ESE 1

$V_{CPU} = 121$ $S_{CPU} = 0.005\ s$

$V_{D1} = 70$ $S_{D1} = 0.030\ s$

$V_{D2} = 50$ $S_{D2} = 0.027\ s$

$\lambda = 0.3\ i/s$

1) $D_i = ?$

$$D_i = V_i S_i = \begin{cases} D_{CPU} = 0.605\ s \\ D_{D1} = 2.1\ s \\ D_{D2} = 1.35\ s \end{cases}$$

2) $X_i, U_i, R_i^{es}, R_i, N_i = ?$

$$X_i = X V_i = \lambda V_i = \begin{cases} X_{CPU} = 36.3\ i/s \\ X_{D1} = 21\ i/s \\ X_{D2} = 15\ i/s \end{cases}$$

$$U_i = X D_i = \lambda D_i = \begin{cases} U_{CPU} = 0.1815 \\ U_{D1} = 0.63 \\ U_{D2} = 0.405 \end{cases}$$

$$R_i^{es} = \frac{D_i}{1 - U_i} = \begin{cases} R_{CPU}^{es} = 0.739\ s \\ R_{D1}^{es} = 5.676\ s \\ R_{D2}^{es} = 2.269\ s \end{cases}$$

$$R_i = \frac{R_i^{es}}{V_i} = \begin{cases} R_{CPU} = 0.006\ s \\ R_{D1} = 0.081\ s \\ R_{D2} = 0.045\ s \end{cases}$$

$$N_i = \frac{U_i}{1 - U_i} = \begin{cases} N_{CPU} = 0.222 \\ N_{D1} = 1.703 \\ N_{D2} = 0.681 \end{cases}$$

3)

$$N_i^{queue} = N_i - U_i = \begin{cases} N_{CPU}^{queue} = 0.041 \\ N_{D1}^{queue} = 0.443 \\ N_{D2}^{queue} = 0.276 \end{cases}$$

$$Q_i = R_i^{es} - D_i = \begin{cases} Q_{CPU} = 0.134\ s \\ Q_{D1} = 3.576\ s \\ Q_{D2} = 0.919\ s \end{cases}$$

$$Q_i^{visit} = R_i - S_i = \frac{Q_i}{V_i} = \begin{cases} Q_{CPU}^{visit} = 0.001\ s \\ Q_{D1}^{visit} = 0.051\ s \\ Q_{D2}^{visit} = 0.018\ s \end{cases}$$

4) $\lambda_{MAX} = ?$

$$\lambda_{MAX} = X_{MAX} = \frac{1}{D_{MAX}} = \frac{1}{D_{D2}} = 0.476\ i/s$$

5) $R, N = ?$

$$R = \sum_i R_i^{es} = 8.624\ s \qquad N = \lambda R = \sum_i N_i = 2.606$$

ESE 5.2

$C = 10000$ $T = 2h = 7200\ s$

$C_A = 2200$ $B_A = 2100\ s$

$C_B = 3000$ $B_B = 1440\ s$

$C_C = 4800$ $B_C = 3300\ s$

Closed model

$Z = 1\ s$

1) $X, \lambda, X_i, U_i, D_i, V_i, \lambda_{sat} = ?$

$X = \frac{C}{T} = 1.389\ j/s$ $\lambda = X = 1.389\ j/s$ $X_i = \frac{C_i}{T} = \begin{cases} X_A = 0.306\ j/s \\ X_B = 0.417\ j/s \\ X_C = 0.667\ j/s \end{cases}$

$U_i = \frac{B_i}{T} = \begin{cases} U_A = 0.292 \\ U_B = 0.2 \\ U_C = 0.458 \end{cases}$ $D_i = \frac{U_i}{X} = \begin{cases} D_A = 0.210\ s \\ D_B = 0.144\ s \\ D_C = 0.330\ s \end{cases}$

$V_i = \frac{X_i}{X} = \frac{C_i}{C} = \begin{cases} V_A = 0.22 \\ V_B = 0.3 \\ V_C = 0.48 \end{cases}$ $\lambda_{sat} = \lambda_{MAX} = X_{MAX} = \frac{1}{D_{MAX}} = \frac{1}{D_C} = 3.030\ j/s$

2) $N_i, R = ?$

$N_i = \frac{U_i}{1 - U_i} = \begin{cases} N_A = 0.412 \\ N_B = 0.25 \\ N_C = 0.845 \end{cases}$ $R_i^{es} = \frac{D_i}{1 - U_i} = \begin{cases} R_A^{es} = 0.297\ s \\ R_B^{es} = 0.180\ s \\ R_C^{es} = 0.609\ s \end{cases}$ $R = \sum_i R_i^{es} = 1.086\ s$

3) Bottleneck = ? For $N = 6$ the system is in heavy-load or light-load = ?

Resource C is the bottleneck, since $\max\{D_i\} = D_C$.

$N^* = \frac{D + Z}{D_{MAX}} = \frac{0.624\ s + 1s}{0.330\ s} = 5.103 \rightarrow$ The system is in heavy-load because $N = 6 > N^*$.

4) Draw and describe analitically the asymptotes X and R = ?

See theory discribed in class and on Lazowska's book.

ESE 5.3

Single-class model with two service centers.

$R_1 = 3\,s$ $\quad X_1 = 3\ tr/s$ $\quad X = 2\ tr/s$

$R_2 = 8\,s$ $\quad X_2 = 4\ tr/s$

$$V_i = \frac{X_i}{X} = \begin{cases} V_1 = 1.5 \\ V_2 = 2 \end{cases}$$

1) $D_i = ?$

$$R_i^{es} = \frac{D_i}{1-U_i} = \frac{D_i}{1-XD_i} \rightarrow R_i^{es}(1-XD_i) = D_i \rightarrow R_i^{es} = D_i(1+XR_i^{es}) \rightarrow$$

$$\rightarrow D_i = \frac{R_i^{es}}{1+XR_i^{es}} = \frac{V_i R_i}{1+XV_iR_i} = \begin{cases} D_1 = \frac{\frac{3}{2}\cdot 3\,s}{1+2\cdot\frac{3}{2}\cdot 3} = \frac{9}{20}\,s = 0.45\,s \\ D_2 = \frac{2\cdot 8\,s}{1+2\cdot 2\cdot 8} = \frac{16}{33}\,s = 0.485\,s \end{cases}$$

2) $S_i = ?$

$$S_i = \frac{D_i}{V_i} = \begin{cases} S_1 = 0.3\,s \\ S_2 = 0.243\,s \end{cases}$$

ESE 5.4

$T = 3600\,s$ $\quad C = 720\,j$ $\quad N_A = 1\,j$ $\quad N_B = 4\,j$ $\quad U_C = 0.90909$ $\quad$ Closed system $Z = 10\,s$

1) $D_i = ?$

$$X = \frac{C}{T} = 0.2\ j/s \qquad R_i^{es} = \frac{D_i}{1-U_i} = \frac{D_i}{1-XD_i} \rightarrow V_iR_i = \frac{D_i}{1-XD_i} \rightarrow \frac{X_i}{X}R_i = \frac{D_i}{1-XD_i} \rightarrow \frac{N_i}{X} = \frac{D_i}{1-XD_i} \rightarrow$$

$$\rightarrow N_i = \frac{XD_i}{1-XD_i} \rightarrow N_i - XN_iD_i = XD_i \rightarrow N_i = X(1+N_i)D_i \rightarrow$$

$$\rightarrow D_i = \frac{N_i}{X(1+N_i)} = \begin{cases} D_A = 2.5\,s \\ D_B = 4\,s \end{cases} \qquad D_C = \frac{U_C}{X} = 4.54545\,s$$

2) $N_C = ?$

$$N_C = \frac{U_C}{1-U_C} = 10\,j$$

3) $U_A, U_B = ?$

$$U_i = XD_i = \begin{cases} U_A = 0.5 \\ U_B = 0.8 \end{cases}$$

4) $R = ?$

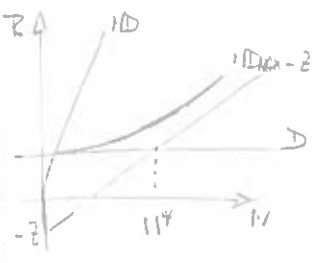
$$R_i^{es} = \frac{D_i}{1-U_i} = \begin{cases} R_A^{es} = 5\,s \\ R_B^{es} = 20\,s \\ R_C^{es} = 50\,s \end{cases} \qquad R = \sum_i R_i^{es} = 75\,s \quad \left(\rightarrow \frac{N}{X} = \frac{\sum N_i}{X}\right)$$

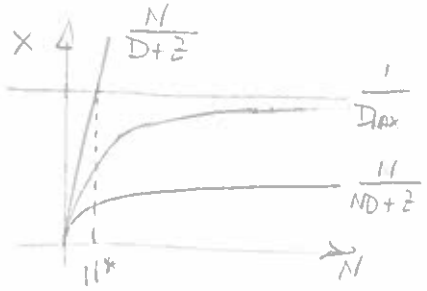
5) Which servers should be replicated to support $\hat{\lambda} = 0.3\ j/s$?

If $\hat{\lambda} = 0.3\ j/s$, then $\hat{U}_i = \hat{\lambda}D_i = \begin{cases} U_A = 0.75 \\ U_B = 1.2 \\ U_C = 1.364 \end{cases}$, servers B and C must be replicated once.

$B \rightarrow B_1$ and B_2 $\quad C \rightarrow C_1$ and C_2

6) Asymptotic bounds = ?





7) $R = 15\,s$ with $N = 7$?

$$N^* = \frac{D+Z}{D_{MAX}} = \frac{11.04545+10}{4.54545} = 4.63$$

$N > N^* \rightarrow R \geq ND_{MAX} - Z = 21.8815\,s \rightarrow$ It's not possible!

Esc 5.5

$T = 600\,s$ $\quad C = 75$

$C_A = 500$ $\quad B_A = 450\,s$

$C_B = 150$ $\quad B_B = 300\,s$

$C_C = 300$ $\quad B_C = 100\,s$

$Z = 10\,s$

1) $U_i = ?$

$$U_i = \frac{B_i}{T} = \begin{cases} U_A = 0.75 \\ U_B = 0.5 \\ U_C = 0.167 \end{cases}$$

2) $D_i = ?$

$$D_i = \frac{B_i}{C} = \begin{cases} D_A = 6\,s \rightarrow \text{Bottleneck} \\ D_B = 4\,s \\ D_C = 1.333\,s \end{cases}$$

3) $V_i = ?$

$$V_i = \frac{C_i}{C} = \begin{cases} V_A = 6.667 \\ V_B = 2 \\ V_C = 4 \end{cases}$$

4) $S_i = ?$

$$S_i = \frac{D_i}{V_i} = \begin{cases} S_A = 0.9\,s \\ S_B = 2\,s \\ S_C = 0.333\,s \end{cases}$$

5) $\lambda_{MAX} = ?$

$$\lambda_{MAX} = X_{MAX} = \frac{1}{D_{MAX}} = \frac{1}{D_A} = 0.167\ i/s$$

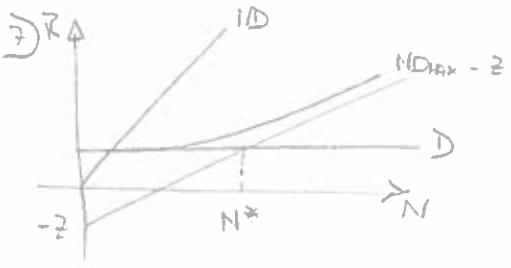
6) $R_i, N_i = ?$

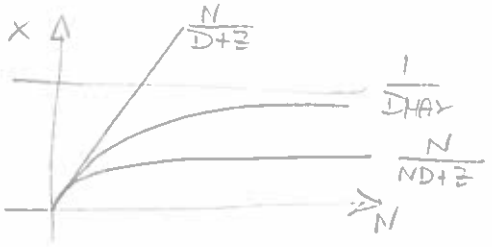
$$R_i^{es} = \frac{D_i}{1 - U_i} = \begin{cases} R_A^{es} = 24\,s \\ R_B^{es} = 8\,s \\ R_C^{es} = 1.6\,s \end{cases}$$

$$R_i = \frac{R_i^{es}}{V_i} = \begin{cases} R_A = 3.6\,s \\ R_B = 4\,s \\ R_C = 0.4\,s \end{cases}$$

$$N_i = \frac{U_i}{1 - U_i} = \begin{cases} N_A = 3 \\ N_B = 1 \\ N_C = 0.2 \end{cases}$$

7)





8) $N^* = ?$

$$N^* = \frac{D + Z}{D_{MAX}} = 3.556$$

9) $R_{min}, R_{MAX} = ?$

$ND_{MAX} - Z \leq R \leq ND \rightarrow N \cdot 6s - 10s \leq R \leq N \cdot 11.333\,s$

Ex 5.0

$T = 600\,s$ $\quad C = 100$ $\quad X_{MAX} = 0.2\ tr/s$

$C_B = 150$ $\quad B_B = 300\,s$

$C_C = 300$ $\quad B_C = 100\,s$

Closed system

$Z = 10s$

1) $D_i = ?$

$$D_i = \frac{B_i}{C} = \begin{cases} D_B = 3\,s \\ D_C = 1\,s \end{cases} \qquad D_A = \frac{1}{X_{MAX}} = 5\,s \rightarrow \text{Bottleneck}$$

2) $U_i = ?$

$$X = \frac{C}{T} = \frac{1}{6}\ i/s \qquad U_i = X D_i = \begin{cases} U_A = \frac{5}{6} \\ U_B = \frac{1}{2} \\ U_C = \frac{1}{6} \end{cases}$$

3) $V_B = ?$

$$V_B = \frac{C_B}{C} = 1.5$$

4) $R_A^{es},\ N_A = ?$

$$R_A^{es} = \frac{D_A}{1 - U_A} = \frac{5\,s}{1 - \frac{5}{6}} = 30\,s \qquad N_A = \frac{U_A}{1 - U_A} = \frac{5/6}{1 - 5/6} = \frac{5/6}{1/6} = 5$$

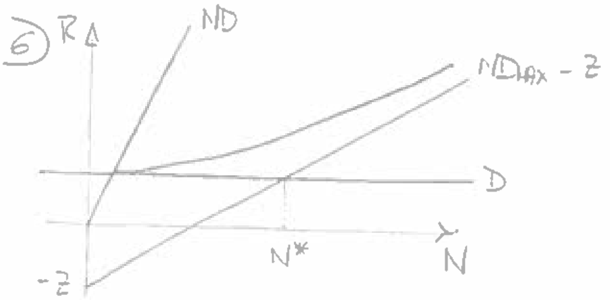
5) How many A in parallel for $R_A^{es} < 18\,s$?

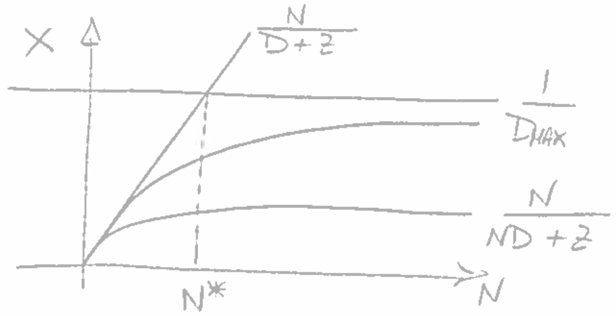
$$R_A^{es} < 18\,s \longrightarrow \frac{D_A}{1 - \frac{U_A}{K}} < 18\,s \longrightarrow \frac{D_A}{1 - \frac{X D_A}{K}} < 18\,s \longrightarrow \frac{D_A \cdot K}{K - X D_A} < 18\,s \longrightarrow$$

$$\longrightarrow D_A \cdot K < 18\,s\,(K - X D_A) \longrightarrow K(D_A - 18s) < -X D_A \cdot 18s \longrightarrow K(18s - D_A) > 18s \cdot X D_A \longrightarrow$$

$$\longrightarrow K > \frac{X D_A \cdot 18\,s}{18s - D_A} = \frac{\frac{1}{6}\,i/s \cdot 5s \cdot 18s}{18s - 5s} = \frac{15\,s}{13\,s} = 1.154 \longrightarrow K = 2$$ is the minimum number of replicas to have $R_A^{es} < 18\,s$.

6)





7) N_{MAX} for $R < 40\,s$?

$$R < 40s \longrightarrow ND < 40s \longrightarrow N < \frac{40\,s}{D} = \frac{40\,s}{9\,s} = 4.444 \longrightarrow N_{MAX} = 4$$