L16 - Hashing

7/18/24

Hashing demo

- hash.py

Hashing applications

- When I download a file, how can I verify that the download was successful (no errors or tampering)?
  - Using a common hash function (e.g., SHA256), the website can post the hash of the file. I can then hash my file and compare
- Websites need to verify my password when I login, but they can’t store it in plaintext (insecure)
  - They store the hash of my plaintext password
- Cryptocurrency/blockchain (simplified example)
  - The blockchain can be thought of as a linked list of blocks where the links are hashes
  - The system generates a “target” hash value (say, a hash where the first 30 bits are all 0) for the current block, and you need to find a value that, after being hashed, is less than that value
  - If so, you get a Bitcoin (and create a new block)
- For us,
  - Hashing is O(1), allowing us to implement some operations such as insert, delete, find in O(1)
  - But ordering operations (findMin, traversal) cannot be done
  - Basic idea, we can index into an array (integer indices) in O(1). What if the key is not an integer? Convert it to an integer by hashing, then use the hash to locate data in an array-like structure

Hash terms

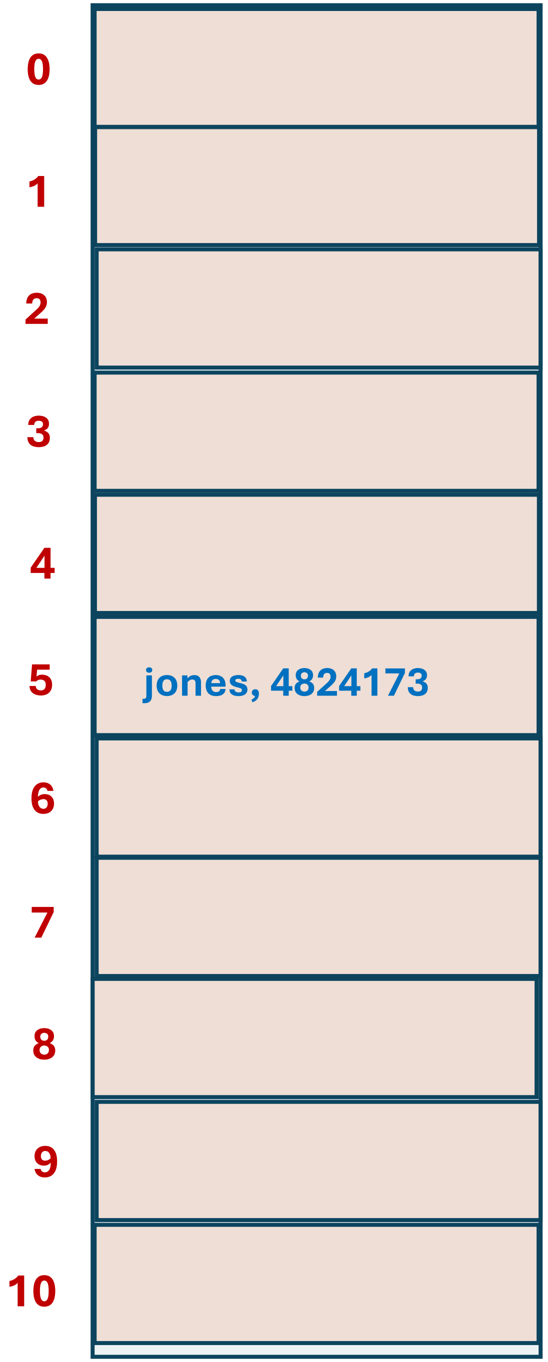
- Hashing is the basic concept of computing an integer (the “hash” or “hash value”) from some data value (the “key”)
- We intend to use that hash integer as an index into an array or table of associated data (keys and associated values)
- Map is an ADT similar to Python’s dict
  - void put(k, v)
  - V get(k)
  - void remove(k)
  - boolean contains(k)
- HashMap is an implementation of Map using a hash function
- Hash table is the array where data is stored

Hash function

- The computation that generates a hash value from a key
  - hash(key) -> int
- Used to implement Map via hash table
  - get(key) generally becomes table[hash(key)] in the implementation

Hash table

- Hash table is an array of key/value pairs
- put(“jones”, 4834173)
  - Suppose hash(“jones”) is 5
  - So we put this K, V pair into array slot 5
- get(“jones”)
  - hash(“jones”) is again 5
  - We look at array slot 5 and retrieve the associated value 4824173
- What if hash is bigger than table size?
  - Use modulus, i.e., index = hash(“jones”) % size
  - May omit the % later, but you should assume it’s there

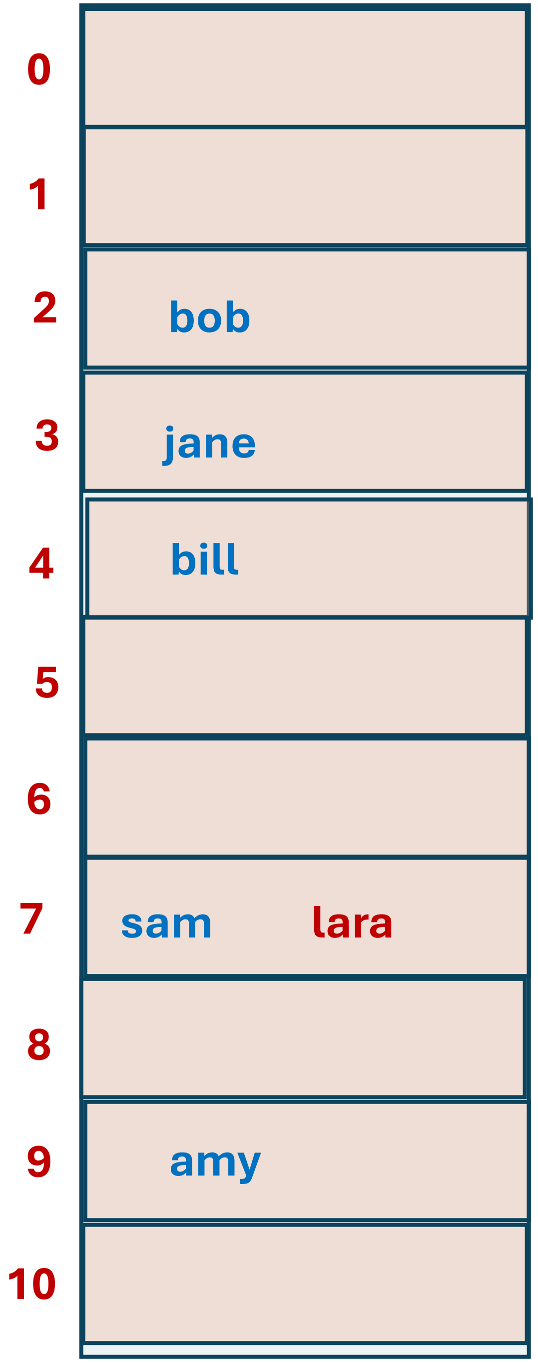


Time complexity

- Since we assume the hash function is O(1) to compute, put, get, and find are O(1)
  - Compute hash
  - Look in array slot
- Find is O(n) for simple array, average-case O(log n) for BST

Collision

- For simplicity, can show hash tables with just the key, but remember for a Map, there can be associated data
- In this table, suppose hash(“sam”) is 7
- Suppose hash(“lara”) is also 7
- Slot already taken
- Collision!



Pigeonhole principle

- Are collisions possible to avoid entirely?
- Would be possible if two distinct keys always get two different hash values
- But we allow our keys to be anything, whereas the hash output is fixed-size
- Pigeonhole principle: if there are 8 chicken boxes and 9 chickens, there must be 2 chickens in some box
- Best we can do is design the hash function so that it distributes keys evenly over the available array subscripts

Hash functions and collisions in practice

- A good hash function makes collisions very rare
- E.g., SHA256 has a 256 bit output, thus 2^{256} possible outputs
- How large is 2^{256}?
  - 10^{78} to 12^{82} atoms in the universe
  - To date, no one has found a collision for SHA256 (doing so would break it for cryptographic purposes)
- But for our purposes, we can't always spare 256 bits of output per key (SHA256 is more for cryptography)
- We have to deal with collisions

Good hash functions

- Hash function must be fast to compute
  - O(1)
  - Really, something like O(K) where K is the key size, but we'll assume keys are fixed-size (e.g., strings have a max length) so that this becomes O(1)
- Hash function must distribute keys evenly over the available range of values
  - For us, the available range is {0, 1, ..., array size-1}
- Incorporates all data of the key
- Decorrelates keys such that if two keys are similar, they should not get similar hash values
- Ideally, two distinct keys should get two different hash values

Probability of collisions

- Probability of collision affected by
  - Quality of hash function
    - How well it evenly distributes keys over the index range
  - Table structure
    - Number of array slots
    - Mathematical properties of the maximum index
      - E.g., if size is prime or not
      - Will show example soon
- For the rest of lecture, assume keys are lowercase Strings with some reasonable maximum upper bound

Example bad hash function

- Suppose our hash function for String is the first letter of the key as its position in the alphabet

```
static int badHash(String key) {
    return (((int) key.charAt(0)) - ((int)
'a'));
  }
```

Why is bad hash function (first letter's position in alphabet) bad?

Nobody has responded yet.

Hang tight! Responses are coming in.

Why is it bad?

- Only 26 different range elements
  - Can only store 26 keys before guaranteed collisions
- First character is not evenly distributed over alphabet
  - Lots of “s”, “m”, “t” words
  - Not many “x”, “z”, “q” words

Better hash function

- Sum all chars, mod by table size

```
static int betterHash(String key, int tabSize) {
    int hval = 0;
    for (int i=0; i<key.length(); i++) {
      hval += key.charAt(i);
    }
    return hval % tabSize;
  }
```

If tabSize is large (say 10007)
And keys short (say 8-15 chars)

Then the sum of the chars is small
and will cluster at low end of table

Come up with a collision for betterHash?

“cat”, “act” (doesn’t matter what table size is)

Nobody has responded yet.

Hang tight! Responses are coming in.

Pretty good hash

- Use multiplication for bigger numbers to avoid clustering
- Use prime multiplications to avoid small cycles
  - Example cycle: [0…9] % 8 = [**<u>0</u>**, 1, 2, 3, 4, 5, 6, 7, **<u>0</u>**, 1…]
- Multiplier and tabSize should be coprime
  - Consider multiplier 2 and tabSize 8, what goes wrong?

```
static int prettyGoodHash(String key, int
tabSize) {
    int hval = 7;
    for (int i = 0; i < key.length(); i++)
{
      hval = 31 * hval + key.charAt(i);
    }
    hval = hval % tabSize;
    if (hval < 0) {
      hval += tabSize;
    }
    return hval;
  }
```

Another idea: multiply by multiplier^i, e.g. 31 * charAt(0), 31^2 * charAt(1), etc.

Table size

- Best to use a prime table size
- Or, for convenience (i.e., don’t have to choose prime number), power of 2 as the table size, but do not use even multipliers for the multiplication
- Load factor
  - # elements in table / table size
  - 500 elements, size 997 table => load factor 500/997
  - Table half-full?
    - Depends on how collisions are handled

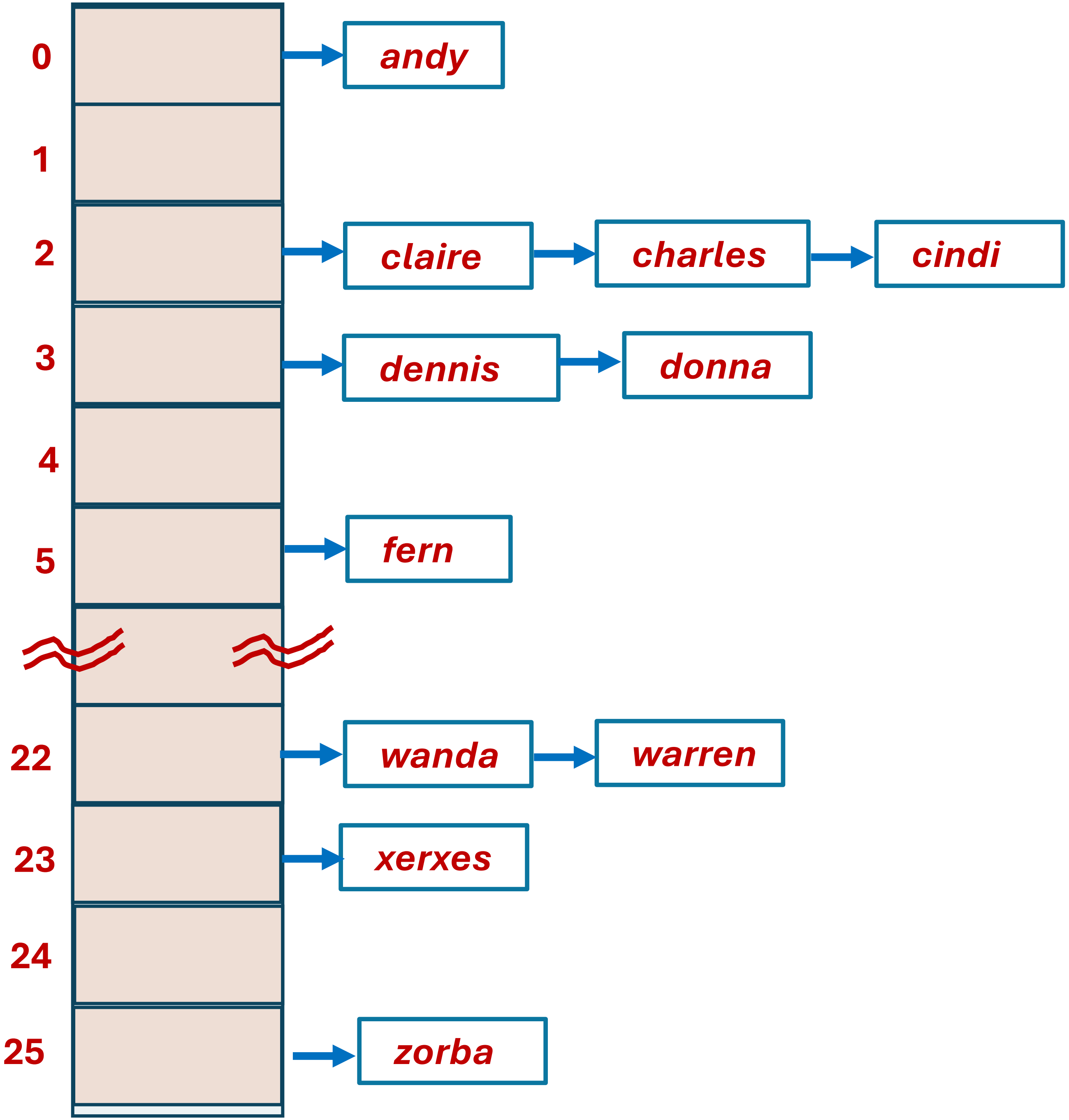
Collision resolution

- Two main forms
- Chaining
  - Each array slot contains not a single element but a list
- Linear probing
  - Each array slot contains one element
  - If we hash to full slot, we have a plan for going to a next slot to try
- How does this affect O(1) of insert and find?

Chaining

- Each entry is null or a list of cells
- If a new key hashes to an empty slot, start a new list with that key data
- If a new key hashes to an occupied slot, add that key data to the list

Example



Keys: *hash*

andy, 0
dennis, 3
zorba, 25
claire, 2
wanda, 22
charles, 2
fern, 5
warren, 22
cindi, 2
xerxes, 23

donna 3

Use bad hash function (first char) for simplicity

Chaining operations

- put(key, value)
  - hash(key) to get table index
  - Look for key in the list at that index in the table
  - If key exists in that list, replace associated value with new value
  - If key does not exist in that list, add key/value pair to that list
- find(key)
  - hash(key) to get table index
  - Look for key in the list at that index in the table
  - If key exists in that list, return associated value
  - Otherwise, does not exist (return null or throw exception, etc.)

Chaining operations

- remove(key)
  - hash(key) to get table index
  - Look for key in the list at that index in the table
  - If key exists in that list, remove key/value pair from list

Chaining operations time complexity

- Get (average and worst)
  - Calculate hash to find right list
    - O(k) => O(1) for bounded key size
  - Traverse list looking for key
    - O(average list size)
    - Average list size == load factor
    - If we resize table when load factor hits a constant limit, this is amortized O(1)
  - **O(1)**

Chaining operations time complexity

- put (average)
  - Calculate hash to find right list
    - O(K) => O(1)
  - Traverse list
    - O(avg. list size) => O(load) => O(1)
  - Insert into list if not found
    - O(1), add to head of list
    - No need for tail pointer
  - **O(1)**
- put (worst)
  - May need to resize table if load limit exceeded to reduce average list size and spread keys out
  - Each existing K needs to be rehashed
  - **O(n)**
  - Best practice: resize when load exceeds **1.0**

Is BST instead of list worth it?

- In the case of a collision where we have to traverse a list to find some element, why don't we store a BST to make traversal faster? That is, log(list length) instead of ½ list length
- BST is over-complicated for little gain, if any
- Prefer to focus on keeping lists short so that we can consider O(list length) to be **O(1)**
- Make table size bigger, make hash function distribute over more slots

Probing

- Table entry is null or a single cell
- If a new key hashes to an empty slot, then store a cell there with that key data
- If a new key hashes to an occupied slot, compute a next slot to try (repeat as needed)

Linear probing

- Linear probing says to try other “nearby” slots
- If table[hash(key)] % size is full
- Try table[hash(key)+1] % size, and if that’s full
- Try table[hash(key)+2] % size
- …
- Until a slot is open

Example

Keys: ***hash***

andy,	0
dennis,	3
zorba,	25
claire,	2
wanda,	22
charles,	2 → 3 → 4
fern,	5
warren,	22 → 23
cindi,	2 → 3 → 4 → 5 → 6
xerxes,	23 → 24
donna	3 → 4 → 5 → 6 → 7

Shows "clustering" or "clumping"

where you get heavily used crowded parts, empty parts...

0	*andy*
1	
2	*claire*
3	*dennis*
4	*charles*
5	*fern*
6	*cindi*
7	*donna*
8	
22	*wanda*
23	*warren*
24	*xerxes*
25	*zorba*

Probing operations

- put(key, value)
  - Hash(key) to get table index
  - If table entry contains correct key or is empty, then replace value or store key/value pair at that spot as appropriate
  - Else, try the next table entry
  - Continue until success
- find(key)
  - Hash(key) to get table index
  - If table entry contains matching key, then return associated value
  - Else, try the next table entry
  - Continue until key is found or empty spot is encountered
  - If empty spot is encountered, then key is not in map

Probing operations - remove

- remove(key)
  - **Not so simple**, can’t just empty the table cell which might create a gap in some probe chain
    - E.g., on slide before previous one, suppose we actually delete ”dennis”
    - Then when attempting to find “charles”, we go 2 -> 3, but 3 is now empty, so we assume “charles” is not in the table and stop finding
  - **Lazy deletion** required
    - Replace removed key with some “inactive” marker
    - On find, “inactive” says “occupied, keep probing”
    - On put, “inactive” says “open, free for use”

Clustering issue

- Clustering slows access
  - It's like having to search a list in hashing to lists
- Solution: larger table size
  - Table space in probing is like the list cells in chaining
- More space means more open slots for initial hash, less hopping to probe
- Load λ should be **½** for probing (assumes well distributed hash function)
  - Probing guaranteed to work if $\lambda < 1.0$ (i.e., not full), but 0.5 for better performance

Clustering solution?

- Need custom hash function?
- Some data may have a form that makes clusters with some hash function, not others
- Consider a hash function that uses first 3 chars
  - McDuff, MacBeth, McBride, McDaniel, MacGraw, MacDonald, MacLean, McKensie, McDermott, ...
  - These will collide
- No hash function is perfect for all data

Clustering solution: probe randomly

- We’ll actually probe “more randomly”
- Probe farther away from collision site and leave some slots near the collision open for future keys
- General probing formula
  - h_i = hash(key) + f(i)
  - f(0) = 0
- Can get different probing patterns using different formulas f(i)

General Probing

Let's formalize this

A key defines a sequence of hash values

$$h_0 , h_1 , h_2 , \ldots h_n , \ldots$$

We try each hash val in sequence until we get an open slot

$$h_i = hash(key) + f(i)$$

$$f(0) = 0$$

this makes

$$h_0 = hash(key)$$ ***the basic hash value***

Linear Probing

We get different probing patterns by defining different $\mathbf{f(i)}$ functions

Linear Probing:

$$h_i = hash(key) + f(i)$$

$$f(0) = 0\,,\quad f(i) = i \quad for \quad i > 0$$

Sequence: ***hash(key)+0***
hash(key)+1
hash(key)+2
hash(key)+3 ... ***% table length***

Quadratic Probing

Probe via skipping by squares

$$h_i = hash(key) + f(i)$$

$$f(0) = 0\,,\ \ f(i) = i^2 \quad for \ \ i > 0$$

Sequence: ***0: hash(key)+0***

1: hash(key)+1^2 is hash(key)+1

2: hash(key)+2^2 is hash(key)+4

3: hash(key)+3^2 is hash(key)+9

…

8: hash(key)+8^2 is hash(key)+64

% table length

Exponential Probing

Probe via skipping by powers of 2

$$h_i = hash(key) + f(i)$$

$$f(0) = 0\,,\ \ f(i) = 2^i \quad for\ \ i > 0$$

Sequence: ***0: hash(key)+0***

1: hash(key)+2^1 is hash(key)+2
2: hash(key)+2^2 is hash(key)+4
3: hash(key)+2^3 is hash(key)+8

...

8: hash(key)+2^8 is hash(key)+256

% table length

Probing performance

- get (average and worst)
  - Hash(key) to find initial slot
    - O(1)
  - Traverse probing sequence looking for key
    - O(avg. cluster size)
    - Avg. cluster size based on load factor
    - If we resize table when load factor hits constant limit, then O(1)
  - **O(1)**

Probing performance

- put (average)
  - Hash(key) to find initial slot
    - O(1)
  - Traverse probing sequence looking for key
    - O(avg. cluster size) = O(load) = O(1)
  - Insert into empty slot O(1)
  - **O(1)**
- put (worst)
  - May need to resize table if load limit exceeded
  - Each existing K rehashed
  - **O(n)**

Practice problem 1

- Using an initially empty HashTable of size 11 and the Hash Function H(k) = k % 11, insert the following keys, in the given order, using the linear probing method: 0, 1, 8, 9, 41, 33, 45, 42, 61, 53

Practice problem 1 solution

- To save work, first apply the hash function to all values (though not what would happen in reality)
- [0, 1, 8, 9, 41, 33, 45, 42, 61, 53] % 11 =
- [0, 1, 8, 9, 8, 0, 1, 9, 6, 9]

Index	Key
0	0
1	1
2	33
3	45
4	42
5	53
6	61
7	
8	8
9	9
10	41

Practice problem 2

- Using an initially empty HashTable of size 11 and the Hash Function H(k) = k % 11, insert the following keys, in the given order, using the quadratic probing method: 0, 1, 8, 9, 41, 33, 45, 42, 61, 53

Practice problem 2 solution

- To save work, first apply the hash function to all values (though not what would happen in reality)
- [0, 1, 8, 9, 41, 33, 45, 42, 61, 53] % 11 =
- [0, 1, 8, 9, 8, 0, 1, 9, 6, 9]

Index	Key
0	0
1	1
2	45
3	53
4	33
5	
6	41
7	61
8	8
9	9
10	42