CSCE 420 Homework Two

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1. (a) Prove that $(A \land B \implies C \land D) \vdash (A \land B \implies C)$ ("conjunctive rule splitting") is a **sound rule-of-inference** using a **truth table**.
   (b) Also prove $(A \land B \implies C \land D) \models (A \land B \implies C)$ using **Natural Deduction**.

   (Hint: it might help to use a ROI for "Implication Introduction". If you have a Horn clause, with 1 positive literal and $n-1$ negative literals, like $(\neg X \lor Z \lor \neg Y)$, you can transform it into a rule by collecting the negative literals as positive antecedents, e.g. $X \land Y \implies Z$. This is a truth-preserving operation (hence sound), which you could prove to yourself using a truth table.)
   (c) Also prove $(A \land B \implies C \land D) \models (A \land B \implies C)$ using **Resolution**.

Solution:

(a) Below, see the truth table.

A	B	C	D	$A \land B$	$C \land D$	$A \land B \implies C \land D$	$A \land B \implies C$
T	F	T	T	F	T	T	T
T	F	T	F	F	F	T	T
T	F	F	T	F	F	T	T
T	F	F	F	F	F	T	T
T	T	T	T	T	T	T	T
T	T	T	F	T	F	F	T
T	T	F	T	T	F	F	F
T	T	F	F	T	F	F	F
F	F	T	T	F	T	T	T
F	F	T	F	F	F	T	T
F	F	F	T	F	F	T	T
F	F	F	F	F	F	T	T
F	T	T	T	F	T	F	T
F	T	T	F	F	F	T	T
F	T	F	T	F	F	T	T
F	T	F	F	F	F	T	T

As the truth table shows, whenever the premise $(A \wedge B \implies C \wedge D)$ is true, so is the conclusion $(A \wedge B \implies C)$. Therefore, $(A \wedge B \implies C \wedge D) \vdash (A \wedge B \implies C)$ is a sound rule-of-inference.

(b) Here, we use implication introduction as suggested to derive the conclusion.

(1)	$(A \wedge B \implies C \wedge D)$	(Premise)
(2)	$\neg(A \wedge B) \vee (C \wedge D)$	(Implication Elim., 1)
(3)	$(\neg A \vee \neg B) \vee (C \wedge D)$	(DeMorgan's, 2)
(4)	$(\neg A \vee \neg B \vee C) \wedge (\neg A \vee \neg B \vee D)$	(Distributivity, 3)
(5)	$(\neg A \vee \neg B \vee C)$	(And Elim., 4)
(6)	$(\mathrm{A} \wedge B \implies C)$	(Implication Intr., 5)

(c) First, we convert the premise to CNF as follows:

$$(A \wedge B \implies C \wedge D) \equiv \neg(A \wedge B) \vee (C \wedge D) \equiv (\neg A \vee \neg B) \vee (C \wedge D).$$

Then, using distributivity, this can be separated into two separate clauses

$$(\neg A \vee \neg B \vee C) \qquad (\neg A \vee \neg B \vee D).$$

Then, we will convert the conclusion to CNF as follows:

$$(A \wedge B \implies C) \equiv \neg(A \vee B) \vee C \equiv (\neg A \vee \neg B \vee C).$$

Now, we negate the CNF of the conclusion as follows:

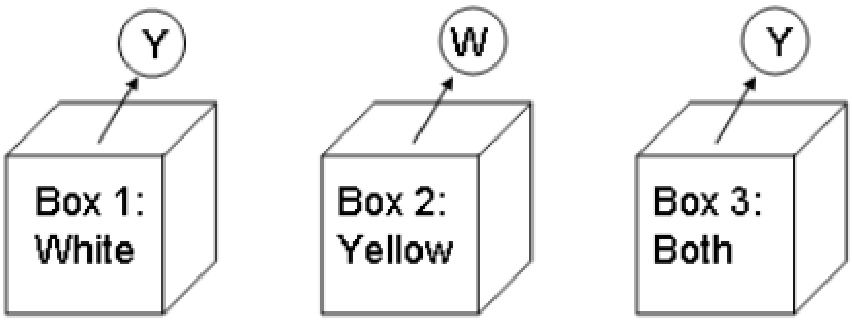
$$\neg(\neg A \vee \neg B \vee C) \equiv (A \wedge B \wedge \neg C).$$

Now, we have all of the information we need to conduct a resolution refutation proof as follows.

(1)	$(\neg A \vee \neg B \vee C)$	(Premise)	(7)	$\neg B \vee C$	(Resolve 1 and 4)
(2)	$(\neg A \vee \neg B \vee D)$	(Premise)	(8)	$\neg B \vee D$	(Resolve 2 and 4)
(3)	$(A \wedge B \wedge \neg C)$	(Negated Query)	(9)	C	(Resolve 5 and 7)
(4)	A	(And Elim., 3)	(10)	D	(Resolve 5 and 8)
(5)	B	(And Elim., 3)	(11)	$\emptyset$	(Resolve 6 and 9)
(6)	$\neg C$	(And Elim., 3)			

Since we have derived the empty clause in step (11), this implies that if $(A \wedge B \implies C \wedge D)$ is true, then it is not possible for $(A \wedge B \implies C)$ to be false. Thus we have proven that $(A \wedge B \implies C \wedge D) \models (A \wedge B \implies C)$ using resolution.

2. You are the proprietor of Sammy's Sport Shop. You have just received a shipment of three boxes filled with tennis balls. One box contains only yellow tennis balls, one box contains only white tennis balls, and one contains both yellow and white tennis balls. You would like to stock the tennis balls in appropriate places on your shelves. Unfortunately, the boxes have been labeled incorrectly; the manufacturer tells you that you have exactly one box of each, but that **each box is definitely labeled wrong**. You draw one ball from each box and observe its color. Given the initial (incorrect) labeling of the boxes above, and the three observations, use Propositional Logic to infer the correct contents of the middle box.



Use propositional symbols in the following form: O1Y means a yellow ball was drawn (observed) from box 1, L1W means box 1 was initially labeled white, C1W means box 1 contains (only) white balls, and C1B means box 1 actually contains both types of tennis balls. Note, there is no 'O1B', etc, because you can't directly "observe both". When you draw a tennis ball, it will either be white or yellow.

The initial facts describing this particular situation are: {O1Y, L1W, O2W, L2Y, O3Y, L3B}.

(a) Using these propositional symbols, write a propositional knowledge base (sammy.kb) that captures the knowledge in this domain (i.e. implications of what different observations or labels mean, as well as constraints inherent in this problem, such as that all boxes have different contents). Do it in a complete and general way, writing down all the rules and constraints, not just the ones needed to make the specific inference about the middle box. Do not include derived knowledge that depends on the particular labeling of this instance shown above; stick to what is stated in the problem description above. Your KB should be general enough to reason about any alternative scenario, not just the one given above (e.g. with different observations and labels and box contents).

(b) Prove that box 2 must contain white balls (**C2W**) using **Natural Deduction**.

(c) Convert your KB to CNF.

(d) Prove C2W using **Resolution**.

Solution:

(a) First of all, each box can only contain one type of ball: either yellow, white, or both. Further, no two boxes can have the same contents. So we have:

- $(C1W \wedge \neg C1Y \wedge \neg C1B) \vee (\neg C1W \wedge C1Y \wedge \neg C1B) \vee (\neg C1W \wedge \neg C1Y \wedge C1B)$
- $(C2W \wedge \neg C2Y \wedge \neg C2B) \vee (\neg C2W \wedge C2Y \wedge \neg C2B) \vee (\neg C2W \wedge \neg C2Y \wedge C2B)$
- $(C3W \wedge \neg C3Y \wedge \neg C3B) \vee (\neg C3W \wedge C3Y \wedge \neg C3B) \vee (\neg C3W \wedge \neg C3Y \wedge C3B)$

A box cannot contain *only* the opposite color ball that was chosen, i.e., if a white ball is observed, the box cannot contain *only* yellow balls.

- $O1Y \implies (\neg C1W \wedge (C1Y \vee C1B))$
- $O1W \implies (\neg C1Y \wedge (C1W \vee C1B))$
- $O2Y \implies (\neg C2W \wedge (C2Y \vee C2B))$
- $O2W \implies (\neg C2Y \wedge (C2W \vee C2B))$
- $O3Y \implies (\neg C3W \wedge (C3Y \vee C3B))$
- $O3W \implies (\neg C3Y \wedge (C3W \vee C3B))$

Perhaps most obviously, if a box definitely contains one type of ball only, then that type of ball will be observed when chosen from the respective box.

- $C1W \implies O1W$
- $C1Y \implies O1Y$
- $C2W \implies O2W$
- $C2Y \implies O2Y$
- $C3W \implies O3W$
- $C3Y \implies O3Y$

Each box *must* be labelled incorrectly.

- $(L1W \wedge \neg C1W) \vee (L1Y \wedge \neg C1Y) \vee (L1B \wedge \neg C1B)$
- $(L2W \wedge \neg C2W) \vee (L2Y \wedge \neg C2Y) \vee (L2B \wedge \neg C2B)$
- $(L3W \wedge \neg C3W) \vee (L3Y \wedge \neg C3Y) \vee (L3B \wedge \neg C3B)$

If a box is labelled a certain type, it must not contain that type and can contain either of the other two types.

- $L1W \implies (\neg C1W \wedge (C1Y \vee C1B))$
- $L1Y \implies (\neg C1Y \wedge (C1W \vee C1B))$
- $L1B \implies (\neg C1B \wedge (C1W \vee C1Y))$
- $L2W \implies (\neg C2W \wedge (C2Y \vee C2B))$
- $L2Y \implies (\neg C2Y \wedge (C2W \vee C2B))$
- $L2B \implies (\neg C2B \wedge (C2W \vee C2Y))$
- $L3W \implies (\neg C3W \wedge (C3Y \vee C3B))$
- $L3Y \implies (\neg C3Y \wedge (C3W \vee C3B))$
- $L3B \implies (\neg C3B \wedge (C3W \vee C3Y))$

All boxes have unique contents/no two boxes have the same contents.

- $C1W \implies \neg C2W \wedge \neg C3W$
- $C2W \implies \neg C1W \wedge \neg C3W$
- $C3W \implies \neg C1W \wedge \neg C2W$
- $C1Y \implies \neg C2Y \wedge \neg C3Y$
- $C2Y \implies \neg C1Y \wedge \neg C3Y$
- $C3Y \implies \neg C1Y \wedge \neg C2Y$

- $C1B \implies \neg C2B \land \neg C3B$
- $C2B \implies \neg C1B \land \neg C3B$
- $C3B \implies \neg C1B \land \neg C2B$

So, our complete KB, sammy.kb, is described as:

$$
\begin{align}
&(C1W \land \neg C1Y \land \neg C1B) \lor (\neg C1W \land C1Y \land \neg C1B) \lor (\neg C1W \land \neg C1Y \land C1B) \tag{1}\\
&(C2W \land \neg C2Y \land \neg C2B) \lor (\neg C2W \land C2Y \land \neg C2B) \lor (\neg C2W \land \neg C2Y \land C2B) \tag{2}\\
&(C3W \land \neg C3Y \land \neg C3B) \lor (\neg C3W \land C3Y \land \neg C3B) \lor (\neg C3W \land \neg C3Y \land C3B) \tag{3}\\
&O1Y \implies (\neg C1W \land (C1Y \lor C1B)) \tag{4}\\
&O1W \implies (\neg C1Y \land (C1W \lor C1B)) \tag{5}\\
&O2Y \implies (\neg C2W \land (C2Y \lor C2B)) \tag{6}\\
&O2W \implies (\neg C2Y \land (C2W \lor C2B)) \tag{7}\\
&O3Y \implies (\neg C3W \land (C3Y \lor C3B)) \tag{8}\\
&O3W \implies (\neg C3Y \land (C3W \lor C3B)) \tag{9}\\
&C1W \implies O1W \tag{10}\\
&C1Y \implies O1Y \tag{11}\\
&C2W \implies O2W \tag{12}\\
&C2W \implies O2W \tag{13}\\
&C3W \implies O3W \tag{14}\\
&C3W \implies O3W \tag{15}\\
&(L1W \land \neg C1W) \lor (L1Y \land \neg C1Y) \lor (L1B \land \neg C1B) \tag{16}\\
&(L2W \land \neg C2W) \lor (L2Y \land \neg C2Y) \lor (L2B \land \neg C2B) \tag{17}\\
&(L3W \land \neg C3W) \lor (L3Y \land \neg C3Y) \lor (L3B \land \neg C3B) \tag{18}\\
&L1W \implies (\neg C1W \land (C1Y \lor C1B)) \tag{19}\\
&L1Y \implies (\neg C1Y \land (C1W \lor C1B)) \tag{20}\\
&L1B \implies (\neg C1B \land (C1W \lor C1Y)) \tag{21}\\
&L2W \implies (\neg C2W \land (C2Y \lor C2B)) \tag{22}\\
&L2Y \implies (\neg C2Y \land (C2W \lor C2B)) \tag{23}\\
&L2B \implies (\neg C2B \land (C2W \lor C2Y)) \tag{24}\\
&L3W \implies (\neg C3W \land (C3Y \lor C3B)) \tag{25}\\
&L3Y \implies (\neg C3Y \land (C3W \lor C3B)) \tag{26}\\
&L3B \implies (\neg C3B \land (C3W \lor C3Y)) \tag{27}\\
&C1W \implies \neg C2W \land \neg C3W \tag{28}\\
&C2W \implies \neg C1W \land \neg C3W \tag{29}\\
&C3W \implies \neg C1W \land \neg C2W \tag{30}\\
&C1Y \implies \neg C2Y \land \neg C3Y \tag{31}\\
&C2Y \implies \neg C1Y \land \neg C3Y \tag{32}\\
&C3Y \implies \neg C1Y \land \neg C2Y \tag{33}\\
&C1B \implies \neg C2B \land \neg C3B \tag{34}\\
&C2B \implies \neg C1B \land \neg C3B \tag{35}\\
&C3B \implies \neg C1B \land \neg C2B \tag{36}
\end{align}
$$

(b) Now, we can perform natural deduction based on the initial facts $\{O1Y, L1W, O2W, L2Y, O3Y, L3B\}$ and the KB as follows:

(37)	$O1Y$	(Premise)
(38)	$L1W$	(Premise)
(39)	$O2W$	(Premise)
(40)	$L2Y$	(Premise)
(41)	$O3Y$	(Premise)
(42)	$L3B$	(Premise)
(43)	$\neg C3W \wedge (C3Y \vee C3B)$	(MP, 8, 41)
(44)	$C3Y \vee C3B$	(And Elim., 43)
(45)	$(\neg C3B \wedge (C3W \vee C3Y))$	(MP, 27, 42)
(46)	$\neg C3B$	(And Elim., 45)
(47)	$C3Y$	(Resolution, 44, 46)
(48)	$\neg C1Y \wedge \neg C2Y$	(MP, 33, 47)
(49)	$\neg C1W \wedge (C1Y \vee C1B)$	(MP, 4, 37)
(50)	$C1Y \vee C1B$	(And Elim., 49)
(51)	$\neg C1Y$	(And Elim., 48)
(52)	$C1B$	(Resolve 49, 51)
(53)	$\neg C2B \wedge \neg C3B$	(MP, 34, 52)
(54)	$\neg C2B$	(And Elim., 53)
(55)	$\neg C2Y \wedge (C2W \vee C2B)$	(MP, 7, 39)
(56)	$C2W \vee C2B$	(And Elim., 55)
(57)	$C2W$	(Resolve 54, 56)

(c) Let us convert the KB to CNF.

$$
\begin{array}{cc}
(1.a) & (\neg C1W \vee \neg O1Y) \\
(1.b) & (C1Y \vee C1B \vee \neg O1Y) \\
(1.c) & (\neg C1W \vee \neg C1B) \\
(1.d) & (C1Y \vee \neg C1B \vee \neg C1W) \\
(1.e) & (\neg C1B \vee C1W \vee \neg C1Y) \\
(1.f) & (\neg C1B \vee \neg C1Y) \\
(1.g) & (C1Y \vee C1W \vee C1B) \\
(2.a) & (\neg C2W \vee \neg O2Y) \\
(2.b) & (C2Y \vee C2B \vee \neg O2Y) \\
(2.c) & (\neg C2W \vee \neg C2B) \\
(2.d) & (C2Y \vee \neg C2B \vee \neg C2W) \\
(2.e) & (\neg C2B \vee C2W \vee \neg C2Y) \\
(2.f) & (\neg C2B \vee \neg C2Y) \\
(2.g) & (C2Y \vee C2W \vee C2B) \\
(3.a) & (\neg C3W \vee \neg O3Y) \\
(3.b) & (C3Y \vee C3B \vee \neg O3Y) \\
(3.c) & (\neg C3W \vee \neg C3B) \\
(3.d) & (C3Y \vee \neg C3B \vee \neg C3W) \\
(3.e) & (\neg C3B \vee C3W \vee \neg C3Y) \\
(3.f) & (\neg C3B \vee \neg C3Y) \\
(3.g) & (C3Y \vee C3W \vee C3B) \\
(4.a) & (\neg C1W \vee \neg O1Y) \\
(4.b) & (C1Y \vee C1B \vee \neg O1Y) \\
(5.a) & (\neg C1Y \vee \neg O1W) \\
(5.b) & (C1W \vee C1B \vee \neg O1W) \\
(6.a) & (\neg C2W \vee \neg O2Y) \\
(6.b) & (C2Y \vee C2B \vee \neg O2Y) \\
(7.a) & (\neg C2Y \vee \neg O2W) \\
(7.b) & (C2W \vee C2B \vee \neg O2W) \\
(8.a) & (\neg C3W \vee \neg O3Y) \\
(8.b) & (C3Y \vee C3B \vee \neg O3Y) \\
(9.a) & (\neg C3Y \vee \neg O3W) \\
(9.b) & (C3W \vee C3B \vee \neg O3W) \\
(10) & \neg C1W \vee O1W \\
(11) & \neg C1Y \vee O1Y \\
(12) & \neg C2W \vee O2W \\
(13) & \neg C2Y \vee O2Y \\
(14) & \neg C3W \vee O3W \\
(15) & \neg C3Y \vee O3Y \\
(16.a) & (L1Y \vee L1W \vee L1B) \\
(16.b) & (\neg C1Y \vee L1W \vee L1B)
\end{array}
$$

(16.c)	$(L1Y \vee \neg C1W \vee L1B)$
(16.d)	$(\neg C1Y \vee \neg C1W \vee L1B)$
(16.e)	$(L1Y \vee L1W \vee \neg C1B)$
(16.f)	$(\neg C1Y \vee L1W \vee \neg C1B)$
(16.g)	$(L1Y \vee \neg C1W \vee \neg C1B)$
(16.h)	$(\neg C1Y \vee \neg C1W \vee \neg C1B)$
(17.a)	$(L2Y \vee L2W \vee L2B)$
(17.b)	$(\neg C2Y \vee L2W \vee L2B)$
(17.c)	$(L2Y \vee \neg C2W \vee L2B)$
(17.d)	$(\neg C2Y \vee \neg C2W \vee L2B)$
(17.e)	$(L2Y \vee L2W \vee \neg C2B)$
(17.f)	$(\neg C2Y \vee L2W \vee \neg C2B)$
(17.g)	$(L2Y \vee \neg C2W \vee \neg C2B)$
(17.h)	$(\neg C2Y \vee \neg C2W \vee \neg C2B)$
(18.a)	$(L3Y \vee L3W \vee L3B)$
(18.b)	$(\neg C3Y \vee L3W \vee L3B)$
(18.c)	$(L3Y \vee \neg C3W \vee L3B)$
(18.d)	$(\neg C3Y \vee \neg C3W \vee L3B)$
(18.e)	$(L3Y \vee L3W \vee \neg C3B)$
(18.f)	$(\neg C3Y \vee L3W \vee \neg C3B)$
(18.g)	$(L3Y \vee \neg C3W \vee \neg C3B)$
(18.h)	$(\neg C3Y \vee \neg C3W \vee \neg C3B)$
(19.a)	$(\neg C1W \vee \neg L1W)$
(19.b)	$(C1Y \vee C1B \vee \neg L1W)$
(20.a)	$(\neg C1Y \vee \neg L1Y)$
(20.b)	$(C1W \vee C1B \vee \neg L1Y)$
(21.a)	$(\neg C1B \vee \neg L1B)$
(21.b)	$(C1W \vee C1Y \vee \neg L1B)$
(22.a)	$(\neg C2W \vee \neg L2W)$
(22.b)	$(C2Y \vee C2B \vee \neg L2W)$
(23.a)	$(\neg C2Y \vee \neg L2Y)$
(23.b)	$(C2W \vee C2B \vee \neg L2Y)$
(24.a)	$(\neg C2B \vee \neg L2B)$
(24.b)	$(C2W \vee C2Y \vee \neg L2B)$
(25.a)	$(\neg C3W \vee \neg L3W)$
(25.b)	$(C3Y \vee C3B \vee \neg L3W)$
(26.a)	$(\neg C3Y \vee \neg L3Y)$
(26.b)	$(C3W \vee C3B \vee \neg L3Y)$
(27.a)	$(\neg C3B \vee \neg L3B)$
(27.b)	$(C3W \vee C3Y \vee \neg L3B)$
(28.a)	$(\neg C2W \vee \neg C1W)$
(28.b)	$(\neg C3W \vee \neg C1W)$
(29.a)	$(\neg C1W \vee \neg C2W)$

(29.b)	$(\neg C3W \vee \neg C2W)$
(30.a)	$(\neg C1W \vee \neg C3W)$
(30.b)	$(\neg C2W \vee \neg C3W)$
(31.a)	$(\neg C2Y \vee \neg C1Y)$
(31.b)	$(\neg C3Y \vee \neg C1Y)$
(32.a)	$(\neg C1Y \vee \neg C2Y)$
(32.b)	$(\neg C3Y \vee \neg C2Y)$
(33.a)	$(\neg C1Y \vee \neg C3Y)$
(33.b)	$(\neg C2Y \vee \neg C3Y)$
(34.a)	$(\neg C2B \vee \neg C1B)$
(34.b)	$(\neg C3B \vee \neg C1B)$
(35a)	$(\neg C1B \vee \neg C2B)$
(35.b)	$(\neg C3B \vee \neg C2B)$
(36.a)	$(\neg C1B \vee \neg C3B)$
(36.b)	$(\neg C2B \vee \neg C3B)$

Remark: At this point, I realized that my initial KB was far too *complicated*. Even though I should be able to derive a contradiction (since we have already proven $C2W$ using natural deduction), there are simply too many clauses when converted to CNF. After banging my head trying to figure out a resolution refutation proof, I have decided to make a simpler KB for this part of the problem and then convert it to CNF and try resolution refutation again. Note, I kept my original KB and converted it to CNF.

Below is my new knowledge base. Notice the statements are much simpler than before, noting observations such as picking a white ball out of a box implies it must either contain white balls or both balls, each box must have unique contents, etc.

$$
\begin{array}{rc}
(1) & O1W \implies C1W \lor C1B \\
(2) & O1Y \implies C1Y \lor C1B \\
(3) & O2W \implies C2W \lor C2B \\
(4) & O2Y \implies C2Y \lor C2B \\
(5) & O3W \implies C3W \lor C3B \\
(6) & O3Y \implies C3Y \lor C3B \\
(7) & L1Y \implies \neg C1Y \\
(8) & L1W \implies \neg C1W \\
(9) & L1B \implies \neg C1B \\
(10) & L2Y \implies \neg C2Y \\
(11) & L2W \implies \neg C2W \\
(12) & L2B \implies \neg C2B \\
(13) & L3Y \implies \neg C3Y \\
(14) & L3W \implies \neg C3W \\
(15) & L3B \implies \neg C3B \\
(16) & C1W \lor C1Y \lor C1B \\
(17) & C2W \lor C2Y \lor C2B \\
(18) & C3W \lor C3Y \lor C3B \\
(19) & C1W \implies \neg C2W \land \neg C3W \\
(20) & C1Y \implies \neg C2Y \land \neg C3Y \\
(21) & C1B \implies \neg C2B \land \neg C3B \\
(22) & C2W \implies \neg C1W \land \neg C3W \\
(23) & C2Y \implies \neg C1Y \land \neg C3Y \\
(24) & C2B \implies \neg C1B \land \neg C3B \\
(25) & C3W \implies \neg C1W \land \neg C2W \\
(26) & C3Y \implies \neg C1Y \land \neg C2Y \\
(27) & C3B \implies \neg C1B \land \neg C2B
\end{array}
$$

Below, we convert the *new* KB to CNF.

(1)	$\neg O1W \vee C1W \vee C1B$	(19.a)	$(\neg C2W \vee \neg C1W)$
(2)	$\neg O1Y \vee C1Y \vee C1B$	(19.b)	$(\neg C3W \vee \neg C1W)$
(3)	$\neg O2W \vee C2W \vee C2B$	(20.a)	$(\neg C2Y \vee \neg C1Y)$
(4)	$\neg O2Y \vee C2Y \vee C2B$	(20.b)	$(\neg C3Y \vee \neg C1Y)$
(5)	$\neg O3W \vee C3W \vee C3B$	(21.a)	$(\neg C2B \vee \neg C1B)$
(6)	$\neg O3Y \vee C3Y \vee C3B$	(21.b)	$(\neg C3B \vee \neg C1B)$
(7)	$\neg L1Y \vee \neg C1Y$	(22.a)	$(\neg C1W \vee \neg C2W)$
(8)	$\neg L1W \vee \neg C1W$	(22.b)	$(\neg C3W \vee \neg C2W)$
(9)	$\neg L1B \vee \neg C1B$	(23.a)	$(\neg C1Y \vee \neg C2Y)$
(10)	$\neg L2Y \vee \neg C2Y$	(23.b)	$(\neg C3Y \vee \neg C2Y)$
(11)	$\neg L2W \vee \neg C2W$	(24.a)	$(\neg C1B \vee \neg C2B)$
(12)	$\neg L2B \vee \neg C2B$	(24.b)	$(\neg C3B \vee \neg C2B)$
(13)	$\neg L3Y \vee \neg C3Y$	(25.a)	$(\neg C1W \vee \neg C3W)$
(14)	$\neg L3W \vee \neg C3W$	(25.b)	$(\neg C2W \vee \neg C3W)$
(15)	$\neg L3B \vee \neg C3B$	(26.a)	$(\neg C1Y \vee \neg C3Y)$
(16)	$C1W \vee C1Y \vee C1B$	(26.b)	$(\neg C2Y \vee \neg C3Y)$
(17)	$C2W \vee C2Y \vee C2B$	(27.a)	$(\neg C1B \vee \neg C3B)$
(18)	$C3W \vee C3Y \vee C3B$	(27.b)	$(\neg C2B \vee \neg C3B)$

(d) Now, we will perform resolution refutation with the *new* KB to prove $C2W$. First, we will negate the conclusion and add it to the KB. So KB (37) is $\neg C2W$.

(28)	$O1Y$	(Premise)
(29)	$L1W$	(Premise)
(30)	$O2W$	(Premise)
(31)	$L2Y$	(Premise)
(32)	$O3Y$	(Premise)
(33)	$L3B$	(Premise)
(34)	$\neg C2W$	(Premise)
(35)	$C1Y \vee C1B$	(Resolve 2, 28)
(36)	$C3Y \vee C3B$	(Resolve 6, 32)
(37)	$\neg C3B$	(Resolve 15, 33)
(38)	$C3Y$	(Resolve 36, 37)
(39)	$\neg C2Y$	(Resolve 26.b, 38)
(40)	$\neg C1Y$	(Resolve 26.a, 38)
(41)	$C1B$	(Resolve 35, 40)
(42)	$\neg C2B$	(Resolve 24.a, 41)
(43)	$C2W \vee C2B$	(Resolve 3, 30)
(44)	$C2W$	(Resolve 42, 43)
(45)	$\emptyset$	(Resolve 34, 44)

Since we derived an empty clause, we arrive at a contradiction when we use the negated query, therefore the query ($C2W$) must be true. As an aside, this problem made me realize that resolution refutation is *definitely* not the most effective proof method (especially by hand)!

3. Do **Forward Chaining** for the *CanGetToWork* KB below.

   You don't need to follow the formal FC algorithm (with agenda/queue and counts array). Just indicate which rules are triggered (in any order), and keep going until all consequences are generated. Show the final list of all inferred propositions at the end. *Is CanGetToWork among them*?

   a. CanBikeToWork $\implies$ CanGetToWork
   b. CanDriveToWork $\implies$ CanGetToWork
   c. CanWalkToWork $\implies$ CanGetToWork
   d. HaveBike $\wedge$ WorkCloseToHome $\wedge$ Sunny $\implies$ CanBikeToWork
   e. HaveMountainBike $\implies$ HaveBike
   f. HaveTenSpeed $\implies$ HaveBike
   g. OwnCar $\implies$ CanDriveToWork
   h. OwnCar $\implies$ MustGetAnnualInspection
   i. OwnCar $\implies$ MustHaveValidLicense
   j. CanRentCar $\implies$ CanDriveToWork
   k. HaveMoney $\wedge$ CarRentalOpen $\implies$ CanRentCar
   l. HertzOpen $\implies$ CarRentalOpen
   m. AvisOpen $\implies$ CarRentalOpen
   n. EnterpriseOpen $\implies$ CarRentalOpen
   o. CarRentalOpen $\implies$ IsNotAHoliday
   p. HaveMoney $\wedge$ TaxiAvailable $\implies$ CanDriveToWork
   q. Sunny $\wedge$ WorkCloseToHome $\implies$ CanWalkToWork
   r. HaveUmbrella $\wedge$ WorkCloseToHome $\implies$ CanWalkToWork
   s. Sunny $\implies$ StreetsDry

   Facts: {Rainy, HaveMoutainBike, EnjoyPlayingSoccer, WorkForUniversity, WorkCloseToHome, HaveMoney, HertzClosed, AvisOpen, McDonaldsOpen}.

   Solution: Not following the algorithm *exactly*, we note when certain rules are triggered (as noted in instructions).

(1)	HaveMountainBike $\implies$ HaveBike	Fact triggers (e)
(2)	AvisOpen $\implies$ CarRentalOpen	Fact triggers (m)
(3)	CarRentalOpen $\implies$ IsNotAHoliday	(2) triggers (o)
(4)	HaveMoney $\wedge$ CarRentalOpen $\implies$ CanRentCar	Fact and (2) triggers (k)
(5)	CanRentCar $\implies$ CanDriveToWork	(4) triggers (j)
(6)	CanDriveToWork $\implies$ CanGetToWork	(5) triggers (b)

   As we can see, CanGetToWork is among the list of inferred propositions.

4. Do **Backward Chaining** for the *CanGetToWork* KB.

   In this case, you should follow the BC algorithm closely (the pseudocode for the propositional version of Back-chaining is given in the lecture slides).

   Important: when you pop a subgoal (proposition) from the goal stack, you should systematically go through all rules that can be used to prove it IN THE ORDER THEY APPEAR IN THE KB. In some cases, this will lead to *back-tracking*, which you should show.

   Also, the sequence of results depends on order in which antecedents are pushed onto the stack. If you have a rule like $A \wedge B \implies C$, and you pop C off the stack, push the antecedents in reverse order, so B goes in first, then A; in the next iteration, A would be the next subgoal popped off the stack.

Solution:

(1)	CanGetToWork	Initialize with query
(2)	CanBikeToWork	Pop CanGetToWork, push ant. of (a)
(3)	Sunny, WorkCloseToHome, HaveBike	Pop CanBikeToWork, push ants. of (d)
Sunny is not provable, so back-track to other rule for CanBikeToWork		
CanBikeToWork unprovable, back-track to next rule for CanGetToWork		
(4)	CanDriveToWork	Pop CanGetToWork, push ant. of (b)
(5)	OwnCar	Pop CanDriveToWork, push ant. of (g)
OwnCar is not provable, so back-track to next rule for CanDriveToWork		
(6)	CanRentCar	Pop CanDriveToWork, push ant. of (j)
(7)	CarRentalOpen, HaveMoney	Pop CanRentCar, push ants. of (k)
(8)	HertzOpen, HaveMoney	Pop CarRentalOpen, push ants. of (l)
HertzOpen is not provable, so back-track to next rule for CarRentalOpen		
(9)	AvisOpen, HaveMoney	Pop CarRentalOpen, push ant. of (m)
(10)	HaveMoney	Pop AvisOpen (fact)
(11)	$\emptyset$	Pop HaveMoney (fact)

Since the stack became empty, we can return success!

5. In what kinds of problems would it be better to use forward-chaining? When would it be better to use backward-chaining?

Solution: Since forward-chaining requires the derivation of *all conclusions* based on a KB and a set of initial facts. Therefore, when the problem is more data-driven, and you would like to know many different conclusions, FC could be more effective. Furthermore, when the KB has a large amount of rules, FC allows more focus on the relevant facts. Also, if the goal of the problem is not well-defined, or one wishes to seek multiple answers to the same problem. FC could be better as it generates all possible conclusions based on the initial facts and the KB (as mentioned previously).

On the other hand, backward-chaining (which is *sort of* opposite of FC) is most likely better for problems with a more well-defined goal. If a problem has a specific end goal, it is wasteful to generate all conclusions if they are irrelevant (FC). BC can also be more effective in problems where there are a large amount of facts and a smaller amount of rules. This is because, each fact gives valuable information to "back-track with" but each additional rule gives more opportunity to "go the wrong direction." Finally, BC is most likely better in problems where only a small portion of the KB has relevant information. This way, this area of the KB will likely be untouched, wasting less computation time.

As seen in problems (3) and (4), FC and BC have their key differences and thus the nature of the problem dictates the effectiveness of the algorithm.