

# WAVELET SCATTERING FOR AUTOMATIC CHORD ESTIMATION

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## ABSTRACT

State-of-the-art automatic chord recognition systems rely on multi-band chroma representations, Gaussian Mixture Model pattern matching, and Viterbi decoding. This paper explores the use of Haar wavelet transforms and scattering in place of multi-band chroma. Wavelets operating across octaves encode sums and differences in chroma bins at different scales. We describe both the Haar wavelet transform and deep wavelet scattering and develop an efficient algorithm for their computation. Potential benefits of wavelet representations, including stability to octave deformations, over multi-band chroma are discussed. Accuracy of wavelet representations used for chord recognition is analyzed over a large vocabulary of chord qualities.

## 1. INTRODUCTION

Along with lyrics and melody, chord sequences provide a succinct description of tonal music. As such, they are often written down under the form of lead sheets, for the use of accompanists and improvisers. Besides its original purpose in music education and transmission, the knowledge of harmonic content has been leveraged in music information research to address higher-level tasks, including cover song identification [5], genre recognition [11], and lyrics-to-audio alignment [9]. We refer to the review of McVicar et al. [10] for a recent state of the art.

As stressed by Humphrey and Bello [6], chord labels are not mutually exclusive categories, but instead follow a relation of partial order. For instance, the tetrad  $A:min7$  is contained in the triad  $A:min$ , which in turn is contained in the power chord  $A:5$ . By setting up equivalence rules for chord labels that belong to a common superset, discrepancies between conflicting annotations can be resolved up to a desired level of specificity. In spite of this variety of settings, all evaluation metrics for automatic chord estimation share the following minimal property: a chord label remains the same if all its components are jointly transposed by one octave, be it upwards or downwards.

In order to comply with this requirement, the vast majority of existing systems rely on the chroma representation, i.e. a 12-dimensional vector derived from the



**Figure 1.** Three possible voicings of the pitch class set  $\{C, E, G, A\}$ , resulting either in the chord  $A:min7$  or  $C:maj6$ . See text for details.

constant-Q spectrum by summing up all frequency bands which share the same pitch class, according to the twelve-tone equal temperament. However, it should be noted that the chroma representation is not only invariant to octave transposition, but also to any permutation of the chord factors – an operation known in music theory as inversion. Yet, although major and minor triads are unchanged by inversion, some rarer chords, such as augmented triads and minor seventh tetrads, are conditional upon the position of the root.

Figure 1 illustrates the importance of disambiguating inversions when transcribing chords. The first two voicings are identical up to octave transposition of all the chord factors, and thus have the same chord label  $A:min7$ . In contrast, the third voicing is labeled as  $C:maj6$  in root position, although its third inversion would correspond to the first voicing.

With the aim of improving automatic chord estimation under fine-grained evaluation metrics, this article introduces two feature extraction methods that are invariant to octave transposition, yet sensitive to chord inversion. The former consists in computing Haar wavelet transform of the constant-Q spectrum along the octave variable and keeping the absolute values of the resulting coefficients, at all scales and positions. The latter iterates the Haar wavelet modulus nonlinear operator over increasing scales, until reaching the full extent of the constant-Q spectrum. Both methods build upon the software of Cho and Bello [3], which holds state-of-the-art performance on the McGill Billboard dataset [1].

Section 2 describes the multi-band chroma features, as introduced by Cho and Bello, and its integration into a multi-stream hidden Markov model. Section 3 defines the Haar wavelet transform across octaves of the constant-Q spectrum. Section 4 defines the deep Haar scattering transform. Section 5 discusses the efficiency of the presented systems over a dataset of 65 Beatles songs, according to different evaluation metrics and parameter settings.



## 2. MULTI-BAND CHROMA FEATURES

A system for automatic chord estimation typically consists of two stages: feature extraction and acoustic modeling. At the first stage, the audio query is converted into a time series of pitch class profiles, which represent the relative salience of pitch classes according to the twelve-tone equal temperament. At the second stage, each frame in the time series is assigned a chord label among a pre-defined vocabulary. This section presents a multi-stream approach to acoustic modeling, as first introduced by Cho and Bello [3].

The constant-Q transform  $\mathbf{X}[t, \gamma]$  is a time-frequency representation whose center frequencies  $2^{\gamma/Q}$  are in a geometric progression. By setting  $Q = 12$ , the log-frequency variable  $\gamma$  is akin to a pitch in twelve-tone equal temperament. Moreover, the Euclidean division  $\gamma = Q \times j + u$  reveals the octave  $u$  and pitch class  $j$ , which play an essential role in music harmony. In all of the following, we reshape the constant-Q transform accordingly, and keep the notation  $\mathbf{X}[t, q, u]$  for simplicity.

Let us recall that the chroma representation consists in summing up octaves  $u$  at fixed time  $t$  and pitch class  $q$ :

$$\mathbf{Y}[t, q] = \sum_u \mathbf{X}[t, q, u]. \quad (1)$$

To address the disambiguation of chords in an extended vocabulary, Cho and Bello have divided the constant-Q spectrum into  $K$  bands, by means of half-overlapping Gaussian windows along the log-frequency axis [3]. The width  $\sigma$  of the windows is inversely proportional to the desired number of bands  $K$ : it is of the order of one octave for  $K = 8$ , and two octaves for  $K = 4$ . The centers of the windows are denoted by  $\gamma_k$ , where the band index  $k$  ranges from 0 to  $K - 1$ . Consequently, the multi-band chroma features are defined as the following three-way tensor:

$$\mathbf{Y}[t, q, k] = \sum_u \mathbf{X}[t, q, u] \mathbf{w}[Q \times j + u - \gamma_k], \quad (2)$$

where  $\mathbf{w}[\gamma] = \exp(-\gamma^2/(2\sigma^2))$  is a Gaussian window of width  $\sigma$ , centered around zero.

Because some chord transitions are more frequent than others, acoustic modeling is classically achieved with a hidden Markov model (HMM) whose states are estimated as mixtures of multivariate Gaussian probability distributions, i.e. Gaussian mixture models (GMM), in dimension  $Q = 12$ . In order to extend this framework to multi-band chroma features, Cho and Bello have trained  $K$  models in parallel, end-to-end, over each feature map  $k$  of the tensor  $\mathbf{Y}[t, q, k]$ . At test time, the emission probability distributions of each model are aggregated such that they are the predicted outputs of a single state sequence.

The computational complexity of resulting  $K$ -stream HMM grows exponentially with the number of streams  $K$ . However, by assuming synchronicity and statistical independence of the streams, the aggregation boils down to a geometric mean, thus with linear complexity in  $K$ . It must be noted that the geometric mean does not yield a true probability distribution, as it does not sum to one. Yet,

it is of widespread use in speech recognition, due to its simplicity and computational tractability.

## 3. HAAR WAVELET TRANSFORM

All subsequent operations apply to the octave variable  $u$ , and are vectorized in terms of time  $t$  and chroma  $q$ . To alleviate notations, we replace the three-way tensor  $\mathbf{X}[t, q, u]$  by a vector  $\mathbf{x}[u]$ , thus leaving the indices  $t$  and  $q$  implicit.

Dating back to 1909, the Haar wavelet  $\psi$  is a piecewise constant, real function of compact support, consisting of two steps of equal length and opposite values. Within a discrete framework, it is defined by the following formula:

$$\forall u \in \mathbb{Z}, \psi[u] = \begin{cases} \frac{-1}{\sqrt{2}} & \text{if } u = 0 \\ \frac{1}{\sqrt{2}} & \text{if } u = 1 \\ 0 & \text{otherwise} \end{cases} \quad (3)$$

The "mother" wavelet  $\psi[u]$  is translated and dilated by powers of two, so as to produce a family of discrete sequences  $\psi_{j,b}[u] = 2^{j/2} \psi[2^j(u - 2b)]$  indexed by the scale parameter  $j \in \mathbb{N}$  and the translation parameter  $b \in \mathbb{Z}$ . After endowing them with the Euclidean inner product

$$\langle \psi_{j,b}, \psi_{j',b'} \rangle = \sum_{u=-\infty}^{+\infty} \psi_{j,b}[u] \psi_{j',b'}[u], \quad (4)$$

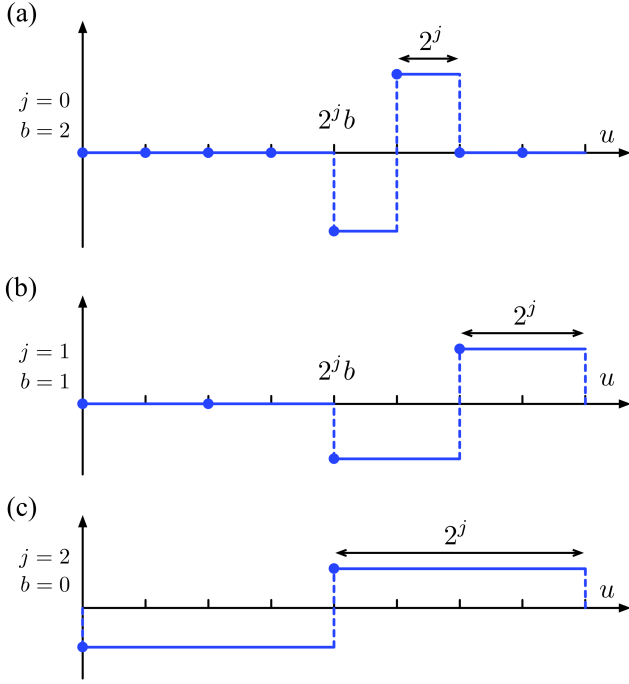
the wavelets  $\{\psi_{j,b}\}_{j,b}$  form an orthonormal basis of finite-energy real sequences. Moreover, the Haar wavelet is the shortest function of compact support such that the family  $\{\psi_{j,b}\}_{j,b}$  satisfies this orthonormality property. On the flip side, it has a poor localization in the Fourier domain, owing to its sharp discontinuities.

It must be noted that, unlike the pseudo-continuous variables of time and frequency, the octave variable is intrinsically discrete, and has no more than 8 coefficients in the audible spectrum. As such, we choose to favor compact support over regularity, i.e. Haar over Daubechies or Gabor wavelets. Elements of the Haar wavelet basis are shown on Figure 2 for various values of  $j$  and  $b$ .

The wavelet transform coefficients of some finite-energy sequence  $\mathbf{x} \in \ell^2(\mathbb{Z})$  are defined by  $\mathbf{W}\mathbf{x}[j, b] = \langle \mathbf{x}, \psi_{j,b} \rangle$ . Since  $\mathbf{x}[u]$  has a finite length  $K = 2^J$ , the above decomposition is informative only for indices  $(j, b)$  such that  $j < J$  and  $2^j b < K$ , that is  $b < 2^{J-j}$ . The number of coefficients in the Haar wavelet transform of  $\mathbf{x}[u]$  is thus equal to  $\sum_{j < J} 2^{J-j} = 2^J - 1$ . For the wavelet representation to preserve energy and allow signal reconstruction, a residual term

$$\mathbf{A}_J \mathbf{x} = \mathbf{x}[0] - \sum_{j,b} \langle \mathbf{x}, \psi_{j,b} \rangle \psi_{j,b}[0] = \sum_{u < K} \mathbf{x}[u] \quad (5)$$

must be appended to the wavelet coefficients. Observe that  $\mathbf{A}_J \mathbf{x}$  computes a delocalized average of all signal coefficients, which can equivalently be formulated as an inner product with the constant function  $\phi[u] = 2^{-J/2}$  over the support  $\llbracket 0; K \rrbracket$ . Henceforth, it corresponds to the traditional chroma representation, where spectrogram bands of the same pitch class  $q$  are summed across all  $K$  octaves.



**Figure 2.** Three elements of the Haar wavelet basis  $\{\psi_{j,b}\}$  for various values of the scale index  $j$  and the translation index  $b$ . See text for details.

Since the wavelet representation amounts to  $K$  inner products in  $\mathbb{R}^K$ , its computational complexity is  $\Theta(K^2)$  if implemented as a matrix-vector product. Implementing these inner products as convolutions would bring the complexity to  $\Theta(K(\log_2 K)^2)$  by using the Fast Fourier Transform (FFT) algorithm. To improve this, Mallat has developed a recursive scheme, called *multiresolution pyramid* [8], which operates as a cascade of convolutions with some pair of quadrature mirror filters ( $g, h$ ) and progressive subsamplings by a factor of two. Denoting by  $g_{\downarrow 2}$  and  $h_{\downarrow 2}$  the corresponding operators of subsampled convolutions, the wavelet transform coefficients rewrite as

$$Wx[j, b] = (h_{\downarrow 2} \circ g_{\downarrow 2} \circ \dots \circ g_{\downarrow 2} x)[b], \quad (6)$$

while the fully delocalized chroma representation is

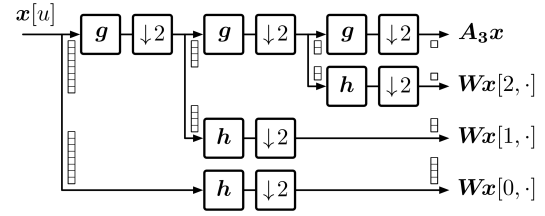
$$A_J x = (g_{\downarrow 2} \circ \dots \circ g_{\downarrow 2} x)[0]. \quad (7)$$

A flowchart of the wavelet transform operations is shown on Figure 3. The interested reader can refer to chapter 7 of Mallat's textbook [7], of which we adopt the mathematical notations.

In the case of Haar wavelets, the low-pass filtering ( $x * g$ ) consists of the sum between adjacent coefficients, whereas the high-pass filtering ( $x * h$ ) is the corresponding difference, up to a renormalization constant:

$$(x * g)[2b] = \frac{x[2b+1] + x[2b]}{\sqrt{2}} = x[2b] - \langle x | \psi_{0,b} \rangle \quad (8)$$

$$(x * h)[2b] = \frac{x[2b+1] - x[2b]}{\sqrt{2}} = \langle x | \psi_{0,b} \rangle. \quad (9)$$



**Figure 3.** Discrete wavelet transform of a signal of length 8, as implemented with a multiresolution pyramid scheme. See text for details.

	operations	memory
Matrix-vector product	$\Theta(K^2)$	$\Theta(K)$
Fast Fourier transform	$\Theta(K(\log K)^2)$	$\Theta(K)$
Multiresolution pyramid	$\Theta(K \log K)$	$\Theta(1)$

**Table 1.** Computational complexity and memory usage of various implementations of the Haar wavelet transform, for a one-dimensional signal of length  $K$ . See text for details.

Besides its improved computational complexity, the multiresolution pyramid scheme has the advantage of being achievable without allocating memory. Indeed, at every scale  $j^{\text{th}}$ , the pair  $(g_{\downarrow 2}, h_{\downarrow 2})$  has  $2^{-j}K$  inputs and  $2^{-j}K$  outputs. By performing the sum and differences in place, and deferring the renormalization to the end of the flowchart, the feature extraction step involved by the wavelet transform procedure can be made extremely efficient.

#### 4. DEEP HAAR SCATTERING

Two neighboring samples  $x[2b]$  and  $x[2b+1]$  can be retrieved, up to a permutation, from the absolute values of the subsampled QMF decomposition  $(g_{\downarrow 2}x, h_{\downarrow 2}x)$ :

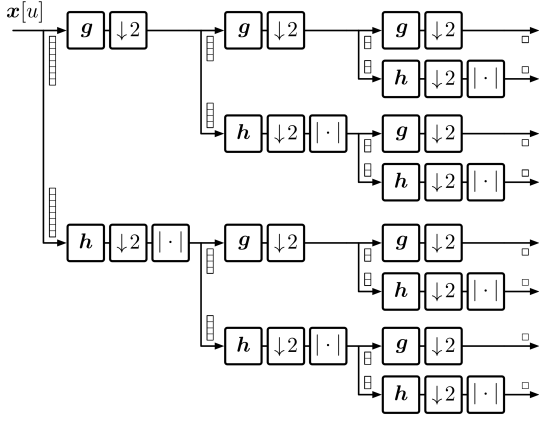
$$\max_{u \in \{2b, 2b+1\}} x[u] = \frac{|g_{\downarrow 2}x[b]| + |h_{\downarrow 2}x[b]|}{\sqrt{2}} \quad (10)$$

$$\min_{u \in \{2b, 2b+1\}} x[u] = \frac{|g_{\downarrow 2}x[b]| - |h_{\downarrow 2}x[b]|}{\sqrt{2}} \quad (11)$$

As opposed to the Haar wavelet transform representation, a Haar scattering representation concatenates sum and difference terms at different scales and then iterates. From the chroma representation  $\chi_{t,q,z}$ , we reshape to a tensor of dimension  $J+2$  called  $\phi$ :

$$\chi_{t,q,z} \rightarrow \phi_{t,q,\sigma_1,\sigma_2,\dots,\sigma_J} \quad (12)$$

where the size of  $\phi$  is  $T \times Q \times 2_1 \times 2_2 \dots \times 2_J$ . At each scale  $j \in [1, J]$ , we first create a copy of  $\phi$  called  $\hat{\phi}$ , and then concatenate the sum of  $\hat{\phi}$  along scale  $\sigma_j$  and the difference of  $\hat{\phi}$  along  $\sigma_j$ , setting the result as  $\phi$ . We then take the modulus  $|\phi|$  and iterate.



**Figure 4.** Deep scattering transform of a signal of length 8, as implemented with a convolutional pyramid scheme. See text for details.

	operations	memory
Matrix-vector product	$\Theta(K^3)$	$\Theta(K^2)$
Fast Fourier transform	$\Theta(K^2(\log K)^2)$	$\Theta(K^2)$
Multiresolution pyramid	$\Theta(K^2 \log K)$	$\Theta(1)$

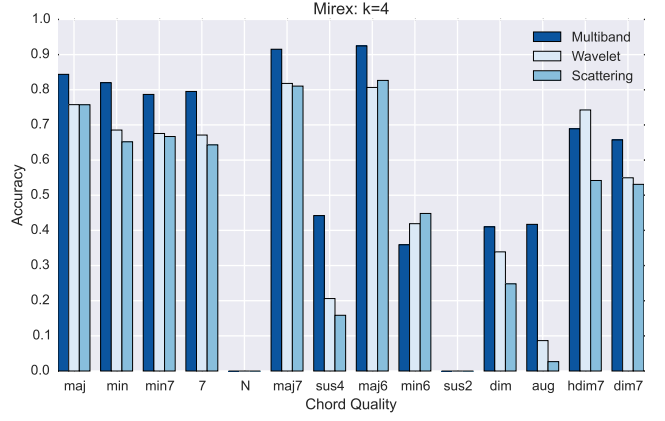
**Table 2.** Computational complexity and memory usage of various implementations of the deep Haar scattering transform, for a one-dimensional signal of length  $K$ . See text for details.

## 5. EXPERIMENTAL SETUP AND EVALUATION

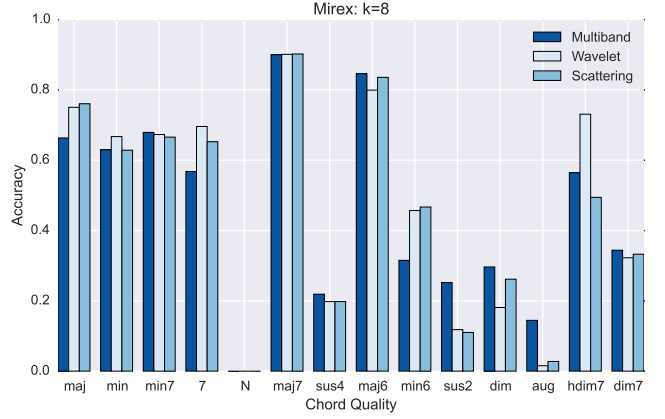
For each experiment, a chord model and Viterbi transition probability matrix are generated from a training set of 451 songs. The ‘band’ ( $K$ ) of any experiment determines the amount of chroma vectors at any given temporal window – equivalent to the number of bands in the multiband chroma representation, and the maximum wavelet scale in the wavelet and scattering representations (i.e. the number of wavelet coefficients). Chord recognition is then carried out on the testing set of 65 songs. Both the training set and testing set of songs are kept constant across all experiments.

After generating estimated chord labels for each song in the test set, a Python script evaluates the results. As per [12], there is “no single right way to compare two sequences of chord labels,” and the *mir\_eval* package computes ACE accuracy based along metrics such as root, triads, maj/min, sevenths, inversions, etc. For simplicity, we evaluate all experiments using the *mirex* measure, which “considers a chord correct if it shares at least three pitch classes in common” [12].

To begin, some baseline trials were run on a training set consisting of 108 songs from the Beatles discography, 99 RWC pop songs, 224 songs from the Billboard dataset, and 20 Queen songs, for a total of 451 songs. Our testing



**Figure 5.** Multiband (chroma), Haar wavelet transform, and deep Haar scattering compared for  $K = 4$  streams. Chord accuracy computed via *mirex*.



**Figure 6.** Multiband (chroma), Haar wavelet transform, and deep Haar scattering compared for  $K = 8$  streams. Chord accuracy computed via *mirex*.

dataset comprised of 65 songs from the Beatles and us-pop datasets that were not part of the training set and that contained a sufficient number of examples of each chord quality.

## 6. RESULTS

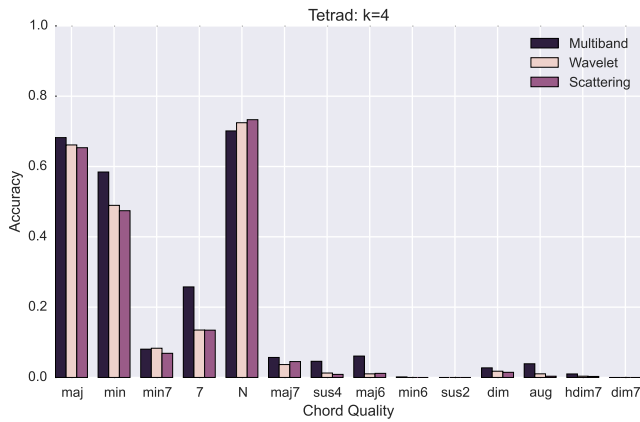
Results and discussion forthcoming.

## 7. DISCUSSION

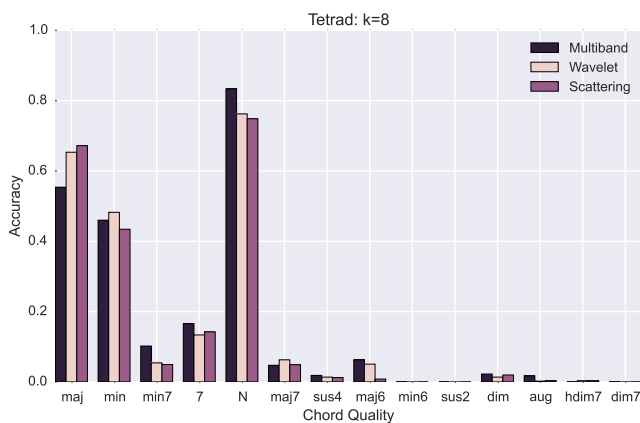
Results and discussion forthcoming.

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**Figure 7.** Multiband (chroma), Haar wavelet transform, and deep Haar scattering compared for  $K = 4$  streams. Chord accuracy computed via tetrad.



**Figure 8.** Multiband (chroma), Haar wavelet transform, and deep Haar scattering compared for  $K = 8$  streams. Chord accuracy computed via tetrad.

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