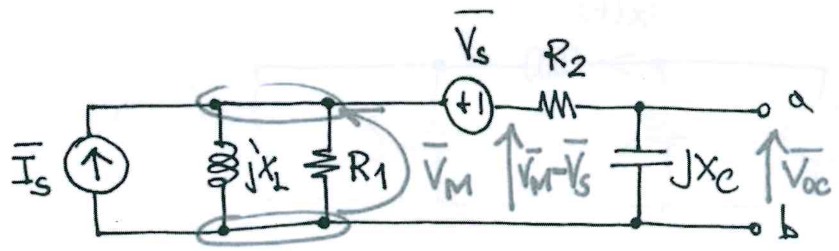
EX



$\overline{V}_s = jV$
$\overline{I}_s = 1A$
$X_L = 8\Omega$
$X_C = -4\Omega$
$R_1 = 8\Omega$
$R_2 = 4\Omega$

Determinare il circuito equivalente di Thevenin e di Norton visto ai morsetti a,b, nel dominio dei fasori

- Tensione a vuoto $\overline{V}_{oc}$

Millman:

$$\overline{V}_M = \frac{\overline{I}_s + \dfrac{\overline{V}_s}{R_2 + jX_C}}{\dfrac{1}{jX_L} + \dfrac{1}{R_1} + \dfrac{1}{R_2 + jX_C}}$$

Partitore di tensione:

$$\overline{V}_{oc} = (\overline{V}_M - \overline{V}_s) \cdot \frac{jX_C}{R_2 + jX_C}$$

Calcoli:

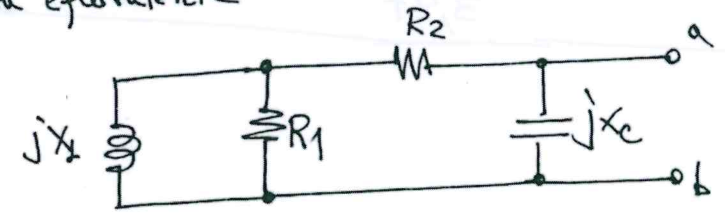
$$\overline{V}_M = \frac{1 + \dfrac{j}{4-j4}\dfrac{4+j4}{4+j4}}{-j\dfrac{1}{8} + \dfrac{1}{8} + \dfrac{1}{4-j4}\dfrac{4+j4}{4+j4}} = \frac{1 + \dfrac{j\cancel{4} - \cancel{4}}{\cancel{32}\,8}}{-j\dfrac{1}{8} + \dfrac{1}{8} + \dfrac{\cancel{4} + j\cancel{4}}{\cancel{32}\,8}} =$$

$$= \frac{\dfrac{7}{8} + j\dfrac{1}{8}}{\dfrac{1}{4}} = \left(\frac{7}{\cancel{8}_2} + j\frac{1}{\cancel{8}_2}\right) \cdot \cancel{4} = \frac{7+j}{2}\ V$$

$$\overline{V}_{oc} = \left(\frac{7}{2} + j\frac{1}{2} - j\right)\frac{-j4}{4-j4} = \frac{(7-j)}{2}\,\frac{-j}{1-j}\,\frac{1+j}{1+j} =$$

$$= \frac{7-j}{2}\,\frac{-j+1}{2} = \frac{-j7+7-1-j}{4} = \frac{-j8+6}{4} = \frac{3}{2} - j2\ V$$

- Impedenza equivalente



$$\overline{Z}_{ab} = \left[(R_1 /\!/ jX_L) + R_2\right] /\!/ jX_C$$

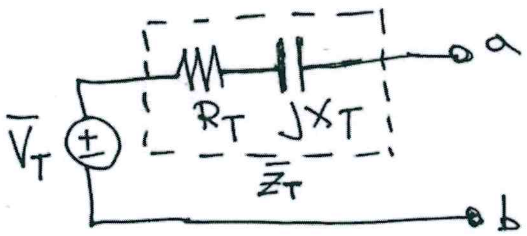
Calcoli:

$$\overline{Z}_{ab} = \left[\frac{\cancel{8}\, j8}{\cancel{8}+j\cancel{8}} + 4 \right] /\!/ (-j4) = \left[\frac{j8}{1+j}\,\frac{1-j}{1-j} + 4 \right] /\!/ (-j4) =$$

$$= \left[\frac{j8+8}{2} + 4 \right] /\!/ (-j4) = (8+j4) /\!/ (-j4) =$$

$$= \frac{(8+j4)(-j4)}{8+\cancel{j4}-\cancel{j4}} = \frac{-j32+16}{8} = 2-j4\ \Omega$$

- CIRCUITO EQUIVALENTE DI THEVENIN



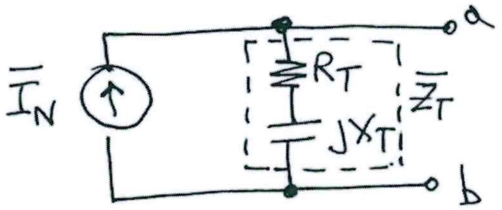
$\overline{V}_T = \overline{V}_{OC} = \frac{3}{2} - j2\ V$

$\overline{Z}_T = \overline{Z}_{ab} = 2 - j4\,\Omega$

$R_T = 2\,\Omega$

$X_T = -4\,\Omega$

- CIRCUITO EQUIVALENTE DI NORTON (per trasformazione di quello di Thevenin)

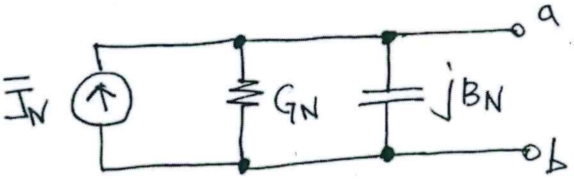


$$\overline{I}_N = \frac{\overline{V}_T}{\overline{Z}_T} = \frac{\frac{3}{2} - j2}{2-j4}\,\frac{2+j4}{2+j4} =$$

$$= \frac{3+j6-j4+8}{20} = \frac{11}{20} + j\frac{1}{10}\ A$$

Non e' ancora il circuito equivalente di Norton. Questo richiede infatti una rappresentazione tipo // dell'impedenza:

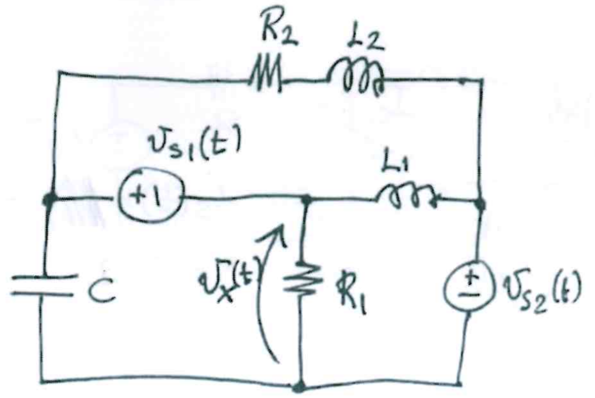
$$\overline{Y}_N = \frac{1}{\overline{Z}_T} = \frac{1}{2-j4}\,\frac{2+j4}{2+j4} = \frac{2+j4}{20} = \underbrace{\frac{1}{10}}_{G_N} + j\underbrace{\frac{1}{5}}_{B_N}\ S$$



$G_N = \frac{1}{10}\ S$ ($R_N = 10\,\Omega$)

$B_N = \frac{1}{5}\ S$ ($X_N = -5\,\Omega$)

EX



$v_{s1}(t)$ = [redacted] $\cos(1000t)$, V

$v_{s2}(t)$ = [redacted] $\cos(1000t)$, V

$R_1 = 1\ \Omega$

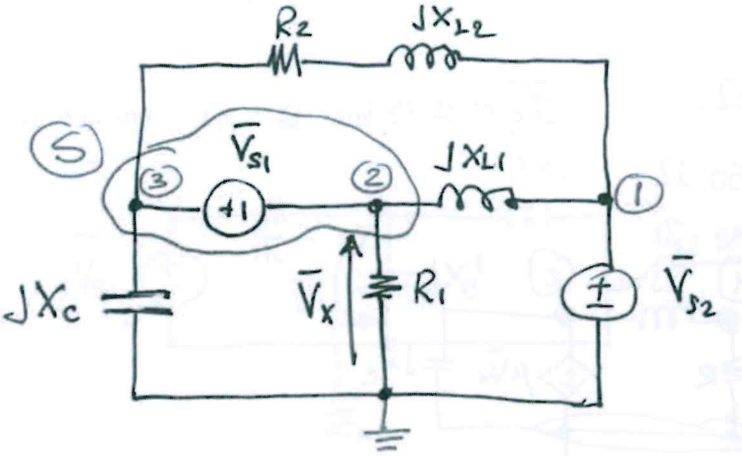
$R_2 = 5\ \Omega$

$L_1 = 10$ mH

$L_2 = 5$ mH

$C = 1$ mF

Determinare $v_x(t)$ a regime.



$\bar{V}_{s1} = \bar{V}_{s2} = 1$ V

$\omega = 1000$ rad/s

$X_{L1} = \omega L_1 = 10\ \Omega$

$X_{L2} = \omega L_2 = 5\ \Omega$

$X_C = -\frac{1}{\omega C} = -1\ \Omega$

Analisi nodale

$\bar{V}_1 = \bar{V}_{s2}$ NOTA!

$\bar{V}_3 = \bar{V}_2 + \bar{V}_{s1}$ VINCOLATA!

$\bar{V}_2$ INCOGNITA!

KCL (S) :
$$\frac{\bar{V}_3}{jX_C} + \frac{\bar{V}_2}{R_1} + \frac{\bar{V}_2 - \bar{V}_1}{jX_{L1}} + \frac{\bar{V}_3 - \bar{V}_1}{R_2 + jX_{L2}} = 0$$

$$\boxed{\frac{\bar{V}_2 + \bar{V}_{s1}}{jX_C} + \frac{\bar{V}_2}{R_1} + \frac{\bar{V}_2 - \bar{V}_{s2}}{jX_{L1}} + \frac{\bar{V}_2 + \bar{V}_{s1} - \bar{V}_{s2}}{R_2 + jX_{L2}} = 0}$$

1 eq. in una incognita

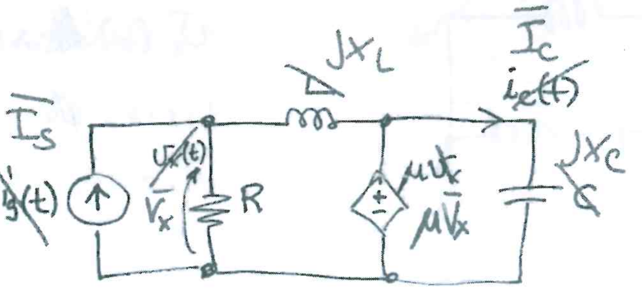
sostituendo:

$$\frac{\bar{V}_2 + 1}{-j} + \frac{\bar{V}_2}{1} + \frac{\bar{V}_2 - 1}{j10} + \frac{\bar{V}_2 + \cancel{1} - \cancel{1}}{5 + j5} = 0$$

$$\bar{V}_2 \left(j + 1 - j\frac{1}{10} + \frac{\cancel{5} - j\cancel{5}}{\cancel{50}\ 10} \right) = -j - j\frac{1}{10} \Rightarrow \bar{V}_2 \left(\frac{11}{10} + j\frac{8}{10} \right) = -j\frac{11}{10}$$

$$\Rightarrow \bar{V}_2 = \frac{-j11}{11 + j8} = 0{,}81\, e^{-j126^\circ} \Rightarrow \bar{V}_x = \bar{V}_2 \Rightarrow \boxed{v_x(t) = \text{[redacted]}\ 0{,}81 \cos(1000t - 126^\circ), \text{V}}$$

EX|



$i_S(t) =$ [redacted] $\cos(1000t)$, A

$R = 2\,\Omega$

$L = 20$ mH

$C = 20\ \mu F$

$\mu = 10$

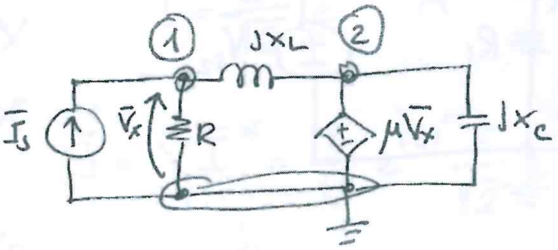
$i_C(t) = ?$ a regime

Passo dal dominio del tempo al dominio dei fasori:
$\omega = 1000$ rad/s

$$X_L = \omega L = 1000 \cdot 20 \cdot 10^{-3} = 20\ \Omega \qquad \bar{I}_S = 1\ A$$

$$X_C = -\frac{1}{\omega C} = -\frac{1}{1000 \cdot 20 \cdot 10^{-6}} = -50\ \Omega$$

Analisi nodale
(con gen. pilotati)



(*) $\bar{V}_2 = \mu \bar{V}_x$ VINCOLATA DAL GENERATORE IDEALE DI TENSIONE (PILOTATO)

$\bar{V}_x = \bar{V}_1$ ESPRESSIONE DELLA PILOTANTE IN FUNZIONE DELLE TENSIONI NODALI

$\bar{V}_1$ INCOGNITA $\Rightarrow$ KCL 1

$$-\bar{I}_S + \frac{\bar{V}_1}{R} + \frac{\bar{V}_1 - \bar{V}_2}{jX_L} = 0$$

SOSTITUISCO (*) $\Rightarrow$ $\boxed{-\bar{I}_S + \frac{\bar{V}_1}{R} + \frac{\bar{V}_1 - \mu\bar{V}_1}{jX_L} = 0}$ 1 EQ. IN 1 INCOGNITA

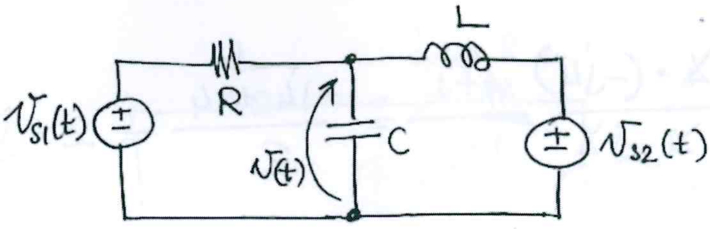
$$-1 + \frac{\bar{V}_1}{2} + \frac{\bar{V}_1 - 10\bar{V}_1}{j20} = 0 \qquad \bar{V}_1\left(\frac{1}{2} + j\frac{9}{20}\right) = 1 \qquad \bar{V}_1 = \frac{20}{10 + j9} =$$

$$= 1{,}49\, e^{-j41{,}99°}\ V$$

$$\Rightarrow \bar{I}_C = \frac{\bar{V}_2}{jX_C} = \frac{1{,}49\, e^{-j41{,}99°} \cdot 10}{50\, e^{-j90°}} = 0{,}298\, e^{j48{,}01°}\ A \cong 298\, e^{j48°}\ mA$$

$$\boxed{i_C(t) = \text{[redacted]} \cdot 298 \cos(1000t + 48°),\ mA}$$

EX



$v_{S1}(t) = \frac{10}{\sqrt{2}} \cos(2t)$, V

$v_{S2}(t) = \frac{10}{\sqrt{2}} \cos(4t - 45°)$, V

$R = 4\,\Omega$

$L = 1\,H$

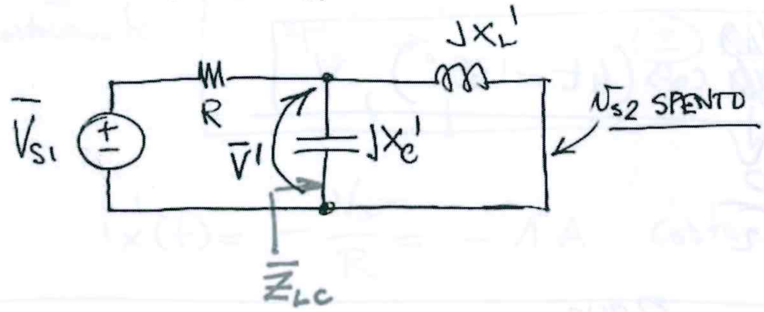
$c = 1/16\,F$

Determinare $v(t)$ a regime

Ci sono 2 PULSAZIONI DIVERSE PER LE SORGENTI: $\omega_1 = 2$ rad/s ; $\omega_2 = 4$ rad/s

$\Rightarrow$ APPLICO <u>SOVRAPPOSIZIONE</u> DI REGIMI SINUSOIDALI (NEL DOMINIO DEL TEMPO!)

- Pulsazione $\omega_1 = 2$ rad/s



$\bar{V}_{S1} = \frac{10}{\sqrt{2}}$ V

$X_L' = \omega_1 L = 2\,\Omega$

$X_C' = -\frac{1}{\omega_1 C} = -8\,\Omega$

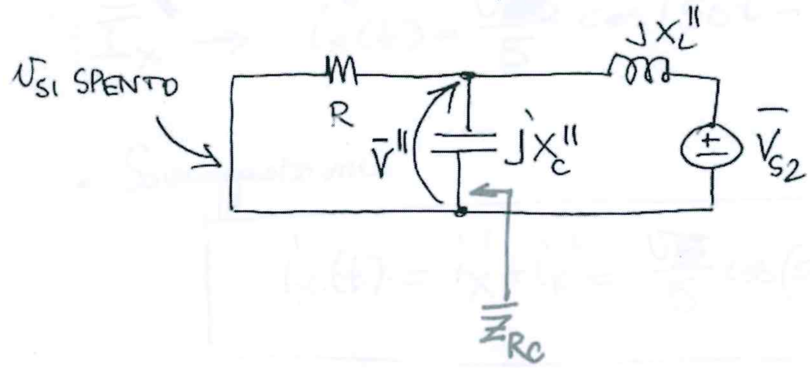
$$\bar{Z}_{LC} = jX_C' \parallel jX_L' = \frac{-j8 \cdot j2}{-j8+j2} = \frac{16}{-j6} = j\frac{8}{3}\,\Omega$$

$$\bar{V}' = \bar{V}_{S1} \cdot \frac{\bar{Z}_{LC}}{\bar{Z}_{LC} + R} = \frac{10}{\sqrt{2}} \cdot \frac{j\frac{8}{3}}{j\frac{8}{3} + 4} = \frac{10}{\sqrt{2}} \frac{j\cancel{8}^{2}}{j\cancel{8}_{2}+\cancel{12}_{3}} \frac{3-j2}{3-j2}$$

$$= \frac{10}{\sqrt{2}} \frac{j6+4}{13} = \frac{10}{\sqrt{2} \cdot 13} \sqrt{6^2+4^2}\, e^{j \operatorname{arctg} \frac{6}{4}} = 3{,}923\, e^{j56{,}31°}\ \text{V}$$

$$\bar{V}' \rightarrow \boxed{v'(t) = 3{,}923 \cos(2t + 56{,}31°),\ \text{V}}$$

- Pulsazione $\omega_2 = 4$ rad/s



$\bar{V}_{S2} = \frac{10}{\sqrt{2}} e^{-j45°}$ V

$X_L'' = \omega_2 L = 4\,\Omega$

$X_C'' = -\frac{1}{\omega_2 C} = -4\,\Omega$

$$\overline{Z}_{RC} = R /\!/ jX_C'' = \frac{\cancel{4}\cdot(-j4)}{\cancel{4}-j\cancel{4}}\,\frac{1+j}{1+j} = \frac{-j4+4}{2} = 2-j2\ \Omega$$

$$\overline{V}'' = \overline{V}_{S2}\cdot\frac{\overline{Z}_{RC}}{\overline{Z}_{RC}+jX_L''} = \frac{10}{\sqrt{2}}\,e^{-j45^\circ}\,\frac{2-j2}{2-j2+j4} = \frac{10}{\sqrt{2}}\,e^{-j45}\,\frac{1-j}{1+j}\,\frac{1-j}{1-j}$$

$$= \frac{10}{\sqrt{2}}\,e^{-j45^\circ}\,\frac{\cancel{1}-\cancel{2}j-\cancel{1}}{\cancel{2}} = \frac{10}{\sqrt{2}}\,e^{-j45^\circ}(-j) = \frac{10}{\sqrt{2}}\,e^{-j45^\circ}e^{-j90^\circ}$$

$$= \frac{10}{\sqrt{2}}\,e^{-j135^\circ}\ \text{V}$$

$$\overline{V}'' \longrightarrow \boxed{v''(t) = \frac{10}{\sqrt{2}}\cos(4t-135^\circ),\ \text{V}}$$

- Sovrapposizione

$$\boxed{v(t) = v'(t)+v''(t) = 3{,}293\cos(2t+56{,}31^\circ)+\frac{10}{\sqrt{2}}\cos(4t-135^\circ),\ \text{V}}$$

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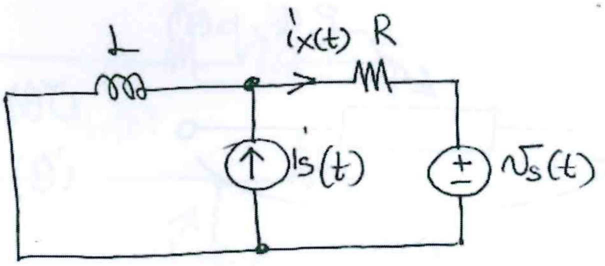


$v_s(t) = 1\ V$

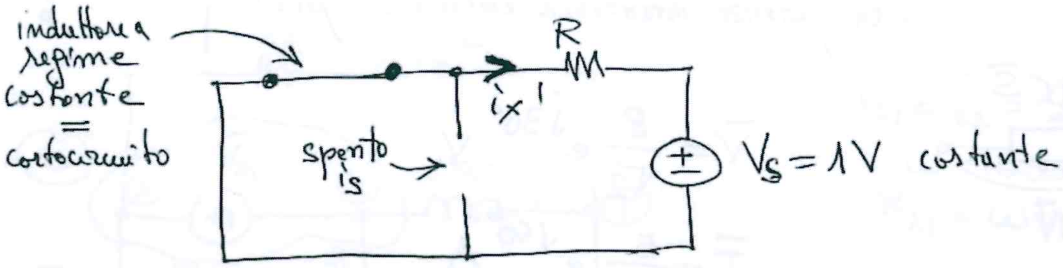
$i_s(t) = [illegible]\sin(50t)$, A

$L = 10\ mH$

$R = 1\ \Omega$

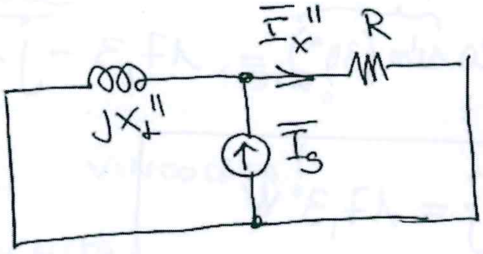
Determinare $i_x(t)$ a regime

- Pulsazione nulla $\omega_1 = 0$ (<u>regime costante</u>)



$$i_x'(t) = -\frac{V_s}{R} = -1\ A \quad \text{costante}$$

- Pulsazione $\omega_2 = 50$ rad/s



$\bar{I}_s = -j\ A$

$X_L'' = \omega_2 L = 50 \cdot 10 \cdot 10^{-3} = 0{,}5\ \Omega$

$$\bar{I}_x'' = \bar{I}_s \cdot \frac{jX_L''}{jX_L'' + R} = -j \frac{j\frac{1}{2}}{j\frac{1}{2} + 1} = -j\frac{j}{j+2}\,\frac{-j+2}{-j+2} = \frac{-j+2}{5}\ A$$

$$= \frac{1}{5}\sqrt{1^2+2^2}\, e^{j\,\mathrm{arctg}\left(-\frac{1}{2}\right)} = \frac{\sqrt{5}}{5} e^{-j26{,}56°}\ A$$

$$\bar{I}_x'' \rightarrow i_x''(t) = \frac{\sqrt{5}}{5}\cos(50t - 26{,}56°),\ A$$

- Sovrapposizione

$$\boxed{i_x(t) = i_x' + i_x'' = \frac{\sqrt{5}}{5}\cos(50t - 26{,}56°) - 1,\ A}$$