CSCE 420 Homework Four

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1. Bayesian Inference

   Consider two factors that influence whether a student passes a given test: a) being smart, and b) studying. Suppose 30% of students believe they are intrinsically smart. But since students do not know a priori whether they are smart enough to pass a test, suppose 40% of will study for it anyway. (assume Smart and Study are independent). The causal relationship of these variables on the probability of actually passing the test can be expressed in a conditional probability table (CPT) as follows:

P(pass | Smart, Study)	¬smart	smart
¬study	0.2	0.7
study	0.6	0.95

   Prior Probabilities: P(smart) = 0.3, P(study) = 0.4.

   (a) Write out the equation for calculating joint probabilities, P(Smart, Study, Pass).

   (b) Calculate all the entries in the full joint probability table (JPT) [a 4x2 matrix, like Fig 12.3 in the textbook; [Note: names of variables are capitalized, lower-case indicates truth value, e.g. 'pass' means Pass=T, and '-pass' means Pass=F.]

   (c) From the JPT, compute the probability that a student is smart, given that they pass the test but did not study.

   (d) From the JPT, compute the probability that a student did not study, given that they are smart but did not pass the test.

   (e) Compute the marginal probability that a student will pass the test given that they are smart.

   (f) Compute the marginal probability that a student will pass the test given that they study.

Solution:

(a) The equation for calculating the joint probabilities is

$$\mathbb{P}(Smart, Study, Pass) = \mathbb{P}(Pass \mid Smart, Study) \cdot \mathbb{P}(Smart) \cdot \mathbb{P}(Study),$$

since $\mathbb{P}(Smart)$ and $\mathbb{P}(Study)$ are independent.

(b) Here are the calculations:

$$\begin{aligned}
\mathbb{P}(Smart, Study, Pass) &= \mathbb{P}(Pass \mid Smart, Study) \cdot \mathbb{P}(Smart) \cdot \mathbb{P}(Study) \\
&= 0.95 \cdot 0.30 \cdot 0.40 \\
&= 0.114 \\
\mathbb{P}(Smart, Study, \neg Pass) &= (1 - \mathbb{P}(Pass \mid Smart, Study)) \cdot \mathbb{P}(Smart) \cdot \mathbb{P}(Study) \\
&= 0.05 \cdot 0.30 \cdot 0.40 \\
&= 0.006 \\
\mathbb{P}(Smart, \neg Study, Pass) &= \mathbb{P}(Pass \mid Smart, \neg Study) \cdot \mathbb{P}(Smart) \cdot \mathbb{P}(\neg Study) \\
&= \mathbb{P}(Pass \mid Smart, \neg Study) \cdot \mathbb{P}(Smart) \cdot (1 - \mathbb{P}(Study)) \\
&= 0.70 \cdot 0.30 \cdot 0.60 \\
&= 0.126 \\
\mathbb{P}(Smart, \neg Study, \neg Pass) &= \mathbb{P}(\neg Pass \mid Smart, \neg Study) \cdot \mathbb{P}(Smart) \cdot \mathbb{P}(\neg Study) \\
&= (1 - \mathbb{P}(Pass \mid Smart, \neg Study) \cdot \mathbb{P}(Smart) \cdot (1 - \mathbb{P}(Study)) \\
&= 0.30 \cdot 0.30 \cdot 0.60 \\
&= 0.054 \\
\mathbb{P}(\neg Smart, Study, Pass) &= \mathbb{P}(Pass \mid \neg Smart, Study) \cdot \mathbb{P}(\neg Smart) \cdot \mathbb{P}(Study) \\
&= \mathbb{P}(Pass \mid \neg Smart, Study) \cdot (1 - \mathbb{P}(Smart)) \cdot \mathbb{P}(Study) \\
&= 0.60 \cdot 0.70 \cdot 0.40 \\
&= 0.168 \\
\mathbb{P}(\neg Pass \mid \neg Smart, Study) &= (1 - \mathbb{P}(Pass \mid \neg Smart, Study)) \cdot (1 - \mathbb{P}(Smart)) \cdot \mathbb{P}(Study) \\
&= 0.40 \cdot 0.70 \cdot 0.40 \\
&= 0.112 \\
\mathbb{P}(\neg Smart, \neg Study, Pass) &= \mathbb{P}(Pass \mid \neg Smart, \neg Study) \cdot \mathbb{P}(\neg Smart) \cdot \mathbb{P}(\neg Study) \\
&= \mathbb{P}(Pass, \neg Smart, \neg Study) \cdot (1 - \mathbb{P}(Smart)) \cdot (1 - \mathbb{P}(Study)) \\
&= 0.20 \cdot 0.70 \cdot 0.60 \\
&= 0.084 \\
\mathbb{P}(\neg Smart, \neg Study, \neg Pass) &= \mathbb{P}(\neg Pass \mid \neg Smart, \neg Study) \cdot \mathbb{P}(\neg Smart) \cdot \mathbb{P}(\neg Study) \\
&= (1 - \mathbb{P}(Pass \mid \neg Smart, \neg Study)) \cdot (1 - \mathbb{P}(Smart)) \\
&\quad \cdot (1 - \mathbb{P}(Study)) \\
&= 0.80 \cdot 0.70 \cdot 0.60 \\
&= 0.336.
\end{aligned}$$

Below, see the full JPT:

	$pass$		$\neg pass$	
	$\neg smart$	$smart$	$\neg smart$	$smart$
$\neg study$	0.084	0.126	0.336	0.054
$study$	0.168	0.114	0.112	0.006

(c) From the JPT, we compute

$$\begin{aligned}
\mathbb{P}(Smart \mid Pass, \neg Study) &= \frac{\mathbb{P}(Smart, Pass, \neg Study)}{\mathbb{P}(Pass, \neg Study)} \\
&= \frac{0.114}{0.084 + 0.126} \\
&= 0.60.
\end{aligned}$$

(d) From the JPT, we compute

$$\begin{aligned}
\mathbb{P}(\neg Study \mid Smart, \neg Pass) &= \frac{\mathbb{P}(\neg Study, Smart, \neg Pass)}{\mathbb{P}(Smart, \neg Pass)} \\
&= \frac{0.0.054}{0.054 + 0.006} \\
&= 0.90.
\end{aligned}$$

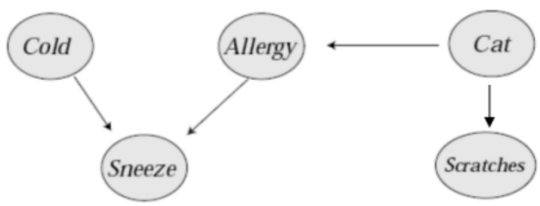
(e) To compute the marginal probability that a student will pass the test given that they are smart, we use the following formula and compute

$$\begin{aligned}
\mathbb{P}(Pass \mid Smart) &= \frac{\mathbb{P}(Smart, Pass)}{\mathbb{P}(Smart)} \\
&= \frac{0.126 + 0.114}{0.126 + 0.114 + 0.054 + 0.006} \\
&= 0.80.
\end{aligned}$$

(f) To compute the marginal probability that a student will pass the test given that they study, we use the following formula and compute

$$\begin{aligned}
\mathbb{P}(Pass \mid Study) &= \frac{\mathbb{P}(Study, Pass)}{\mathbb{P}(Study)} \\
&= \frac{0.168 + 0.114}{0.168 + 0.114 + 0.112 + 0.006} \\
&= 0.705.
\end{aligned}$$

2. Bayesian Networks. Here is a probabilistic model that describes what it might mean when a person sneezes, e.g. depending on whether they have a cold, or whether a cat is present and they are allergic. Scratches on the furniture would be evidence that a cat had been present.



Cold = False	Cold = True
.95	.05

Cat Presence = False	Cat Presence = True
.98	.02

Cat Presence	Allergic Reaction = False	Allergic Reaction = True
False	.95	.05
True	.25	.75

Cold	Allergic Reaction	Sneeze = False	Sneeze = True
False	False	.99	.01
False	True	.3	.7
True	False	.2	.8
True	True	.1	.9

Cat Presence	Scratches = False	Scratches = True
False	.95	.05
True	.5	.5

Cold = False	**Cold = True**
.95	.05

Cat Presence = False	**Cat Presence = True**
.98	.02

Cat Presence	**Allergic Reaction = False**	**Allergic Reaction = True**
False	.95	.05
True	.25	.75

Cold	**Allergic Reaction**	**Sneeze = False**	**Sneeze = True**
False	False	.99	.01
False	True	.30	.70
True	False	.20	.80
True	True	.10	.90

Cat Presence	**Scratches = False**	**Scratches = True**
False	.95	.05
True	.50	.50

(a) Using Equation 13.2 in the textbook (p. 415), write out the expression for the joint probability for any state (combination of truth values for the 5 variables). [Note: Use capital letters for names of variables, and lower-case to indicate truth value, e.g. ‘cold’ means Cold=T, and ‘-cold’ means Cold=F.]

(b) Use the joint prob. equation to calculate the probability for all 32 entries in the JPT (you might want to write a little script to do this).

Cold	Sneeze	Allergy	Scratch	Cat	Joint Prob.
0	0	0	0	0	?
0	0	0	0	1	?
...	...	...	...	...	...
1	1	1	1	1	0.000338

(c) Calculate the probability that someone is allergic, given that they sneezed but do not have a cold, and it is unknown whether a cat has been present but there are scratches on the furniture,
$P(allergic \mid sneeze, \neg cold, scratches)$. Do this calculation numerically using entries in the JPT. (hint: you will have to marginalize over cat presence.)

Solution:

(a) Equation 13.2 from the book states that, for some random variables x_i

$$\mathbb{P}(x_1, \ldots, x_n) = \prod_{i=1}^{n} \mathbb{P}(x_i \mid parents(X_i)).$$

Therefore, we have that

$$\begin{aligned}\mathbb{P}(cold, cat, allergy, scratches, sneeze) &= \mathbb{P}(cat) \cdot \mathbb{P}(cold) \cdot \mathbb{P}(sneeze \mid cold, allergy) \\ &\quad \cdot \mathbb{P}(allergy \mid cat) \cdot \mathbb{P}(scratches \mid cat).\end{aligned}$$

(b) The following JPT was created using a simple Python script.

Cold	**Sneeze**	**Allergy**	**Scratch**	**Cat**	**Joint Prob.**
0	0	0	0	0	0.831825
0	0	0	1	0	0.043780
0	1	0	0	0	0.008402
0	1	0	1	0	0.000442
0	0	1	0	0	0.013267
0	0	1	1	0	0.000698
0	1	1	0	0	0.030956
0	1	1	1	0	0.001629
0	0	0	0	1	0.002351
0	0	0	1	1	0.002351

0	1	0	0	1	0.000024
0	1	0	1	1	0.000024
0	0	1	0	1	0.002137
0	0	1	1	1	0.002137
0	1	1	0	1	0.004987
0	1	1	1	1	0.004987
1	0	0	0	0	0.008844
1	0	0	1	0	0.000466
1	1	0	0	0	0.035378
1	1	0	1	0	0.001862
1	0	1	0	0	0.000233
1	0	1	1	0	0.000012
1	1	1	0	0	0.002095
1	1	1	1	0	0.000110
1	0	0	0	1	0.000025
1	0	0	1	1	0.000025
1	1	0	0	1	0.000100
1	1	0	1	1	0.000100
1	0	1	0	1	0.000038
1	0	1	1	1	0.000038
1	1	1	0	1	0.000338
1	1	1	1	1	0.000338

(c) Since we do not know whether or not a cat is present, we must marginalize this variable out. We know that

$$\mathbb{P}(\mathit{allergic} \mid \mathit{sneeze}, \neg \mathit{cold}, \mathit{scratches}) = \frac{\mathbb{P}(\mathit{allergic}, \mathit{sneeze}, \neg \mathit{cold}, \mathit{scratches})}{\mathbb{P}(\mathit{sneeze}, \neg \mathit{cold}, \mathit{scratches})}.$$

First, we seek to find the joint probability of sneeze, no cold, scratches, and allergic from the JPT (among both cat present and cat not present).

$$\mathbb{P}(\mathit{sneeze}, \neg \mathit{cold}, \mathit{scratches}, \mathit{allergic}, \neg \mathit{cat}) = 0.001629 \qquad \text{(Row 8)}$$
$$\mathbb{P}(\mathit{sneeze}, \neg \mathit{cold}, \mathit{scratches}, \mathit{allergic}, \mathit{cat}) = 0.004987. \qquad \text{(Row 16)}$$

Summing up the probabilities yields

$$\mathbb{P}(\mathit{allergic}, \mathit{sneeze}, \neg \mathit{cold}, \mathit{scratches}) = 0.006616.$$

Next, we seek to find the joint probability of sneeze, no cold, and scratches from the JPT (among both cat present and cat not present).

$$\mathbb{P}(\mathit{sneeze}, \neg \mathit{cold}, \neg \mathit{allergic}, \mathit{scratches}, \neg \mathit{cat}) = 0.000442 \qquad \text{(Row 4)}$$
$$\mathbb{P}(\mathit{sneeze}, \neg \mathit{cold}, \mathit{allergic}, \mathit{scratches}, \neg \mathit{cat}) = 0.001629 \qquad \text{(Row 8)}$$
$$\mathbb{P}(\mathit{sneeze}, \neg \mathit{cold}, \neg \mathit{allergic}, \mathit{scratches}, \mathit{cat}) = 0.000024 \qquad \text{(Row 12)}$$
$$\mathbb{P}(\mathit{sneeze}, \neg \mathit{cold}, \mathit{allergic}, \mathit{scratches}, \mathit{cat}) = 0.004987. \qquad \text{(Row 16)}$$

Summing up the probabilities yields

$$\mathbb{P}(\mathit{sneeze}, \neg \mathit{cold}, \mathit{scratches}) = 0.007082.$$

Now, the conditional probability (as previously mentioned) can be calculated as

$$\begin{aligned}\mathbb{P}(\mathit{allergic} \mid \mathit{sneeze}, \neg \mathit{cold}, \mathit{scratches}) &= \frac{\mathbb{P}(\mathit{allergic}, \mathit{sneeze}, \neg \mathit{cold}, \mathit{scratches})}{\mathbb{P}(\mathit{sneeze}, \neg \mathit{cold}, \mathit{scratches})} \\ &= \frac{0.006616}{0.007082} \\ &= 0.934210.\end{aligned}$$

So the probability that someone is allergic, given that they sneezed but do not have a cold (and it is not known whether a cat has been present) and there are scratches present is approximately 0.934210.

3. PDDL and Situation Calculus

Starting a car: You have to be at the car and have the key, and the car has to have a charged battery and the tank has to have gas. Afterwards, the car will be running, and you will still be at the car and have the key after starting the engine.

(a) Write a PDDL operator to describe this action.

(note: you can express this ego-centrically – you don't have to refer explicitly to the person starting the car; but the operator should take the car being started as an argument)

(b) Describe the same operator using Situation Calculus (remember to add a situation argument to your predicates).

(c) Add a Frame Axiom that says that starting this car will not changes whether any other car is out of gas (tank empty).

Solution:

(a) The PDLL operator for this action can be expressed as follows:

$Action(StartCar(car, key))$,
PRECOND: $At(car) \wedge HasKey(key) \wedge BatteryCharged(car)$
$\wedge\, hasGas(car)$
EFFECT: $Running(car)$

(b) The Situation Calculus describing the $StartCar$ action can be decribed as follows:

$$\begin{aligned}\forall s, c, k\ [&(At(c,s) \wedge HasKey(k,s) \wedge BatteryCharged(c,s) \wedge hasGas(c,s)) \\ &\implies (Running(c, do(StartCar(c,k),s)) \wedge At(c, do(StartCar(c,k),s)) \\ &\wedge HasKey(k, do(StartCar(c,k),s)))]\end{aligned}$$

(c) Below is the added Frame Axiom specifying this information.

$$\forall s, c, k, c'\ [(c' \neq c) \implies (OutOfGas(c', s) \iff OutOfGas(c', do(StartCar(c,k),s)))]$$