CSCE 420 Homework Three

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April 4, 2023

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1. **Translate** the following sentences into First-Order Logic. Remember to break things down to simple concepts (with short predicate and function names), and make use of quantifiers. For example, "$tasteDelicious(someRedTomatos)$", is not broken down enough; instead we would be looking for a formulation such as: "$\exists x\ tomato(x) \land red(x) \land taste(x, delicious)$". See the lecture slides for more examples and guidance.

   (a) Bowling balls are sporting equipment.

   (b) All domesticated horses have an owner.

   (c) The rider of a horse can be different than the owner.

   (d) Horses move faster than frogs.

   (e) A finger is any digit on a hand other than the thumb.

   (f) An isosceles triangle is defined as a polygon with 3 edges, which are connected at 3 vertices, where 2 (but not 3) edges have the same length.

Solution:

(a) $\forall x\ (\text{bowlingBall}(x) \implies \text{sportingEquipment}(x))$

(b) $\forall x\ (\text{horse}(x) \land \text{domesticated}(x)) \implies \exists y\ (\text{person}(y) \land \text{owns}(x, y))$

(c) $\exists x, y, z\ (\text{horse}(x) \land \text{person}(y) \land \text{person}(z) \land \text{owns}(x, y) \land \text{rides}(x, z))$

(d) $\forall h, f\ (\text{horse}(h) \land \text{frog}(f) \implies \text{speed}(h) < \text{speed}(f))$

(e) $\forall x\ (\text{finger}(x) \iff (\text{digit}(x) \land (\exists y\ (\text{hand}(y) \land \text{partOf}(x, y)) \land \neg\text{thumb}(x)))$

(f) $\forall t\ (\text{triangle}(t) \land \text{isosceles}(t) \iff (\text{polygon}(t) \land \text{edges}(t) = 3$
$\land(\exists u, v, w, x, y, z(\text{edge}(u) \land \text{edge}(v) \land \text{edge}(w) \land \text{vertex}(x) \land \text{vertex}(y) \land \text{vertex}(z) \land$
$\text{connected}(t, u)))))$

2. Convert the following first-order logic sentence to CNF:

$$\forall x \text{ person}(x) \wedge [\exists z \text{ petOf}(x,z) \wedge \forall y \text{ petOf}(x,y) \implies \text{dog}(y)] \implies \text{doglover}(x)$$

Solution: First, let's add parentheses to make it more clear which way we must parse the sentence:

$$\begin{aligned}&\forall x \text{ person}(x) \wedge [\exists z \text{ petOf}(x,z) \wedge \forall y \text{ petOf}(x,y) \implies \text{dog}(y)] \implies \text{doglover}(x)\\ &\equiv \forall x \ [\text{person}(x) \wedge [(\exists z \text{ petOf}(x,z) \wedge \forall y \ (\text{petOf}(x,y) \implies \text{dog}(y)))] \implies \text{doglover}(x)].\end{aligned}$$

Next, eliminate implications:

$$\equiv \forall x \ \neg[\text{person}(x) \wedge [(\exists z \text{ petOf}(x,z) \wedge \forall y \ (\neg\text{petOf}(x,y) \vee \text{dog}(y)))] \vee \text{doglover}(x)].$$

Now, we move the negations inwards:

$$\begin{aligned}&\equiv \forall x \ [\neg\text{person}(x) \wedge \neg[(\exists z \text{ petOf}(x,z) \wedge \forall y \ (\neg\text{petOf}(x,y) \vee \text{dog}(y)))] \vee \text{doglover}(x)]\\ &\equiv \forall x \ [\neg\text{person}(x) \wedge [(\forall z \ \neg\text{petOf}(x,z) \vee \exists y \ (\neg\neg\text{petOf}(x,y) \wedge \neg\text{dog}(y)))] \vee \text{doglover}(x)].\end{aligned}$$

Now, we can drop the universal quantifiers:

$$\equiv \neg\text{person}(x) \wedge (\neg\text{petOf}(x,z) \vee \exists y \ (\text{petOf}(x,y) \wedge \neg\text{dog}(y))) \vee \text{doglover}(x).$$

There are no repeat variables, so we don't have to standardize our variables in this case. Therefore, our next step is to Skolemize our existentially quantified variables as follows:

$$\equiv \neg\text{person}(x) \wedge (\neg\text{petOf}(x,z) \vee (\text{petOf}(x,F(x)) \wedge \neg\text{dog}(F(x)))) \vee \text{doglover}(x).$$

Finally, we distribute to get the sentence into CNF:

$$\begin{aligned}&\equiv \neg\text{person}(x) \vee (\neg\text{petOf}(x,z) \vee (\text{petOf}(x,F(x)) \wedge \neg\text{dog}(F(x)))) \vee \text{doglover}(x)\\ &\equiv \neg\text{person}(x) \vee ((\neg\text{petOf}(x,z) \vee \text{petOf}(x,F(x))) \wedge (\neg\text{petOf}(x,z) \vee \neg\text{dog}(F(x))))\\ &\qquad \vee \text{doglover}(x)\\ &\equiv (\neg\text{person}(x) \vee \neg\text{petOf}(x,z) \vee \text{petOf}(x,F(x))) \wedge\\ &\qquad (\neg\text{person}(x) \vee \neg\text{petOf}(x,z) \vee \neg\text{dog}(F(x))) \vee \text{doglover}(x)\\ &\equiv (\neg\text{person}(x) \vee \neg\text{petOf}(x,z) \vee \text{petOf}(x,F(x)) \vee \text{doglover}(x))\\ &\qquad \wedge (\neg\text{person}(x) \vee \neg\text{petOf}(x,z) \vee \neg\text{dog}(F(x)) \vee \text{doglover}(x)).\end{aligned}$$

3. Determine whether or not the following pairs of predicates are **unifiable**. If they are, give the most-general unifier and show the result of applying the substitution to each predicate. If they are not unifiable, indicate why. *Capital letters represent variables*; constants and function names are lowercase. For example, '$loves(A, hay)$' and '$loves(horse, hay)$' are

unifiable, the unifier is $u = \{A/horse\}$, and the unified expression is '$loves(horse, hay)$' for both.

(a) $\text{owes}(\text{owner}(X), \text{citibank}, \text{cost}(X)) \qquad \text{owes}(\text{owner}(ferrari), Z, \text{cost}(Y))$

(b) $\text{gives}(\mathit{bill}, jerry, \mathit{book}21) \qquad \text{gives}(X, \text{brother}(X), Z)$

(c) $\text{opened}(X, \text{result}(\text{open}(X), s_0)) \qquad \text{opened}(\mathit{toolbox}, Z)$

Solution:

(a) $\text{owner}(X)$ and $\text{owner}(ferrari)$ are unifiable with the substitution $\{X/ferrari\}$. *citibank* and Z are unifiable with the substitution $\{Z/\mathit{citibank}\}$. $\text{cost}(X)$ and $\text{cost}(Y)$ are unifiable with the substitution $\{Y/X\}$. So the most general unifier is

$$MGU = \{X/ferrari, Z/\mathit{citibank}, Y/X\}.$$

Applying the substitition to the first predicate gives us

$$\text{owes}(\text{owner}(ferrari), \mathit{citibank}, \text{cost}(ferrari)).$$

To the second predicate:

$$\text{owes}(\text{owner}(ferrari), \mathit{citibank}, \text{cost}(ferrari)).$$

(b) These two predicates are not unifiable. Surely, we could make the substitution $\{X/\mathit{bill}\}$ and $\{Z/\mathit{book}21\}$ to satisfy the first part and last part of both predicates, but the second (middle) part messes things up. We can only substitute *variables* to functions or constants. Since we only have $jerry$ (a constant) and $\text{brother}(X)$ (a function), there is no substitution that will make the two predicates semantically identical. Thus, they are not unifiable.

(c) X and *toolbox* are unifiable with the substitution $\{X/\mathit{toolbox}\}$. Since X can be freely substituted with *toolbox*, then $\text{result}(\text{open}(X), s_0) \rightarrow$ $\text{result}(\text{open}(\mathit{toolbox}), s_0)$. Therefore, we can make the substitution $\{Z/\text{result}(\text{open}(\mathit{toolbox}), s_0)\}$, which will make the two statements identical, after substitution. So the most general unifier is

$$MGU = \{X/\mathit{toolbox}, Z/\text{result}(\text{open}(\mathit{toolbox}), s_0)\}.$$

Applying the substitution to the first predicate gives us:

$$\text{opened}(\mathit{toolbox}, \text{result}(\text{open}(\mathit{toolbox}), s_0)).$$

To the second predicate:

$$\text{opened}(\mathit{toolbox}, \text{result}(\text{open}(\mathit{toolbox}), s_0)).$$

As we can see, this substitution makes the two predicates identical, and therefore they are unifiable.

4. Consider the following situation:

   Marcus is a Pompeian.
   All Pompeians are Romans.
   Caesar is a ruler.
   All Romans are either loyal to Caesar or hate Caesar (but not both).
   Everyone is loyal to someone.
   People only try to assassinate rulers they are not loyal to.
   Marcus tries to assassinate Caesar.

   (a) *Translate* these sentences to first-order logic.
   (b) Prove that **Marcus hates Caesar** using Natural Deduction. In the same style as the examples in the lecture slides, label all derived sentences with the ROI used, which prior sentences were used, and what unifier was used.
   (c) Convert all sentences into CNF.
   (d) Prove that **Marcus hates Caesar** using Resolution Refutation.

Solution:

(a) We will translate each sentence to FOL:

(1) *Marcus is a Pompeian.*

$$\text{pompeian}(Marcus)$$

(2) *All Pompeians are Romans.*

$$\forall x\ (\text{pompeian}(x) \implies \text{roman}(x))$$

(3) *Caesar is a ruler.*

$$\text{ruler}(Caesar)$$

(4) *All Romans are either loyal to Caesar or hate Caesar (but not both).*

$$\forall x\ (roman(x) \implies (\neg\text{loyal}(x, caesar) \land \text{hate}(x, caesar)) \lor (\text{loyal}(x, caesar) \land \neg\text{hate}(x, caesar)))$$

(5) *Everyone is loyal to someone.*

$$\forall x\ \exists y\ \text{person}(x) \land \text{person}(y) \land \text{loyal}(x, y)$$

(6) *People only try to assassinate rulers they are not loyal to.*

$$\forall x, y\ (\text{person}(x) \land \text{person}(y) \land \text{assassinate}(x, y) \implies (\text{ruler}(y) \land \neg\text{loyal}(x, y)))$$

(7) *Marcus tries to assassinate Caesar.*

$$\text{person}(Marcus) \wedge \text{person}(Caesar) \wedge \text{assassinate}(Marcus, Caesar)$$

(b) Now, we will perform natural deduction to prove **Marcus hates Caesar**.

(8) By Modus Ponens and 1, 2 with $\theta = \{x/Marcus\}$, obtain

$$\text{roman}(Marcus)$$

(9) By Modus Ponens and 4, 8 with $\theta = \{x/Marcus\}$, obtain

$$(\neg\text{loyal}(Marcus, Caesar) \wedge \text{hate}(Marcus, Caesar))$$
$$\vee(\text{loyal}(Marcus, Caesar) \wedge \neg\text{hate}(Marcus, Caesar)$$

(10) By Universal Instantiation on 5 and $\theta = \{x/Marcus\}$, obtain

$$\exists y \ (\text{person}(Marcus) \wedge \text{person}(y) \wedge \text{loyal}(Marcus, y))$$

(11) By Existential Instantiation on 10 and $\theta = \{y/p_1\}$, obtain

$$\text{person}(Marcus) \wedge \text{person}(p_1) \wedge \text{loyal}(Marcus, p_1)$$

(12) By Modus Ponens on 6, 7 with $\theta = \{x/Marcus, y/Caesar\}$, obtain

$$\text{ruler}(Caesar) \wedge \neg\text{loyal}(Marcus, Caesar)$$

(13) By And Elimination on 12, obtain

$$\neg\text{loyal}(Marcus, Caesar)$$

(14) By resolution on 9 with 12, obtain

$$\neg\text{loyal}(Marcus, Caesar) \wedge \text{hate}(Marcus, Caesar)$$

(15) And Elimination on 14 yields

$$\text{hate}(Marcus, Caesar)$$

So we have proved that **Marcus hates Caesar** using natural deduction.

(c) Now, we will convert all sentences to CNF:

(1) This is already in CNF.

$$\text{pompeian}(Marcus)$$

(2) Translating $\forall x\ (\text{pompeian}(x) \implies \text{roman}(x))$ into CNF:

$$\forall x\ (\text{pompeian}(x) \implies \text{roman}(x))$$
$$\forall x\ (\neg\text{pompeian}(x) \lor \text{roman}(x))$$
$$\neg\text{pompeian}(x) \lor \text{roman}(x)$$

(3) $\text{ruler}(Caesar)$ is already in CNF.

(4) Convert $\forall x\ (roman(x) \implies (\neg\text{loyal}(x, caesar) \land \text{hate}(x, caesar)) \lor (\text{loyal}(x, caesar) \land \neg\text{hate}(x, caesar)))$ into CNF:

$\equiv \forall x\ (\neg\text{roman}(x) \lor ((\neg\text{loyal}(x, caesar) \land \text{hate}(x, caesar))$
$\lor (\text{loyal}(x, caesar) \land \neg\text{hate}(x, caesar)))$
$\equiv \neg\text{roman}(x) \lor ((\neg\text{loyal}(x, Caesar) \lor \text{loyal}(x, Caesar)) \land (\neg\text{hate}(X, Caesar)$
$\lor \neg\text{loyal}(x, Caesar)) \land (\text{hate}(x, Caesar) \lor \neg\text{hate}(x, Caesar)) \land (\text{hate}(x, Caesar)$
$\lor \text{loyal}(X, Caesar))))$
$\equiv \neg\text{roman}(x) \lor (\neg\text{hate}(X, Caesar) \lor \neg\text{loyal}(x, Caesar))$
$\land (\text{hate}(x, Caesar) \lor \text{loyal}(X, Caesar))))$
$\equiv (\neg\text{roman}(x) \lor \neg\text{loyal}(x, Caesar) \lor \neg\text{hate}(x, Caesar))$
$\land (\neg\text{roman}(x) \lor \text{hate}(x, Caesar) \lor \text{loyal}(x, Caesar))$

(5) Convert $\forall x\ \exists y\ \text{person}(x) \land \text{person}(y) \land \text{loyal}(x, y)$ into CNF. All we need to do is Skolemize the existential quantifier variables as follows:

$$\equiv \forall x\ \text{person}(x) \land \text{person}(F(x)) \land \text{loyal}(x, F(x))$$
$$\equiv \text{person}(x) \land \text{person}(F(x)) \land \text{loyal}(x, F(x))$$

(6) Convert $\forall x, y\ (\text{person}(x) \land \text{person}(y) \land \text{assassinate}(x, y) \implies (\text{ruler}(y) \land \neg\text{loyal}(x, y)))$ into CNF as follows:

$\equiv \forall x, y\ [\neg(\text{person}(x) \land \text{person}(y) \land \text{assassinate}(x, y)) \lor (\text{ruler}(y) \land \neg\text{loyal}(x, y))]$
$\equiv \forall x, y\ [(\neg\text{person}(x) \lor \neg\text{person}(y) \lor \neg\text{assassinate}(x, y)) \lor (\text{ruler}(y) \land \neg\text{loyal}(x, y))]$
$\equiv (\neg\text{person}(x) \lor \neg\text{person}(y) \lor \neg\text{assassinate}(x, y) \lor \text{ruler}(y))$
$\quad \land (\neg\text{person}(x) \lor \neg\text{person}(y) \lor \neg\text{assassinate}(x, y) \lor \neg\text{loyal}(x, y))$

(7) $\text{person}(Marcus) \land \text{person}(Caesar) \land \text{assassinate}(Marcus, Caesar)$ is already in CNF.

(d) Given this KB in CNF, we will prove **Marcus hates Caesar** using Resolution Refutation.

(1) $\text{pompeian}(Marcus)$

(2) $\neg\text{pompeian}(x) \lor \text{roman}(x)$

(3) ruler($Caesar$)

(4) $\neg$roman(x) $\vee$ $\neg$loyal($x, Caesar$) $\vee$ $\neg$hate($x, Caesar$)

(5) $\neg$roman(x) $\vee$ hate($x, Caesar$) $\vee$ loyal($x, Caesar$)

(6) person(x) $\wedge$ person($F(x)$) $\wedge$ loyal($x, F(x)$)

(7) $\neg$person(x) $\vee$ $\neg$person(y) $\vee$ $\neg$assassinate(x, y) $\vee$ ruler(y)

(8) $\neg$person(x) $\vee$ $\neg$person(y) $\vee$ $\neg$assassinate(x, y) $\vee$ $\neg$loyal(x, y)

(9) person($Marcus$)

(10) person($Caesar$)

(11) assassinate($Marcus, Caesar$)

(12) $\neg$hate($Marcus, Caesar$) (**Negated query**)

(13) With $x = Marcus$, resolve (1) and (2) to obtain

$$\text{roman}(Marcus)$$

(14) With $x = Marcus, y = Caesar$, resolve (7) with (9), (10), and (11) to obtain

$$\text{ruler}(Caesar)$$

(15) With $x = Marcus, y = Caesar$, resolve (8) with (9), (10), and (11) to obtain

$$\neg\text{loyal}(Marcus, Caesar)$$

(16) With $x = Marcus$, resolve (4) with (13) to obtain

$$\neg\text{loyal}(Marcus, Caesar) \vee \neg\text{hate}(Marcus, Caesar)$$

(17) With $x = Marcus$, resolve (5) with (13) to obtain

$$\text{hate}(Marcus, Caesar) \vee \text{loyal}(Marcus, Caesar)$$

(18) Resolve (15) with (17) to obtain

$$\text{hate}(Marcus, Caesar)$$

(19) Resolve (13) with (18) to obtain

$$\varnothing$$

Thus, we have obtained the empty clause, meaning we have derived a contradiction and thus we have proved that Marcus hates Caesar using resolution refutation.

5. Write a KB in First-Order Logic with rules/axioms for...

(a) **Map-coloring** - every state must be exactly 1 color, and adjacent states must be different colors. Assume possible colors are states are defined using unary predicate like color(red) or state(WA). To say a state has a color, use a binary predicate, e.g. 'color(WA, red)'.

(b) **Sammy's Sport Shop** – include sentences for specific facts like obs($1, W$) or label($2, B$), as well as for general constraints about the boxes and colors. Use binary predicate 'contains(x, q)' to represent that box x contains tennis balls of color q (where q could be W, Y, or B).

(c) **Wumpus World** - Write rules for which rooms are 'stenchy', 'breezy', and 'safe'. (Hint: Define a helper concept called 'adjacent(x,y,p,q)' which defines when a room at coordinates (x,y) is adjacent to another room at (p,q), and x,y,p,q are integers 1-4.

(d) **4-Queens** – assume row(1)...row(4) and col(1)...col(4) are facts; write rules that describe configurations of 4 queens such that none can attack each other, using '$queen(r, c)$' to represent that there is a queen in row r and col c.

Solution:

(a) Below, we write a KB for Map-coloring in FOL (this is largely based on the lecture slides).

- Each state has a color.

$$\forall s \ [\text{state}(s) \implies (\exists c \ \text{color}(c) \wedge \text{hasColor}(s, c))]$$

- Each state is only one color.

$$\forall s, c_1, c_2 \ [(\text{color}(c_1) \wedge \text{color}(c_2) \wedge \text{state}(s)) \implies c_1 = c_2]$$

- No two neighboring states have the same color.

$$\forall s_1, s_2, c \ [(\text{state}(s_1) \wedge \text{state}(s_2) \wedge \text{neighbor}(s_1, s_2) \wedge \text{color}(c) \wedge \text{hasColor}(s_1, c)) \\ \implies \neg\text{hasColor}(s_2, c)]$$

(b) Below, we write a KB for Sammy's Sport Shop in FOL. First, we lay out the facts given in the prompt for HW2:

- obs($1, Y$)
- label($1, W$)
- obs($2, W$)
- label($2, Y$)
- obs($3, Y$)
- label($3, B$)

Now, we lay out the rules for the constraints.

- Each box is labelled incorrectly.

$$\forall b, c\ [(\text{box}(b) \wedge \text{color}(c) \wedge \text{label}(b, c)) \implies \neg\text{contains}(b, c)]$$

- There is precisely one box of each color.

$$\forall x, y, z\ [(\text{contains}(x, W) \wedge \text{contains}(y, Y) \wedge \text{contains}(z, B) \\ \implies ((x \neq y) \wedge (x \neq z) \wedge (y \neq z))]$$

- If a box contains both colors, then a white or yellow color may be observed.

$$\forall b\ [(\text{box}(b) \wedge \text{contains}(b, B)) \implies (\text{obs}(b, W) \vee \text{obs}(b, Y))]$$

- The contents of a box is what will be observed (as long as the color is not equal to B).

$$\forall b, c\ [(\text{box}(b) \wedge \text{color}(c) \wedge c \neq B \wedge \text{contains}(b, c)) \implies \text{obs}(b, c)]$$

(c) Below, we write a KB for Wumpus World in FOL. First, we define the following predicates:

- wumpus(x, y): Wumpus is in room (x, y), where $x, y \in \mathbb{Z}$.
- pit(x, y): There is a pit in room (x, y), where $x, y \in \mathbb{Z}$.
- gold(x, y): There is gold in room (x, y), where $x, y \in \mathbb{Z}$.
- stench(x, y): There is a stench in room (x, y), where $x, y \in \mathbb{Z}$.
- breeze(x, y): There is a breeze in room (x, y), where $x, y \in \mathbb{Z}$.
- safe(x, y): Room (x, y) is safe, where $x, y \in \mathbb{Z}$.
- adjacent(x, y, p, q): Room (x, y) is adjacent to room (p, q), where $x, y, p, q \in \mathbb{Z}$.

Next, we can write the given facts *after visiting rooms* $(1, 1)$, $(1, 2)$, *and* $(2, 1)$, in FOL based on the example from the textbook/lecture slides. **Note**: This was my best guess on what the instructions meant by *include specific facts*.

(1) stench$(1, 2)$

(2) breeze$(2, 1)$

(3) safe$(1, 1)$

(4) safe$(1, 2)$

(5) safe$(2, 1)$

Now, we will write the KB in FOL.

- The Wumpus is located in a valid room.

$$\forall x, y\ [\text{wumpus}(x, y) \implies ((1 \leq x \leq 4) \land (1 \leq y \leq 4))]$$

- The pit is located in a valid room.

$$\forall x, y\ [\text{pit}(x, y) \implies ((1 \leq x \leq 4) \land (1 \leq y \leq 4))]$$

- Define adjacency.

$$\forall x, y, p, q\ [\text{adjacent}(x, y, p, q) \iff (((x = p) \land (y = q + 1)) \\ \lor ((x = p) \land (y = q - 1)) \lor ((y = q) \land (x = p + 1)) \lor ((y = q) \land (x = p - 1))]$$

- Room (x, y) has a stench if and only if a Wumpus is in an adjacent room.

$$\forall x, y,\ [\text{stench}(x, y) \iff (\exists p, q\ (\text{adjacent}(x, y, p, q) \land \text{wumpus}(p, q)))]$$

- Room (x, y) has a breeze if and only if there is a pit in an adjacent room.

$$\forall x, y,\ [\text{breeze}(x, y) \iff (\exists p, q\ (\text{adjacent}(x, y, p, q) \land \text{pit}(p, q)))]$$

- Room (x, y) is safe if and only if there is no Wumpus and no pit in it.

$$\forall x, y\ [\text{safe}(x, y) \iff (\neg\text{wumpus}(x, y) \land \neg\text{pit}(x, y))]$$

With this information in our KB, we have enough information to infer which rooms are 'stenchy', 'breezy,' and 'safe.'

(d) Assuming $\text{row}(1), \text{col}(1), \ldots, \text{row}(4), \text{col}(4)$ are facts, we a KB that contains the rules necessary for describing the configurations of 4 queens such that none can attack one another. First, let us define the following predicates:

- $\text{queen}(r, c)$: A queen resides at row r and column c.
- $\text{row}(r)$: The r^{th} row on the board.
- $\text{col}(c)$: The c^{th} column on the board.

Now, for the rules.

- No two queens can reside in the same row.

$$\forall r, c_1, c_2[(\text{row}(r) \land \text{col}(c_1) \land \text{col}(c_2) \land \text{queen}(r, c_1) \land \text{queen}(r, c_2)) \\ \implies c_1 = c_2]$$

- No two queens can reside in the same column.

$$\forall c, r_1, r_2[(\text{col}(c) \land \text{row}(r_1) \land \text{row}(r_2) \land \text{queen}(c, r_1) \land \text{queen}(c, r_2)) \\ \implies r_1 = r_2]$$

- No two queens can be placed in the same diagonal (since the grid is a square, this is easy to represent since the difference between the row numbers cannot be the same as the difference between the column numbers, namely, the absolute value of their differences).

  $$\forall r_1, r_2, c_1, c_2[(\text{row}(r_1) \wedge \text{row}(r_2) \wedge \text{col}(c_1) \wedge \text{col}(c_2) \wedge \text{queen}(r_1, c_1) \wedge \text{queen}(r_2, c_2)) \implies (|r_1 - r_2| \neq |c_1 - c_2|)]$$

This KB holds the necessary and sufficient rules and facts in order to determine a valid placement of queens on a 4×4 grid, such that none of them can attack one another.