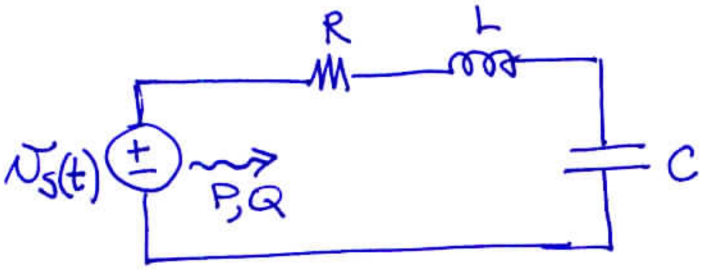
EX



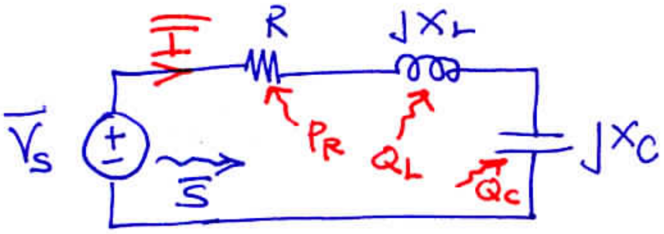
$v_S(t) = 4\cos(10t)$, V

$R = 2\,\Omega$

$L = 50$ mH

$C = 0{,}1$ F

Determinare la potenza attiva P e la potenza reattiva Q uscenti dal generatore ideale di tensione



$$\bar{V}_S = \frac{4}{\sqrt{2}}\ \text{V}$$

$$X_L = \omega L = 10 \cdot 50 \cdot 10^{-3} = 0{,}5\,\Omega$$

$$X_C = -\frac{1}{\omega C} = -\frac{1}{10 \cdot 0{,}1} = -1\,\Omega$$

$$\bar{I} = \frac{\bar{V}_S}{R + jX_L + jX_C} = \frac{4/\sqrt{2}}{2 + j\frac{1}{2} - j} = \frac{1}{\sqrt{2}} \frac{4}{\frac{4-j}{2}} = \frac{1}{\sqrt{2}} \frac{8}{4-j} \frac{4+j}{4+j}$$

$$= \frac{1}{\sqrt{2}} \frac{32 + j8}{17}$$

$\bar{S} = \bar{V}\bar{I}^*$ (conv. generatori, potenza uscente)

$$= \frac{4}{\sqrt{2}} \cdot \frac{1}{\sqrt{2}} \frac{32 - j8}{17} = \frac{64 - j16}{17} = 3{,}76 - j\,0{,}94\ \text{VA}$$

$$\Rightarrow \boxed{\begin{array}{l} P = 3{,}76\ \text{W} \\ Q = -0{,}94\ \text{VAR} \end{array}}$$

VERIFICA: Per il teorema di conservazione della potenza complessa (th. di Boucherot) deve risultare

$$P = P_R \quad ; \quad Q = Q_L + Q_C$$

Con le formule specifiche delle P e Q per resistore, induttore, condensatore:

$$P_R = R I^2 ; \quad Q_L = X_L I^2 ; \quad Q_C = X_C I^2$$

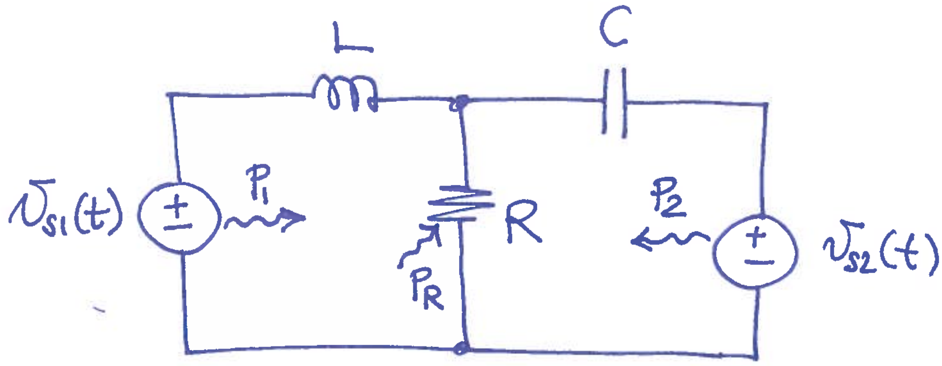
$$I = |\bar{I}| = \frac{1}{\sqrt{2}\cdot 17} \sqrt{32^2+8^2} = 1{,}37 \text{ A}$$

$P_R = 2 \cdot 1{,}37^2 = 3{,}75 \text{ W}$ $\quad P = P_R$ ok! (*)

$$\left.\begin{array}{l} Q_L = 0{,}5 \cdot 1{,}37^2 = 0{,}93 \text{ VAR} \\ Q_C = -1 \cdot 1{,}37^2 = -1{,}88 \text{ VAR} \end{array}\right\} Q_L + Q_C = -0{,}95 = Q \quad \text{ok! } (*)$$

(*) le differenze sono dovute ad approssimazioni numeriche

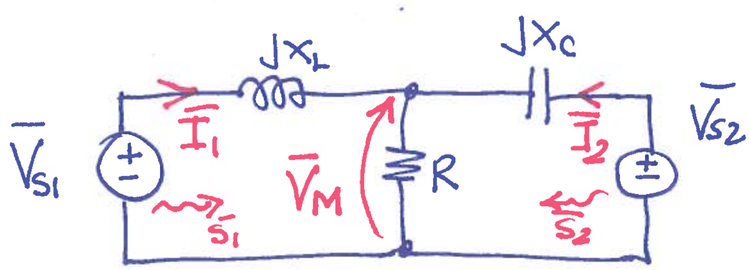
EX



$v_{s1}(t) = \cos(10t)$, V $\qquad L = 1\,H \qquad R = 10\,\Omega$

$v_{s2}(t) = -\sin(10t)$, V $\qquad C = 1/50\,F$

Determinare le potenze medie (potenze attive) P_1, P_2, P_R e verificare la conservazione delle potenze (th. di Boucherot).



$\bar{V}_{S1} = \frac{1}{\sqrt{2}}$ V ; $\bar{V}_{S2} = j\frac{1}{\sqrt{2}}$ V

$X_L = \omega L = 10\,\Omega$

$X_C = -\frac{1}{\omega C} = -5\,\Omega$

Millman:

$$\bar{V}_M = \frac{\frac{\bar{V}_{S1}}{jX_L} + \frac{\bar{V}_{S2}}{jX_C}}{\frac{1}{jX_L} + \frac{1}{jX_C} + \frac{1}{R}} = \frac{1}{\sqrt{2}} \frac{\frac{1}{j10} + \frac{j}{(-j5)}}{\frac{1}{j10} + \frac{1}{(-j5)} + \frac{1}{10}} =$$

$$= \frac{1}{\sqrt{2}} \frac{-\frac{1}{5} - j\frac{1}{10}}{-j\frac{1}{10} + j\frac{1}{5} + \frac{1}{10}} = \frac{1}{\sqrt{2}} \frac{\frac{-2-j}{\cancel{10}}}{\frac{-j+2j+1}{\cancel{10}}} = \frac{1}{\sqrt{2}} \frac{-2-j}{j+1} \frac{1-j}{1-j} =$$

$$= \frac{1}{\sqrt{2}} \frac{-2+j2-j-1}{2} = \frac{1}{2\sqrt{2}}(-3+j)\ \text{V}$$

$$\bar{I}_1 = \frac{\bar{V}_{S1} - \bar{V}_M}{jX_L} = \frac{1}{\sqrt{2}} \frac{1+(3-j)/2}{j10} = \frac{1}{\sqrt{2}} \frac{\frac{5}{2} - j\frac{1}{2}}{j10} = \frac{1}{\sqrt{2}}(-j)\frac{\frac{5}{2}-j\frac{1}{2}}{10}$$

$$= \frac{1}{\sqrt{2}\cdot 20}(-1-j5)\ \text{A}$$

$$\bar{I}_2 = \frac{\bar{V}_{S2} - \bar{V}_M}{jX_C} = \frac{1}{\sqrt{2}} \frac{j + (3-j)/2}{-j5} = \frac{1}{\sqrt{2}} \frac{\frac{3}{2} + j\frac{1}{2}}{-j5} =$$

$$= \frac{1}{10\sqrt{2}} j(3+j) = \frac{1}{\sqrt{2}\cdot 10}(-1+j3)\ A$$

- $\bar{S}_1 = \bar{V}_{S_1}\bar{I}_1^* = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{2}\cdot 20}(-1+j5) = \frac{1}{40}(-1+j5)\ VA$

$$\Rightarrow \boxed{P_1 = \mathrm{Re}\{\bar{S}_1\} = -\frac{1}{40}\ W}$$

- $\bar{S}_2 = \bar{V}_{S2}\bar{I}_2^* = \frac{j}{\sqrt{2}} \frac{1}{\sqrt{2}\cdot 10}(-1-j3) = \frac{1}{20}(-j+3)\ VA$

$$\Rightarrow \boxed{P_2 = \mathrm{Re}\{\bar{S}_2\} = \frac{3}{20}\ W}$$

- $P_R = \frac{V_M^2}{R}$ $\qquad V_M = \frac{1}{2\sqrt{2}}\sqrt{3^2+1^2} = \frac{\sqrt{10}}{2\sqrt{2}} = \frac{\sqrt{5}}{2}\ V$

$$\boxed{P_R} = \frac{(\sqrt{5}/2)^2}{10} = \frac{5}{40} = \boxed{\frac{1}{8}\ W}$$

- Th. di Boucherot $\qquad \sum P_{entranti} = 0$

$$-P_1 - P_2 + P_R = 0 \qquad +\frac{1}{40} - \frac{3}{20} + \frac{1}{8} = \frac{1-6+5}{40} = 0 \quad \text{ok!}$$

Per casa: determinare tutte le potenze reattive e verificare il teorema di Boucherot. Si ottiene:

$Q_1 = \mathrm{Im}\{\bar{S}_1\} = \frac{1}{8}$ VAR (uscente)

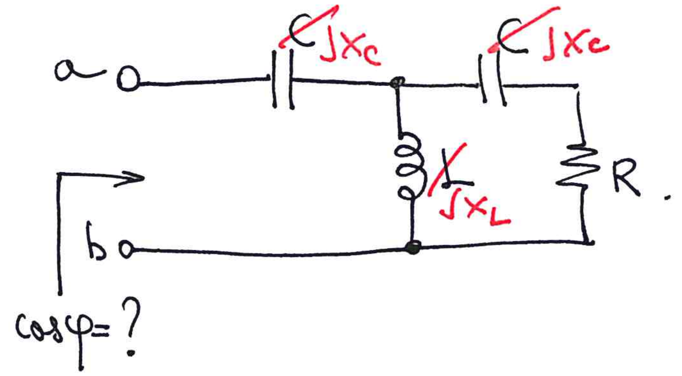
$Q_2 = \mathrm{Im}\{\bar{S}_2\} = -\frac{1}{20}$ VAR "

$Q_L = X_L I_1^2 = \frac{13}{40}$ VAR (entrante)

$Q_C = X_C I_2^2 = -\frac{1}{4}$ VAR (entrante)

$$-Q_1 - Q_2 + Q_L + Q_C =$$
$$= -\frac{1}{8} + \frac{1}{20} + \frac{13}{40} - \frac{1}{4} =$$
$$= \frac{-5+2+13-10}{40} = 0 \quad \text{ok!}$$

EX



$f = 50\ Hz$

$C = 2\ mF$

$L = 8\ mH$

$R = 4\ \Omega$

Determinare il fattore di potenza del bipolo di morsetti a, b

$$X_C = -\frac{1}{\omega C} = -\frac{1}{2\pi \cdot 50 \cdot 2 \cdot 10^{-3}} = -1{,}59\ \Omega$$

$$X_L = \omega L = 2\pi \cdot 50 \cdot 8 \cdot 10^{-3} = 2{,}51\ \Omega$$

$$\bar{Z}_{ab} = jX_C + \frac{jX_L(R + jX_C)}{jX_L + R + jX_C} = -j1{,}59 + \frac{j2{,}51(4 - j1{,}59)}{j2{,}51 + 4 - j1{,}59} =$$

$$= -j1{,}59 + \frac{4 + j10{,}04}{4 + j0{,}92} = -j1{,}59 + \frac{\sqrt{4^2 + 10{,}04^2}\ e^{j\,\mathrm{arctg}\left(\frac{10{,}04}{4}\right)}}{\sqrt{4^2 + 0{,}92^2}\ e^{j\,\mathrm{arctg}\left(\frac{0{,}92}{4}\right)}}$$

$$= -j1{,}59 + \frac{10{,}81\ e^{j68{,}28^\circ}}{4{,}10\ e^{j12{,}95^\circ}} = -j1{,}59 + 2{,}64\ e^{j55{,}33^\circ} =$$

$$= -j1{,}59 + 2{,}64\cos(55{,}33^\circ) + j\,2{,}64\sin(55{,}33^\circ) =$$

$$= -j1{,}59 + 1{,}58 + j\,2{,}17 \simeq 1{,}58 + j\,0{,}58\ \Omega$$

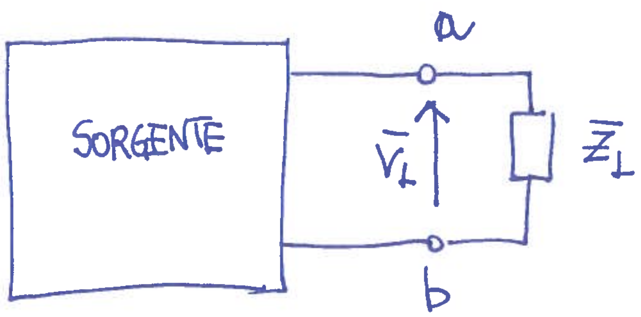
$$\varphi = \angle \bar{Z}_{ab} = \mathrm{arctg}\frac{0{,}58}{1{,}58} = 20{,}16^\circ$$

$$\boxed{\cos\varphi = 0{,}939 \quad (\text{rit.})}$$

BIPOLO ~~[illegible]~~ RESISTIVO - INDUTTIVO

CORRENTE IN RITARDO DI 20,16° RISPETTO ALLA TENSIONE

EX



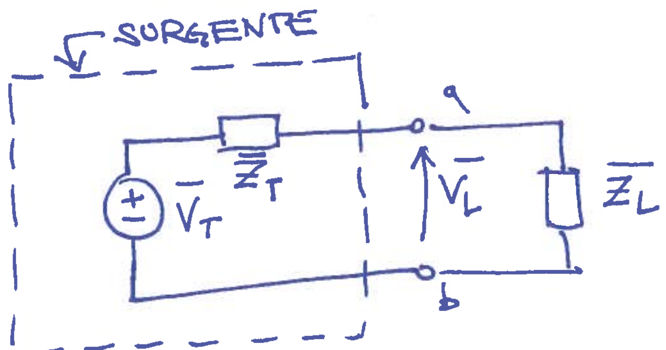
In laboratorio, si osserva che

(a) La sorgente ha una tensione a vuoto $\bar{V}_{oc} = 120\,V$ ai morsetti: a, b

(b) La sorgente fornisce una tensione $\bar{V}_L = 47{,}1\, e^{j11{,}3°}\,V$ ad un carico $\bar{Z}_L = 50 - j50\ \Omega$

- Determinare la potenza media disponibile (massima potenza attiva che la sorgente può erogare) della sorgente
- Determinare il carico $\bar{Z}_L$ che assorbe tale potenza attiva.

Per il teorema di Thevenin:



$$\boxed{\bar{V}_T = \bar{V}_{oc} = 120\,V}$$

$$\bar{V}_L = \bar{V}_T \cdot \frac{\bar{Z}_L}{\bar{Z}_L + \bar{Z}_T} \qquad 47{,}1\, e^{j11{,}3°} = 120 \cdot \frac{50 - j50}{\bar{Z}_T + 50 - j50}$$

$$(\bar{Z}_T + 50 - j50)\, 47{,}1\, e^{j11{,}3°} = 120 \cdot (50 - j50)$$

$$\boxed{\bar{Z}_T} = \frac{120\cdot(50-j50)}{47{,}1\,e^{j11{,}3°}} - 50 + j50 =$$

$$= \frac{120\sqrt{50^2+50^2}\,e^{j\,\mathrm{arctg}(-1)}}{47{,}1\,e^{j11{,}3°}} - 50 + j50 =$$

$$= 180{,}15\,e^{-j56{,}3°} - 50 + j50 = 180{,}15\cos(56{,}3°) - j180{,}15\sin(56{,}3°) - 50 + j50 =$$

$$\cong 100 - j150 - 50 + j50 = \boxed{50 - j100\ \Omega}$$

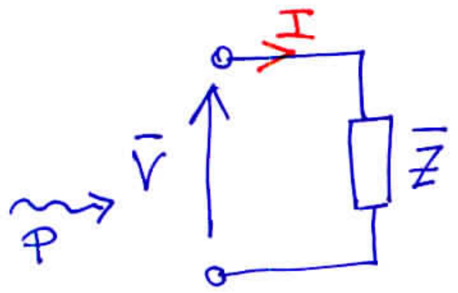
Per il teorema del massimo trasferimento di potenza attiva:

$$\boxed{\bar{Z}_L = \bar{Z}_T^{*} = 50 + j100}$$

è il carico che assorbe tutta la potenza attiva disponibile della sorgente, che vale

$$\boxed{P_{MAX}} = \frac{V_T^2}{4\,\mathrm{Re}\{\bar{Z}_T\}} = \frac{120^2}{4\cdot 50} = \boxed{72\,W}$$

EX



$V = 220\,V$ val. eff.

$\cos\varphi = 0{,}8$ (rit.)

$P = 3\ kW$

Determinare l'impedenza del bipolo $\bar{Z}$.

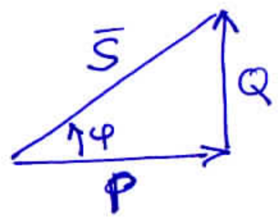
Essendo cos φ. di tipo ritardo ($0° < \varphi < 90°$) l'impedenza e' RESISTIVA - INDUTTIVA.

$$P = VI\cos\varphi \rightarrow I = \frac{P}{V\cos\varphi} = \frac{3\cdot10^3}{220\cdot 0{,}8} = 17{,}05\ A$$

$$P = RI^2 \rightarrow R = \frac{P}{I^2} = \frac{3\cdot10^3}{17{,}05^2} = 10{,}32\ \Omega$$

$$Q = P\,\mathrm{tg}\,\varphi = P\,\mathrm{tg}\left[\arccos 0{,}8\right] =$$

$$= 3\cdot \mathrm{tg}\left[36{,}87°\right] = 2{,}25\ kVAR$$



$$Q = XI^2 \rightarrow X = \frac{Q}{I^2} = \frac{2{,}25\cdot10^3}{17{,}05^2} = 7{,}74\ \Omega$$

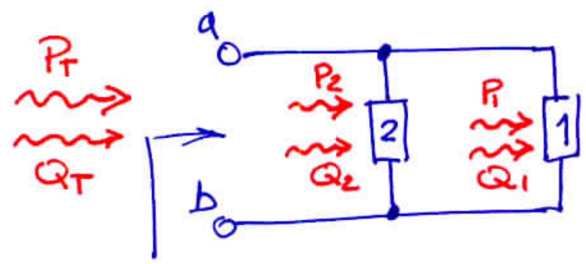
Quindi l'impedenza e'

$$\boxed{\bar{Z} = R + jX = 10{,}32 + j7{,}74\ \Omega}$$

Per casa: Verificare che l'angolo dell'impedenza $\angle\bar{Z}$ e' pari all'angolo della potenza complessa φ

$$\angle\bar{Z} = \mathrm{arctg}\,\frac{7{,}74}{10{,}32} = 36{,}87° = \varphi \qquad \text{OK}$$

EX



$\cos\varphi = ?$

I carichi 1 e 2 presentano i seguenti dati

$$\#1 \begin{cases} P_1 = 20 \text{ kW} \quad \text{assorbita} \\ \cos\varphi_1 = 0{,}8 \ (\text{rit.}) \end{cases}$$

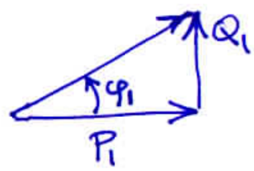
$$\#2 \begin{cases} P_2 = 10 \text{ kW} \quad \text{assorbita} \\ \cos\varphi_2 = 0{,}7 \ (\text{ant.}) \end{cases}$$

Determinare il fattore di potenza complessivo ai morsetti a, b.

- Per il teorema di Boucherot $\begin{cases} P_T = P_1 + P_2 \\ Q_T = Q_1 + Q_2 \end{cases}$

$$\boxed{P_T = 20 + 10 = 30 \text{ kW}}$$

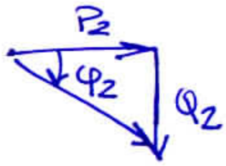
$Q_1 = P_1 \, tg\,\varphi_1$ con $\varphi_1 = \arccos(0{,}8) = 36{,}87°$



$= 20 \, tg(36{,}87°) = 15 \text{ kVAR}$

ant.

$Q_2 = P_2 \, tg\,\varphi_2$ con $\varphi_2 = -\arccos(0{,}7) = -45{,}57°$



$= 10 \, tg(-45{,}57°) = -10{,}2 \text{ kVAR}$

$$\boxed{Q_T = 15 - 10{,}2 = 4{,}8 \text{ kVAR}}$$

$$S_T = \sqrt{P_T^2 + Q_T^2} = \sqrt{30^2 + 4{,}8^2} = 30{,}38 \text{ kVA}$$

$$\boxed{\cos\varphi = \frac{P_T}{S_T} = \frac{30}{30{,}38} = 0{,}987 \ (\text{rit.})}$$