Disclaimer

These extra solved exercises come from old exams of the Performance Evaluation course when open models were considered there. They have been superficially checked and seem to be correct and compatible with the current course, but keep in mind that:

- There could be some question using some formula / theoretical elements that are missing or it wre presented in a different way in the current course
- Some mistake in the solution might be present

Please use this as an additional study material, and if something in the solution do not convince you, you might be right and the problem could be due to one of the previous notes.

Please also note that the method used for computing visits in that course was slightly different from the one seen here: so please, in the solutions, refer only to the final values and not to the steps followed for finding them!!!!

3. Question 3 - Exercise - (**10 points**)

   Consider the open queuing network in Figure 1, with probabilities and service times given in figure.

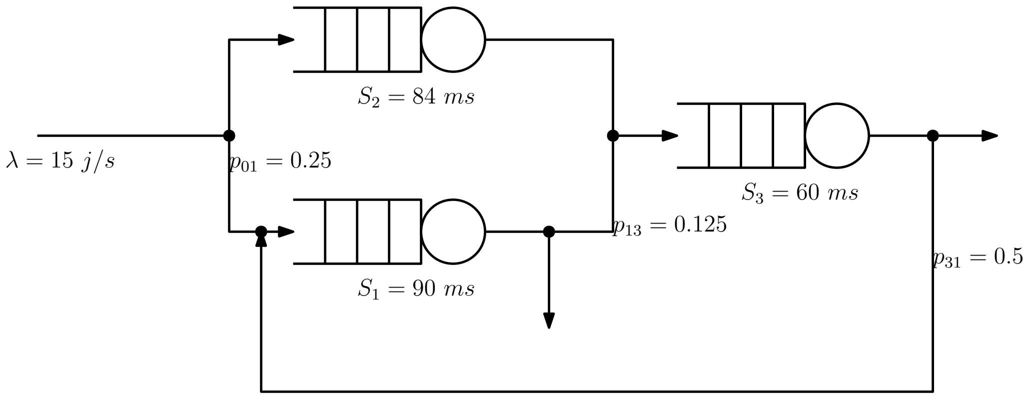


Figure 1: **Closed queuing model**

   Determine:

   (a) the visits at each station, v_1, v_2 and v_3;

   (b) the service demand of each queuing center, D_1, D_2 and D_3;

   (c) the bottleneck of the system;

   (d) the system throughput, X;

   (e) the utilization of each resource, U_1, U_2 and U_3;

   (f) the response time of each resource, Φ_1, Φ_2, Φ_3;

   (g) the system response time, R;

   (h) the average number of customers in the system, N;

   (i) if the system administrator is right when he/she claims this system can handle $\lambda = 17$ j/s.

   SOLUTIONS:

   (a) $\begin{cases} \lambda_1 = \lambda \cdot p_{01} + \lambda_3 \cdot p_{31} \\ \lambda_2 = \lambda \cdot p_{02} \\ \lambda_3 = \lambda_2 \cdot p_{23} + \lambda_1 \cdot p_{13} \end{cases}$ $\begin{cases} \lambda_1 = 0.25\lambda + 0.5\lambda_3 \\ \lambda_2 = 0.75\lambda \\ \lambda_3 = \lambda_2 + 0.125\lambda \end{cases}$ $\begin{cases} \lambda_1 = 0.25\lambda + 0.5\lambda_3 \\ \lambda_2 = 0.75\lambda \\ \lambda_3 = 0.75\lambda + 0.03125\lambda + 0.0625\lambda_3 \end{cases}$

   $\begin{cases} \lambda_1 = 0.25\lambda + 0.5\lambda_3 \\ \lambda_2 = 0.75\lambda \\ 0.9375\lambda_3 = 0.78125\lambda \end{cases}$ $\begin{cases} \lambda_1 = \frac{2}{3}\lambda \\ \lambda_2 = \frac{3}{4}\lambda \\ \lambda_3 = \frac{5}{6}\lambda \end{cases}$ $\begin{cases} v_1 = \frac{2}{3} \\ v_2 = \frac{3}{4} \\ v_3 = \frac{5}{6} \end{cases}$

   (b) $D_k = v_k \cdot S_k = \begin{cases} D_1 = 60\ ms \\ D_2 = 63\ ms \\ D_3 = 50\ ms \end{cases}$

(c) The bottleneck of the system is resource 2 since it has the maximum service demand.

(d) $X = \lambda = 15$ j/s

(e) $U_k = X \cdot D_k = \begin{cases} U_1 = 0.9 \\ U_2 = 0.945 \\ U_3 = 0.75 \end{cases}$

(f) $\Phi_k = \frac{S_k}{1-U_k} = \begin{cases} \Phi_1 = 0.9\ s \\ \Phi_2 = 1.527\ s \\ \Phi_3 = 0.24\ s \end{cases}$

(g) $R_k = v_k \cdot \Phi_k = \begin{cases} R_1 = 0.6\ s \\ R_2 = 1.145\ s \\ R_3 = 0.2\ s \end{cases} \quad R = \sum_k R_k = R_1 + R_2 + R_3 = 1.945\ s$

(h) $N = X \cdot R = 29.175$

(i) System administrator is wrong because:
$\lambda_{max} = \frac{1}{D_{max}} = \frac{1}{D_2} = 15.873\ j/s < 17\ j/s$

2. Question 2 - Exercise - (**8 points**)

   Consider the following open queuing network, whose routing probabilities and the average service times are shown in the figure.

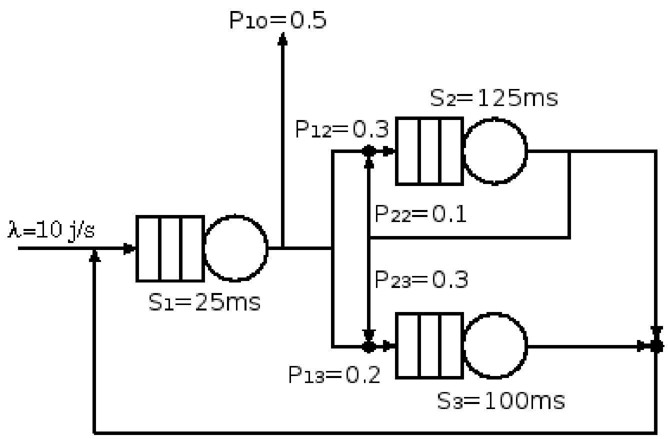


Figure 1: **Open queuing model**

   (a) Determine the visits at each station.

   (b) Compute the throughput of each station.

   (c) Compute the service demands of each resource.

   (d) Compute the utilization of the resources.

   (e) Compute the system response time, the system throughput and the average number of jobs in the system.

   SOLUTIONS:

   (a) $\begin{cases} \lambda_1 = \lambda + 0.6\lambda_2 + \lambda_3 \\ \lambda_2 = 0.3\lambda_1 + 0.1\lambda_2 \\ \lambda_3 = 0.2\lambda_1 + 0.3\lambda_2 \end{cases} \begin{cases} \lambda_1 = \lambda + 0.2\lambda_1 + 0.3\lambda_1 \\ \lambda_2 = \frac{1}{3}\lambda_1 \\ \lambda_3 = 0.2\lambda_1 + 0.1\lambda_1 = 0.3\lambda_1 \end{cases} \begin{cases} \lambda_1 = \frac{\lambda}{0.5} = 20j/s \\ \lambda_2 = \frac{1}{3}\lambda_1 = 6.667j/s \\ \lambda_3 = 0.3\lambda_1 = 6j/s \end{cases}$

   $\begin{cases} v_1 = \frac{\lambda_1}{\lambda} = 2 \\ v_2 = \frac{\lambda_2}{\lambda} = 0.6667 \\ v_3 = \frac{\lambda_3}{\lambda} = 0.6 \end{cases}$

   (b) $X_k = v_k \cdot \lambda = \begin{cases} X_1 = 20j/s \\ X_2 = 6.667j/s \\ X_3 = 6/s \end{cases}$

   (c) $D_k = v_k \cdot S_k = \begin{cases} D_1 = 0.05\ s \\ D_2 = 0.0833\ s \\ D_3 = 0.06\ s \end{cases}$

(d) $U_k = \lambda D_k = \begin{cases} U_1 = 0.5 \\ U_2 = 0.83 \\ U_3 = 0.6 \end{cases}$

(e) $R_k = \frac{D_k}{1-U_k} = \begin{cases} R_1 = 0.1\ s \\ R_2 = 0.499\ s \quad \rightarrow R = \sum_k R_k = 0.749\ s \\ R_3 = 0.15\ s \end{cases}$

Since this is an open and stable system, $X = \lambda = 10\ j/s$

$N = XR = 7.49$

2. Question 2 - Exercise - (**9 points**)

Consider the open queuing network in Figure 1, with the following service times, routing probabilities and inter-arrival rate:

- $S_1 = 100$ ms $\quad S_2 = 500$ ms $\quad S_3 = 150$ ms
- $P_{01} = 0.6 \quad P_{02} = 0.4 \quad P_{12} = 0.2 \quad P_{13} = 0.8 \quad P_{23} = 0.6$
- $\lambda = 0.6$ j/s

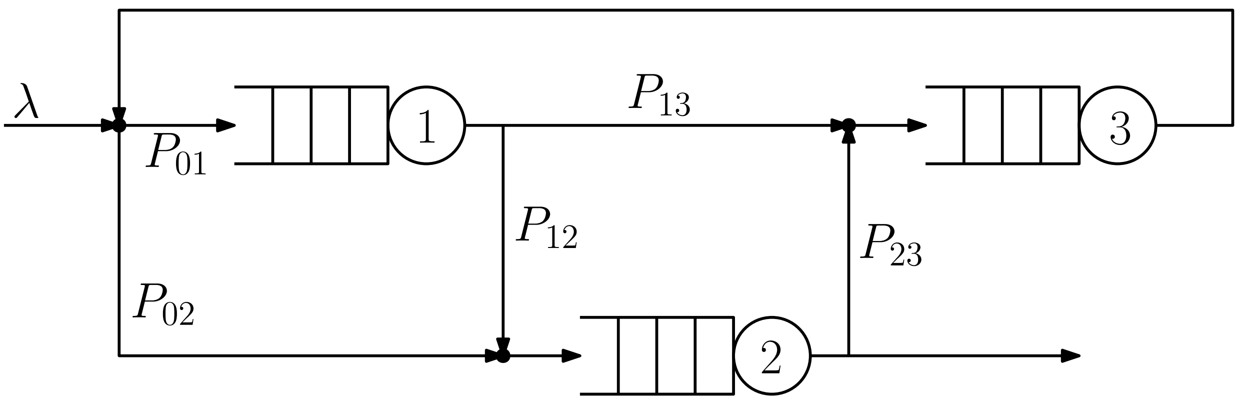


Figure 1: **Open queuing model**

Compute:

(a) the visits at each station, v_1, v_2 and v_3;

(b) the utilization of each resource, U_1, U_2 and U_3;

(c) the service demand of each queuing center, D_1, D_2 and D_3;

(d) the number of customers in each resource, N_1, N_2 and N_3.

Now, assuming the same system is modeled as an interactive closed queuing network, with think time $Z = 10$ seconds, number of customers into the system $N = 15$ and the same service demands obtained at point (c).

(e) Compute the system response time and the system throughput, using the approximate MVA and stopping after the third iteration of the algorithm.

SOLUTIONS:

(a) $\begin{cases} \lambda_1 = 0.6\lambda + \lambda_3 \\ \lambda_2 = 0.4\lambda + 0.2\lambda_1 \\ \lambda_3 = 0.8\lambda_1 + 0.6\lambda_2 \end{cases} \begin{cases} \lambda_1 = 0.84\lambda + 0.92\lambda_1 \\ \lambda_2 = 0.4\lambda + 0.2\lambda_1 \\ \lambda_3 = 0.8\lambda_1 + 0.24\lambda + 0.12\lambda_1 \end{cases} \begin{cases} \lambda_1 = 10.5\lambda \\ \lambda_2 = 0.4\lambda + 2.1\lambda = 2.5\lambda \\ \lambda_3 = 9.66\lambda + 0.24\lambda = 9.9\lambda \end{cases}$

$\begin{cases} v_1 = 10.5 \\ v_2 = 2.5 \\ v_3 = 9.9 \end{cases}$

(b) $U_k = X_k \cdot S_k = \begin{cases} U_1 = 6.3\ j/s \cdot 0.1\ s = 0.63 \\ U_2 = 1.5\ j/s \cdot 0.5\ s = 0.75 \\ U_3 = 5.94\ j/s \cdot 0.15\ s = 0.891 \end{cases}$

(c) $D_k = v_k \cdot S_k = \begin{cases} D_1 = 1.05\ s \\ D_2 = 1.25\ s \\ D_3 = 1.485\ s \end{cases}$

(d) $N_k = \frac{U_k}{1-U_k} = \begin{cases} N_1 = 1.703 \\ N_2 = 3 \\ N_3 = 8.174 \end{cases}$

(e)

i	R_1	R_2	R_3	R	X	N_1	N_2	N_3
0						5	5	5
1	5.950	7.083	8.415	21.448	0.477	2.838	3.379	4.014
2	3.831	5.192	7.048	16.071	0.575	2.203	2.985	4.053
3	3.209	4.733	7.102	15.044	0.599	1.922	2.835	4.254

$X = 0.599\ j/s$
$R = 15.044\ ms$

3. Question 3 - Exercise - (**9 points**)

A website is modeled as an open network with three different resources: a web server WS and two database servers DB_1 and DB_2. The following data have been collected after monitoring the website for $T = 10$ minutes:

- the web server busy time $B_{WS} = 477$ seconds;
- the first database busy time $B_{DB_1} = 585$ seconds;
- the second database busy time $B_{DB_2} = 180$ seconds;
- the number of requests completed by the system $C = 900$.

Compute:

(a) the service demand of each resource, D_{WS}, D_{DB_1} and D_{DB_2};

(b) the utilization of each resource, U_{WS}, U_{DB_1} and U_{DB_2};

(c) the residence time of each resource, R_{WS}, R_{DB_1} and R_{DB_2};

(d) the average number of users that is waiting in queue for each resource, N_{WS}^{queue}, $N_{DB_1}^{queue}$ and $N_{DB_2}^{queue}$;

Now, assume the system is modeled through an interactive closed network with think time $Z = 1$ seconds and the service demands derived for the open model. When $N = 10$:

(e) the administrator of the system claims the throughput is larger than 2 j/s; is he right? Motivate your answer using asymptotic bounds.

(f) balancing the load between the two database servers, it is possible to improve the performance of the network without buying faster components. Prove it through computations, knowing the average service times of the database servers are $S_{DB_1} = 130$ ms and $S_{DB_2} = 200$ ms.

SOLUTIONS:

(a) $D_k = \frac{U_k}{X} = \frac{B_k}{C} = \begin{cases} D_{WS} = 0.53\ s \\ D_{DB_1} = 0.65\ s \\ D_{DB_2} = 0.2\ s \end{cases}$

(b) $U_k = \frac{B_k}{T} = \begin{cases} U_{WS} = 0.795 \\ U_{DB_1} = 0.975 \\ U_{DB_2} = 0.3 \end{cases}$

(c) $R_k = \frac{D_k}{1-U_k} = \begin{cases} R_{WS} = 2.5854\ s \\ R_{DB_1} = 26\ s \\ R_{DB_2} = 0.286\ s \end{cases}$

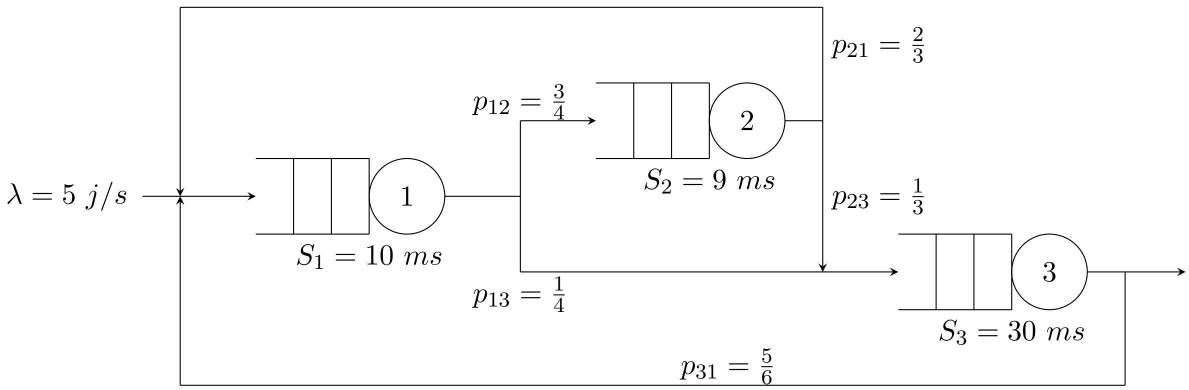
(d) $N_k^{queue} = N_k - U_k = \frac{U_k}{1-U_k} - U_k = \frac{U_k^2}{1-U_k} = \begin{cases} N_{WS}^{queue} = 3.083 \\ N_{DB_1}^{queue} = 38.025 \\ N_{DB_2}^{queue} = 0.129 \end{cases}$

(e) Since $N^* = \frac{D+Z}{D_{max}} = \frac{2.38\ s}{0.65\ s} = 3.662$, when $N = 10$ the system is in heavy-load regime. Thus:
$X_{min} = \frac{N}{ND+Z} = 0.675\ j/s$
$X_{max} = \frac{1}{D_{max}} = 1.538\ j/s$
The throughput cannot be $X = 2\ j/s$.

(f) $v_{DB_1} = \frac{D_{DB_1}}{S_{DB_1}} = \frac{650\ ms}{130\ ms} = 5$
$v_{DB_2} = \frac{D_{DB_2}}{S_{DB_2}} = \frac{200\ ms}{200\ ms} = 1$
$v_{DB_1} + v_{DB_2} = 6 \quad \rightarrow \quad \frac{v_{DB_1} \cdot S_{DB_1}}{S_{DB_1}} + \frac{v_{DB_2} \cdot S_{DB_2}}{S_{DB_2}} = 6 \quad \rightarrow \quad \frac{D_{DB_1}}{S_{DB_1}} + \frac{D_{DB_2}}{S_{DB_2}} = 6$
Now, setting $D_{DB_1} = D_{DB_2} = D_{DB}$:
$D_{DB}\left(\frac{1}{S_{DB_1}} + \frac{1}{S_{DB_2}}\right) = 6 \quad \rightarrow \quad D_{DB} = \frac{6 \cdot S_{DB_1} \cdot S_{DB_2}}{S_{DB_1} + S_{DB_2}} = 472.727\ ms.$
Thus, visits must be set to $v_{DB_1} = \frac{D_{DB}}{S_{DB_1}} = 3.636$ and $v_{DB_2} = \frac{D_{DB}}{S_{DB_2}} = 2.364$ in order to balance the load between the two database servers.

2. Question 2 - Exercise - (**7 points**)

Consider the following open queuing network, whose arrival rate, routing probabilities and the average service times are shown in the figure.



(a) Determine the visits at each station.

(b) Compute the throughput of the resources.

(c) Compute the utilization of the resources.

(d) Compute the system response time and the system throughput.

(e) Compute the arrival rate for which the system saturates.

SOLUTIONS:

(a) $\begin{cases} \lambda_1 = \lambda + \frac{2}{3}\lambda_2 + \frac{5}{6}\lambda_3 \\ \lambda_2 = \frac{3}{4}\lambda_1 \\ \lambda_3 = \frac{1}{4}\lambda_1 + \frac{1}{3}\lambda_2 \end{cases}$ $\begin{cases} \lambda_1 = \lambda + \frac{1}{2}\lambda_1 + \frac{5}{12}\lambda_1 \\ \lambda_2 = \frac{3}{4}\lambda_1 \\ \lambda_3 = \frac{1}{4}\lambda_1 + \frac{1}{4}\lambda_1 = \frac{1}{2}\lambda_1 \end{cases}$ $\begin{cases} \lambda_1 = 12\lambda \\ \lambda_2 = 9\lambda \\ \lambda_3 = 6\lambda \end{cases}$

$\begin{cases} v_1 = \frac{\lambda_1}{\lambda} = 12 \\ v_2 = \frac{\lambda_2}{\lambda} = 9 \\ v_3 = \frac{\lambda_3}{\lambda} = 6 \end{cases}$

(b) $X_i = X \cdot v_i = \lambda_i = \begin{cases} X_1 = 60\ j/s \\ X_2 = 45\ j/s \\ X_3 = 30\ j/s \end{cases}$

(c) $U_i = X_i S_i = \begin{cases} U_1 = 0.6 \\ U_2 = 0.405 \\ U_3 = 0.9 \end{cases}$

(d) $R_i = \frac{D_i}{1-U_i} = \frac{v_i S_i}{1-U_i} = \begin{cases} R_1 = 0.3\ s \\ R_2 = 0.136\ s \\ R_3 = 1.8\ s \end{cases} \rightarrow R = \sum_i R_i = 2.236\ s$

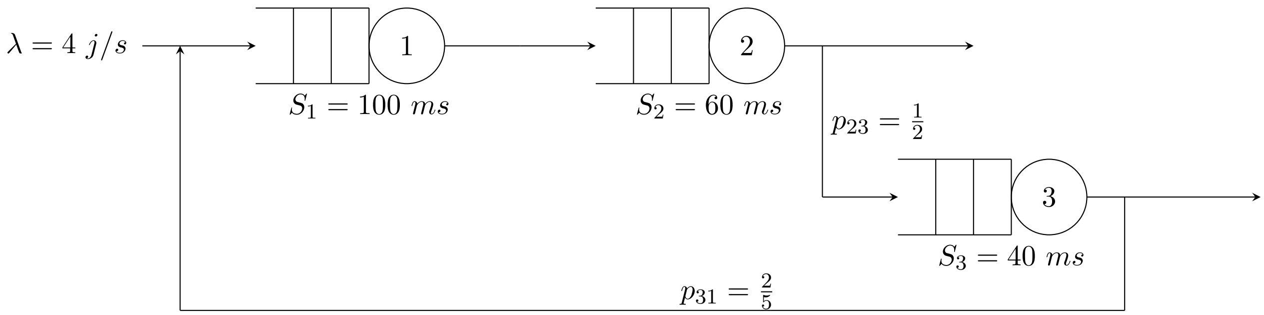
$X = \lambda = 5\ j/s$

(e) $D_i = v_i S_i = \begin{cases} D_1 = 0.12\ s \\ D_2 = 0.081\ s \\ D_3 = 0.18\ s \end{cases}$

$\lambda_{sat} = \frac{1}{D_{max}} = \frac{1}{D_3} = \frac{1}{0.18} = 5.56\ j/s$

2. Question 2 - Exercise - (**8 points**)

   Consider the following open queuing network, whose arrival rate, routing probabilities and the average service times are shown in the figure.



   (a) Determine the visits at each station.
   (b) Compute the service demands of all the resources.
   (c) Compute the system throughput and the throughput of each resource.
   (d) Compute the utilization of the resources.
   (e) Compute the system response time.
   (f) Compute the arrival rate for which the system saturates.
   (g) Assume the system is closed with $N = 120$, $Z = 1$ *sec.* and the service demands computed for the open system. Determine the system throughput and the system response time using approximate MVA. Stop after three iterations of the algorithm.
   NB: specify the value of the two output measures.

   SOLUTIONS:

   (a) $\begin{cases} \lambda_1 = \lambda + \frac{2}{5}\lambda_3 \\ \lambda_2 = \lambda_1 \\ \lambda_3 = \frac{1}{2}\lambda_2 \end{cases} \begin{cases} \lambda_1 = \lambda + \frac{1}{5}\lambda_1 \\ \lambda_2 = \lambda_1 \\ \lambda_3 = \frac{1}{2}\lambda_1 \end{cases} \begin{cases} \lambda_1 = \frac{5}{4}\lambda \\ \lambda_2 = \frac{5}{4}\lambda \\ \lambda_3 = \frac{5}{8}\lambda \end{cases}$

   $\begin{cases} v_1 = \frac{\lambda_1}{\lambda} = 1.25 \\ v_2 = \frac{\lambda_2}{\lambda} = 1.25 \\ v_3 = \frac{\lambda_3}{\lambda} = 0.625 \end{cases}$

   (b) $D_i = v_i S_i = \begin{cases} D_1 = 0.125\ s \\ D_2 = 0.075\ s \\ D_3 = 0.025\ s \end{cases}$

   (c) Since we are working with an open system: $X = \lambda = 4\ j/s$

   $X_i = X \cdot v_i = \lambda_i = \begin{cases} X_1 = 5\ j/s \\ X_2 = 5\ j/s \\ X_3 = 2.5\ j/s \end{cases}$

(d) $U_i = X_i S_i = X D_i = \begin{cases} U_1 = 0.5 \\ U_2 = 0.3 \\ U_3 = 0.1 \end{cases}$

(e) $R_i = \frac{D_i}{1-U_i} = \begin{cases} R_1 = 0.25\ s \\ R_2 = 0.107\ s \\ R_3 = 0.0278\ s \end{cases} \rightarrow R = \sum_i R_i = 385\ ms$

(f) $\lambda_{sat} = \frac{1}{D_{max}} = \frac{1}{D_1} = \frac{1}{0.125} = 8\ j/s$

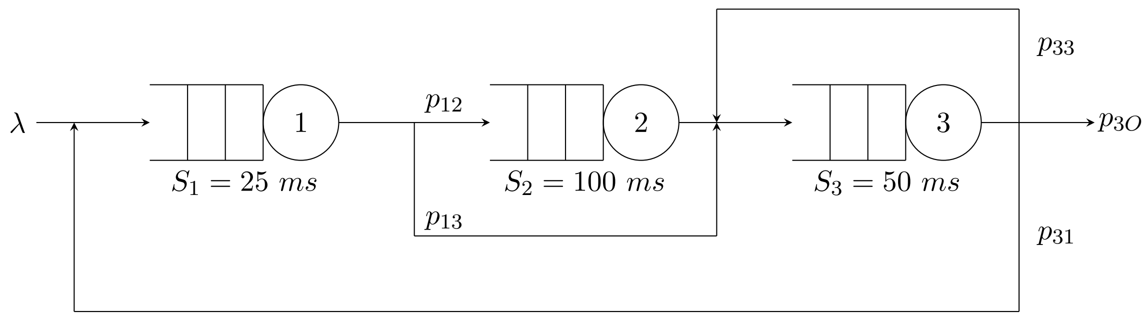
(g)

iter.	R_1	R_2	R_3	R	X	N_1	N_2	N_3
0						40	40	40
1	5.083	3.050	1.017	9.150	11.823	60.099	36.059	12.020
2	7.575	2.757	0.323	10.655	10.296	77.992	28.386	3.326
3	9.793	2.186	0.107	12.086	9.170	89.798	20.047	0.985

$R = 12.086\ s$
$X = 9.170\ j/s$

2. Question 2 - Exercise - (**6 points**)

Consider the open queuing network in the figure:



with the service times given in the figure and the following routing probabilities and arrival rate:

p_{12}	p_{31}	p_{33}	λ
0.75	0.3	0.2	5 j/s

Compute:

(a) p_{13}, p_{3O} and the visits at each station.

(b) the utilization of the resources. Is the system stable?

(c) the system throughput.

(d) the average number of users in the system.

SOLUTIONS:

(a) $\begin{cases} \lambda_1 = \lambda + \lambda_3 p_{31} \\ \lambda_2 = \lambda_1 p_{12} \\ \lambda_3 = \lambda_1 p_{13} + \lambda_2 + \lambda_3 p_{33} \end{cases}$ $\begin{cases} \lambda_1 = \lambda + \lambda_1 \frac{p_{31}(p_{13}+p_{12})}{1-p_{33}} \\ \lambda_2 = \lambda_1 p_{12} \\ \lambda_3 = \lambda_1 \frac{p_{13}+p_{12}}{1-p_{33}} \end{cases}$ $\begin{cases} \lambda_1 = \lambda \frac{1-p_{33}}{p_{3O}} = 8\ r/s \\ \lambda_2 = \lambda \frac{p_{12}(1-p_{33})}{p_{3O}} = 6\ r/s \\ \lambda_3 = \lambda \frac{1}{p_{3O}} = 10\ r/s \end{cases}$

$\begin{cases} v_1 = \lambda_1/\lambda = 1.6 \\ v_2 = \lambda_2/\lambda = 1.2 \\ v_3 = \lambda_3/\lambda = 2 \end{cases}$

(b) $U_k = \lambda_k S_k = \begin{cases} U_1 = 0.2 \\ U_2 = 0.6 \\ U_3 = 0.5 \end{cases}$

The system is stable because $U_k < 1\ \forall k$

(c) The system is open and stable, thus $X = \lambda = 5\ j/s$

(d) $N_k = \frac{U_k}{1-U_k} = \begin{cases} N_1 = 0.25 \\ N_2 = 1.5 \\ N_3 = 1 \end{cases} \quad \rightarrow \quad N = \sum_k N_k = 2.75$

2. Question 2 - Exercise - (**7 points**)

Consider the following open queuing network, whose routing probabilities and the average service times are shown in the figure.

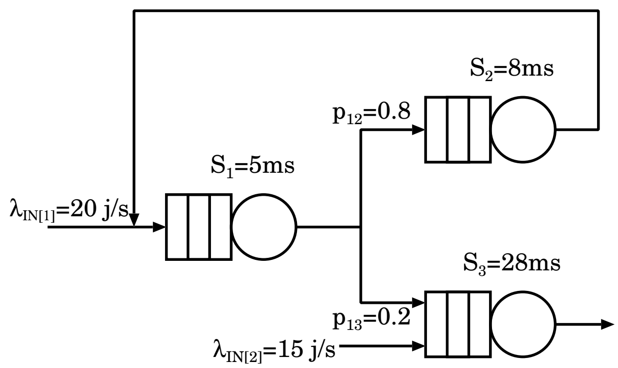


Figure 1: **Open queuing model**

(a) Determine the visits at each station.

(b) Compute the utilization of the resources.

(c) Compute the average residence time of the resources.

(d) Compute the maximum arrival rate at station 3, $\lambda_{IN[2]}$, that the system can handle.

SOLUTIONS:

(a) $\begin{cases} \lambda_1 = \lambda_{IN[1]} + \lambda_2 \\ \lambda_2 = 0.8\lambda_1 \\ \lambda_3 = \lambda_{IN[2]} + 0.2\lambda_1 \end{cases}$ $\begin{cases} \lambda_1 = \lambda_{IN[1]} + \lambda_2 \\ \lambda_2 = 0.8\lambda_{IN[1]} + 0.8\lambda_2 \\ \lambda_3 = \lambda_{IN[2]} + 0.2\lambda_1 \end{cases}$ $\begin{cases} \lambda_1 = 20\ j/s + 80\ j/s = 100\ j/s \\ \lambda_2 = \frac{0.8}{0.2}\lambda_{IN[1]} = 4 \cdot 20\ j/s = 80\ j/s \\ \lambda_3 = 15\ j/s + 20\ j/s = 35\ j/s \end{cases}$

$\begin{cases} v_1 = \frac{\lambda_1}{\lambda} = \frac{100}{35} = 2.857 \\ v_2 = \frac{\lambda_2}{\lambda} = \frac{80}{35} = 2.286 \\ v_3 = \frac{\lambda_3}{\lambda} = \frac{35}{35} = 1 \end{cases}$ where $\lambda = \lambda_{IN[1]} + \lambda_{IN[2]} = 35\ j/s$

(b) $U_1 = \lambda_1 \cdot S_1 = 100\ j/s \cdot 0.005\ s = 0.5$

$U_2 = \lambda_2 \cdot S_2 = 80 j/s \cdot 0.008\ s = 0.64$

$U_3 = \lambda_3 \cdot S_3 = 35\ j/s \cdot 0.028\ s = 0.98$

(c) $D_1 = v_1 \cdot S_1 = 2.857 \cdot 0.005\ s = 0.0143\ s$

$D_2 = v_2 \cdot S_2 = 2.286 \cdot 0.008\ s = 0.0183 s$

$D_3 = v_3 \cdot S_3 = 1 \cdot 0.028\ s = 0.028\ s$

$$R_k = \frac{D_k}{1-U_k} = \begin{cases} R_1 = 0.0286\ s \\ R_2 = 0.0508\ s \\ R_3 = 1.4\ s \end{cases}$$

(d) $\lambda_3 \cdot S_3 < 1 \rightarrow (\lambda_{IN[2]} + 0.2\lambda_1)S_3 < 1 \rightarrow 0.028\lambda_{IN[2]} + 0.56 < 1 \rightarrow$

$\rightarrow \lambda_{IN[2]} < \frac{1-0.56}{0.028} = 15.714\ j/s$