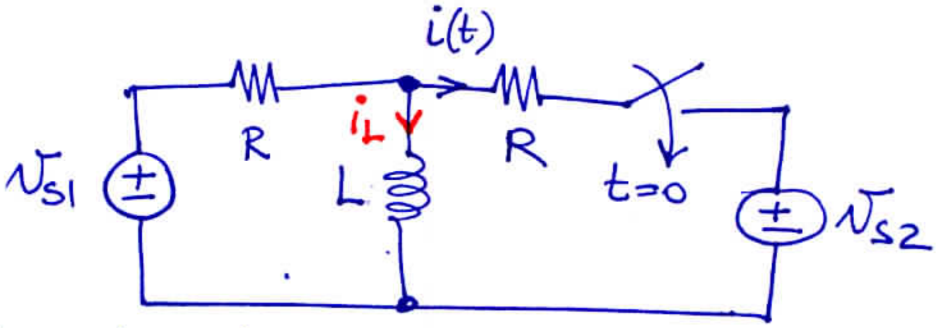
EX

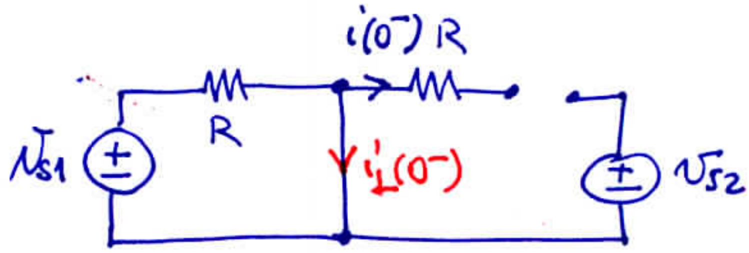


L=2H
$v_{S1}=12V$
$v_{S2}=20V$
$R=4\Omega$

L'interruttore e' aperto da lungo tempo e si chiude in $t=0$.
Determinare $i(t)$ per $t>0$.

1) COND. INIZIALI

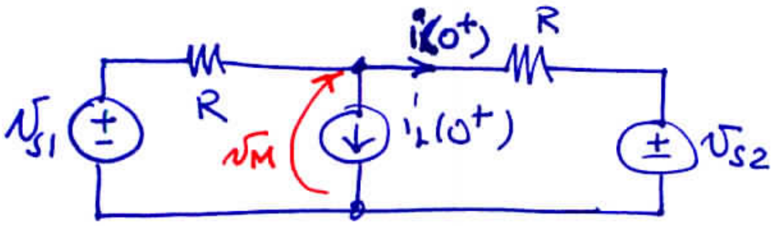
• Per $t=0^-$



$$i(0^-)=0 \qquad i_L(0^-)=\frac{v_{S1}}{R}=3\ A$$

• Per $t=0^+$

$$i_L(0^+)=i_L(0^-)=3\ A$$ (continuita' della variabile di stato)



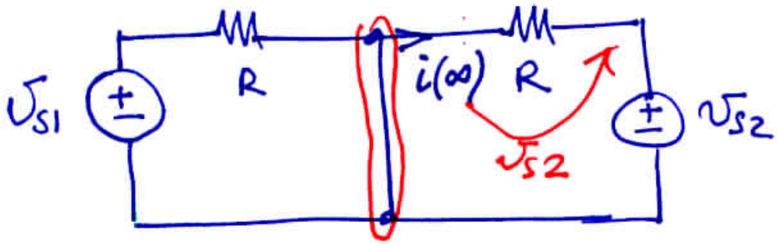
Millman:

$$v_M=\frac{\frac{v_{S1}}{R}+\frac{v_{S2}}{R}-i_L(0^+)}{\frac{1}{R}+\frac{1}{R}}=\frac{3+5-3}{\frac{1}{2}}$$

$$=10\ V$$

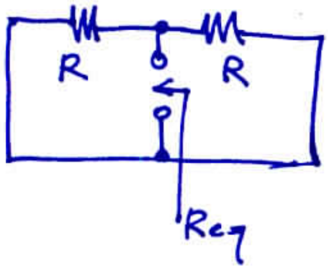
$$i(0^+)=\frac{v_M-v_{S2}}{R}=\frac{10-20}{4}=-\frac{10}{4}=-\frac{5}{2}A$$

2) REGIME per $t \to \infty$



$$i(\infty) = -\frac{v_{S2}}{R} = -5\,A$$

3) COSTANTE DI TEMPO



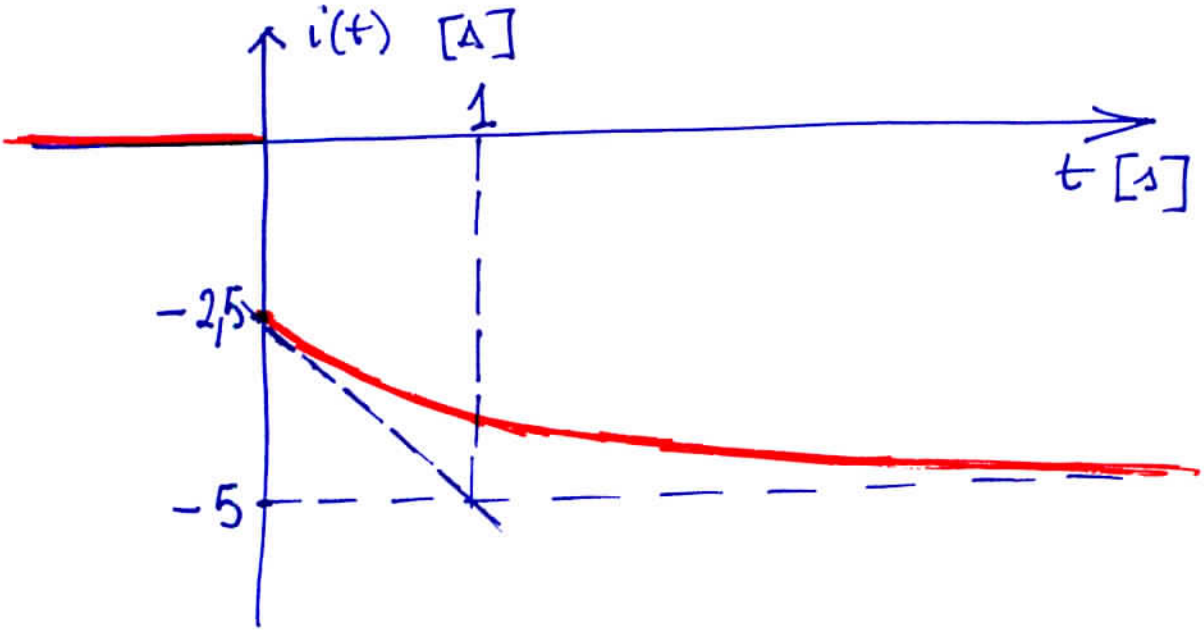
$$R_{eq} = R /\!/ R = R/2 = 2\,\Omega$$

$$\tau = \frac{L}{R_{eq}} = 1\,s$$

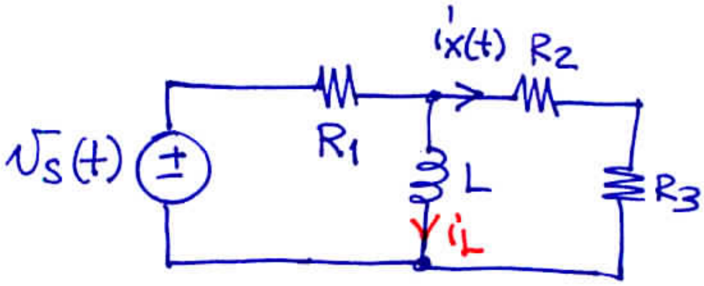
4) SOLUZIONE

$$i(t) = \left[i(0^+) - i(\infty)\right] e^{-t/\tau} + i(\infty) =$$

$$= 2{,}5\, e^{-t} - 5 \;,\; A \quad \text{per } t > 0$$



Ex



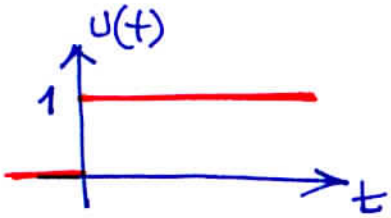
$v_s = 12\,u(t)$

$R_1 = 6\,k\Omega$

$R_2 = 10\,k\Omega$

$R_3 = 20\,k\Omega$

$L = 3\,mH$

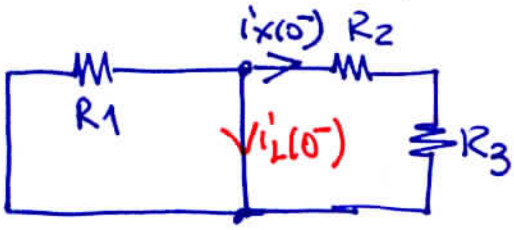


$u(t)$ "funzione gradino unitario" o "unit step"

- Determinare $i_x(t)$ per $t > 0$
- Determinare l'energia W_{R3} dissipata dal resistore R_3 da $t = 0$ a $t \to \infty$

1) CONDIZIONI INIZIALI

- Per $t = 0^-$ $\quad u(0^-) = 0 \Rightarrow v_s(t) = 0$

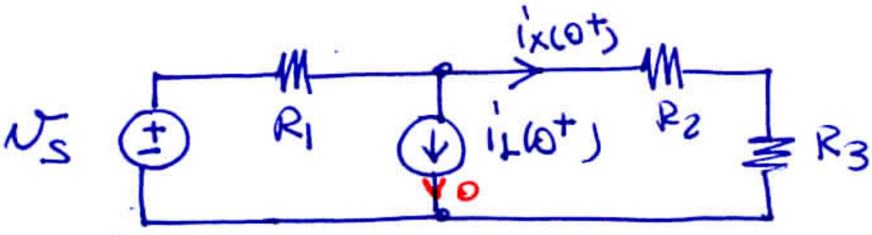


$i_x(0^-) = 0\ A$

$i_L(0^-) = 0\ A$

- Per $t = 0^+$ $\quad i_L(0^+) = i_L(0^-) = 0\ A$

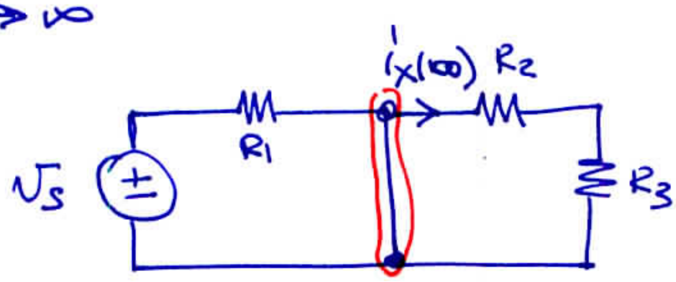
$u(0^+) = 1 \Rightarrow v_s(t) = 12\,V$



$$i_x(0^+) = \frac{v_s}{R_1 + R_2 + R_3} = \frac{12}{6 + 10 + 20} = \frac{1}{3}\,mA$$

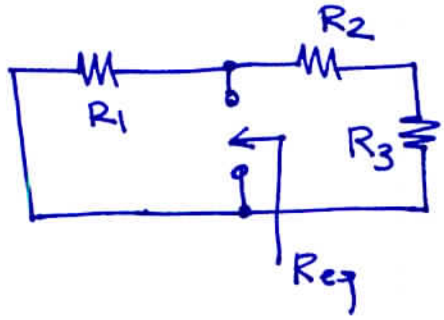
2) REGIME

Per $t \to \infty$



$i_x(\infty) = 0\ A$

3) COSTANTE DI TEMPO

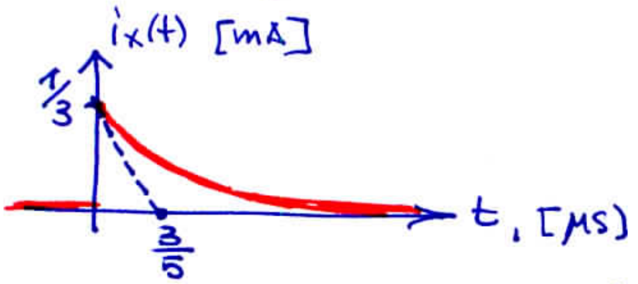


$$R_{eq} = R_1 // (R_2 + R_3) = 6 // 30 = \frac{6 \cdot 30}{36} = 5\ k\Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{3 \cdot 10^{-3}}{5 \cdot 10^{3}} = \frac{3}{5}\ \mu s$$

4) SOLUZIONE

$$i_x(t) = \left[i_x(0^+) - i_x(\infty) \right] e^{-t/\tau} + i_x(\infty)$$
$$= \frac{1}{3} e^{-\frac{5}{3} \cdot 10^6 t}, \quad mA \quad \text{per } t > 0$$

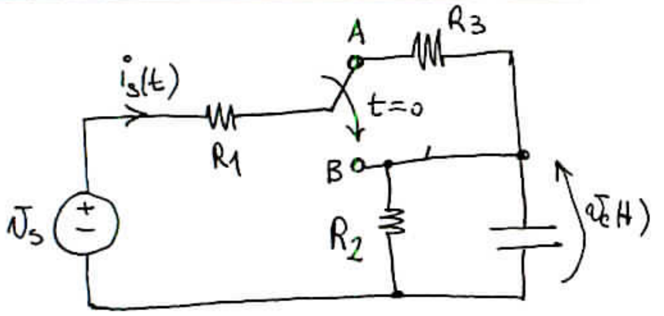


$$p_{R3} = R_3 i_x^2(t) \qquad W_{R3} = \int_0^\infty p_{R3}(t)\, dt$$

$$W_{R3} = \int_0^\infty R_3 i_x^2(t)\, dt = \int_0^\infty 20 \cdot 10^3 \cdot \frac{1}{9} \cdot 10^{-6} e^{-\frac{10}{3} \cdot 10^6 t}\, dt$$

$$= \frac{20}{9} \cdot 10^{-3} \left(\frac{1}{-10/3 \cdot 10^6} \right) \left[e^{-\frac{10}{3} \cdot 10^6} \right]_0^\infty = -\frac{20}{9} \frac{3}{10} 10^{-9} \left[0 - 1 \right]$$

$$= \frac{2}{3}\ nJ$$



Il circuito è a regime per $t<0$.
L'interruttore passa dalla posizione A alla posizione B in $t=0$

Determinare $v_c(t)=?$ $i_s(t)=?$

$v_s = 12\,V$; $R_1 = 20\,k\Omega$; $R_2 = R_3 = 10\,k\Omega$

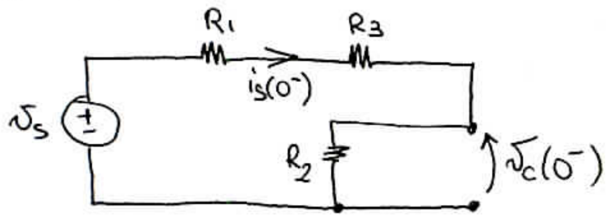
$C = 1\,\mu F$

Determinare l'energia immagazzinata nel condensatore $W_c(0) = ?$ e $W_c(t\to\infty) = ?$

Il condensatore si carica o si scarica?

Soluzione

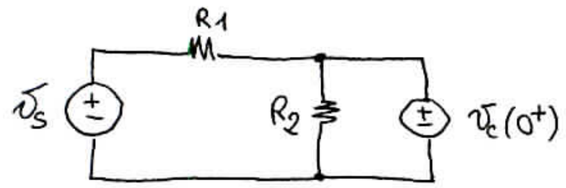
I) Per $t=0^-$



$$i_s(0^-) = \frac{v_s}{R_1+R_2+R_3} = \frac{12}{40} = \frac{3}{10}\ mA$$

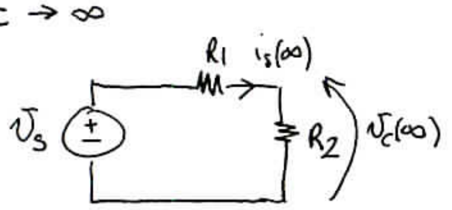
$$v_c(0^-) = R_2\, i_s(0^-) = 3\,V$$

Per $t=0^+$ $\quad v_c(0^+) = v_c(0^-) = 3\,V$



$$i_s(0^+) = \frac{v_s - v_c(0^+)}{R_1} = \frac{9}{20}\ mA$$

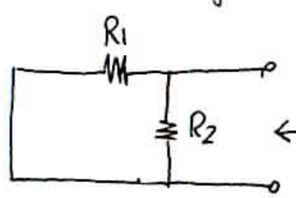
II) Per $t\to\infty$



$$v_c(\infty) = v_s \frac{R_2}{R_1+R_2} = 4\,V$$

$$i_s(\infty) = \frac{v_s}{R_1+R_2} = \frac{2}{5}\ mA$$

III) Costante di tempo

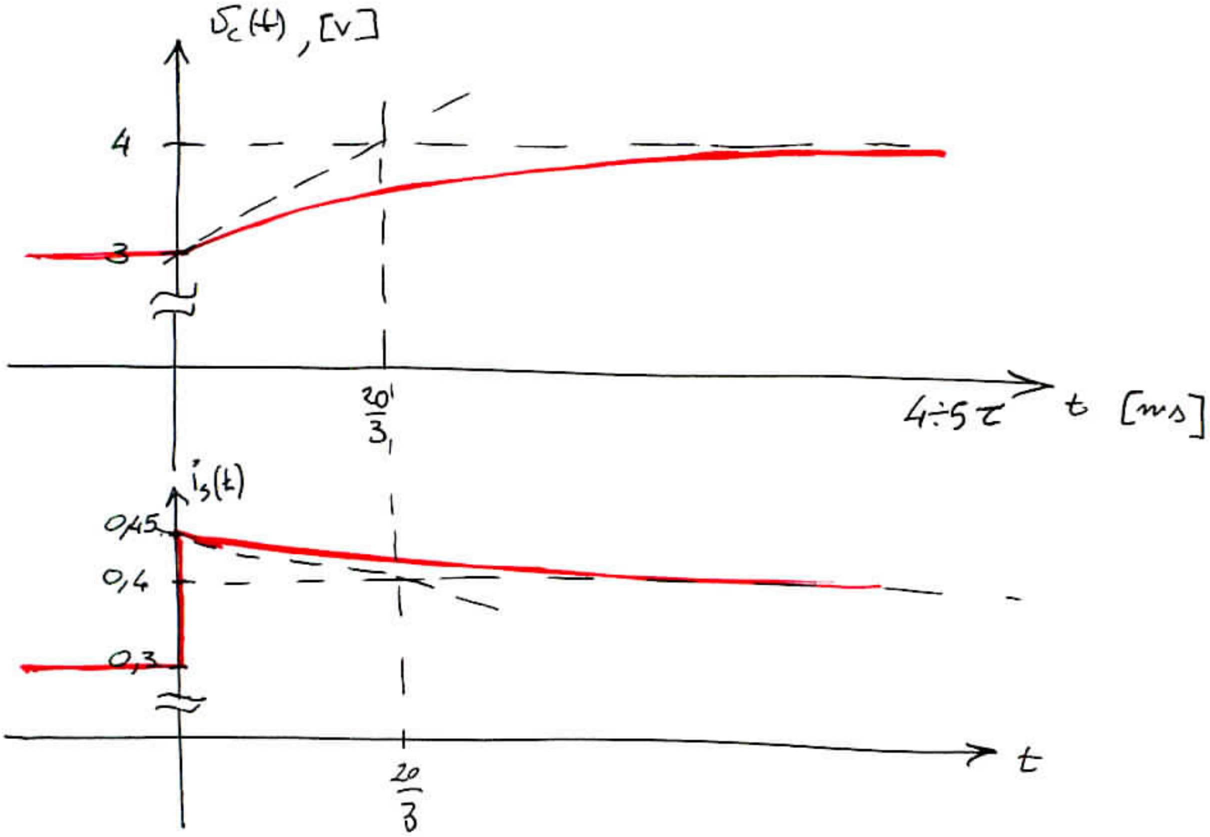


$$\leftarrow R_{eq} = R_1 /\!/ R_2 = \frac{R_1 R_2}{R_1+R_2} = \frac{20}{3}\,k\Omega$$

$$\tau = R_{eq} C = \frac{20}{3}\,ms$$

$$v_c(t) = [v_c(0^+) - v_c(\infty)]\,e^{-t/\tau} + v_c(\infty) = 4 - e^{-\frac{3}{20}\cdot 10^3 t},\ V \quad \text{per } t \geq 0$$

$$i_s(t) = [i_s(0^+) - i_s(\infty)]\,e^{-t/\tau} + i_s(\infty) = \frac{2}{5} + \frac{1}{20}\,e^{-\frac{3}{20}\cdot 10^3 t},\ mA \quad \text{per } t>0$$



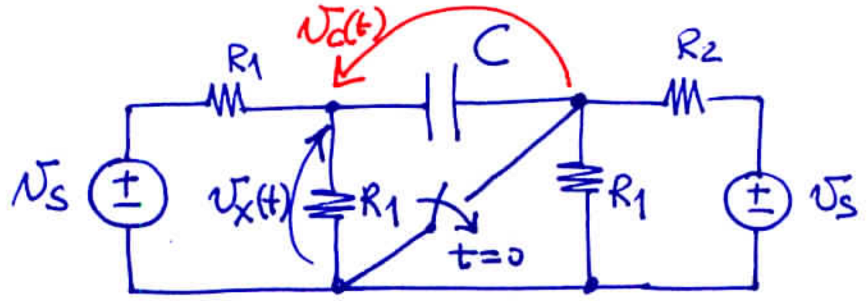
Energia immagazzinata nel condensatore

$$W_c(0) = \frac{1}{2} C v^2(0) = \frac{1}{2} \cdot 1 \cdot 10^{-6} \cdot 3^2 = \frac{9}{2}\ \mu J = 4{,}5\ \mu J$$

$$W_c(t\to\infty) = \frac{1}{2} C v^2(\infty) = \frac{1}{2} \cdot 10^{-6} \cdot 4^2 = 8\ \mu J$$

Il condensatore si carica $\quad \Delta W = W_c(t\to\infty) - W_c(0) = 3{,}5\ \mu J > 0$

EX

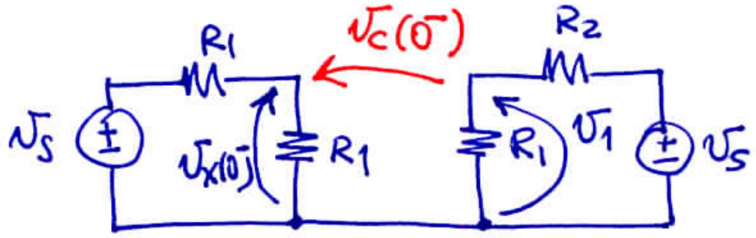


$v_S = 12\,V$
$R_1 = 4\,k\Omega$
$R_2 = 12\,k\Omega$
$C = 1\,\mu F$

L'interruttore e' aperto da lungo tempo e si chiude in $t=0$
Determinare $v_x(t)$ per $t>0$

1) CONDIZIONI INIZIALI

• Per $t=0^-$

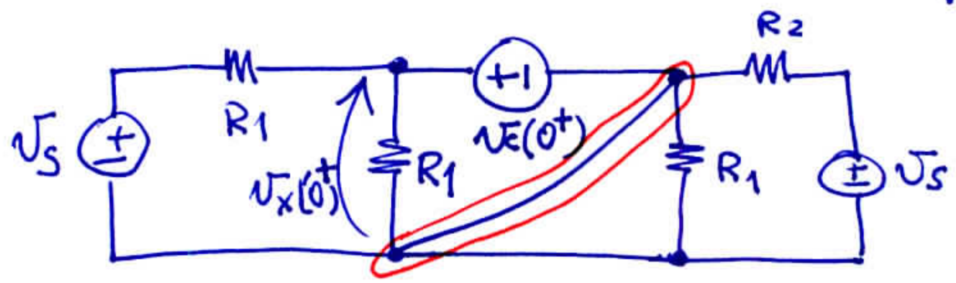


$$v_x(0^-) = v_S \cdot \frac{R_1}{R_1+R_1} = v_S/2 = 6\,V$$

$$v_1 = v_S \cdot \frac{R_1}{R_1+R_2} = 12 \cdot \frac{4}{16} = 3\,V$$

KVL: $v_c(0^-) = v_x(0^-) - v_1 = 6-3 = 3\,V$

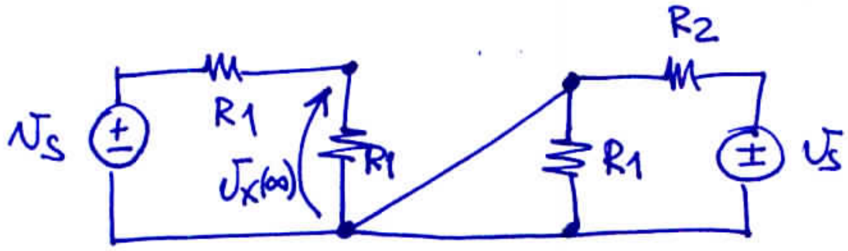
• Per $t=0^+$ $\quad v_c(0^+) = v_c(0^-) = 3\,V$ (continuità della variabile di stato)



$$v_x(0^+) = v_c(0^+) = 3\,V$$

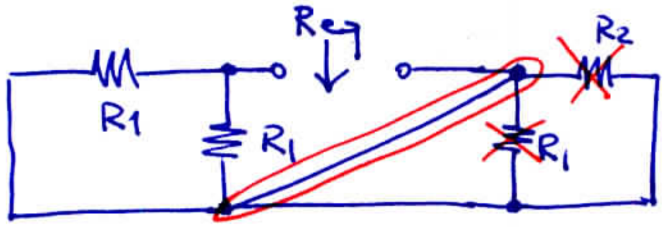
2) REGIME

Per $t \to \infty$



$$v_X(\infty) = v_S \frac{R_1}{R_1+R_1} = \frac{v_S}{2} = 6 \text{ V}$$

3) COSTANTE DI TEMPO



$R_{eq} = R_1 // R_1 =$
$= R_1/2 = 2\,k\Omega$

$$\tau = R_{eq}\, C = 2 \cdot 10^{3} \cdot 1 \cdot 10^{-6} = 2 \text{ ms}$$

4) SOLUZIONE

$$v_X(t) = [v_X(0^+) - v_X(\infty)]\, e^{-t/\tau} + v_X(\infty) =$$
$$= 6 - 3\, e^{-500t}, \text{V} \quad \text{per } t > 0$$