Computer Science

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A glossary reference for Computer Science relevant terms which I have came across.

Glossary

Bottom ($\perp$) is the least element of a poset. Sometimes also known as *zero*, **0**

ω-Chain is an infinite sequence in a poset $(S, \leq)$ of the form $s_0, s_1, s_2, \ldots$ where $\forall i \in \mathbb{N}, s_i \in S$ and $s_i \leq s_{i+1}$.

ω-Complete Partial Order (ω-CPO) is a poset in which every *ω-chain* has a *supremum*.

Note: If such a poset also has minimum element, it is called a *strict ω-CPO* and such an element is commonly known as *bottom*, $\perp$.

Note: A strict ω-CPO is a *flat* CPO if for any d and d', $d \neq d'$, $d \neq \perp$, $d' \neq \perp$ and $d \not\lesssim d'$.

Context-Free Grammar A **context-free grammar** G is a context-free rewrite system where:

- The alphabet A is the union of disjoint sets T (set of *terminals* denoted by lowercase letters) and N (set of *nonterminals* denoted by uppercase letters).
- There exists a distinguished nonterminal known as the *start symbol.*
- Every production has the form $V \to w$ where V is a single nonterminal and w is any string in A^* (w can contain terminals, nonterminals or both).
- The language generated by G is the set $\{w \in T^* | S \overset{*}{\Rightarrow} w\}$.

Context-Free Rewrite System is a rewrite system with the property that for every production of the form $w \to x$, w is a string of length 1 (effectively a single letter).

As a 2-category: Let $\mathscr{W}$ be a context-free rewrite system on the alphabet A. The 2-category $\mathbf{C}(\mathscr{W})$ is defined as follows:

- $\mathbf{C}(\mathscr{W})$ has one 0-cell.
- 1-cells of $\mathbf{C}(\mathscr{W})$ are the strings in A^*.
- A 2-cell $\alpha : w/rightarrowx$ is the derivation forest of a derivation of x from w using productions of W.
- If $\alpha : w \to x$ and $\beta : w' \to x'$ are 2-cells, then $\alpha * \beta : ww' \to xx'$ is the derivation forest corresponding to deriving ww' to xw' using α, then from xw' to xx' using β.
- If $\alpha : w \to x$ and $\gamma : x \to y$ are derivation forests, then $\gamma \circ \alpha : w \to y$ is the derivation forest obtained by using α then γ

Kleene Closure Induced Homormophism For a function $f : A \to B$, between sets A and B, $f^* : A^* \to B^*$ is a monoid homomorphism defined by $f^*((a_1, a_2, \ldots, a_k)) = (f(a_1), f(a_2), \ldots, f(a_k))$.

Note: This function f^* is commonly known as *map f*, for lists/arrays in programming current languages.

Rewrite System A **rewrite system** G consists of a finite set of *productions* which are symbols of the form $y \to z$ where $y, z \in A^*$. Here, A is a finite alphabet and A^* is the kleene closure of A, consisting of strings formed by letters in A.

Directly Derive: A string x can be *directly derived* from w, denoted as $w \Rightarrow x$, if for strings $u, v \in A^*$, $y \to z, w = uyv$ and $x = uzv$.

Example: If G has a production $ab \to baa$, then $baba \Rightarrow bbaaa, abb \Rightarrow baab$ and $ab \Rightarrow baa$.

Derive to: is an operation that is the reflexive transitive closure of $\Rightarrow$ denoted as $\overset{*}{\Rightarrow}$. That is for any string w:

- $w \overset{*}{\Rightarrow} w$
- $(w \overset{*}{\Rightarrow} x) \Leftrightarrow (\exists w' \text{ st. } w \Rightarrow w' \text{ and } w' \Rightarrow x)$

Scott-continuous Function For two posets S and T, a function $f : S \to T$ is **Scott-continuous** if whenever s is the supremum of a chain $\mathscr{C} = (c_0, c_1, c_2, \dots) \in S$, then $f(s)$ is the supremum of the image $f(\mathscr{C}) = \{f(c_i) | i \in \mathbb{N}\}$ (ie: Scott-continuous functions preserves the supremum).

Note: Such a function is *strict* if it also preserves the bottom element between strict ω-CPOs.

Top ($\top$) is the greatest element of a poset. Sometimes also known as *unit*, **1**