Computer Science

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A glossary reference for Computer Science relevant terms which I have came across. [1]

Glossary

Basis A subset E of a complete lattice D is a **basis** of D if $x = \bigsqcup\{e | e \in E, e \leq x\}$ for all $x \in D$ and $\bigsqcup E' \in E$ for all finite $E' \subseteq E$.

Note: If the condition $\bigsqcup E' \in E$ for all finite $E' \subseteq E$ is absent, the E is a *subbasis* of D.

Bottom ($\perp$) is the least element of a poset. Sometimes also known as *zero*, **0**

Chain A poset P is a **chain** (also known as *totally* or *linearly ordered set*) if for all $x, y \in P$, $x \leq y$ or $y \leq x$.

Note: Every subset of a chain is a chain.

ω-Chain is an infinite sequence in a poset $(S, \leq)$ of the form $s_0, s_1, s_2, \ldots$ where $\forall i \in \mathbb{N}, s_i \in S$ and $s_i \leq s_{i+1}$.

ω-Complete Partial Order (ω-CPO) is a poset in which every ω-*chain* has a *supremum*.

Note: If such a poset also has minimum element, it is called a *strict ω-CPO* and such an element is commonly known as *bottom*, $\perp$.

Note: A strict ω-CPO is a *flat CPO* if for any d and d', $d \neq d'$, $d \neq \perp$, $d' \neq \perp$ and $d \not\leq d'$.

Context-Free Grammar A **context-free grammar** G is a context-free rewrite system where:

- The alphabet A is the union of disjoint sets T (set of *terminals* denoted by lowercase letters) and N (set of *nonterminals* denoted by uppercase letters).
- There exists a distinguished nonterminal known as the *start symbol*.
- Every production has the form $V \to w$ where V is a single nonterminal and w is any string in A^* (w can contain terminals, nonterminals or both).
- The language generated by G is the set $\{w \in T^* | S \stackrel{*}{\Rightarrow} w\}$.

Context-Free Rewrite System is a rewrite system with the property that for every production of the form $w \to x$, w is a string of length 1 (effectively a single letter).

As a 2-category: Let $\mathscr{W}$ be a context-free rewrite system on the alphabet A. The 2-category $\mathbf{C}(\mathscr{W})$ is defined as follows:

- $\mathbf{C}(\mathscr{W})$ has one 0-cell.
- 1-cells of $\mathbf{C}(\mathscr{W})$ are the strings in A^*.
- A 2-cell $\alpha : w/rightarrowx$ is the derivation forest of a derivation of x from w using productions of W.
- If $\alpha : w \to x$ and $\beta : w' \to x'$ are 2-cells, then $\alpha * \beta : ww' \to xx'$ is the derivation forest corresponding to deriving ww' to xw' using α, then from xw' to xx' using β.
- If $\alpha : w \to x$ and $\gamma : x \to y$ are derivation forests, then $\gamma \circ \alpha : w \to y$ is the derivation forest obtained by using α then γ

Continuous Function A function $f : D \to D'$ between complete lattices is **continuous** if $f(\bigsqcup X) = \bigsqcup\{f(x)|x \in X\}$ for all directed $X \subseteq D$.
Note: All monotonic functions on finite lattices are continuous.
Continuous Lattice is a complete lattice D such that for all $y \in D$, $y = \bigsqcup\{x \in D|x \ll y\}$.
Note: Every point in a continuous lattice that is not isolated is a limit point.

Directed Set is a set X if every finite subset of X has an upper bound in X. That is, if $A \subseteq X$ is finite then there is an element $b \in X$ such that $a \leq b$ for all $a \in A$.
Note: Finite subsets of X need not have least upper bounds in X.
Note: Directed sets are not empty since the empty et is one of its finite subsets, and thus directed sets must contain an element that is an upper bound of the empty set.
Note: A directed set is *interesting* if it does not contain its least upper bound, ie: $\bigsqcup X \notin X$.
Domain (Domain Theory) is a continuous complete lattice that has countable basis. Functions between such domains are continuous functions.
Note: Products, sums and the function space on a domain are domains.

Fixed Point Operator (fix) For any domain D, the function $fix : (D \to D) \to D$ is a fixed point operator defined by $fix(f) = \bigsqcup_{n=0}^{\infty} f^n(\bot)$.
Note: If $f : D \to D$ is a continuous function, $fix(f)$ is the least fixed point of f.
Note: When applied to a function, fixed point operators produce a fixed point of the function, ie: $fix(f)$ is the fixed point of f.
Flat Lattice is a lattice in which all elements other than $\top$ and $\bot$ are incomparable (have no ordering) with each other.

Isolated Point A point $a \in D$ of a complete lattice is **isolated** if $a \ll a$.
Note: An isolated point cannot be the limit point of any interesting directed set.

Kleene Closure Induced Homormophism For a function $f : A \to B$, between sets A and B, $f^* : A^* \to B^*$ is a monoid homomorphism defined by $f^*((a_1, a_2, \ldots, a_k)) = (f(a_1), f(a_2), \ldots, f(a_k))$.
Note: This function f^* is commonly known as *map f*, for lists/arrays in programming current languages.

Limit Point is an element $x \in D$ of a complete lattice if that $x = \bigsqcup X$ for some interesting directed $X \subseteq D$.

Product Lattice If $D_1, D_2, \ldots, D_n$ are complete lattices, $D_1 \times D_2 \times \cdots \times D_n$ is the **product lattice** (also a complete lattice), whose elements are of the form $\langle x_1, x_2, \ldots, x_n\rangle$ where $x_i \in D_i$ for $1 \leq i \leq n$. Two such elements are partially ordered by $\langle x_1, x_2, \ldots, x_n\rangle \leq \langle y_1, y_2, \ldots, y_n\rangle$ if $x_i \leq y_i$ for all $1 \leq i \leq n$.

Rewrite System A **rewrite system** G consists of a finite set of *productions* which are symbols of the form $y \to z$ where $y, z \in A^*$. Here, A is a finite alphabet and A^* is the kleene closure of A, consisting of strings formed by letters in A.
Directly Derive: A string x can be *directly derived* from w, denoted as $w \Rightarrow x$, if for strings $u, v \in A^*$, $y \to z, w = uyv$ and $x = uzv$.
Example: If G has a production $ab \to baa$, then $baba \Rightarrow bbaaa, abb \Rightarrow baab$ and $ab \Rightarrow baa$.

Derive to: is an operation that is the reflexive transitive closure of $\Rightarrow$ denoted as $\overset{*}{\Rightarrow}$. That is for any string w:

- $w \overset{*}{\Rightarrow} w$
- $(w \overset{*}{\Rightarrow} x) \Leftrightarrow (\exists w'$ st. $w \Rightarrow w'$ and $w' \Rightarrow x)$

Scott-continuous Function For two posets S and T, a function $f : S \to T$ is **Scott-continuous** if whenever s is the supremum of a chain $\mathscr{C} = (c_0, c_1, c_2, \dots) \in S$, then $f(s)$ is the supremum of the image $f(\mathscr{C}) = \{f(c_i) | i \in \mathbb{N}\}$ (ie: Scott-continuous functions preserves the supremum).

Note: Such a function is *strict* if it also preserves the bottom element between strict ω-CPOs.

Sum Lattice If $D_1, D_2, \dots, D_n$ are complete lattices, $D_1 + D_2 + \cdots + D_n$ is the **sum lattice** (also a complete lattice), which is the disjoint union of all the elements of $D_1, D_2, \dots, D_n$ with a new $\top$ and $\bot$. $\bot \leq x \leq \top$ for all $x \in D_1 + D_2 + \cdots + D_n$. If x is an element of D_i and y is an element of D_j, then $x \leq y$ in $D_1 + D_2 + \cdots + D_n$ if and only if $i = j$ and $x \leq y$ in D_i.

Note: The sum lattice construction with a new $\top$ and $\bot$ is also called a *separated sum*.

Note: An alternative sum lattice construction is to make $\top$ and $\bot$ in the sum domain the images of $\top$ and $\bot$ of each of the component domains. This is also known as a *coalesced sum*.

Note: It is possible for the same lattice to appear as two distinct summands of a sum lattice. In this case, each element of this lattice will appear twice in the sum lattice.

Top ($\top$) is the greatest element of a poset. Sometimes also known as *unit*, **1**

Well Below ($\ll$) For x, y in a continous lattice D, x is **well below** y, $x \ll y$ if whenever $y \leq \bigsqcup Z$ for some directed $Z \subseteq D$ then $x \leq z$ for some $z \in Z$.

References

[1] Stoy J.E. *Denotational semantics: The Scott-Strachey approach to programming language theory*. MIT, 1977.