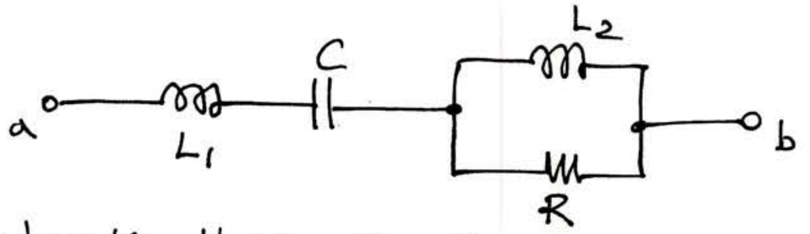
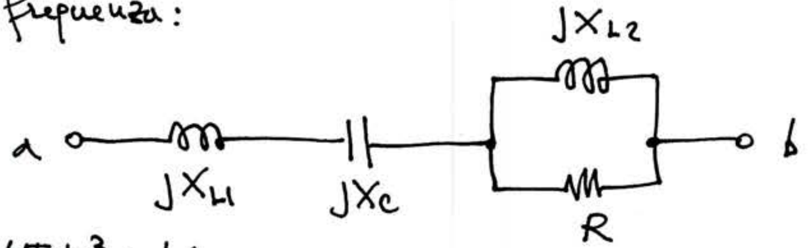
EX



$f = 2\,kHz$; $L_1 = 10\ mH$; $L_2 = 30\ mH$
$R = 60\,\Omega$; $C = 1\,\mu F$

- Determinare $\overline{Z}_{ab}$ (impedenza del bipolo di morsetti a, b)
  $\overline{Y}_{ab}$ (ammettenza " " " " " ")
- Rappresentare il circuito equivalente visto ai morsetti a, b

Dominio della frequenza:



$\omega = 2\pi f = 4\pi \cdot 10^3$ rad/s

$X_{L1} = \omega L_1 = 4\pi \cdot \cancel{10^3} \cdot 10 \cdot \cancel{10^{-3}} = 125,66\ \Omega$

$X_{L2} = \omega L_2 = 4\pi \cdot \cancel{10^3} \cdot 30 \cdot \cancel{10^{-3}} = 376,99\ \Omega$

$X_C = -\frac{1}{\omega C} = -\frac{1}{4\pi \cdot 10^3 \cdot 1 \cdot 10^{-6}} = -79,58\ \Omega$

$$\boxed{\overline{Z}_{ab} = jX_{L1} + jX_C + \frac{jX_{L2} \cdot R}{jX_{L2} + R}}$$

IMPEDENZA

Calcoli:

$$\overline{Z}_{ab} = j\,(125,66 - 79,58) + \frac{j\,376,99 \cdot 60}{j\,376,99 + 60} \cdot \underbrace{\frac{60 - j\,376,99}{60 - j\,376,99}}_{\text{RAZIONALIZZO}} =$$

$$= j\,46,08 + \frac{j\,376,99 \cdot 60^2 + 376,99^2 \cdot 60}{376,99^2 + 60^2} = \frac{376,99^2 \cdot 60}{376,99^2 + 60^2} + j\left(46,08 + \frac{376,99 \cdot 60^2}{376,99^2 + 60^2}\right)$$

$$= 58,52 + j\,55,39\ \Omega$$

In alternativa, potevo eseguire il rapporto di numeri complessi passando tutti i numeri in forma esponenziale:

$$\overline{Z}_{ab} = j\,(125{,}66 - 79{,}58) + \frac{376{,}99 \cdot 60\; e^{j90^\circ}}{\sqrt{376{,}99^2 + 60^2}\; e^{j\underbrace{\mathrm{arctg}\left(\frac{376{,}99}{60}\right)}_{80{,}95^\circ}}}$$

$$= j46{,}08 + 59{,}25\; e^{j\underbrace{(90^\circ - 80{,}95^\circ)}_{9{,}05^\circ}} =$$

$$= j46{,}08 + 59{,}25 \cos(9{,}05^\circ) + j\, 59{,}25 \sin(9{,}05^\circ)$$

$$= 58{,}51 + j\,55{,}40\ \Omega$$

(medesimo risultato ... a meno di approssimazioni numeriche ...)

$$\overline{Y}_{ab} = \frac{1}{\overline{Z}_{ab}} = \frac{1}{58{,}52 + j55{,}39} = \frac{e^{j0^\circ}}{\sqrt{58{,}52^2 + 55{,}39^2}\; e^{j\,\mathrm{arctg}\left(\frac{55{,}39}{58{,}52}\right)}}$$

$$= \frac{e^{j0^\circ}}{80{,}58\; e^{j43{,}43^\circ}} = 0{,}0124\, e^{j(0^\circ - 43{,}43^\circ)} = 0{,}0124\, e^{-j43{,}43^\circ}\ S$$

$$= 12{,}4\, e^{-j43{,}43^\circ}\ mS$$

$[S] = [\Omega^{-1}]$ "SIEMENS"

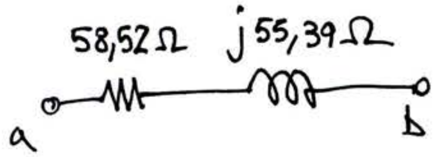
in forma cartesiana:

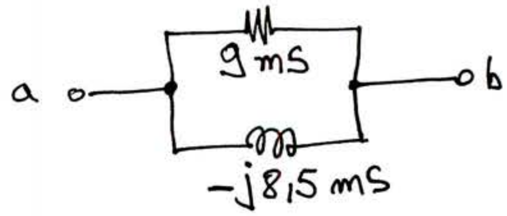
$$\overline{Y}_{ab} = 12{,}4 \cos(-43{,}43^\circ) + j\,12{,}4 \sin(-43{,}43^\circ)$$

$$= 12{,}4 \cos(43{,}43^\circ) - j\,12{,}4 \sin(43{,}43^\circ)$$

$$= 9 - j8{,}5\ mS$$

- CIRCUITI EQUIVALENTI (BIPOLO RESISTIVO-INDUTTIVO, $\mathrm{Im}\{\overline{Z}_{ab}\} > 0$ $\mathrm{Im}\{\overline{Y}_{ab}\} < 0$)





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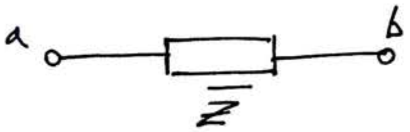
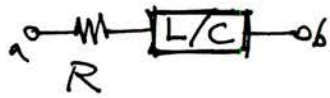


$\overline{Z} = 5 - j10\,\Omega$

$f = 1\,kHz$

Determinare il circuito equivalente (per la frequenza f) costituito da

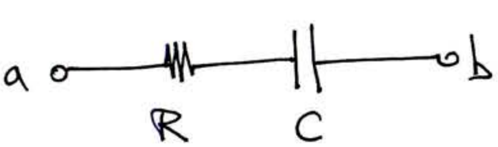
(a) UN RESISTORE E UN BIPOLO DINAMICO IN SERIE



(b) " " " " " " " PARALLELO

(a) $\mathrm{Im}\{\overline{Z}\} < 0 \longrightarrow$ IMPEDENZA RESISTIVA-CAPACITIVA

$\longrightarrow$ IL BIPOLO DINAMICO DEVE ESSERE UN CONDENSATORE



RESISTENZA

REATTANZA CAPACITIVA

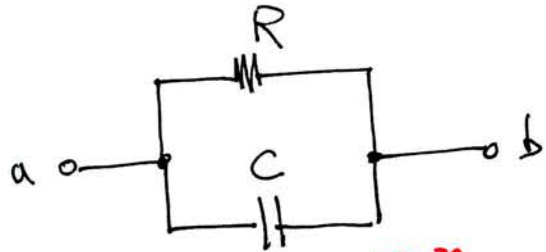
$$\overline{Z}_{eq} = R - j\frac{1}{\omega C} = 5 - j10$$

$$\begin{cases} R = 5 \\ \dfrac{-1}{\omega C} = -10 \end{cases}$$

$\boxed{R = 5\,\Omega}$

$$\boxed{C} = \frac{1}{2\pi \cdot 1 \cdot 10^3 \cdot 10} = 1{,}59 \cdot 10^{-5} = \boxed{15{,}9\ \mu F}$$

(b)



CONDUTTANZA

SUSCETTANZA CAPACITIVA

$$\overline{Y}_{eq} = \frac{1}{R} + j\omega C = \frac{1}{5 - j10} \cdot \frac{5 + j10}{5 + j10} = \frac{5 + j10}{125} = \frac{1}{25} + j\frac{2}{25}$$

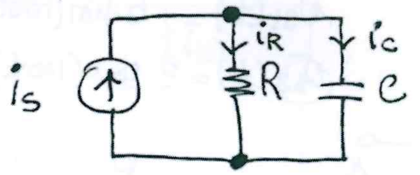
RAZIONALIZZO

$$\begin{cases} \dfrac{1}{R} = \dfrac{1}{25} \\ \omega C = \dfrac{2}{25} \end{cases}$$

$\boxed{R = 25\,\Omega}$

$$\boxed{C} = \frac{2}{25 \cdot 2 \cdot \pi \cdot 1 \cdot 10^3} = 1{,}273 \cdot 10^{-5} = \boxed{12{,}73\ \mu F}$$

EX)



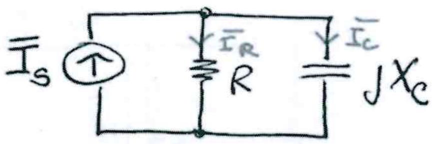
$i_s(t) = \cos(10^5 t)$, A

$R = 10\,\Omega$

$C = 1\,\mu F$

- Determinare $i_R(t)$, $i_c(t)$ a regime.
- Grafico nel piano complesso

CIRCUITO NEL DOMINIO DEI FASORI:



$\omega = 10^5$ rad/s

$\bar{I}_s = \frac{1}{\sqrt{2}}$ A

$$X_C = -\frac{1}{\omega C} = -\frac{1}{10^5 \cdot 1 \cdot 10^{-6}} = -10\,\Omega$$

Partitore di corrente:

$$\bar{I}_R = \bar{I}_s \frac{jX_c}{R + jX_c} = \frac{1}{\sqrt{2}} \cdot \frac{-j10}{10 - j10} \frac{1+j}{1+j} = \frac{1}{\sqrt{2}} \frac{-j+1}{2} = \frac{1-j}{2\sqrt{2}} \text{ A}$$

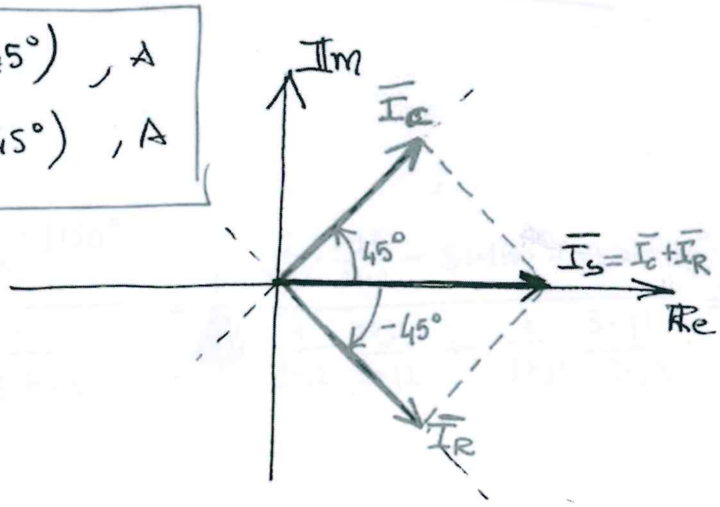
$$\bar{I}_R = \sqrt{\left(\frac{1}{2\sqrt{2}}\right)^2 + \left(\frac{1}{2\sqrt{2}}\right)^2}\, e^{j\,\mathrm{arctg}(-1)} = \frac{1}{2} e^{-j45^\circ} \text{ A}$$

KCL:

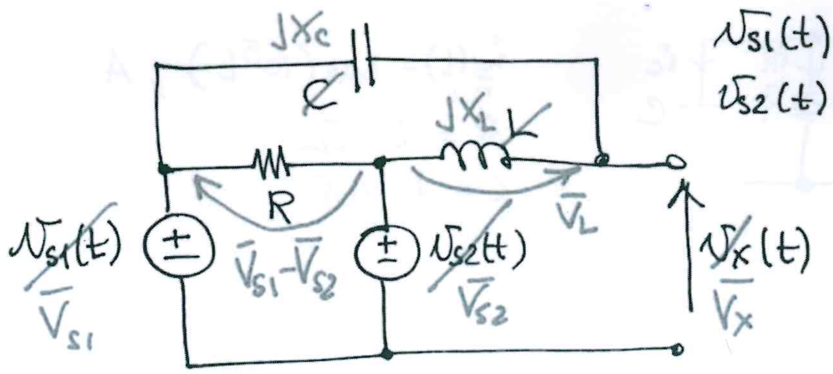
$$\bar{I}_c = \bar{I}_s - \bar{I}_R = \frac{1}{\sqrt{2}} - \frac{1-j}{2\sqrt{2}} = \frac{1+j}{2\sqrt{2}} \text{ A}$$

$$\bar{I}_c = \sqrt{\left(\frac{1}{2\sqrt{2}}\right)^2 + \left(\frac{1}{2\sqrt{2}}\right)^2}\, e^{j\,\mathrm{arctg}\,1} = \frac{1}{2} e^{j45^\circ} \text{ A}$$

$$\Rightarrow \boxed{\begin{aligned} i_R(t) &= \frac{1}{\sqrt{2}} \cos(10^5 t - 45^\circ), \text{ A} \\ i_c(t) &= \frac{1}{\sqrt{2}} \cos(10^5 t + 45^\circ), \text{ A} \end{aligned}}$$



EX



$v_{S1}(t) = -5\sin(100t)$, V
$v_{S2}(t) = 3\cos(100t)$, V

$R = 1\,\Omega$
$L = 20\,mH$
$C = 10\,mF$

Determinare $v_x(t)$ a regime

$$\bar{V}_{S1} = \frac{5}{\sqrt{2}} e^{j90°} = j\frac{5}{\sqrt{2}}\ V \qquad \left[\text{infatti, } -\sin(100t) = \cos(100t + 90°)\right]$$

$$\bar{V}_{S2} = \frac{3}{\sqrt{2}} e^{j0°} = \frac{3}{\sqrt{2}}\ V$$

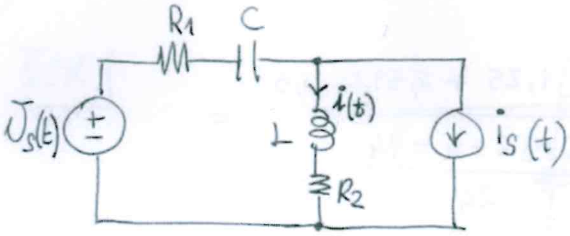
$$X_C = -\frac{1}{\omega C} = -\frac{1}{100 \cdot 10 \cdot 10^{-3}} = -1\,\Omega \quad ; \quad X_L = \omega L = 100 \cdot 20 \cdot 10^{-3} = 2\,\Omega$$

$$\bar{V}_L = (\bar{V}_{S1} - \bar{V}_{S2}) \cdot \frac{jX_L}{jX_C + jX_L} = \frac{1}{\sqrt{2}}(j5 - 3)\frac{j2}{-j + j2} = \frac{1}{\sqrt{2}}(j10 - 6)\ V$$

KVL: $\bar{V}_x = \bar{V}_L + \bar{V}_{S2} = \frac{1}{\sqrt{2}}(j10 - 6 + 3) = \frac{1}{\sqrt{2}}(j10 - 3)\ V$

$$\bar{V}_x = \frac{1}{\sqrt{2}}\sqrt{10^2 + 3^2}\, e^{j\left[\arctan\left(\frac{10}{-3}\right) \pm 180°\right]} = \frac{10{,}44}{\sqrt{2}} e^{j106{,}69°}\ V$$

$$\boxed{v_x(t) = 10{,}44\cos(100t + 106{,}70°)\ ,\ V}$$



$v_s(t) = 5\cos(2000t)$, V

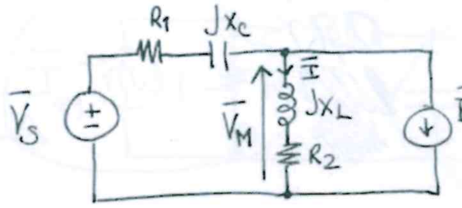
$i_s(t) = 3\sin(2000t - 60°)$, A

$R_1 = 2\,\Omega$ $\quad R_2 = 3\,\Omega$

$L = 3/2$ mH $\quad C = 1/4$ mF $\quad \boxed{i(t) = ?}$

Soluzione:

Circuito nel dominio dei fasori



FASORI:

$$\bar{V}_S = \frac{5}{\cancel{\sqrt{2}}}\ V$$

$$\bar{I}_S = \frac{3}{\cancel{\sqrt{2}}}\, e^{-j150°}\ A$$

(infatti $i_s(t) = 3\cos(2000t - 60° - 90°)$, A)

REATTANZE:

$$X_L = \omega L = \cancel{2}\cdot\cancel{10^3}\cdot\frac{3}{\cancel{2}}\cdot\cancel{10^{-3}} = 3\,\Omega$$

$$X_C = -\frac{1}{\omega C} = -\frac{1}{2\cdot\cancel{10^3}\cdot\frac{1}{4}\cdot\cancel{10^{-3}}} = -2\,\Omega$$

Soluzione del circuito (Millman)

$$\bar{V}_M = \frac{\frac{\bar{V}_s}{R_1 + jX_c} - \bar{I}_s}{\frac{1}{R_1 + jX_c} + \frac{1}{R_2 + jX_L}}$$

$$\bar{I} = \frac{\bar{V}_M}{R_2 + jX_L}$$

Calcoli:

$$\bar{V}_M = \frac{1}{\sqrt{2}}\,\frac{\frac{5}{2-j2} - 3e^{-j150°}}{\frac{1}{2-j2} + \frac{1}{3+j3}} = \cancel{\frac{1}{\sqrt{2}}}\,\frac{\frac{5}{2-j2}\cdot\frac{2+j2}{2+j2} - 3\cos 150° + j3\sin 150°}{\frac{1}{2-j2}\cdot\frac{2+j2}{2+j2} + \frac{1}{3+j3}\cdot\frac{3-j3}{3-j3}} =$$

$$= \frac{1}{\cdot} \; \frac{\frac{\cancel{10}^{6}+j\cancel{10}^{5}}{\cancel{8}\,4} + 2{,}598 + j1{,}5}{\frac{\cancel{2}+j\cancel{2}}{\cancel{8}\,4} + \frac{\cancel{3}-j\cancel{3}}{\cancel{18}\,6}} = \frac{1}{\cdot} \; \frac{1{,}25 + j1{,}25 + 2{,}598 + j1{,}5}{\frac{6 + j6 + 4 - j4}{24}} =$$

$$= \frac{24}{\cdot} \cdot \frac{3{,}84 + j2{,}75}{10 + j2} = \frac{24}{\cdot} \; \frac{\sqrt{3{,}84^2 + 2{,}75^2}\; e^{j\,\mathrm{arctg}\left(\frac{2{,}75}{3{,}84}\right)}}{\sqrt{10^2 + 2^2}\; e^{j\,\mathrm{arctg}\left(2/10\right)}} =$$

$$= \frac{24}{\cdot} \; \frac{4{,}72\, e^{j35{,}61^\circ}}{10{,}2\, e^{j11{,}31^\circ}} = 1{,}10\, e^{j24{,}3^\circ} \quad V$$

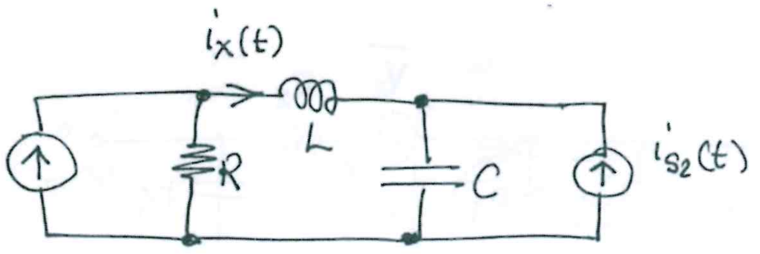
$$\bar{I} = \frac{7{,}85\, e^{j24{,}3^\circ}}{3 + j3} = \frac{7{,}85\, e^{j24{,}3^\circ}}{\sqrt{18}\; e^{j45^\circ}} = 2{,}62\, e^{-j20{,}7^\circ} \quad A$$

<u>Soluzione nel dominio del tempo</u>

~~i(t) = √2 · 7,85 · cos(2000t ...) A~~

$$\boxed{i(t) = 2{,}62 \cos\left(2000t - 20{,}7^\circ\right),\ A}$$

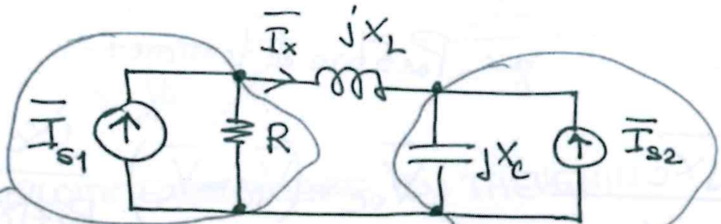
Ex 1



$i_{s1}(t) = 3\sin(2t)$, A $\qquad R_1 = 1\,\Omega \qquad C = \frac{1}{4}$ F

$i_{s2}(t) = 5\sin(2t+30°)$, A $\qquad L = 2$ H

Determinare $i_x(t)$ a regime.



$$\bar{I}_{s1} = \frac{3}{\cancel{\sqrt{2}}} e^{-j90°} = -j\frac{3}{\cancel{\sqrt{2}}} \text{ A}$$

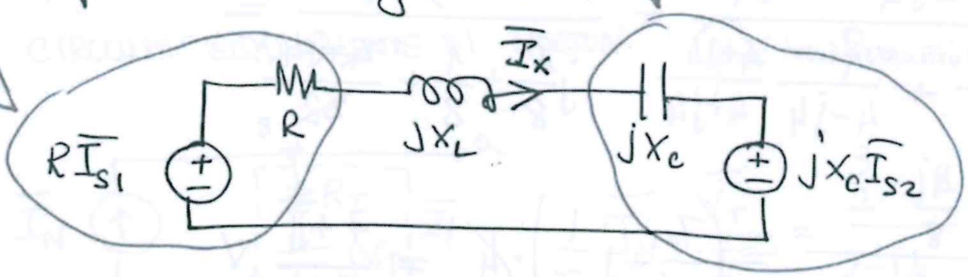
$$\bar{I}_{s2} = \frac{5}{\cancel{\sqrt{2}}} e^{-j60°} \text{ A}$$

(infatti $\cos(2t+30°-90°) = \sin(2t+30°)$)

$$X_L = \omega L = 2\cdot 2 = 4\,\Omega$$

$$X_C = -\frac{1}{\omega C} = -\frac{1}{2\cdot\frac{1}{4}} = -2\,\Omega$$

Trasformazione dei generatori:



$$\bar{I}_x = \frac{R\bar{I}_{s1} - jX_C\bar{I}_{s2}}{R + jX_L - jX_c} = \frac{1}{\cancel{\sqrt{2}}}\,\frac{-j3 + j2\cdot 5e^{-j60°}}{1 + j4 - j2} = \frac{1}{\cancel{\sqrt{2}}}\,\frac{-j3 + j10\overbrace{\cos(60°)}^{1/2} + 10\overbrace{\sin(60°)}^{\sqrt{3}/2}}{1+2j}$$

$$= \frac{1}{\cancel{\sqrt{2}}}\,\frac{8,66 + j2}{1+j2} = \frac{1}{\cancel{\sqrt{2}}}\cdot\frac{8,89\,e^{j13°}}{\sqrt{5}\,e^{j63,43°}} = \cancel{2,81}\ 3,97\,e^{-j50,43°} \text{ A}$$

$$\Rightarrow \boxed{i_x(t) = \underbrace{\cancel{\sqrt{2}\cdot 2,81}}_{3,97}\cos(2t - 50,43°), \text{ A}}$$