POLITECNICO DI MILANO

Elettrotecnica

Parte 5: Doppi Bipoli notevoli e Amplificatori Operazionali

Prof . Ing. Giambattista Gruosso, Ph. D.

Dipartimento di Elettronica, Informazione e Bioingegneria

POLITECNICO DI MILANO

Prof. G. Gruosso

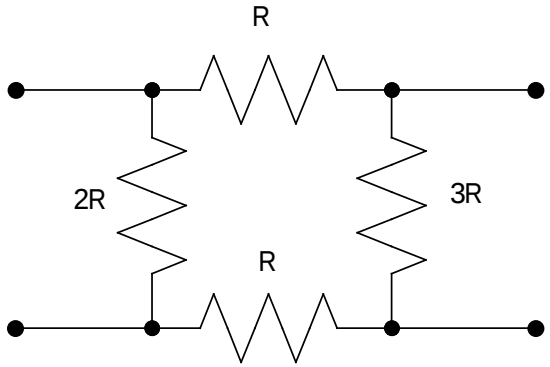
Indice

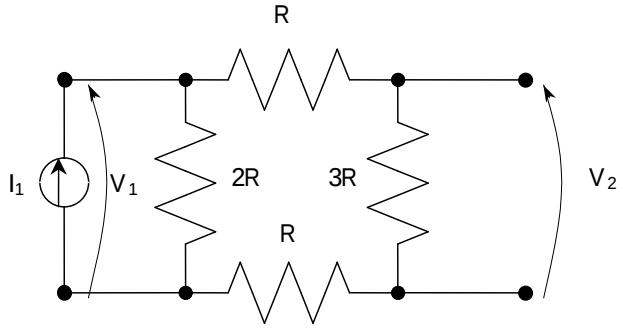
- Generatori Pilotati
- Trasformatori Ideali
- Amplificatori Operazionali

POLITECNICO DI MILANO

Prof. G. Gruosso

Doppi Bipoli Esempio 3





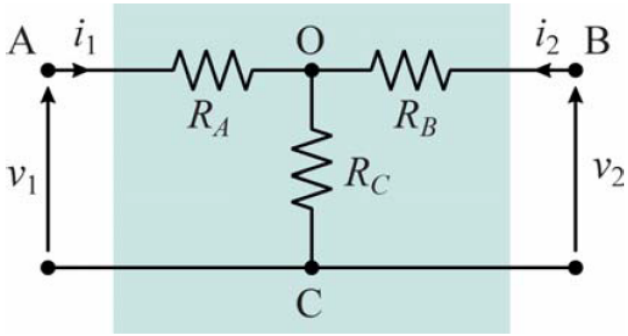
$$R_{21} = \frac{V_2}{I_1} = \frac{(3R)\cdot\left(\frac{2R}{7R}\right)I_1}{I_1} = \frac{6}{7}R$$

$$R_{11} = \frac{V_1}{I_1} = \frac{((R+3R+R)//2R)I_1}{I_1} = ((R+3R+R)//2R) = \frac{10}{7}R$$

Doppi Bipoli a T (o collegamente a stella di Bipoli)

POLITECNICO DI MILANO

Prof. G. Gruos

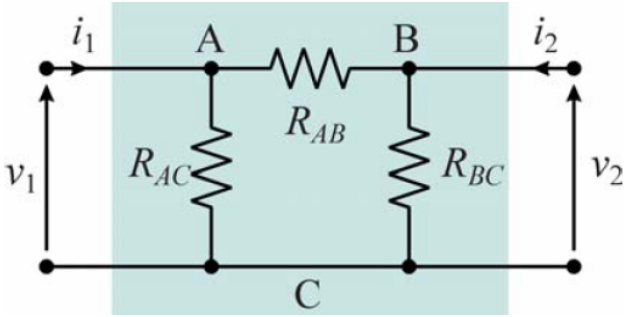


$$\mathbf{R} = \begin{bmatrix} R_A + R_C & R_C \\ R_C & R_B + R_C \end{bmatrix}$$

Doppi Bipoli a Pi (o collegamente a Triangolo di Bipoli)

POLITECNICO DI MILANO

Prof. G. Gruosso

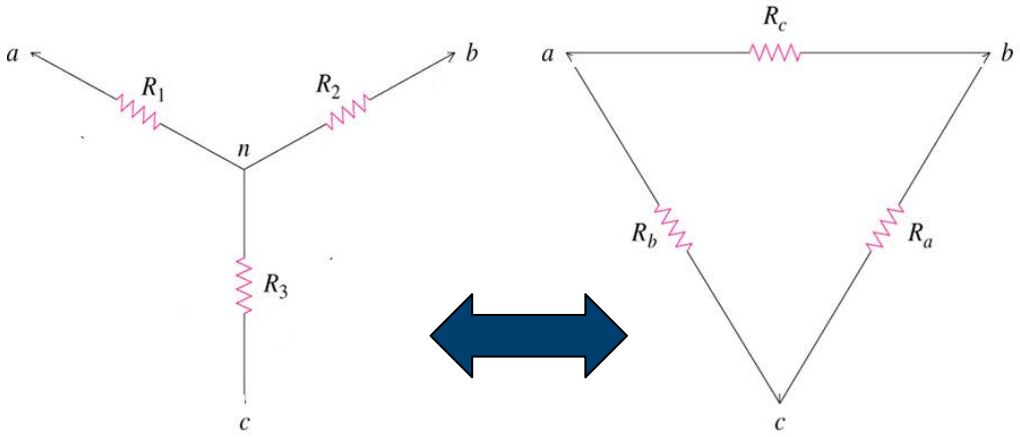


$$\mathbf{R} = \begin{bmatrix} \dfrac{R_{AB}R_{AC} + R_{AC}R_{BC}}{R_{AB} + R_{AC} + R_{BC}} & \dfrac{R_{AC}R_{BC}}{R_{AB} + R_{AC} + R_{BC}} \\ \dfrac{R_{AC}R_{BC}}{R_{AB} + R_{AC} + R_{BC}} & \dfrac{R_{AB}R_{BC} + R_{AC}R_{BC}}{R_{AB} + R_{AC} + R_{BC}} \end{bmatrix}$$

Collegamenti a Stella e a Triangolo

POLITECNICO DI MILANO

Prof. G. Gruosso



$$R_1 = \frac{R_b R_c}{(R_a + R_b + R_c)} \qquad R_a = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_1}$$

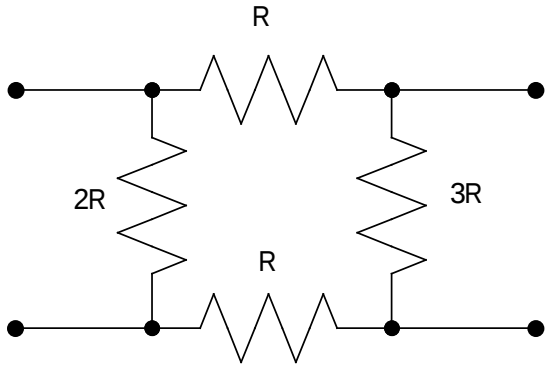
$$R_2 = \frac{R_c R_a}{(R_a + R_b + R_c)} \qquad R_b = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_2}$$

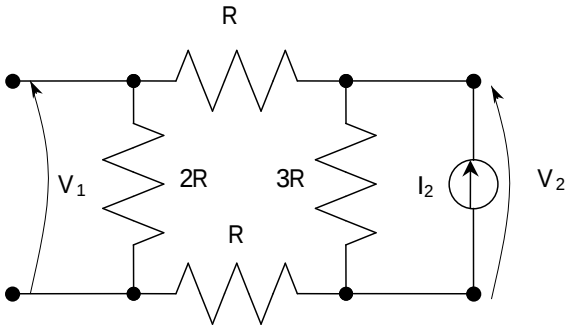
$$R_3 = \frac{R_a R_b}{(R_a + R_b + R_c)} \qquad R_c = \frac{R_1 R_2 + R_2 R_3 + R_3 R_1}{R_3}$$

POLITECNICO DI MILANO

Prof. G. Gruosso

Doppi Bipoli Esempio 3





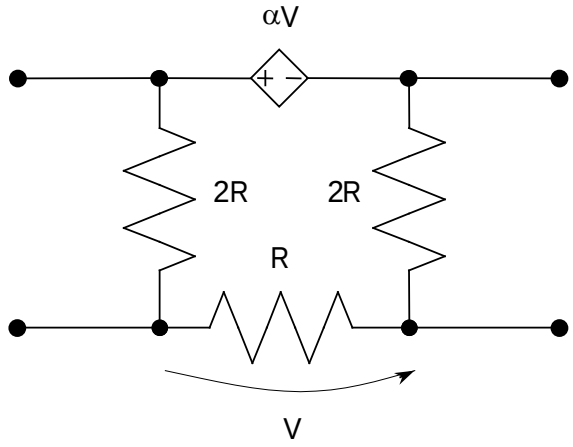
$$R_{22} = \frac{V_2}{I_2} = \frac{\left((R + 2R + R)\,//\,(3R)\right) I_2}{I_2} = \frac{12}{7} R$$

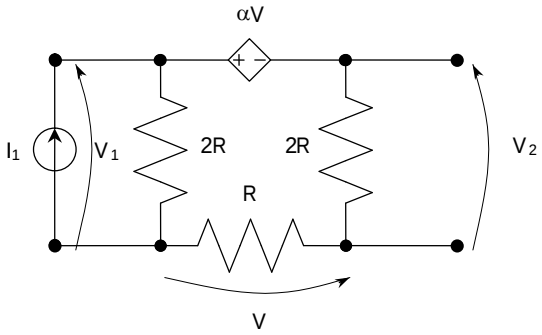
$$R_{12} = \frac{V_1}{I_2} = \frac{(2R) \cdot \left(\frac{3R}{7R}\right) I_2}{I_2} = \frac{6}{7} R$$

POLITECNICO DI MILANO

Prof. G. Gruosso

Doppi Bipoli Esempio 4





$$R_{11} = \frac{V_1}{I_1}$$

$$R_{21} = \frac{V_2}{I_1}$$

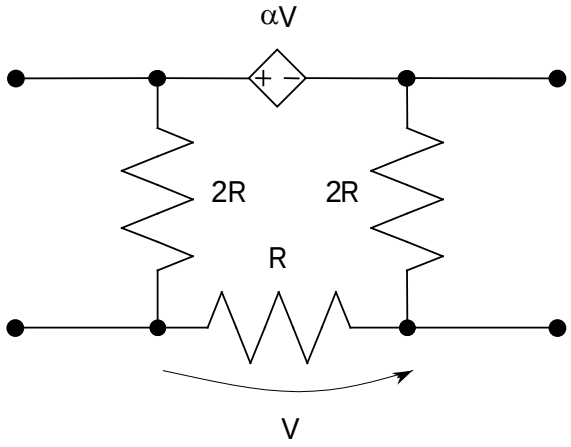
$$R_{11} = -\frac{(6+2\alpha)}{(5+\alpha)}R$$

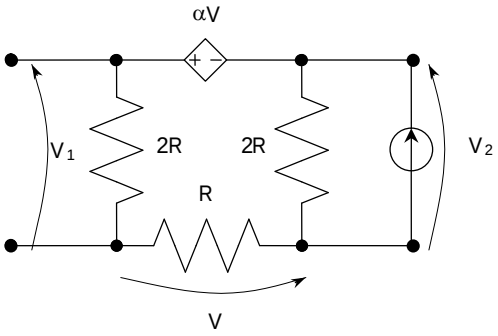
$$R_{21} = \frac{4}{(5+\alpha)}\frac{R}{5}$$

POLITECNICO DI MILANO

Prof. G. Gruosso

Doppi Bipoli Esempio 4





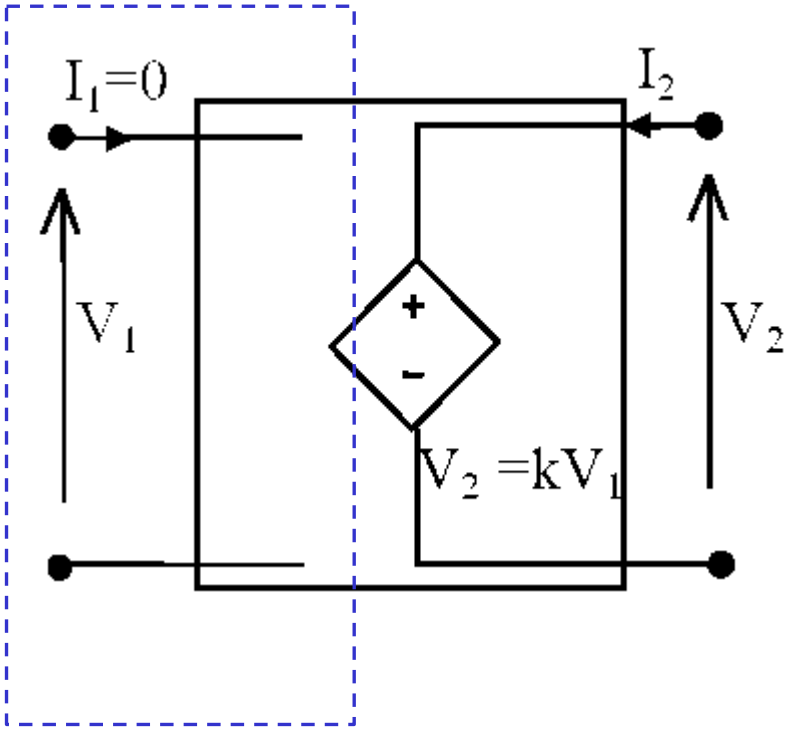
$$R_{12} = \frac{V_1}{I_2} \qquad R_{22} = \frac{V_2}{I_2}$$

$$R_{12} = \frac{(30+2\alpha)}{(5+\alpha)}\frac{R}{5} \qquad R_{22} = \frac{(20+8\alpha)}{(5+\alpha)}\frac{R}{5}$$

Doppi Bipoli notevoli: Generatori Controllati

POLITECNICO DI MILANO

Prof. G. Gruosso



$$I_1 = h'_{11} V_1 + h'_{12} I_2$$
$$V_2 = h'_{21} V_1 + h'_{22} I_2$$

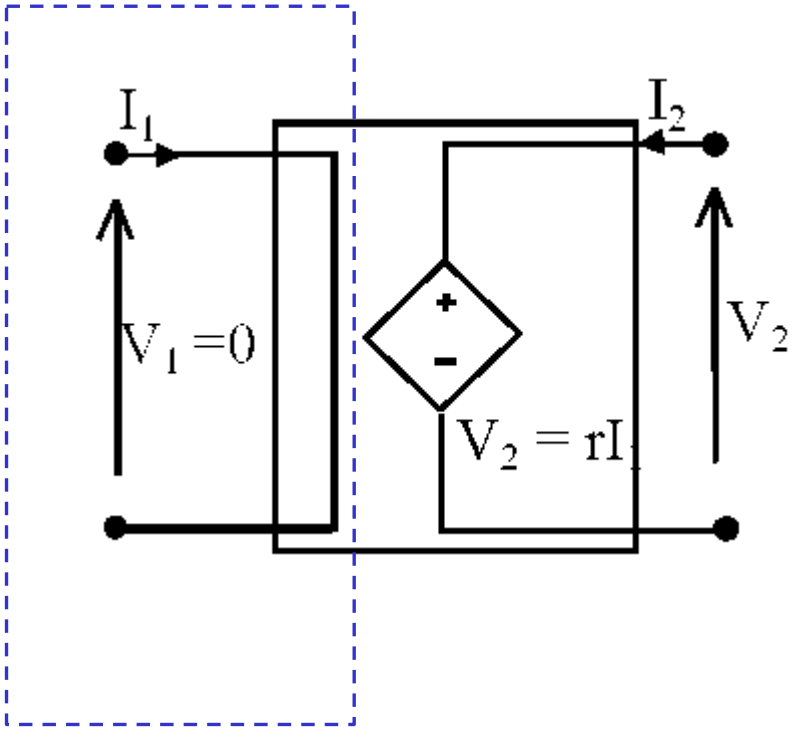
$$I_1 = 0$$
$$V_2 = kV_1$$
$$\begin{pmatrix} I_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ k & 0 \end{pmatrix} \begin{pmatrix} V_1 \\ I_2 \end{pmatrix}$$

Circuito Aperto

Doppi Bipoli notevoli: Generatori Controllati

POLITECNICO DI MILANO

Prof. G. Gruosso



Corto Circuito

$$V_1 = R_{11} I_1 + R_{12} I_2$$
$$V_2 = R_{21} I_1 + R_{22} I_2$$

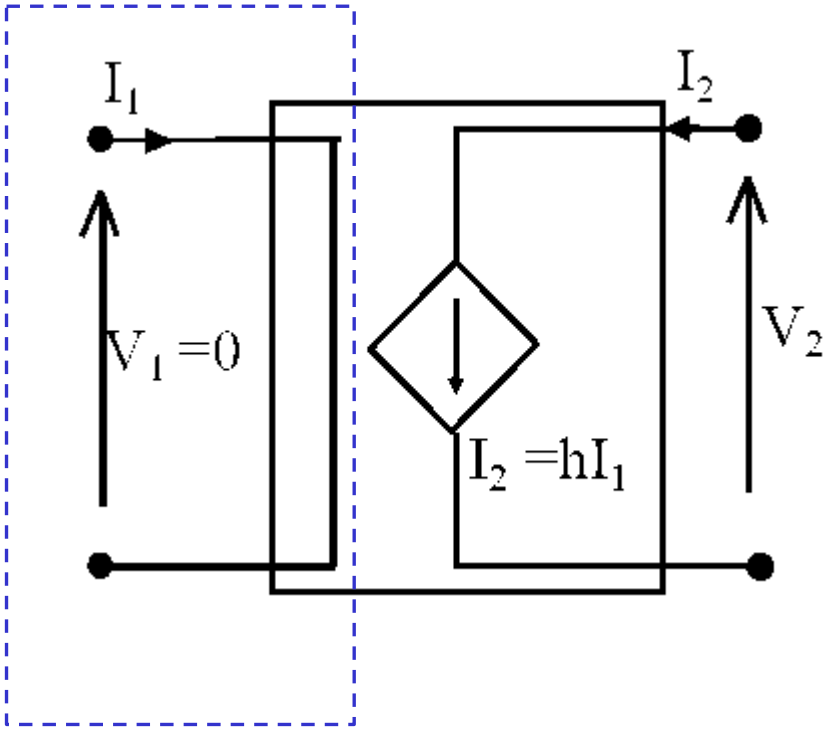
$$V_1 = 0$$
$$V_2 = r I_1$$

$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ r & 0 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

POLITECNICO DI MILANO

Prof. G. Gruosso

Doppi Bipoli notevoli: Generatori Controllati



$$V_1 = 0$$

$$I_2 = hI_1$$

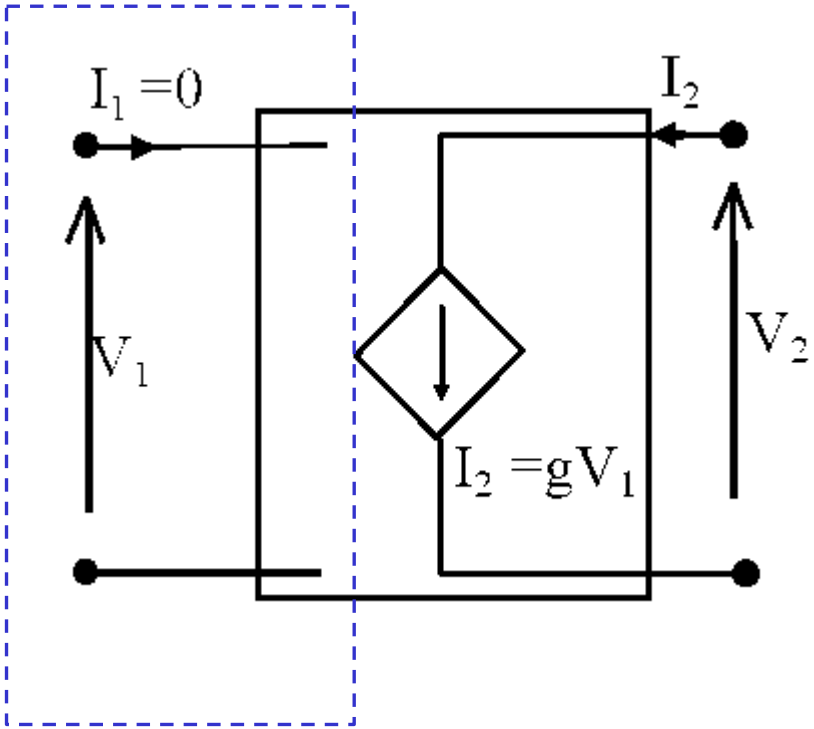
$$\begin{pmatrix} V_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ h & 0 \end{pmatrix} \begin{pmatrix} I_1 \\ V_2 \end{pmatrix}$$

Corto Circuito

POLITECNICO DI MILANO

Prof. G. Gruosso

Doppi Bipoli notevoli: Generatori Controllati



$$I_1 = 0$$

$$I_2 = gV_1$$

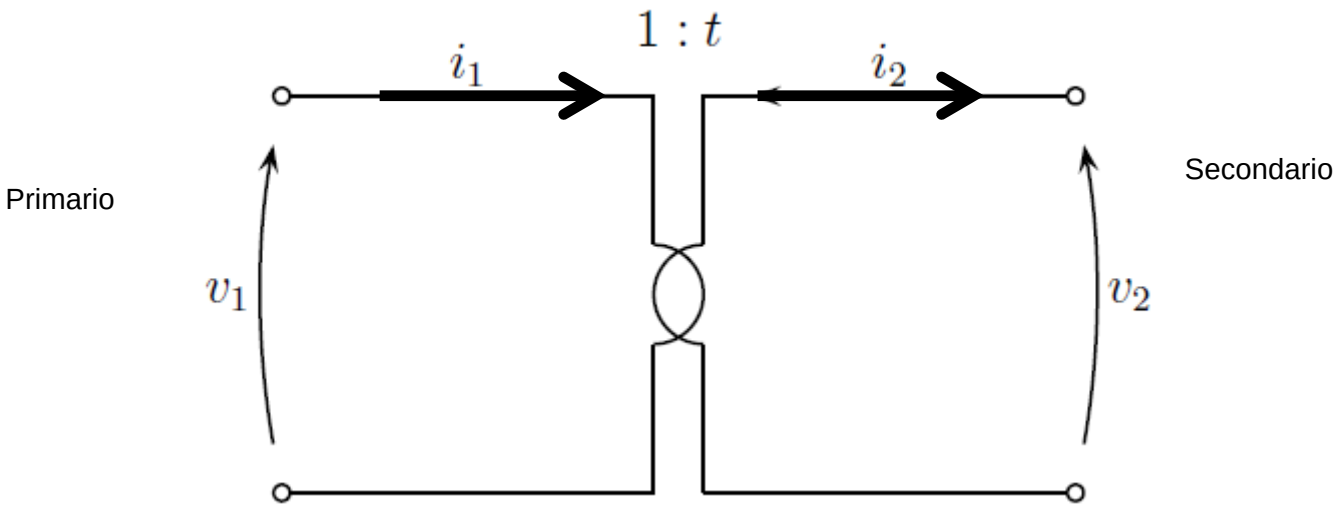
$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ g & 0 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix}$$

Circuito Aperto

POLITECNICO DI MILANO

Prof. G. Gruosso

Doppi Bipoli notevoli: Trasferitore ideale o Trasformatore Ideale



Convenzione utilizzatori Convenzione generatori

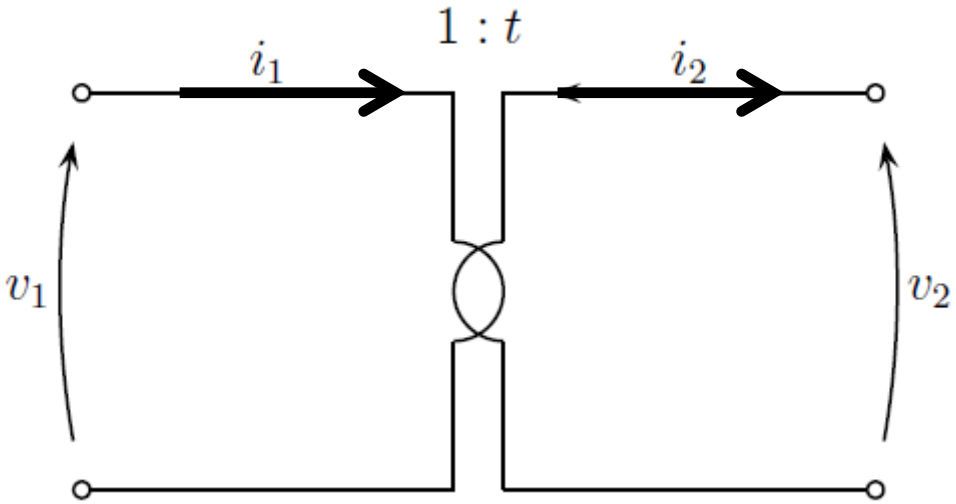
$$v_1 : v_2 = 1 : t$$

$$i_1 : i_2 = t : 1$$

POLITECNICO DI MILANO

Prof. G. Gruosso

Doppi Bipoli notevoli: Trasformatore Ideale



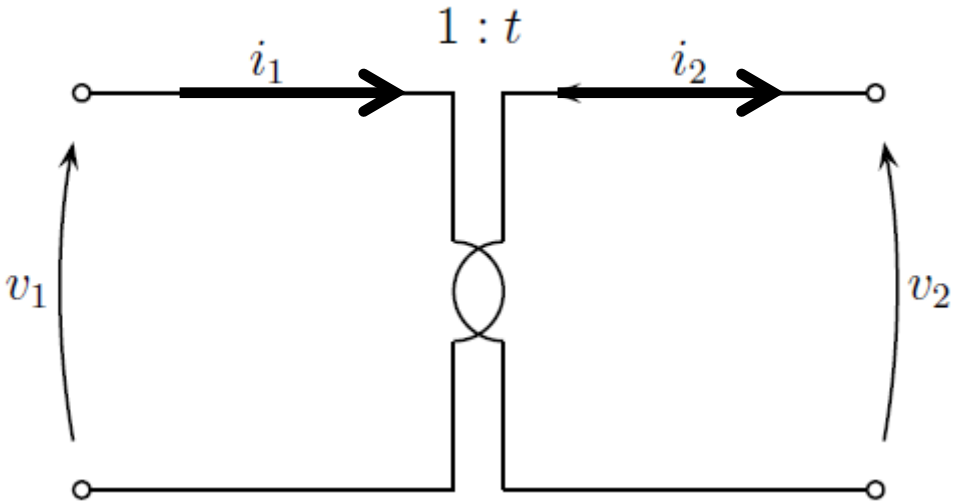
Convenzione utilizzatori Convenzione generatori

$$\begin{cases} v_1 = \dfrac{1}{t} v_2 \\ i_1 = t i_2 \end{cases} \Rightarrow \begin{Bmatrix} v_1 \\ i_1 \end{Bmatrix} = \begin{bmatrix} \dfrac{1}{t} & 0 \\ 0 & t \end{bmatrix} \begin{Bmatrix} v_2 \\ i_2 \end{Bmatrix}$$

POLITECNICO DI MILANO

Prof. G. Gruosso

Doppi Bipoli notevoli: Trasformatore Ideale



Convenzione utilizzatori Convenzione generatori

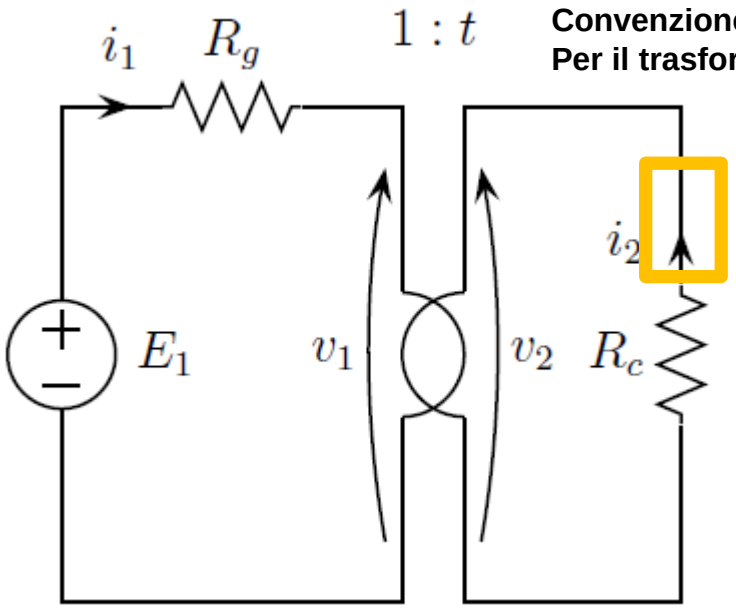
$$p_{ass} = p_{ass\_1} - p_{gen\_2}$$

$$p_{ass} = v_1 i_1 - v_2 i_2 = v_1 i_1 - v_1 t \frac{i_1}{t} = 0$$

POLITECNICO DI MILANO

Prof. G. Gruosso

Trasformatore Ideale – Esempio 1



Convenzione Utilizzatori
Per il trasformatore

$$v_2 = t v_1$$

$$i_2 = -\frac{i_1}{t}$$

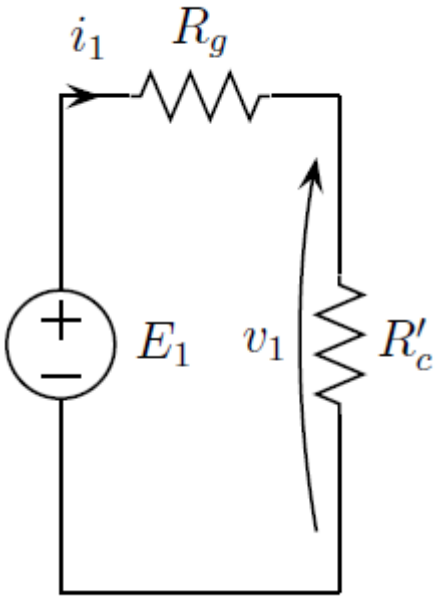
$$v_2 = -R_c i_2$$

$$v_1 = \frac{v_2}{t} = \frac{-R_c i_2}{t} = \frac{R_c}{t^2} \cdot i_1$$

POLITECNICO DI MILANO

Prof. G. Gruosso

Trasformatore Ideale – Esempio 1



$$v_1 = \frac{v_2}{t} = \frac{-R_c i_2}{t} = \frac{R_c}{t^2} i_2$$

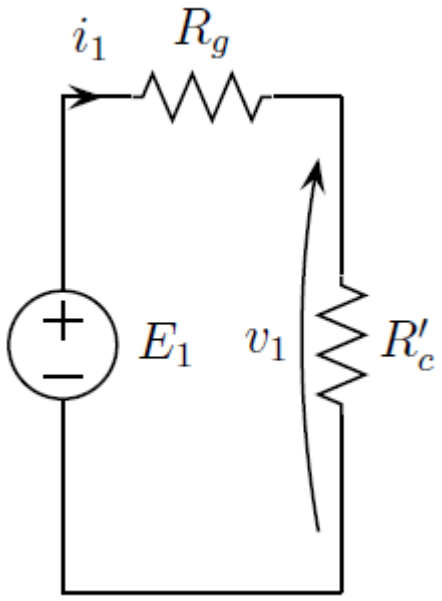
$$R'_c = R_c/t^2$$

Resistenza di secondario riportata al primario

POLITECNICO DI MILANO

Prof. G. Gruosso

Trasformatore Ideale – Esempio 1



$$i_1 = \frac{E_1}{R'_c + R_g}$$

$$v_1 = E_1 - R_g i_1$$

Al secondario si avrà quindi

$$v_2 = t v_1$$

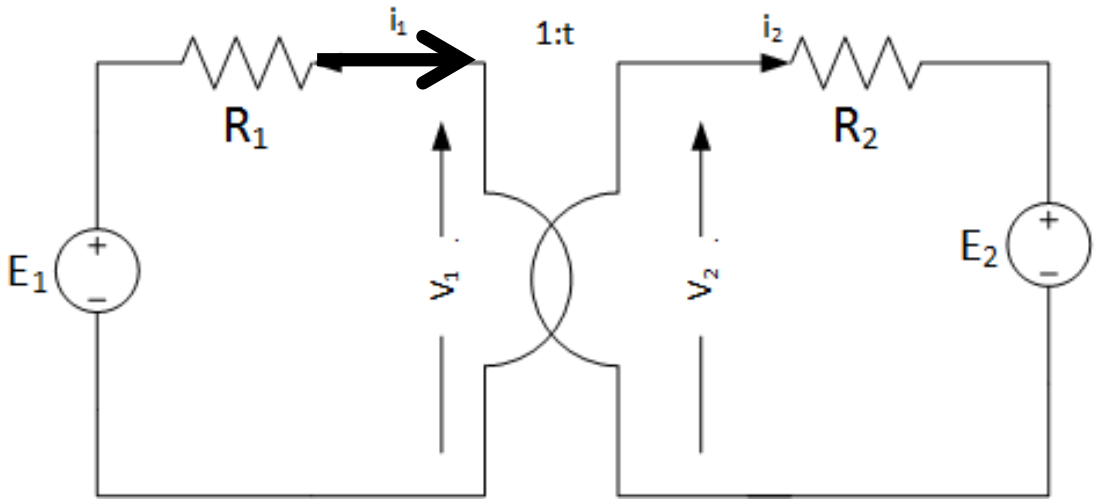
$$i_2 = -\frac{v_1}{t}$$

Nb: se $R'_c = R_g$ allora avrò max trasferimento di potenza. Si usa il trasformatore per fare adattamento di Resistenza

POLITECNICO DI MILANO

Prof. G. Gruosso

Trasformatore Ideale – Esempio 2



$$\begin{cases} v_1 = \dfrac{v_2}{t} \\ i_1 = i_2 t \end{cases}$$

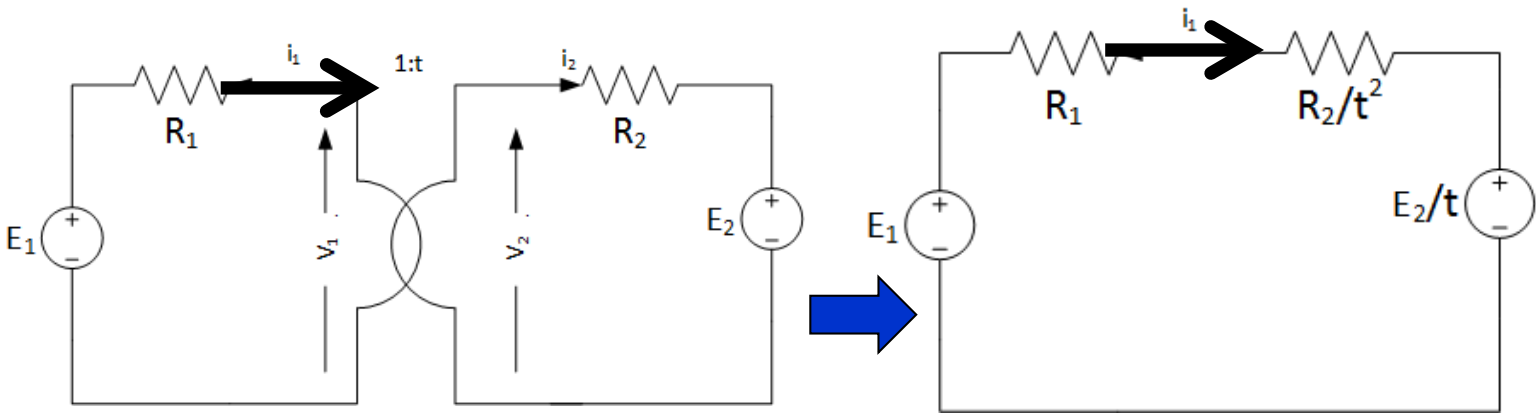
$$v_2 = R_2 i_2 + E_2 \quad \Rightarrow \quad t v_1 = R_2 \frac{i_1}{t} + E_2$$

$$v_1 = R_2 \frac{i_1}{t^2} + \frac{E_2}{t}$$

Trasformatore Ideale – Esempio 2

POLITECNICO DI MILANO

Prof. G. Gruosso



$$v_1 = R_2 \frac{i_1}{t^2} - \frac{E_2}{t}$$

$$i_1 = \frac{E_1 - \frac{E_2}{t}}{R_1 + \frac{R_2}{t^2}}$$

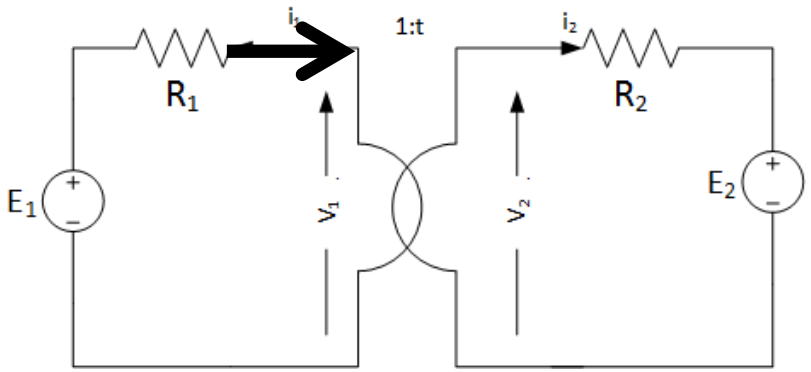
$$i_2 = \frac{i_1}{t}$$

POLITECNICO DI MILANO

Prof. G. Gruosso

Trasformatore Ideale – Riepilogo Trasformazioni

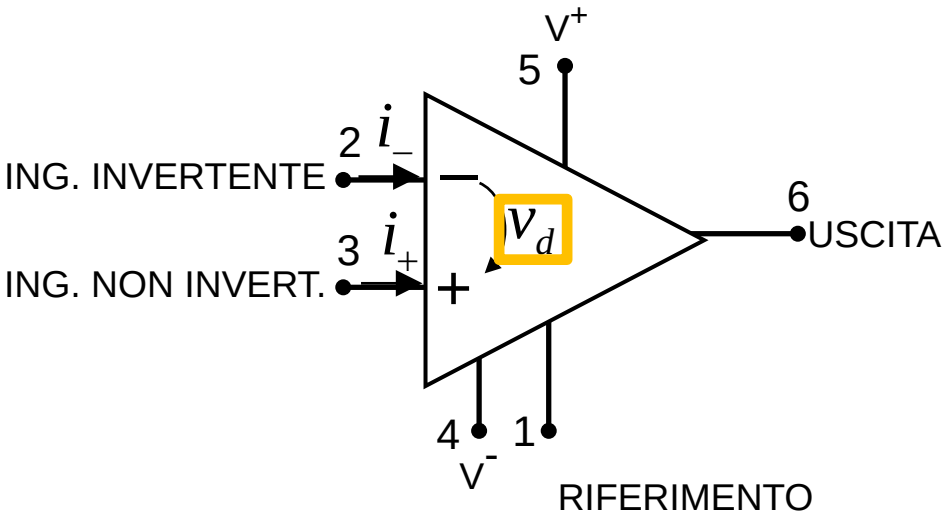
Componente	Da Primario a Secondario	Da Secondario a Primario
R	$R*t^2$	R/t^2
E	$E*t$	E/t
I	I/t	$I*t$

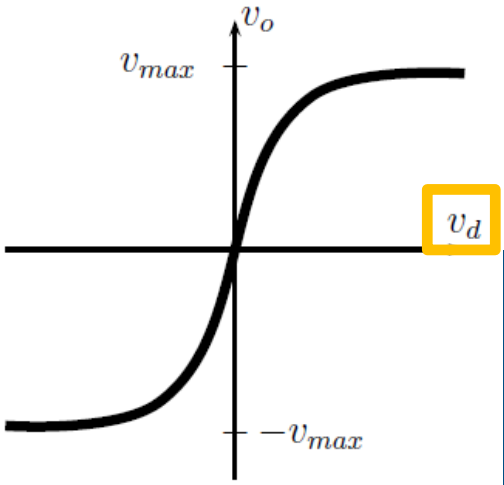


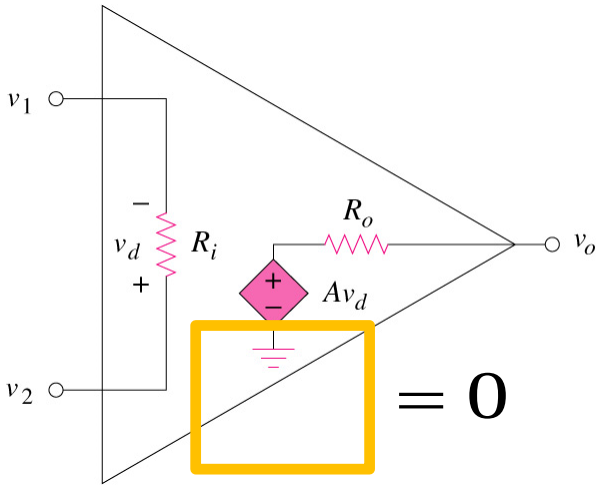
POLITECNICO DI MILANO

Prof. G. Gruosso

Amplificatore operazionale







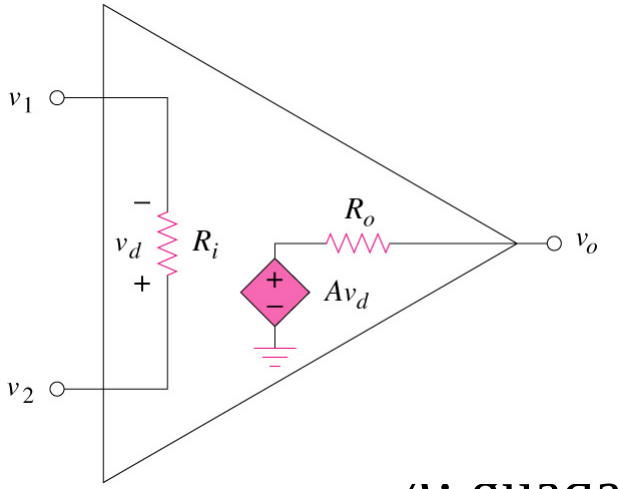
$$v_d = v_2 - v_1$$

$$v_o = A \cdot v_d = A \cdot (v_2 - v_1)$$

POLITECNICO DI MILANO

Prof. G. Gruosso

Amplificatore operazionale



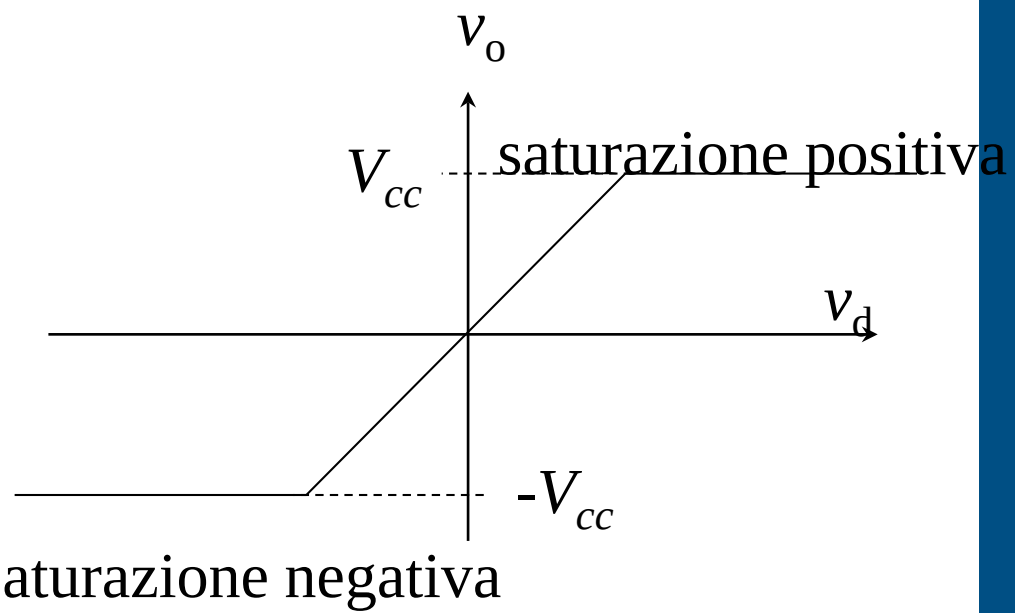
$$v_d = v_2 - v_1$$

$$v_o = A \cdot v_d = A \cdot (v_2 - v_1)$$

A: guadagno di tensione ad anello aperto

valori tipici

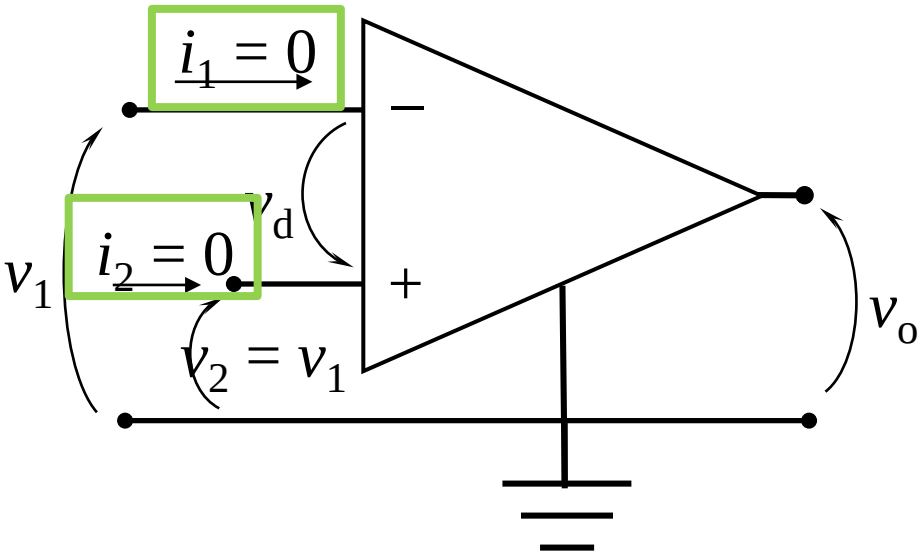
A	$10^5 \div 10^8$
R_i	$10^6 \div 10^{13}\ \Omega$
R_o	$10 \div 100\ \Omega$
V_{cc}	$5 \div 24$ V

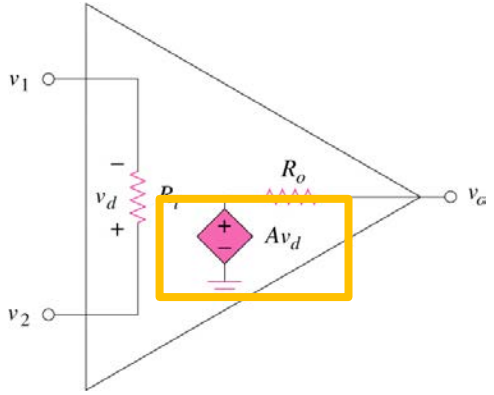


Amplificatore operazionale ideale

POLITECNICO DI MILANO

Prof. G. Gruosso





$$v_d = v_2 - v_1$$

$$v_o = A \cdot v_d = A \cdot (v_2 - v_1)$$

$$\begin{cases} A = \infty \\ R_i = \infty \\ R_o = 0 \end{cases}$$

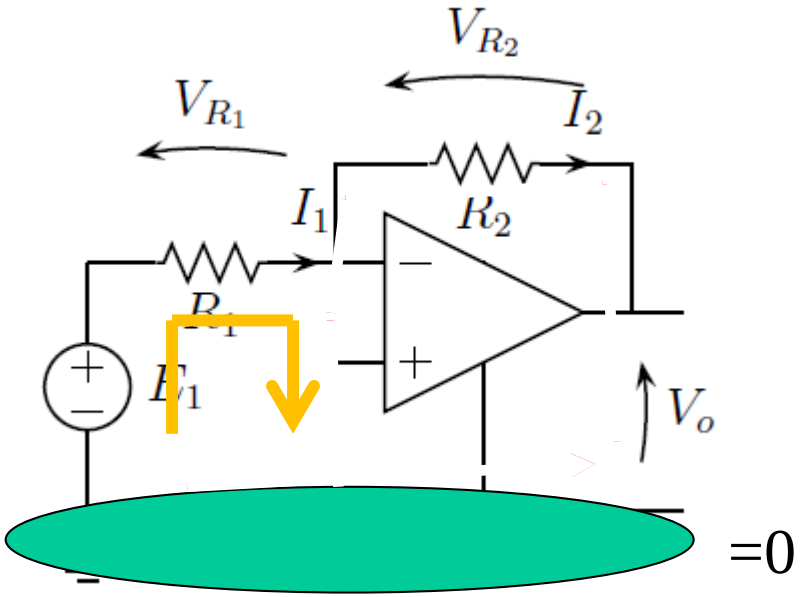
$$\frac{-v_{max}}{A} < v_d < \frac{v_{max}}{A}$$

$$\Rightarrow \begin{cases} i_1 = 0 \\ i_2 = 0 \\ v_d = v_2 - v_1 = 0 \\ v_2 = v_1 \end{cases}$$

POLITECNICO DI MILANO

Prof. G. Gruosso

Amplificatore operazionale ideale – Esempio 1



$$V_- - V_{R_2} - V_0 = 0$$

$$i_- = i_+ = 0 \qquad I_1 = I_2 \qquad E_1 - V_{R_1} - V_- = 0$$

$$V_- = V_+ \qquad V_- = V_+ = 0 \qquad E_1 - R_1 I_1 = 0$$

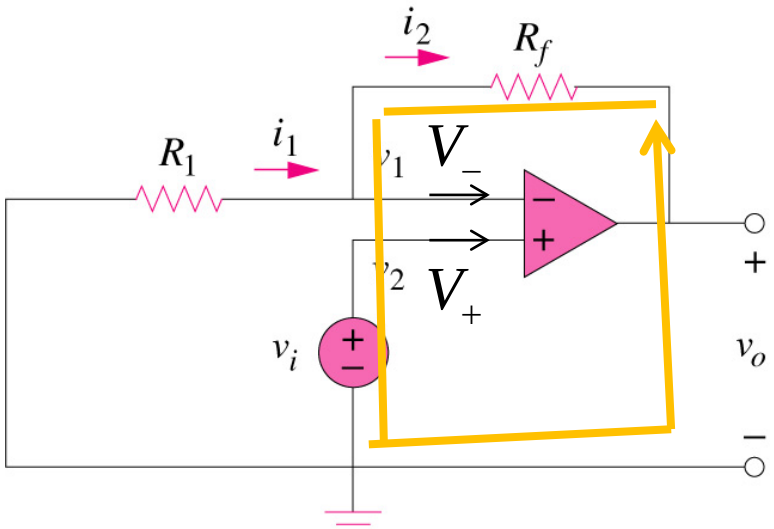
$$V_0 = -\frac{R_2}{R_1} E_1$$

Amplificatore invertente

POLITECNICO DI MILANO

Prof. G. Gruosso

Amplificatore operazionale ideale – Esempio 2



$$i_- = i_+ = 0$$

$$V_- = V_+$$

$$v_1 = v_2 = v_i \qquad i_1 = i_2 = -\frac{v_i}{R_1}$$

$$i_1 = i_2 = -\frac{v_i}{R_1} \qquad V_- - R_f i_2 - v_o = 0$$

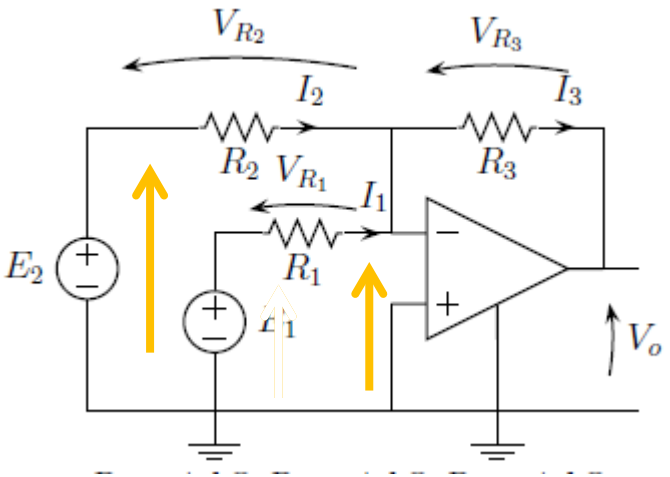
$$v_o = v_i + \frac{R_f}{R_1} v_i$$

Amplificatore non invertente

Amplificatore operazionale ideale – Esempio 3

POLITECNICO DI MILANO

Prof. G. Gruosso



$$i_- = i_+ = 0$$

$$V_- = V_+$$

$$V_o + V_{R_3} - V_- = 0$$

$$I_1 + I_2 - I_3 = 0$$

$$I_1 = \frac{E_1 - V_-}{R_1} = \frac{E_1}{R_1}$$

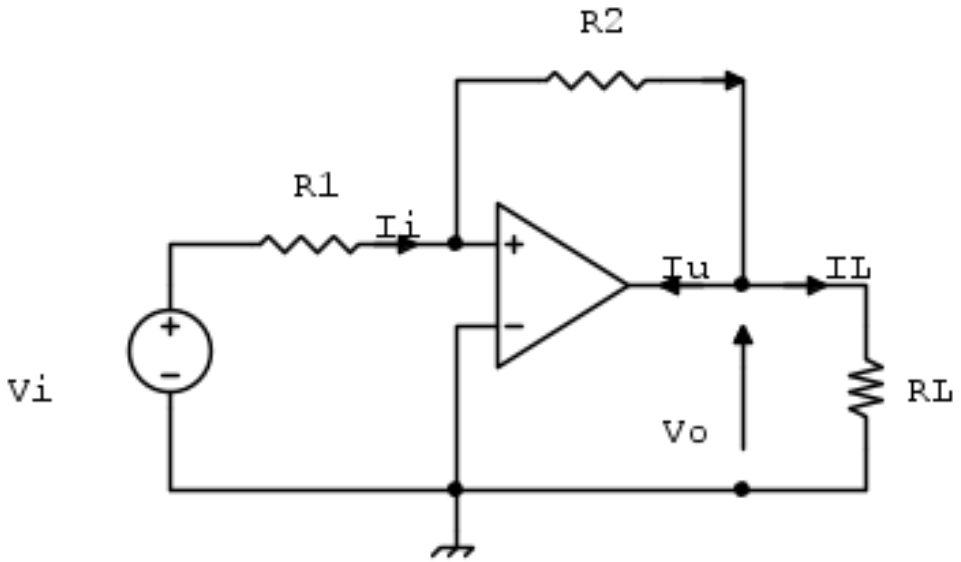
$$I_2 = \frac{E_2 - V_-}{R_2} = \frac{E_2}{R_2}$$

$$V_o = -R_3\left(\frac{E_1}{R_1} + \frac{E_2}{R_2}\right)$$

Calcolo Corrente di uscita e Potenza assorbita di Amplificatore

POLITECNICO DI MILANO

Prof. G. Gruosso



$$I_u + I_L - I_i = 0 \qquad I_u = -I_L + I_i$$

$$V_i I_i - R_1 I_i^2 - \mathrm{R_2 I_i^2} - \mathrm{R_L I_L^2} - \mathrm{P}_{amp} = 0$$

0

$$+\mathrm{V_o I_i} - \mathrm{V_o I_L} - \mathrm{P}_{amp} = 0 \qquad \mathrm{P}_{amp} = V_o I_u$$