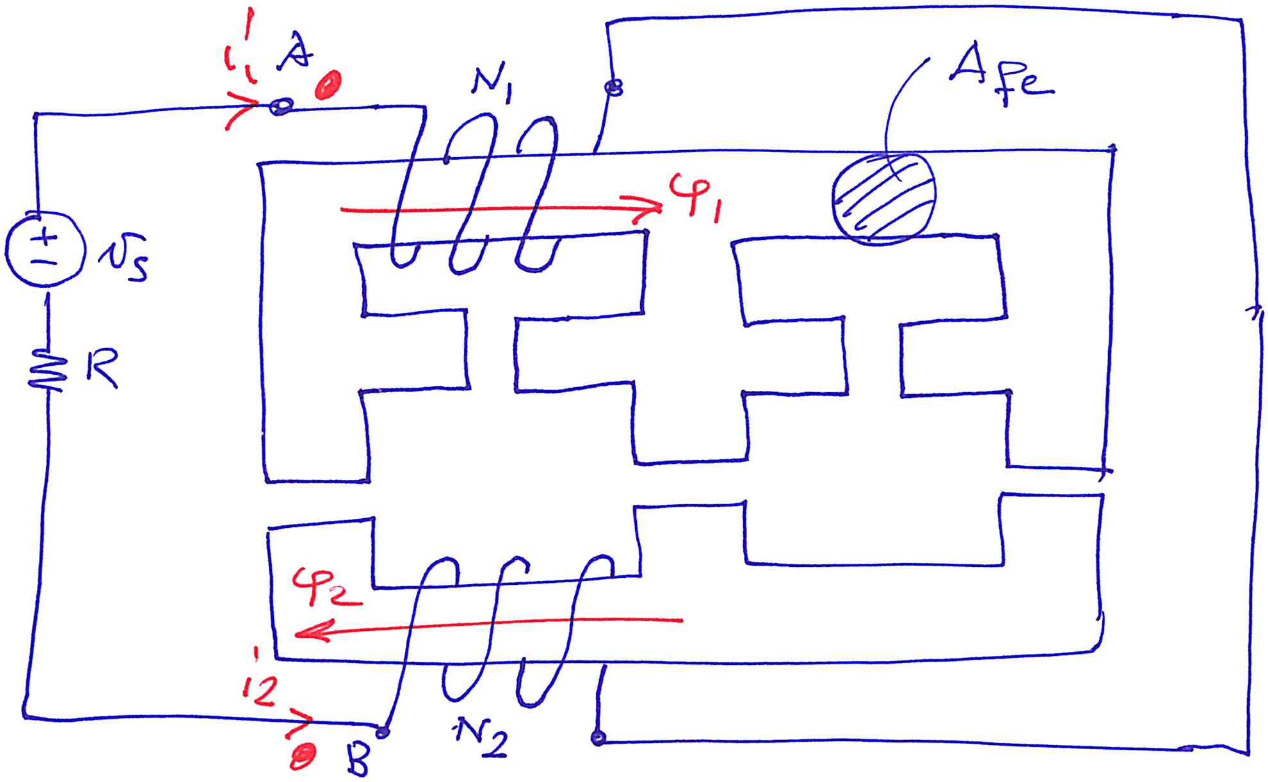
EX



$\delta = 1\,mm$ $\quad N_1 = 100 \quad R = 2\,\Omega$

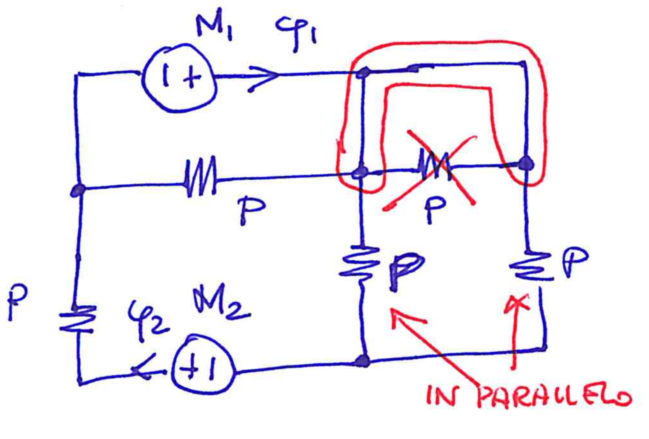
$A_{fe} = 100\,cm^2 \quad N_2 = 200 \quad v_S(t) = 4\,V$ (costante!)

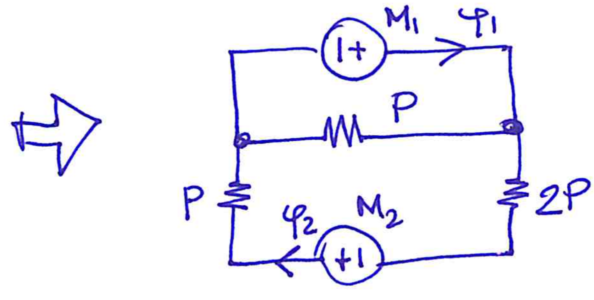
Determinare • L_1, L_2, L_M (mutuo induttore)

- L_{eq} vista ai morsetti a, b
- L'energia immagazzinata (a regime) nel mutuo induttore

1) POSIZIONO I MORSETTI CONTRASSEGNATI ●, INDICO i_1, i_2, φ_1, φ_2
(Meglio farlo in modo tale che $L_M > 0$!)
Vedi istruzioni date nel file pdf. circuiti magnetici

2) CIRCUITO MAGNETICO. Permeanze $P = \mu_0 \dfrac{A_{fe}}{\delta} = 4\pi \cdot 10^{-7} \cdot \dfrac{100 \cdot 10^{-4}}{1 \cdot 10^{-3}} =$
$= 12{,}566 \cdot 10^{-6} = 12{,}566\ \mu H$

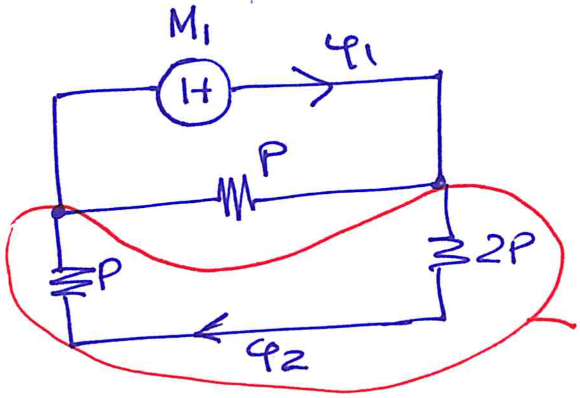




3) RELAZIONE COSTITUTIVA $\begin{bmatrix}\varphi_1\\ \varphi_2\end{bmatrix} = \begin{bmatrix}P_{11} & P_{12}\\ P_{21} & P_{22}\end{bmatrix}\begin{bmatrix}M_1\\ M_2\end{bmatrix}$

$P_{11} = \left.\dfrac{\varphi_1}{M_1}\right|_{M_2=0}$

$P_{21} = \left.\dfrac{\varphi_2}{M_1}\right|_{M_2=0}$



SERIE P e 2P: $\dfrac{P \cdot 2P}{P+2P} = \dfrac{2}{3}P$

$\boxed{P_{11}} = P + \frac{2}{3}P = \boxed{\frac{5}{3}P}$ (permeanza equivalente vista da M_1)

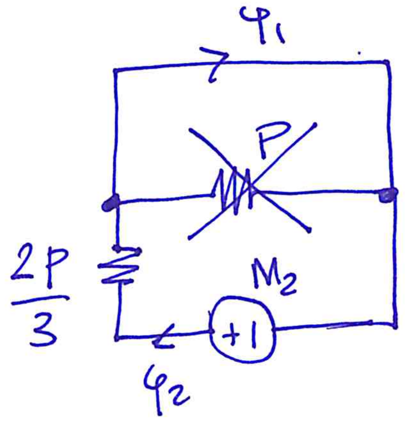
Partitore di flusso:

$$\varphi_2 = \varphi_1 \cdot \frac{\frac{2}{3}P}{\frac{2}{3}P + P} = \varphi_1 \frac{2}{5} = \underbrace{P_{11} M_1}_{\varphi_1} \frac{2}{5} = \frac{\cancel{5}}{3} P \frac{2}{\cancel{5}} M_1 = \frac{2}{3} P M_1$$

$\boxed{P_{21}} = \dfrac{\varphi_2}{M_1} = \boxed{\dfrac{2}{3}P}$

$P_{22} = \left.\dfrac{\varphi_2}{M_2}\right|_{M_1=0}$

$P_{12} = \left.\dfrac{\varphi_1}{M_2}\right|_{M_1=0}$



$\boxed{P_{22} = \frac{2}{3}P}$ (permeanza equivalente vista da M_2)

$\varphi_2 = \varphi_1$

$\boxed{P_{12}} = \dfrac{\varphi_1}{M_2} = \dfrac{\varphi_2}{M_2} = P_{22} = \boxed{\dfrac{2}{3}P}$

Verifica: DEVE SEMPRE RISULTARE $P_{12} = P_{21}$ OK!

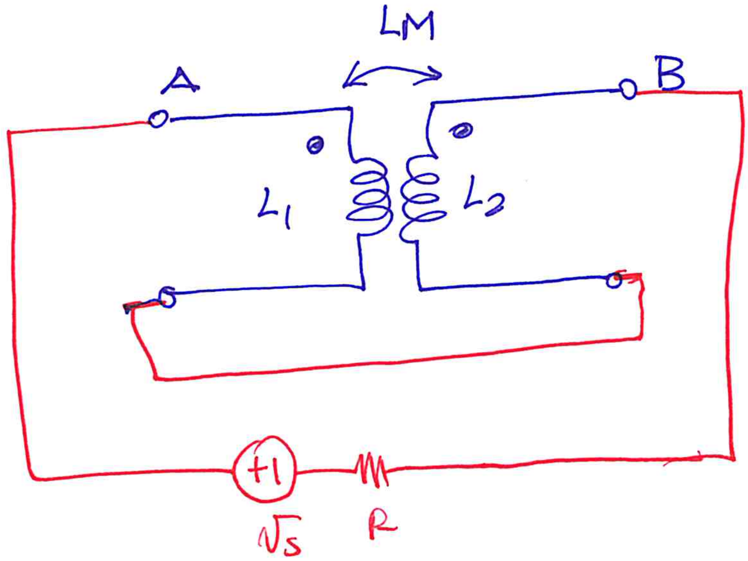
4) MUTUO INDUTTORE:

$$L_1 = N_1^2 P_{11} = N_1^2 \frac{5}{3} P = 0,2094 \, H$$

$$L_2 = N_2^2 P_{22} = N_2^2 \frac{2}{3} P = 0,335 \, H$$

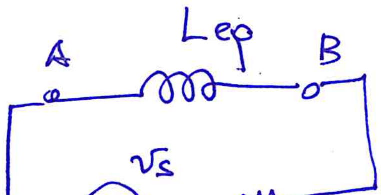
$$L_M = N_1 N_2 P_{21} = N_1 N_2 P_{12} = 0,168 \, H$$

5) SOLUZIONE DEL CIRCUITO PROPOSTO

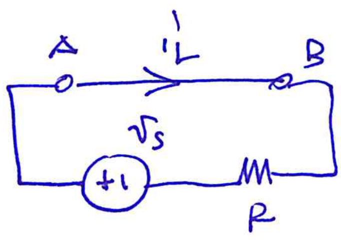


Collego il mutuo induttore come nel testo del problema

Serie Controversa! $L_{eq} = L_1 + L_2 - 2L_M = 0,2094 \, H$



Regime costante:

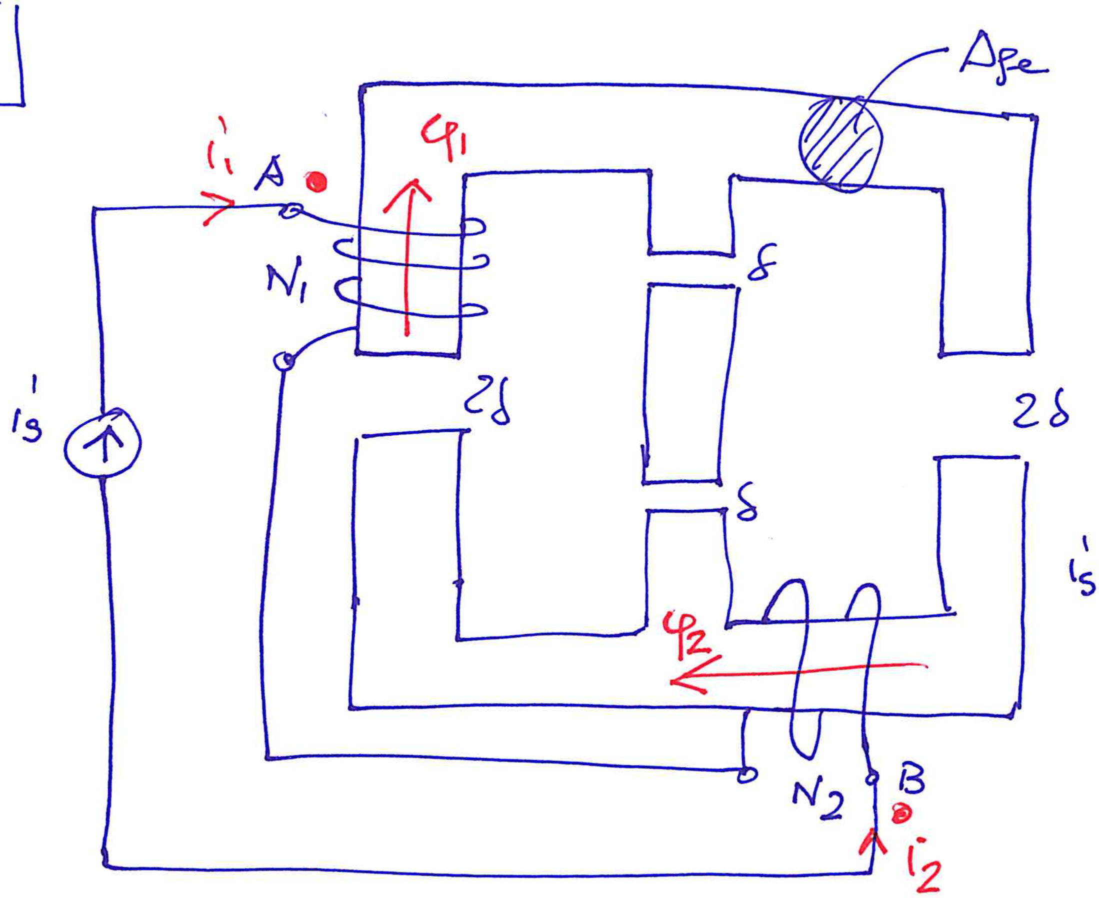


$$i_L = \frac{v_s}{R} = 2A$$

ENERGIA:

$$W_L = \frac{1}{2} L_{eq} i_L^2 = 0,42 \, J$$

EX

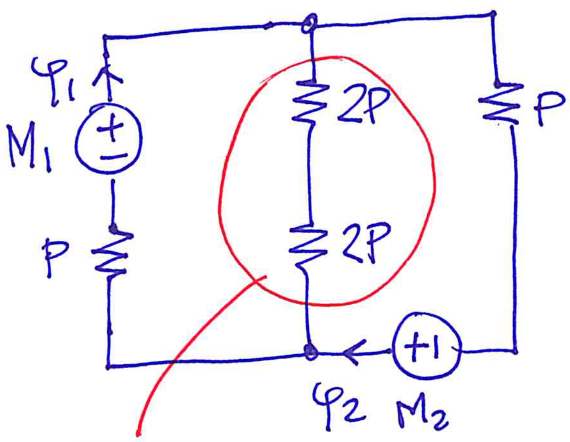


$A_{fe} = 100\ cm^2$
$\delta = 2\ mm$
$N_1 = 200$
$N_2 = 100$
$i_s'(t) = 5\cos(10t)$,

Determinare:
- L_1, L_2, L_M (mutuo induttore)
- L_{eq} vista ai morsetti A, B
- $v_{AB}(t)$ a regime

1) POSIZIONO I MORSETTI CONTRASSEGNATI ●, $\varphi_1, \varphi_2, i_1', i_2'$

2) CIRCUITO MAGNETICO



$$\frac{2P \cdot 2P}{2P+2P} = P$$

Permeanza per 2δ:

$$P = \frac{\mu_0 A_{fe}}{2\delta} = \frac{4\pi \cdot 10^{-7} \cdot 100 \cdot 10^{-4}}{2 \cdot 2 \cdot 10^{-3}} =$$

$$= 3{,}142\ \mu H$$

Permeanza per δ: $2P$

3) RELAZIONE COSTITUTIVA

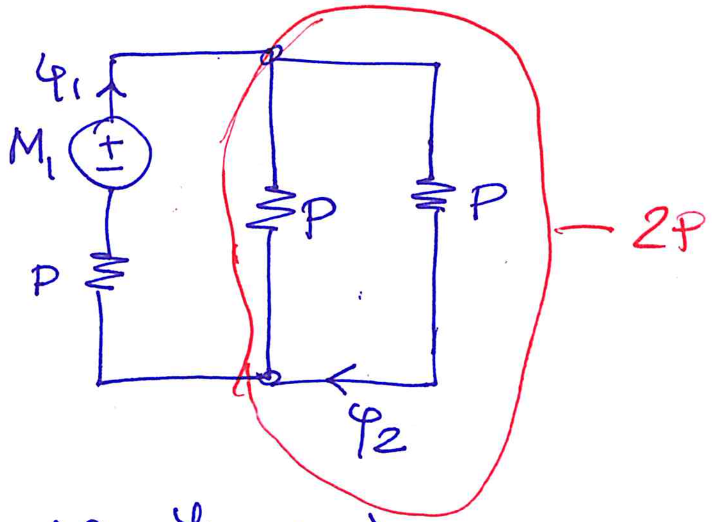
$$P_{12} = P_{21} \quad \text{(VALE SEMPRE!)}$$

$$P_{11} = P_{22} \quad \text{(VALE IN QUESTO PROBLEMA PER SIMMETRIA)}$$

Trovo P_{22}, P_{11} :

$$P_{11} = \left.\frac{\varphi_1}{M_1}\right|_{M_2=0}$$

$$P_{21} = \left.\frac{\varphi_2}{M_1}\right|_{M_2=0}$$



$$\boxed{P_{11}} = \frac{P \cdot 2P}{P+2P} = \boxed{\frac{2}{3}P}$$

$$\varphi_2 = \frac{\varphi_1}{2} = \frac{1}{\cancel{2}} \cdot \frac{\cancel{2}}{3} P\, M_1 = \frac{1}{3} P M_1$$

$$\boxed{P_{21}} = \frac{\varphi_2}{M_1} = \boxed{\frac{1}{3}P}$$

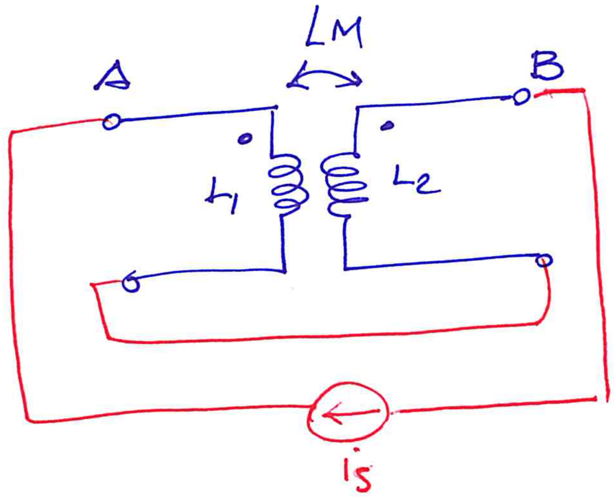
4) MUTUO INDUTTORE

$$L_1 = N_1^2 P_{11} = 200^2 \cdot \frac{2}{3} P = 0{,}084\ H = 84\ mH$$

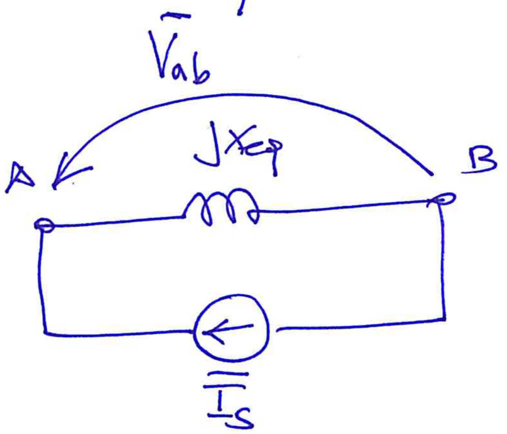
$$L_2 = N_2^2 P_{22} = N_2^2 P_{11} = 100^2 \cdot \frac{2}{3} P = 0{,}021\ H = 21\ mH$$

$$L_M = N_1 N_2 P_{21} = 200 \cdot 100 \cdot \frac{1}{3} P = 0{,}021\ H = 21\ mH$$

5) CIRCUITO



Serie controversa : $L_{eq} = L_1 + L_2 - 2L_M = 63 \text{ mH}$

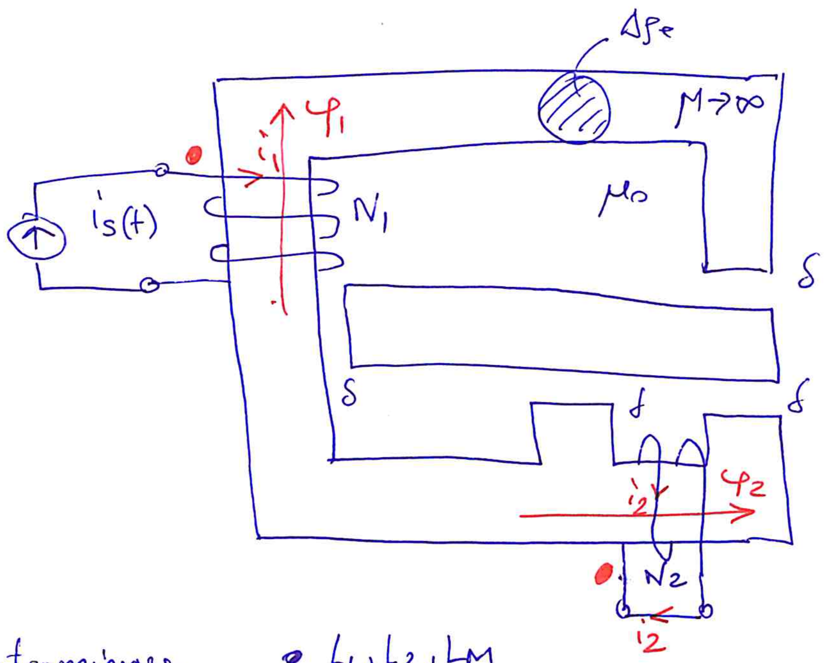


$\overline{I_s} = \frac{5}{\sqrt{2}}$ A

$X_{eq} = \omega L_{eq} = 0{,}63\ \Omega$

$\overline{V_{ab}} = jX_{eq}\overline{I_s} = j2{,}22\ V$

$$\boxed{v_{ab}(t) = \sqrt{2} \cdot 2{,}22 \cos(10t + 90^\circ)\ ,V}$$



$A_{fe} = 100\,cm^2$

$N_1 = 200$

$N_2 = 100$

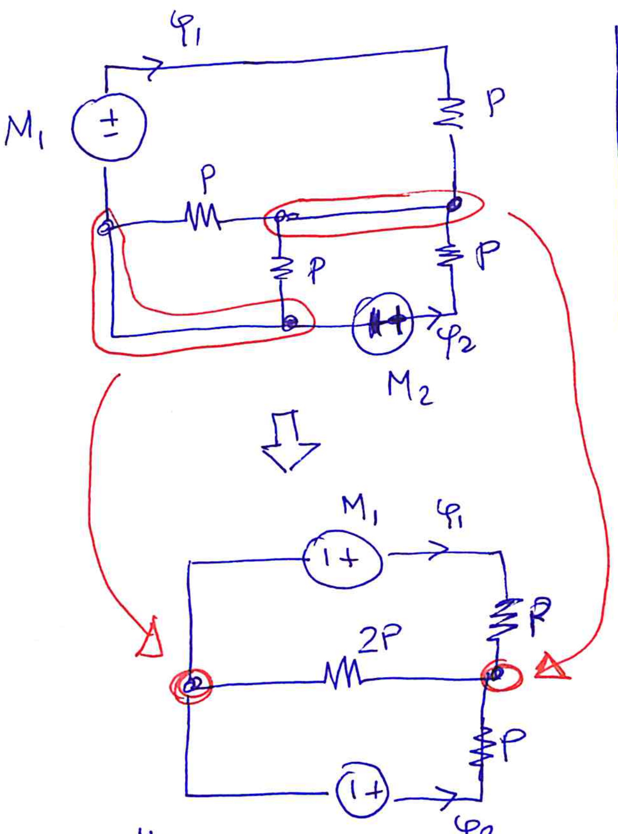
$\delta = 1\,mm$

$i_s(t) = \sqrt{2}\cos(\omega t + \frac{3}{4}\pi)$, A

Determinare

- L_1, L_2, L_M
- l'induttanza equivalente vista dal generatore di corrente
- la corrente che circola a regime nel secondo avvolgimento cortocircuitato

$$P = \mu_0 \frac{A_{fe}}{\delta} = 4\pi \cdot 10^{-7} \cdot \frac{100 \cdot 10^{-4}}{1 \cdot 10^{-3}} = 12{,}566\ \mu H$$



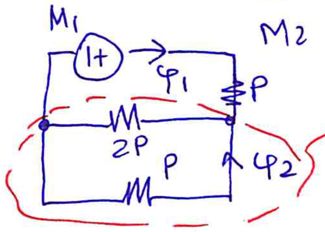
Nota bene:

Ho scelto a caso i morsetti contrassegnati senza preoccuparmi di ragionare se $L_M \gtrless 0$! Vedremo dopo come risulta.

$P /\!/ P = 2P$

$$P_{11} = \frac{\varphi_1}{M_1}\Big|_{M_2=0}$$

$$P_{21} = \frac{\varphi_2}{M_1}\Big|_{M_2=0}$$



$2P /\!/ P = 3P$

$\boxed{P_{11}} = P_{eq} = \frac{3P \cdot P}{3P+P} = \boxed{\frac{3}{4}P}$

Partitore di flusso:

$$\varphi_2 = -\varphi_1 \frac{P}{2P+P} = -P_{11} M_1 \frac{1}{3} = -\frac{3}{4}P\frac{1}{3}M_1 = -\frac{1}{4}P M_1$$

$$\boxed{P_{21} = -\frac{1}{4}P}$$

E' negativa! E' dovuto alla particolare disposizione dei morsetti contrassegnati!

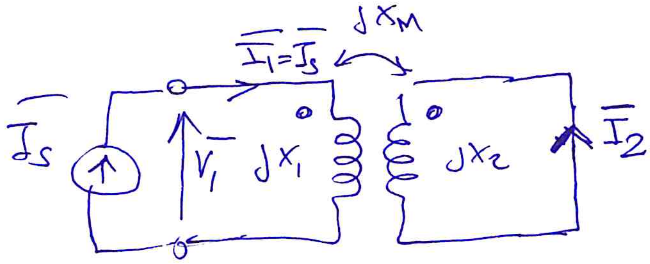
Per simmetria/reciprocità: $P_{22} = P_{11}$

$P_{21} = P_{12}$

Induttanze:

$$\boxed{\begin{aligned} L_1 &= N_1^2 P_{11} = 377 \text{ mH} \\ L_2 &= N_2^2 P_{22} = 94{,}25 \text{ mH} \\ L_M &= N_1 N_2 P_{12} = -62{,}83 \text{ mH} \end{aligned}}$$

Mutua induttanza NEGATIVA!



$$0 = jX_2 \bar{I}_2 + jX_M \bar{I}_S \qquad \bar{I}_2 = -\frac{X_M}{X_2}\bar{I}_S = -\frac{\cancel{\omega} L_M}{\cancel{\omega} L_2}\bar{I}_S$$

$$\bar{V}_1 = jX_1 \bar{I}_S + jX_M \bar{I}_2 = \underbrace{\left[jX_1 - j\frac{X_M^2}{X_2}\right]}_{jX_{eq}} \bar{I}_S$$

$$X_{eq} = X_1 - \frac{X_M^2}{X_2} = \omega L_1 - \frac{\omega^{\cancel{2}} L_M^2}{\cancel{\omega} L_2}$$

$$\boxed{L_{eq} = \frac{X_{eq}}{\omega} = L_1 - \frac{L_M^2}{L_2} = 335 \text{ mH}}$$

$$\boxed{i_2(t) = -\frac{L_M}{L_2} i_S(t) = \overbrace{\frac{62{,}83}{94{,}25}}^{2/3}\sqrt{2}\cos\left(\omega t + \frac{3}{4}\pi\right) = \sqrt{2}\cdot\frac{2}{3}\cos\left(\omega t + \frac{3}{4}\pi\right), \text{ A}}$$