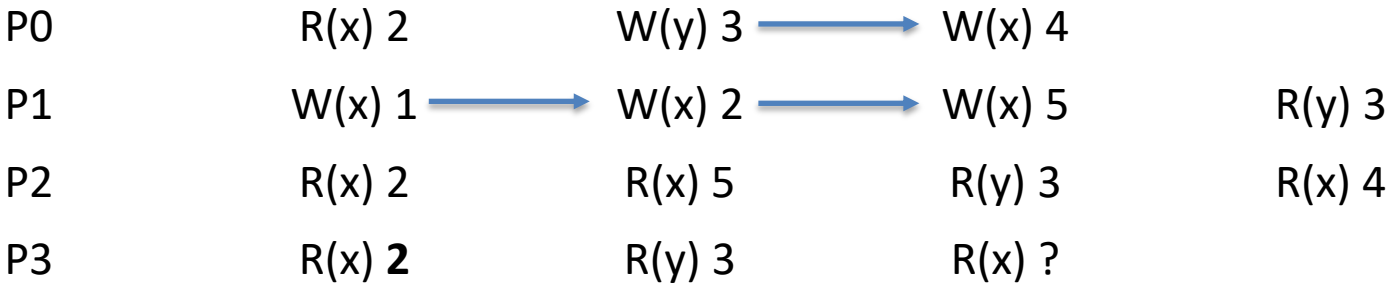
- Which values can P3 read at the end of the schedule in the case of FIFO, causal, and sequential consistency, for K = 2, 4, 5?

Exercise 6

P0	R(x) 2	W(y) 3	W(x) 4	
P1	W(x) 1	W(x) 2	W(x) 5	R(y) 3
P2	R(x) 2	R(x) 5	R(y) 3	R(x) 4
P3	R(x) **2**	R(y) 3	R(x) ?	

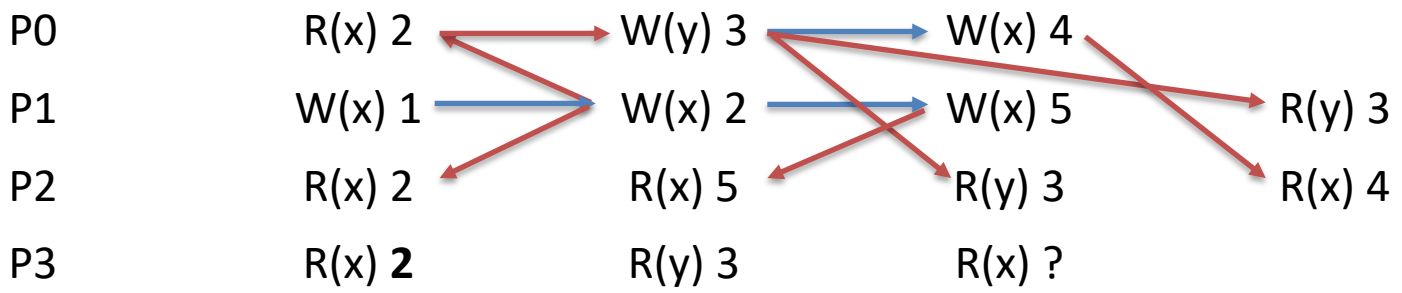
K = 2	FIFO	
	Causal	
	Sequential	
K = 4	FIFO	
	Causal	
	Sequential	
K = 5	FIFO	
	Causal	
	Sequential	

Exercise 6



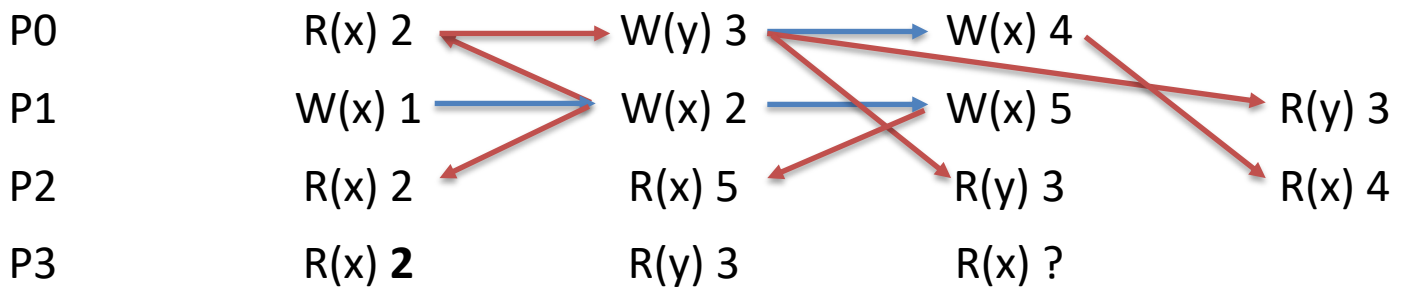
K = 2	FIFO	2, 4, 5
	Causal	
	Sequential	
K = 4	FIFO	
	Causal	
	Sequential	
K = 5	FIFO	
	Causal	
	Sequential	

Exercise 6



K = 2	FIFO	2, 4, 5
	Causal	2, 4, 5
	Sequential	
K = 4	FIFO	
	Causal	
	Sequential	
K = 5	FIFO	
	Causal	
	Sequential	

Exercise 6



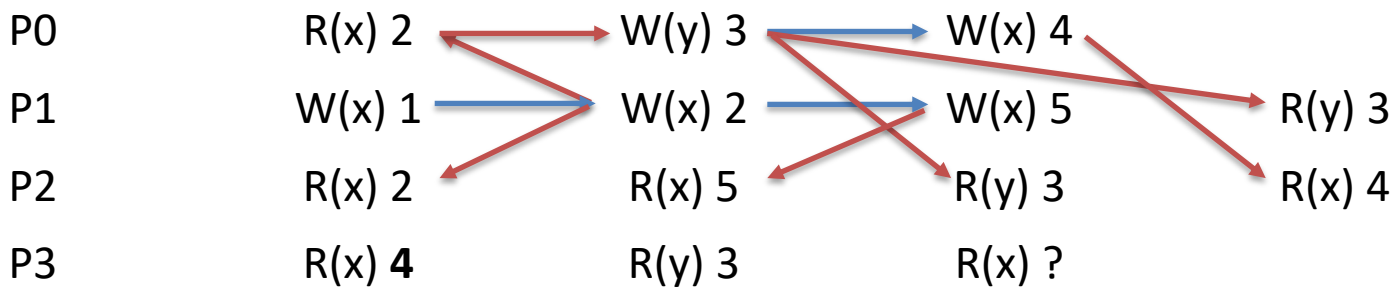
K = 2	FIFO	2, 4, 5
	Causal	2, 4, 5
	Sequential	2, 4, 5
K = 4	FIFO	
	Causal	
	Sequential	
K = 5	FIFO	
	Causal	
	Sequential	

Exercise 6

P0	R(x) 2	W(y) 3 →	W(x) 4	
P1	W(x) 1 →	W(x) 2 →	W(x) 5	R(y) 3
P2	R(x) 2	R(x) 5	R(y) 3	R(x) 4
P3	R(x) **4**	R(y) 3	R(x) ?	

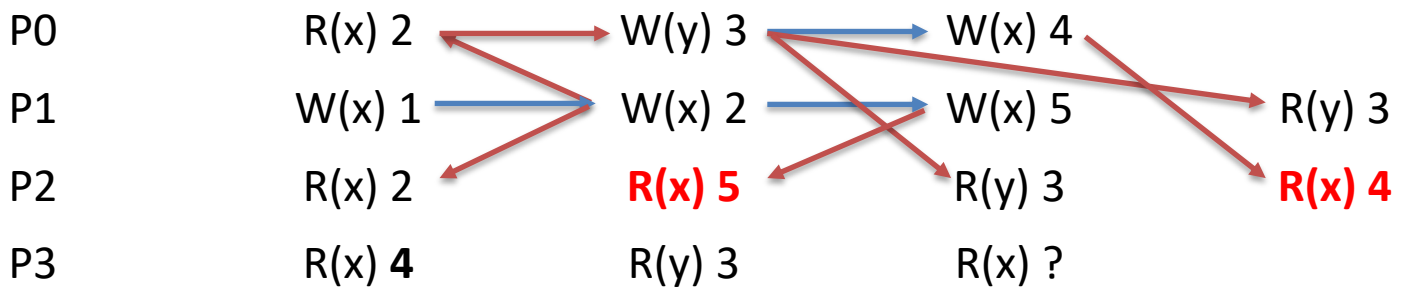
K = 2	FIFO	2, 4, 5
	Causal	2, 4, 5
	Sequential	2, 4, 5
K = 4	FIFO	1, 2, 4, 5
	Causal	
	Sequential	
K = 5	FIFO	
	Causal	
	Sequential	

Exercise 6



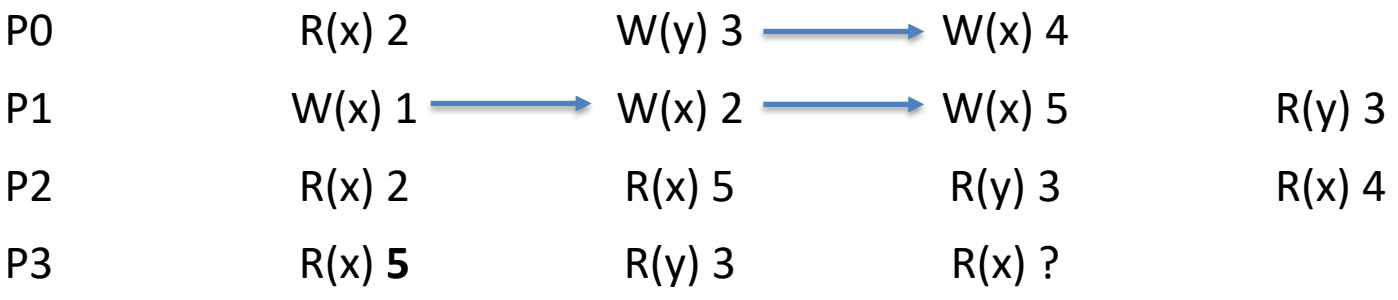
K = 2	FIFO	2, 4, 5
	Causal	2, 4, 5
	Sequential	2, 4, 5
K = 4	FIFO	1, 2, 4, 5
	Causal	4, 5
	Sequential	
K = 5	FIFO	
	Causal	
	Sequential	

Exercise 6



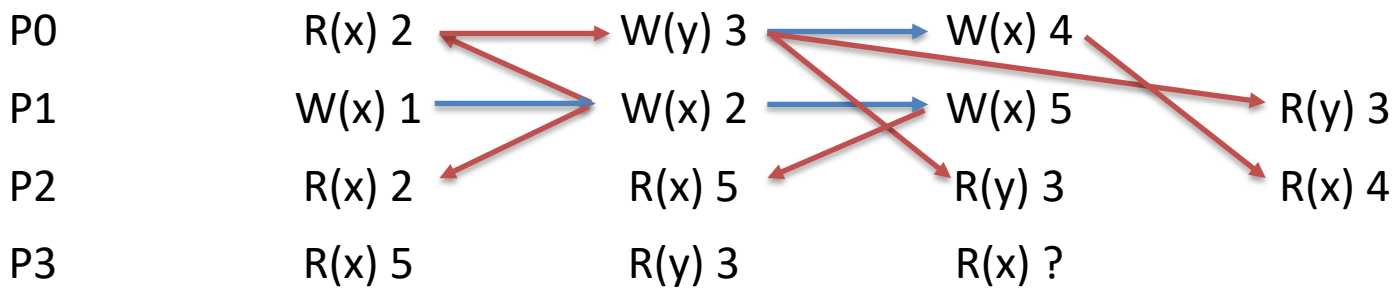
K = 2	FIFO	2, 4, 5
	Causal	2, 4, 5
	Sequential	2, 4, 5
K = 4	FIFO	1, 2, 4, 5
	Causal	4, 5
	Sequential	4
K = 5	FIFO	
	Causal	
	Sequential	

Exercise 6



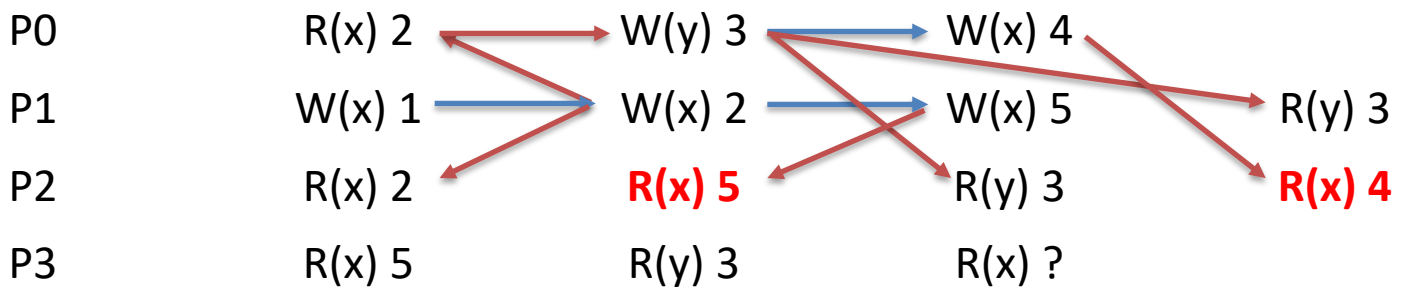
K = 2	FIFO	2, 4, 5
	Causal	2, 4, 5
	Sequential	2, 4, 5
K = 4	FIFO	1, 2, 4, 5
	Causal	4, 5
	Sequential	4
K = 5	FIFO	4, 5
	Causal	
	Sequential	

Exercise 6



K = 2	FIFO	2, 4, 5
	Causal	2, 4, 5
	Sequential	2, 4, 5
K = 4	FIFO	1, 2, 4, 5
	Causal	4, 5
	Sequential	4
K = 5	FIFO	4, 5
	Causal	4, 5
	Sequential	

Exercise 6



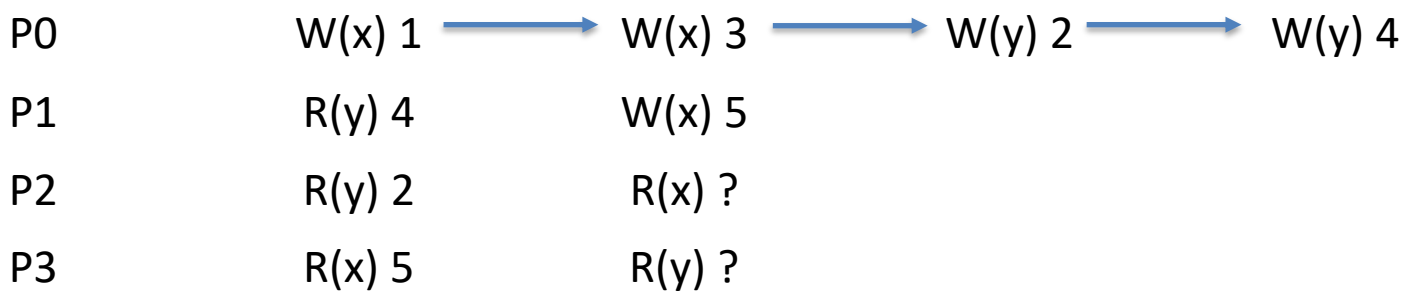
K = 2	FIFO	2, 4, 5
	Causal	2, 4, 5
	Sequential	2, 4, 5
K = 4	FIFO	1, 2, 4, 5
	Causal	4, 5
	Sequential	4
K = 5	FIFO	4, 5
	Causal	4, 5
	Sequential	4, 5

Exercise 7

P0	W(x) 1	W(x) 3	W(y) 2	W(y) 4
P1	R(y) 4	W(x) 5		
P2	R(y) 2	R(x) ?		
P3	R(x) 5	R(y) ?		

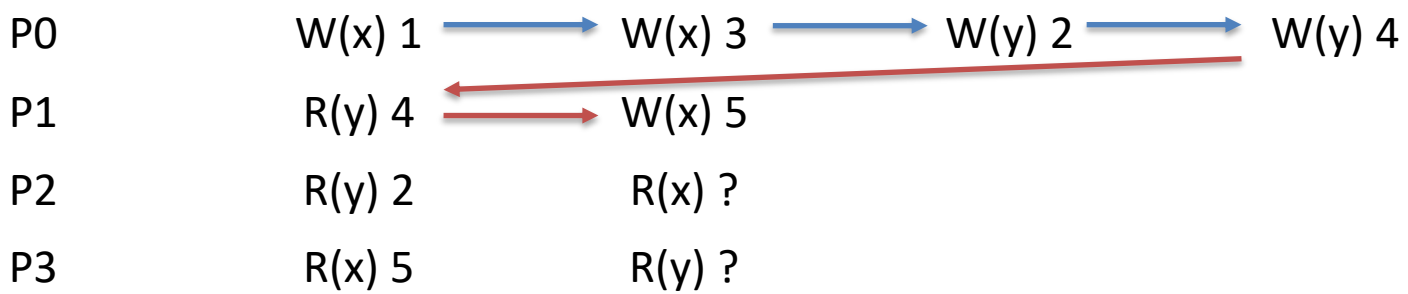
- Which values are P2 and P3 allowed to read in the cases of FIFO, causal, and sequential consistency models?

Exercise 7



- FIFO
  - P2 can read 3, 5 since W(x) 1 → W(x) 3 → W (y) 2
  - P3 can read 2, 4 (no constraints with x=5)

Exercise 7



- Causal
  - P2 can read 3, 5 since W(x) 1 → W(x) 3 → W(y) 2
  - P3 can read 4 since W(y) 2 → W(y) 4 → W(x) 5

- Same for sequential

Exercise 8

P0	W(y) 1	R(x) 1	R(x) 3	R(y) 4
P1	R(y) 1	W(x) 1	R(y) 3	W(x) 3
P2	W(x) 2	R(x) 2	W(y) 3	W(y) 4
P3	R(y) 1	W(y) 2	R(x) 1	R(x) 3
P4	R(x) 0	R(y) 1	R(x) 3	R(y) 2

- Is the schedule FIFO consistent?
- Is the schedule causally consistent?
- Is the schedule sequentially consistent?

Exercise 8

P0	W(y) 1	R(x) 1	R(x) 3	R(y) 4
P1	R(y) 1	W(x) 1	R(y) 3	W(x) 3
P2	W(x) 2	R(x) 2	W(y) 3	W(y) 4
P3	R(y) 1	W(y) 2	R(x) 1	R(x) 3
P4	R(x) 0	R(y) 1	R(x) 3	R(y) 2

- FIFO constraints
  - x=1 → x=3
  - x=2 → y=3 → y=4
- Satisfied!

Exercise 8

P0	W(y) 1	R(x) 1	R(x) 3	R(y) 4
P1	R(y) 1	W(x) 1	R(y) 3	W(x) 3
P2	W(x) 2	R(x) 2	W(y) 3	W(y) 4
P3	R(y) 1	W(y) 2	R(x) 1	R(x) 3
P4	R(x) 0	R(y) 1	R(x) 3	R(y) 2

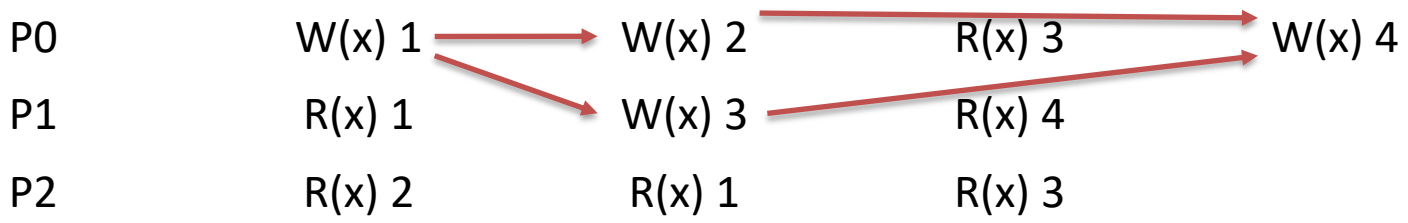
- FIFO constraints
  - x=1 → x=3
  - x=2 → y=3 → y=4
- Causal constraints
  - y=1 → x=1
  - y=1 → x=3
  - y=3 → x=3
  - y=1 → y=2

Exercise 8

P0	W(y) 1	R(x) 1	R(x) 3	R(y) 4
P1	R(y) 1	W(x) 1	R(y) 3	W(x) 3
P2	W(x) 2	R(x) 2	W(y) 3	W(y) 4
P3	R(y) 1	W(y) 2	R(x) 1	R(x) 3
P4	R(x) 0	R(y) 1	R(x) 3	R(y) 2

- Sequential consistency
- R(x)0 W(y)1 R(y)1 R(y)1 R(y)1 W(x)2 R(x)2 W(y)3 W(x)1 R(y)3 R(x)1 W(y)2 R(x)1 W(x)3 R(x)3 R(x)3 R(y)2 R(x)3 W(y)4 R(y)4
- It is possible to find a valid sequence, so the schedule is sequentially consistent

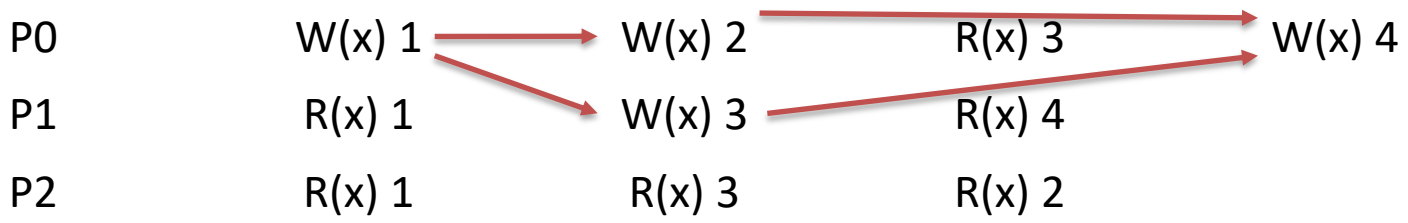
Causal consistency



1. Try to determine all happens-before relations between writes
   - 1 → 2, 1 → 4, 1 → 3, 2 → 4, 3 → 4

2. Check that all the reads satisfy those relations
   - 2 → 1
   - NOT causal

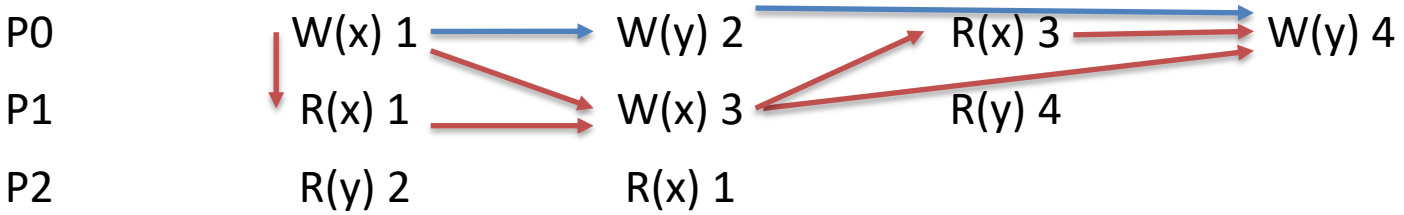
Causal consistency



1. Try to determine all happens-before relations between writes
   - 1 → 2, 1 → 4, 1 → 3, 2 → 4, 3 → 4

2. Check that all the reads satisfy those relations
   - 3 → 2 is NOT a problem
   - The schedule is causal

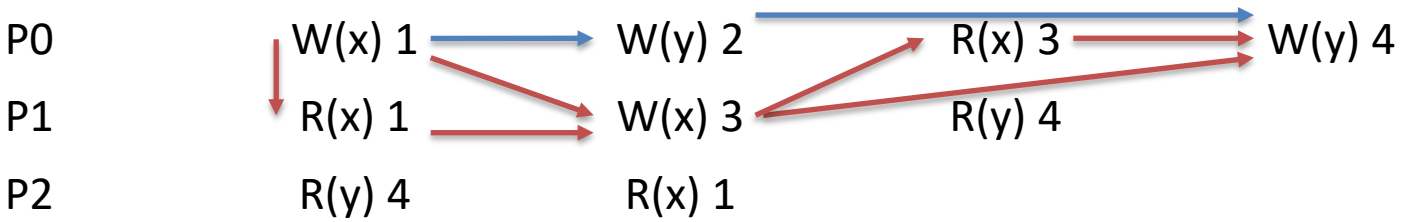
Causal consistency



1. Try to determine all happens-before relations between writes
   - **x1 → y2**, x1 → y4, x1 → x3, y2 → y4, x3 → y4

2. Check that all the reads satisfy those relations
   - y2 → x1 is ***not*** a problem
     - When y becomes 2, x can still be 1
   - The schedule is causal

Causal consistency



1. Try to determine all happens-before relations between writes
   - x1 → y2, x1 → y4, **x1 → x3**, y2 → y4, **x3 → y4**

2. Check that all the reads satisfy those relations
   - y4 → x1 ***is*** a problem
     - When y becomes 4, x should have become 3
   - The schedule is ***not*** causal

Consistency protocols

- Describe the difference between passive and active replication

- Present three different approaches to active replication and discuss their benefits and limitations

Consistency protocols

- Does FIFO consistency allow a process to operate (read and write) from its local replica even if it is disconnected from the other replicas?
  - If yes, under which assumptions?
  - Explain why
- What about causal consistency?
- What about sequential consistency?

Eventual consistency

- Does eventual consistency imply sequential consistency?
- Does eventual consistency imply causal consistency?
- Does eventual consistency imply FIFO consistency?

Eventual consistency

- Describe CRDTs
  - Why they are useful in the context of eventual consistency?
  - What are their limitations?

Eventual consistency

- Consider a messaging system implemented on top of an eventually consistent datastore.

- How could you make the system FIFO consistent?

- How could you you make the system causally consistent?