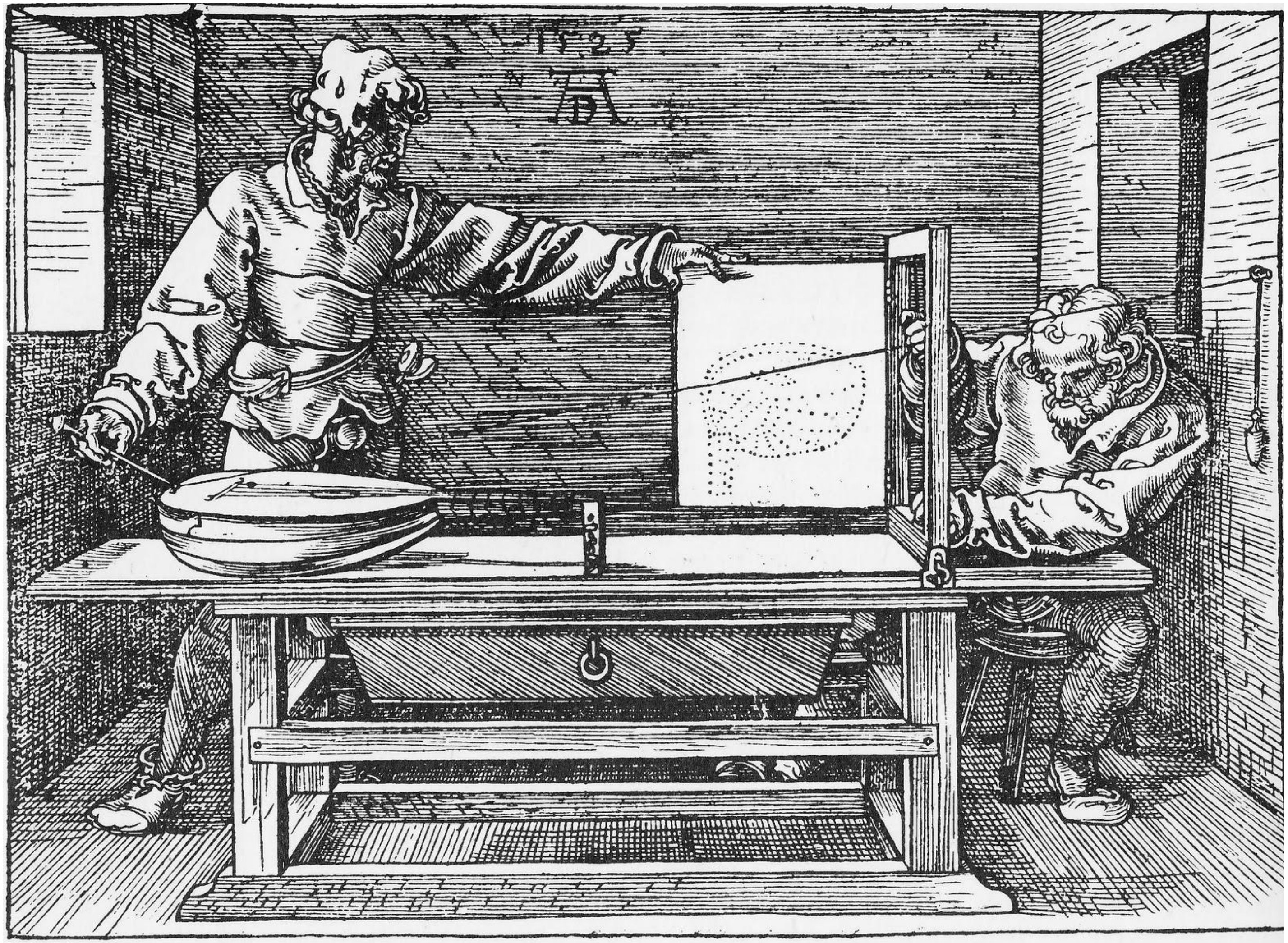
Fundamentals of Image Formation

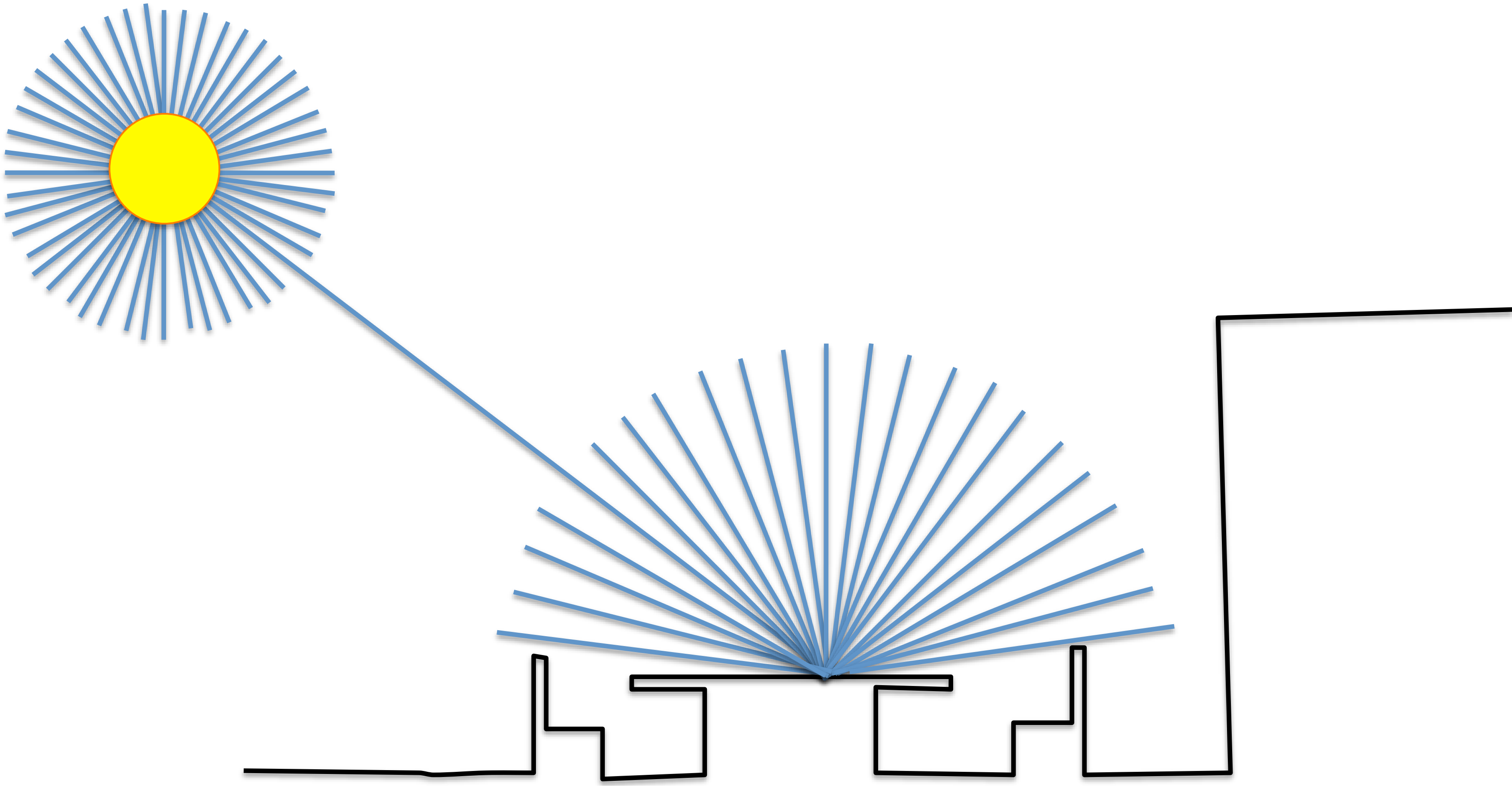


Mechanical creation of a perspective image,
Albrecht Dürer, 1525

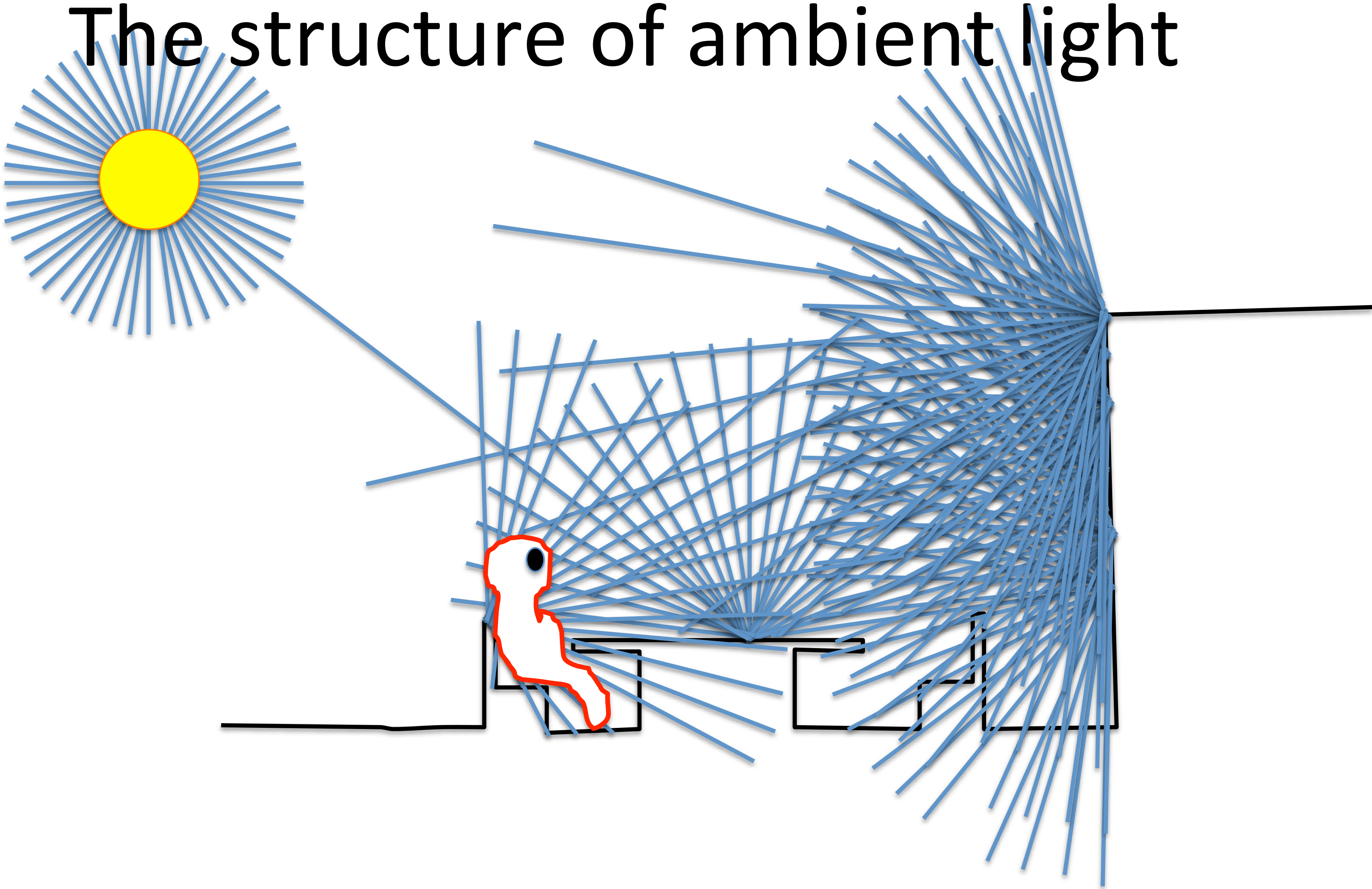
Alexei Efros
CS280, Spring 2024

With slides from Seitz, Malik, Torralba textbook

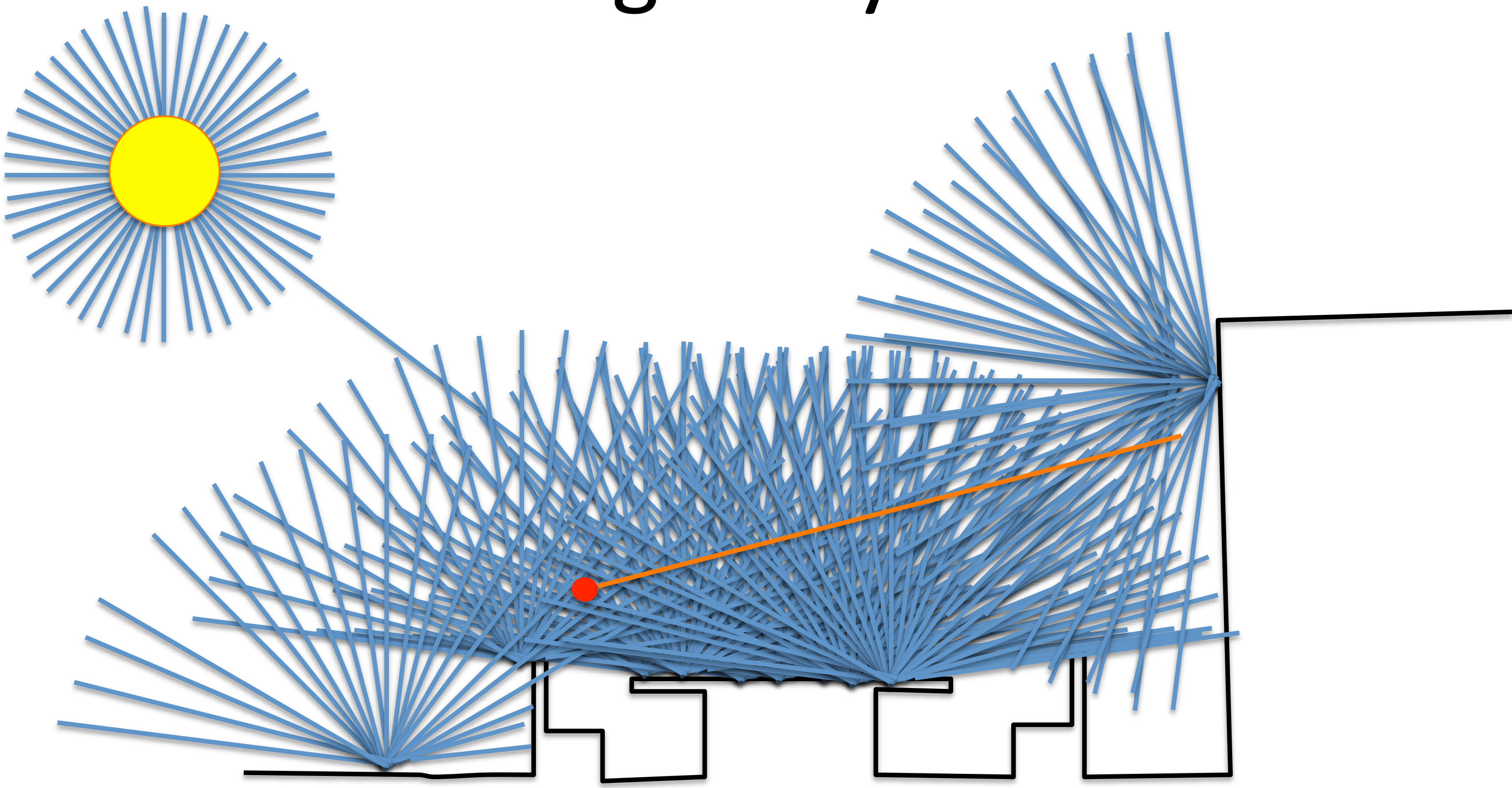
The structure of ambient light



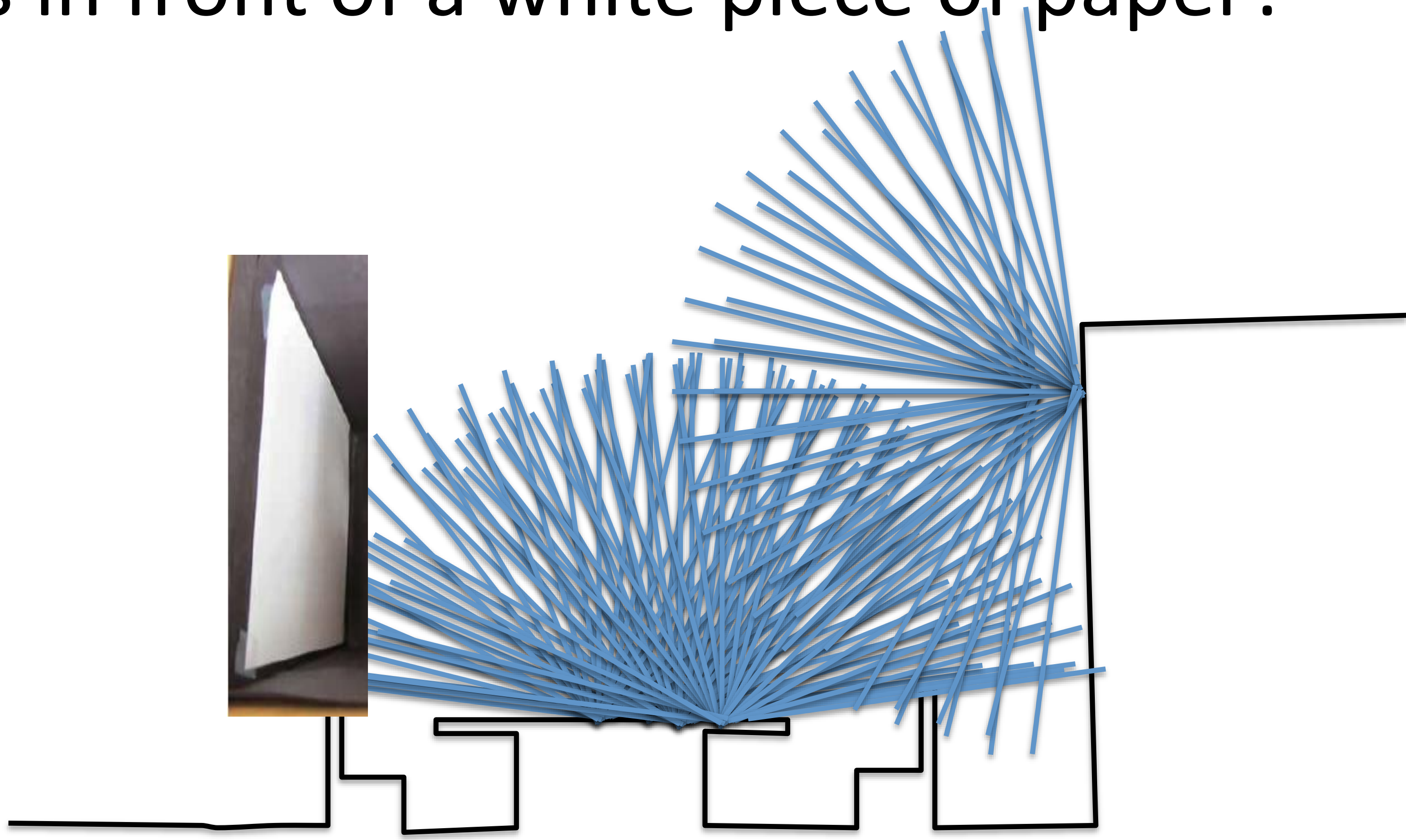
The structure of ambient light



All light rays

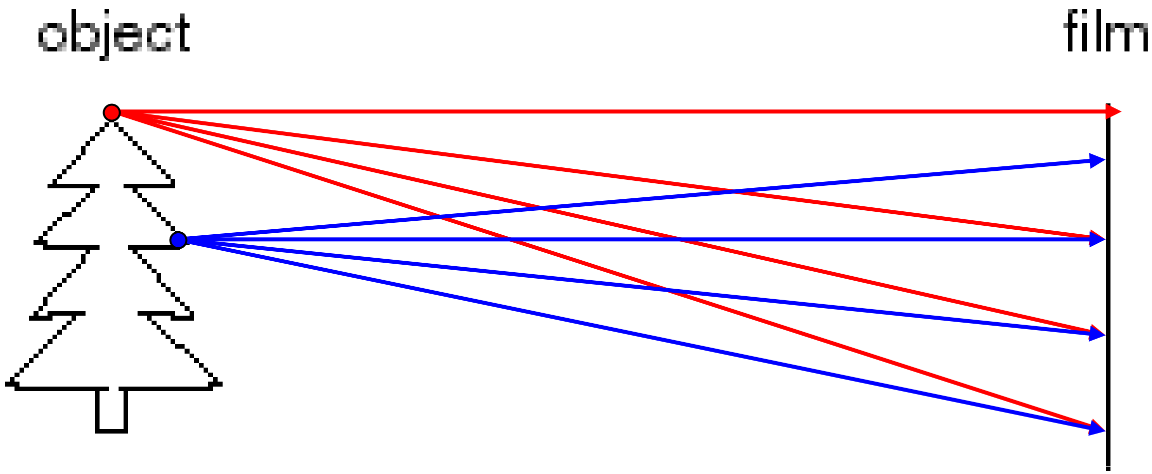


Why don't we generate an image when an object is in front of a white piece of paper?



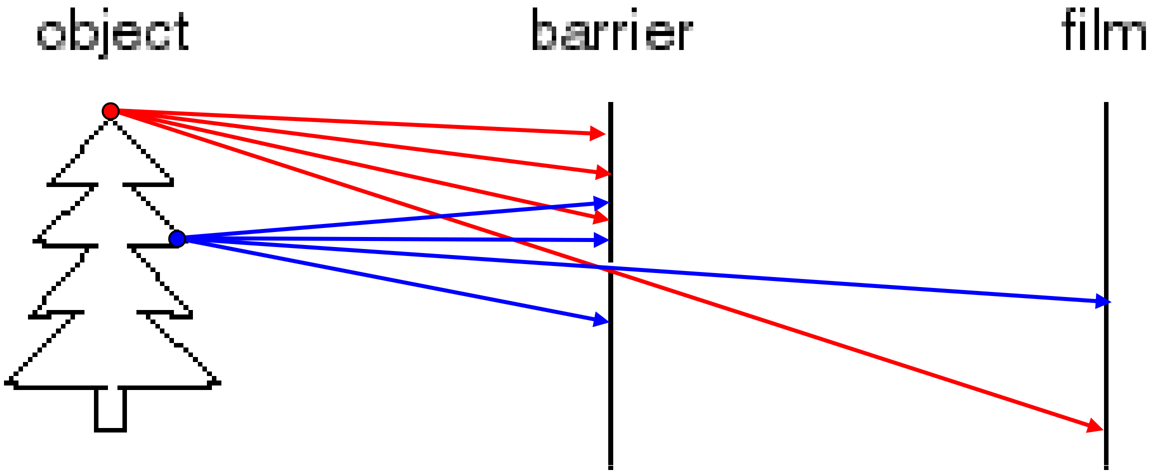
Why is there no picture appearing on the paper?

Let's design a camera



Slide by Steve Seitz

Pinhole camera



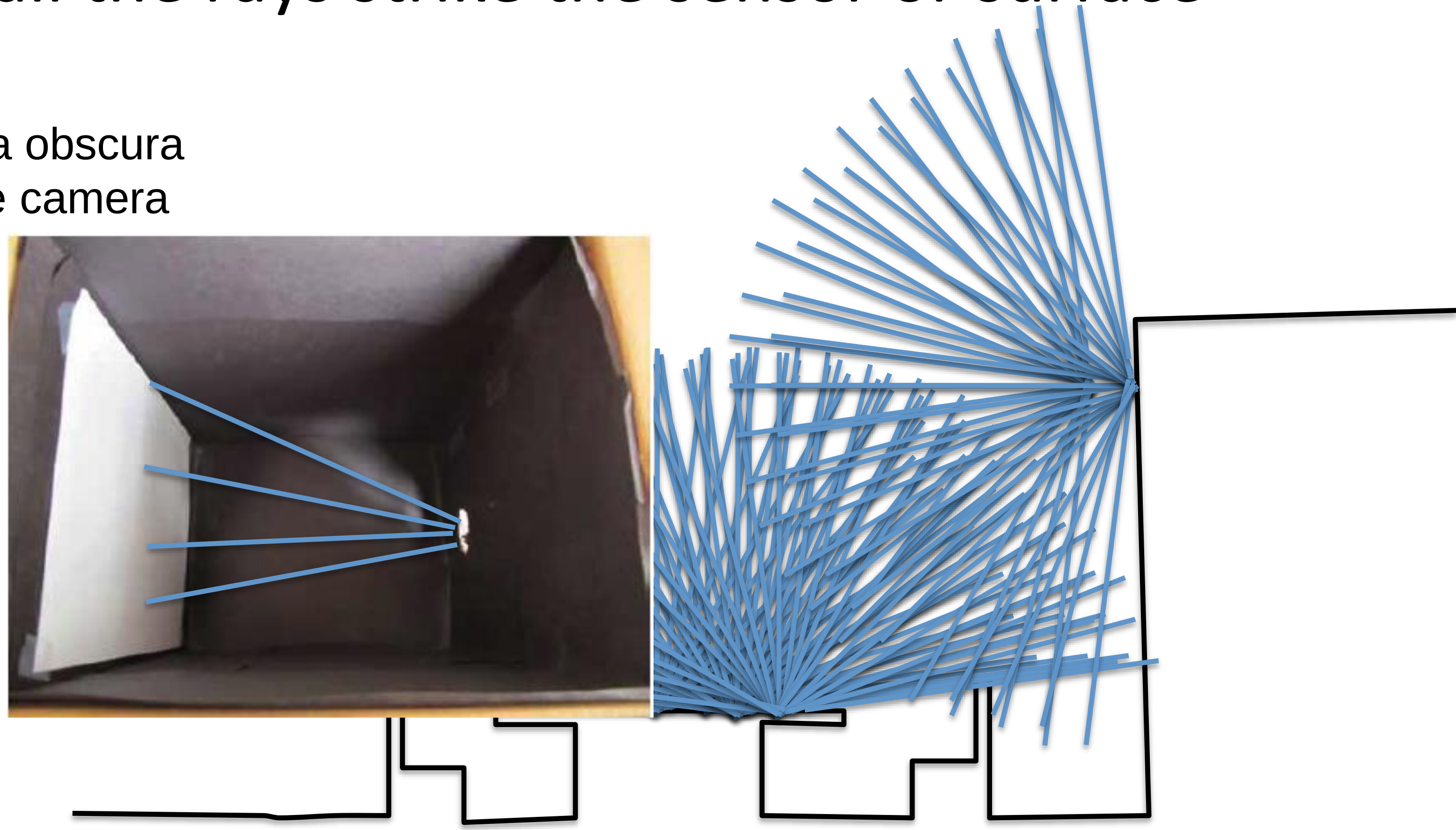
Add a barrier to block off most of the rays

- The opening known as the **aperture**

Slide by Steve Seitz

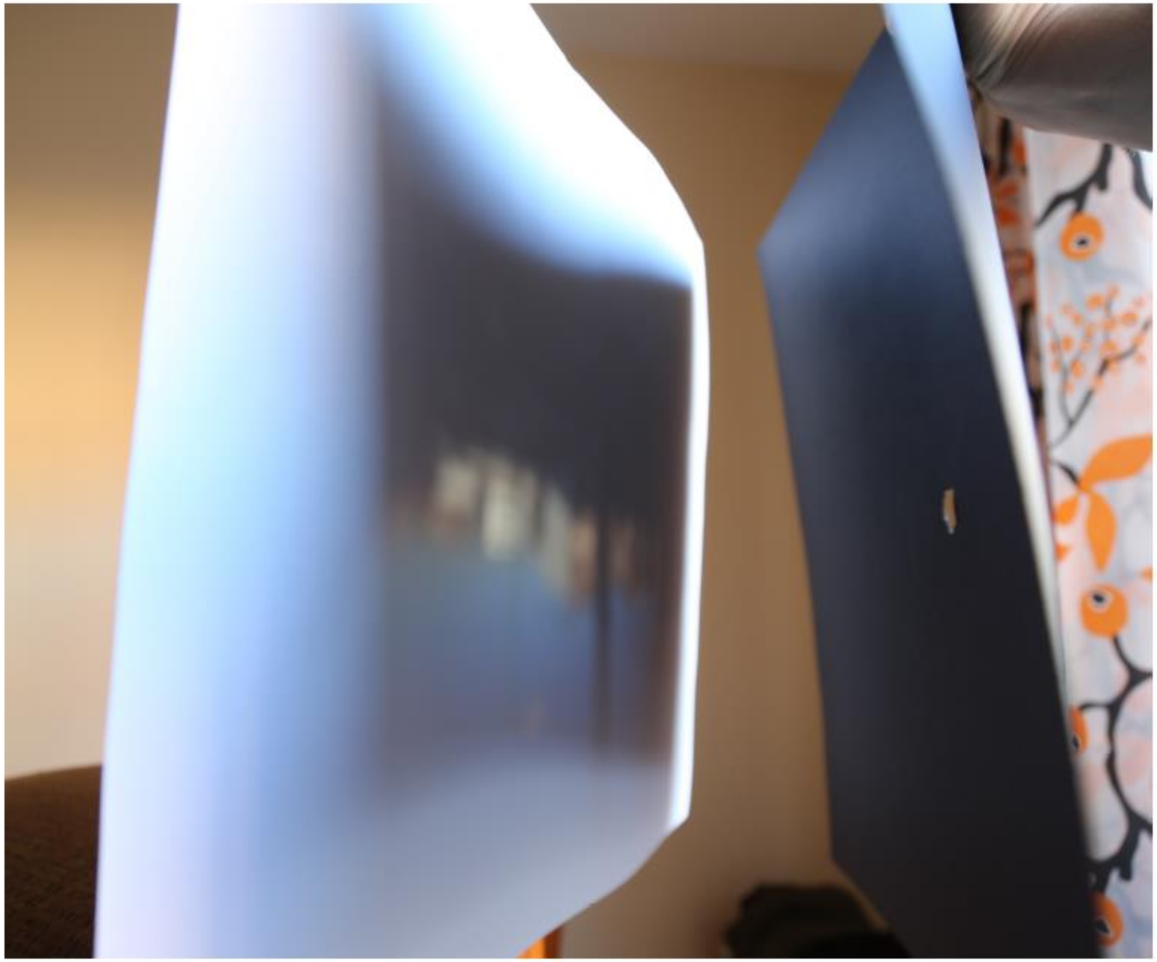
To make an image, we need to have only a subset of all the rays strike the sensor or surface

The camera obscura
The pinhole camera

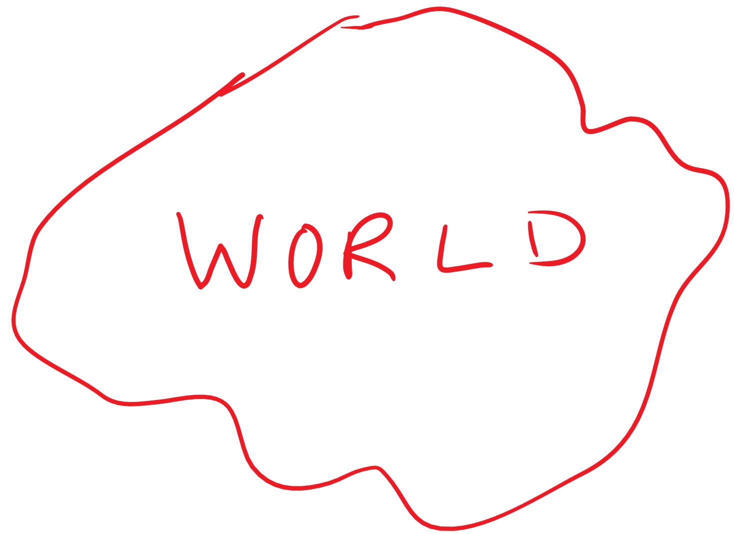


DIY Pinhole Camera

Figure 5.4: A simple setting for creating images on a white piece of paper. In front of the white piece of paper we place another piece of black paper with a hole in the middle. The black paper projects a shadow on the white paper and, in the middle of the shadow, appears a picture of the scene in front of the hole. By making the hole large you will get a brighter, but blurrier image.



A camera creates an image …

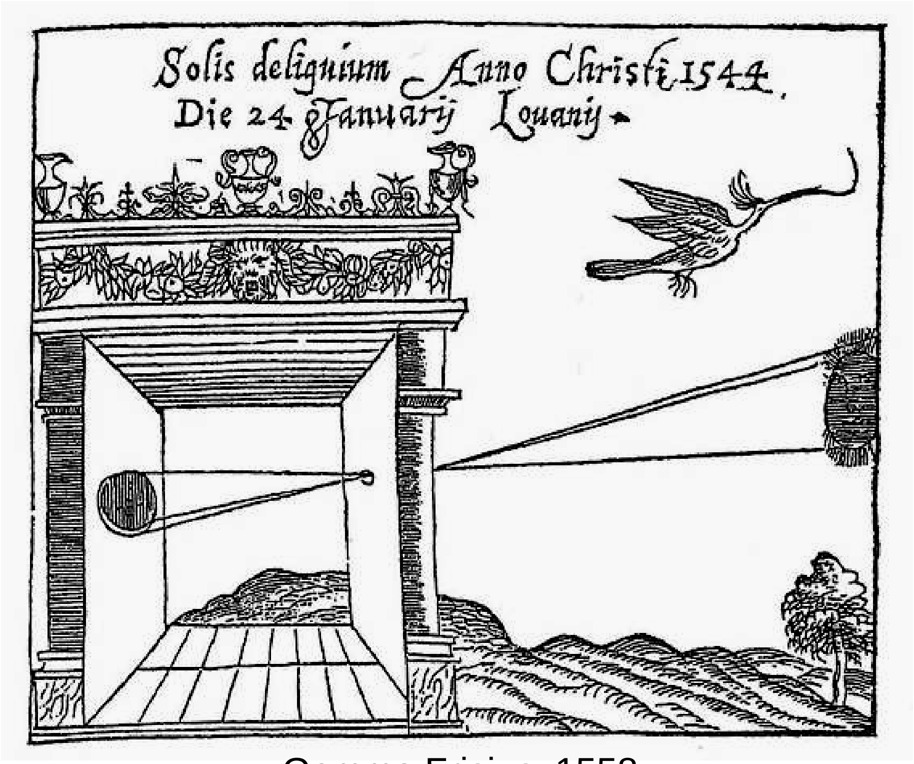


The image I(x,y) measures how much light is captured at pixel (x,y)

We want to know:

- **Where does a point (X,Y,Z) in the world get imaged?**
- What is the brightness at the resulting point (x,y)?

Camera Obscura: a very old idea

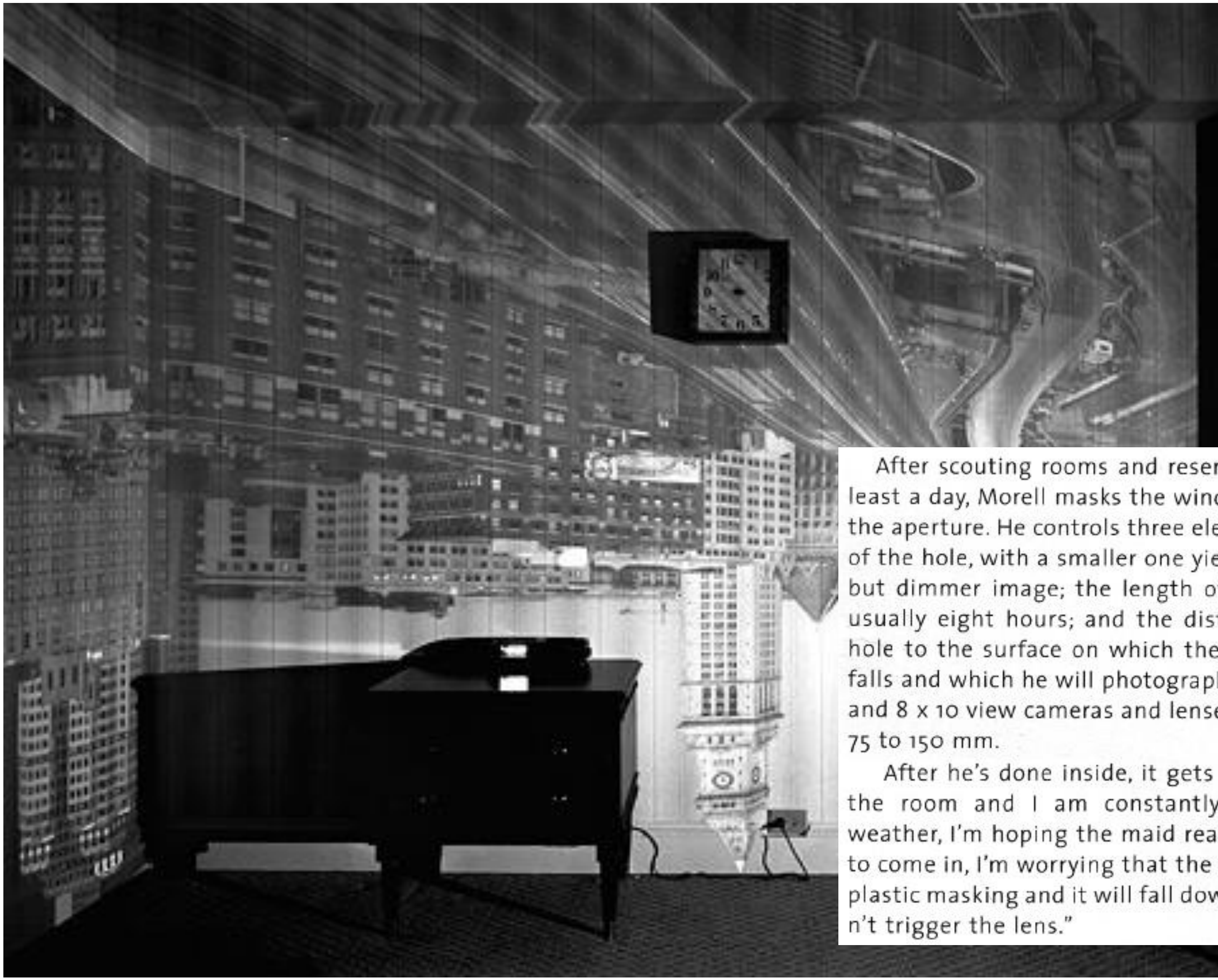


Gemma Frisius, 1558

- First Idea: Mo-Ti, China (470-390 BC)
- First build: Al Hacen, Iraq/Egypt (965-1039 AD)
- Drawing aid for artists: described by Leonardo da Vinci (1452-1519)

Camera Obscura near Cliff House

8-hour exposure (Abelardo Morell)



After scouting rooms and reserving one for at least a day, Morell masks the windows except for the aperture. He controls three elements: the size of the hole, with a smaller one yielding a sharper but dimmer image; the length of the exposure, usually eight hours; and the distance from the hole to the surface on which the outside image falls and which he will photograph. He used 4 x 5 and 8 x 10 view cameras and lenses ranging from 75 to 150 mm.

After he's done inside, it gets harder. "I leave the room and I am constantly checking the weather, I'm hoping the maid reads my note not to come in, I'm worrying that the sun will hit the plastic masking and it will fall down, or that I didn't trigger the lens."

http://www.abelardomorell.net/books/books_m02.html

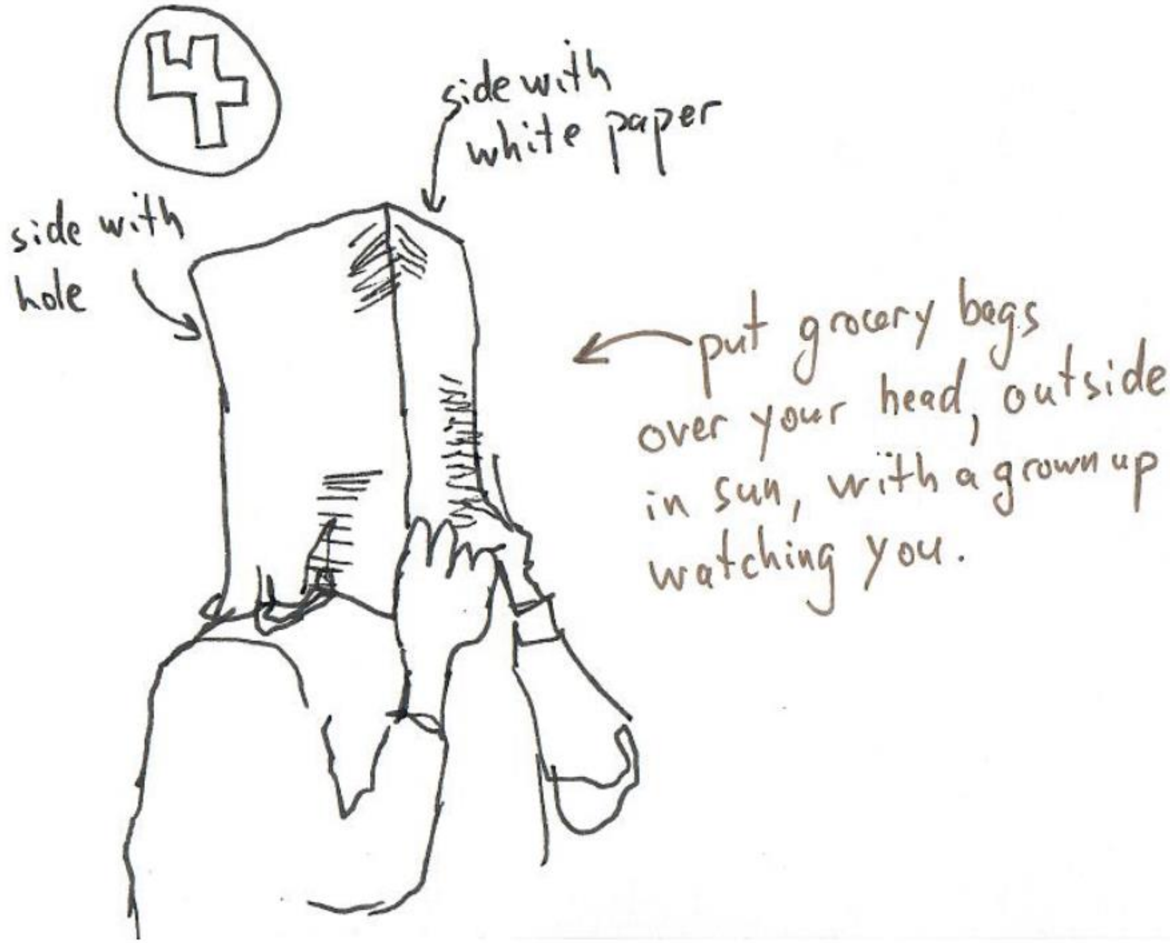
From *Grand Images Through a Tiny Opening*, **Photo District News,** February 2005

“Trashcam” Project

http://petapixel.com/2012/04/18/german-garbage-men-turn-dumpsters-into-giant-pinhole-cameras/

grocery bag pinhole camera

grocery bag pinhole camera



grocery bag pinhole camera

view from outside the bag

http://www.youtube.com/watch?v=FZyCFxsyx8o

view from inside the bag

http://youtu.be/-rhZaAM3F44

me, with GoPro

Recording from GoPro

Accidental pinhole cameras

My hotel room, contrast enhanced.

The view from my window

Accidental pinholes produce images that are unnoticed or misinterpreted as shadows

A. Torralba and W. Freeman, Accidental Pinhole and Pinspeck Cameras, CVPR 2012

Pinhole cameras are everywhere

Figure 3.6: The shadow of this tree on a normal day projects dozens of pictures of the sun on the ground.

Proof!

Tree shad

phot

http://v

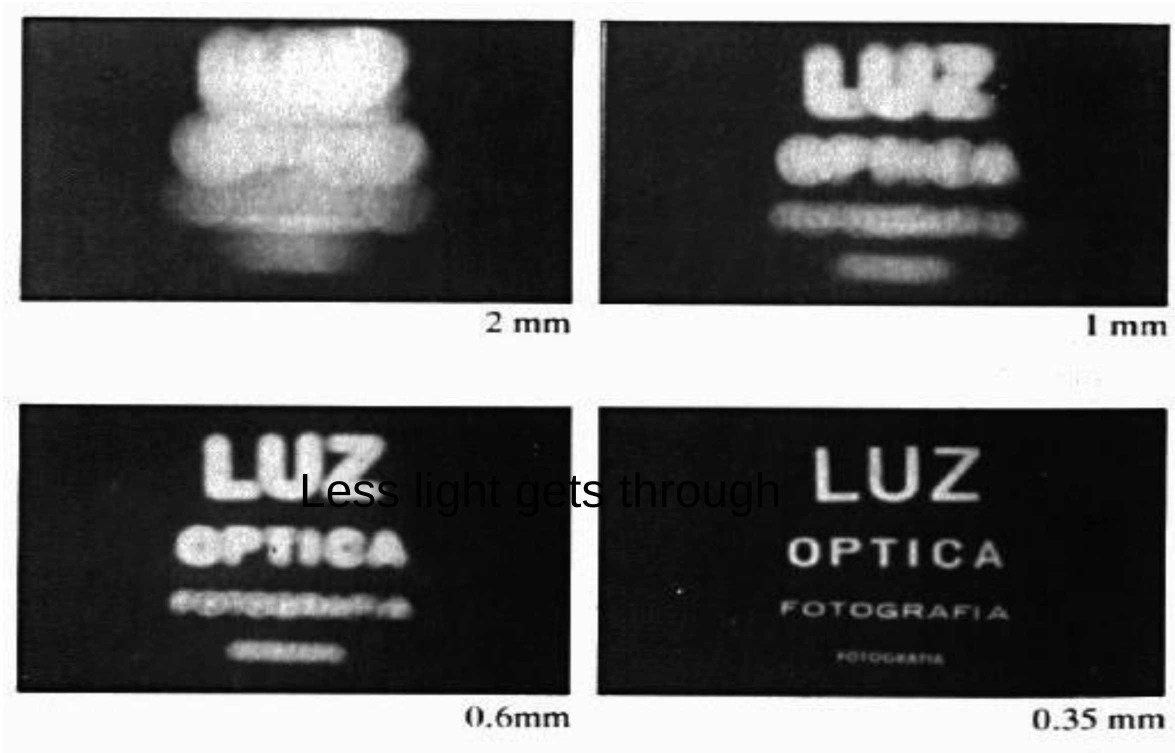
© Trina Singley

Another way to make pinhole camera

Why so blurry?

http://www.debevec.org/Pinhole/

Shrinking the aperture

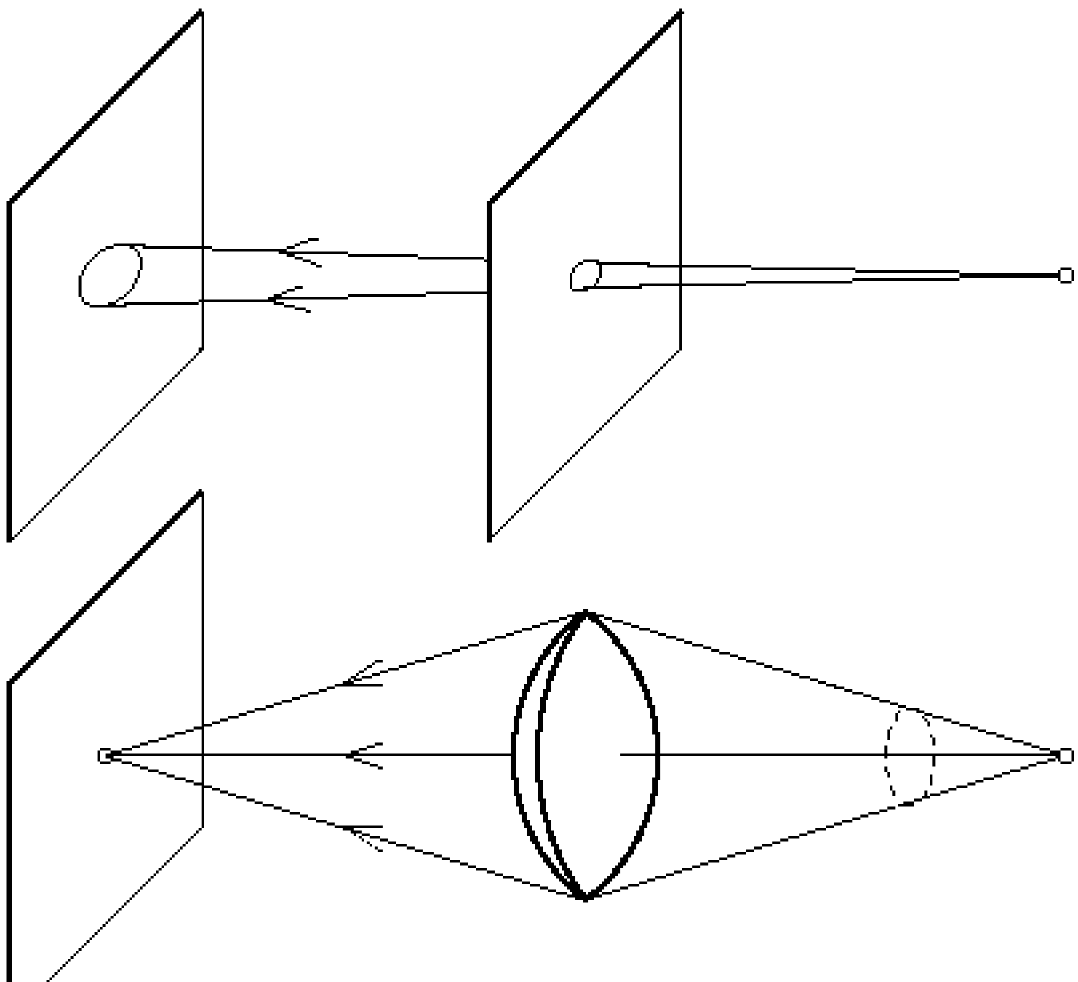


Why not make the aperture as small as possible?

- Less light gets through
- Diffraction effects…

Shrinking the aperture

Aside: the reason for lenses

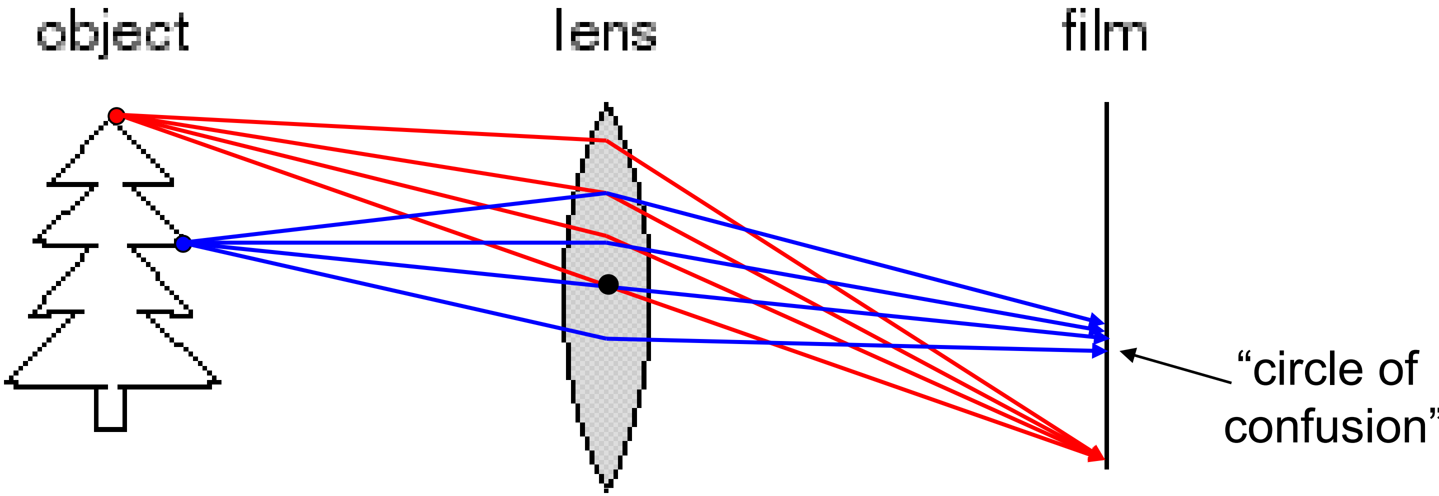


Slide by Steve Seitz

Replacing pinholes with lenses

Photography,
London et al

Aside: Focus and Defocus

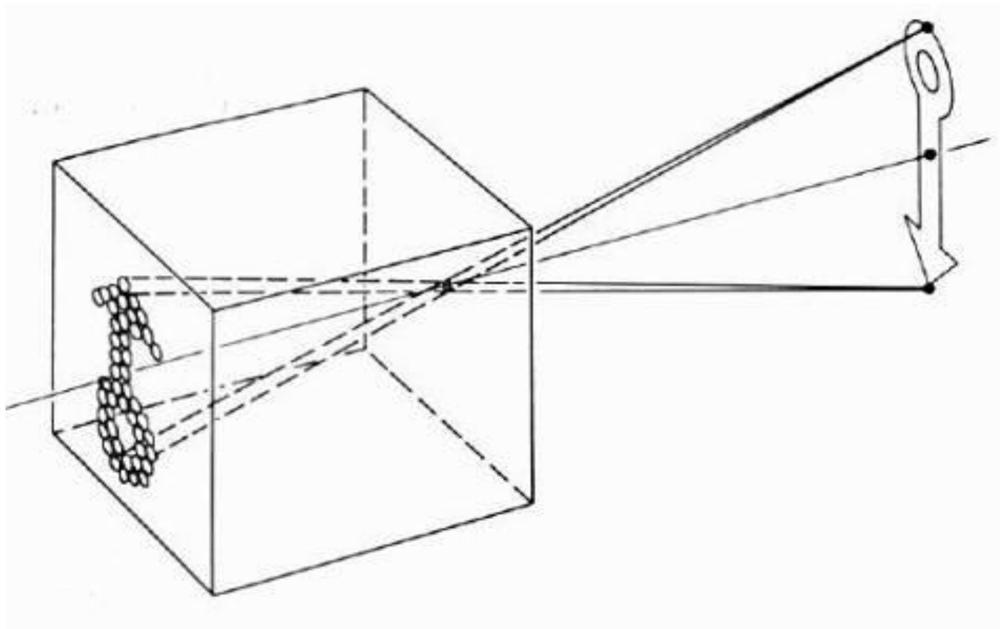


A lens focuses light onto the film

- There is a specific distance at which objects are “in focus”
  – other points project to a “circle of confusion” in the image
- Changing the shape of the lens changes this distance

Slide by Steve Seitz

Back to pinhole camera model

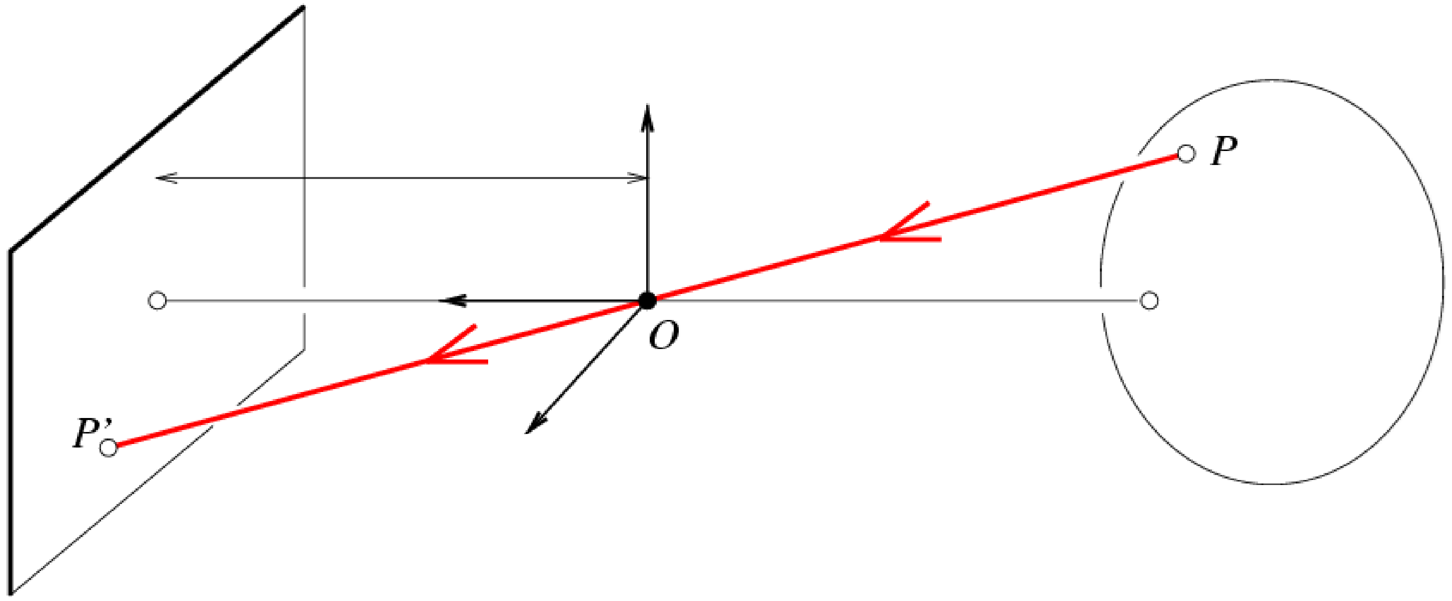


Pinhole model:

- Captures **pencil of rays** – all rays through a single point
- The point is called **Center of Projection (COP)**
- The image is formed on the **Image Plane**
- **Effective focal length *f*** is distance from COP to Image Plane

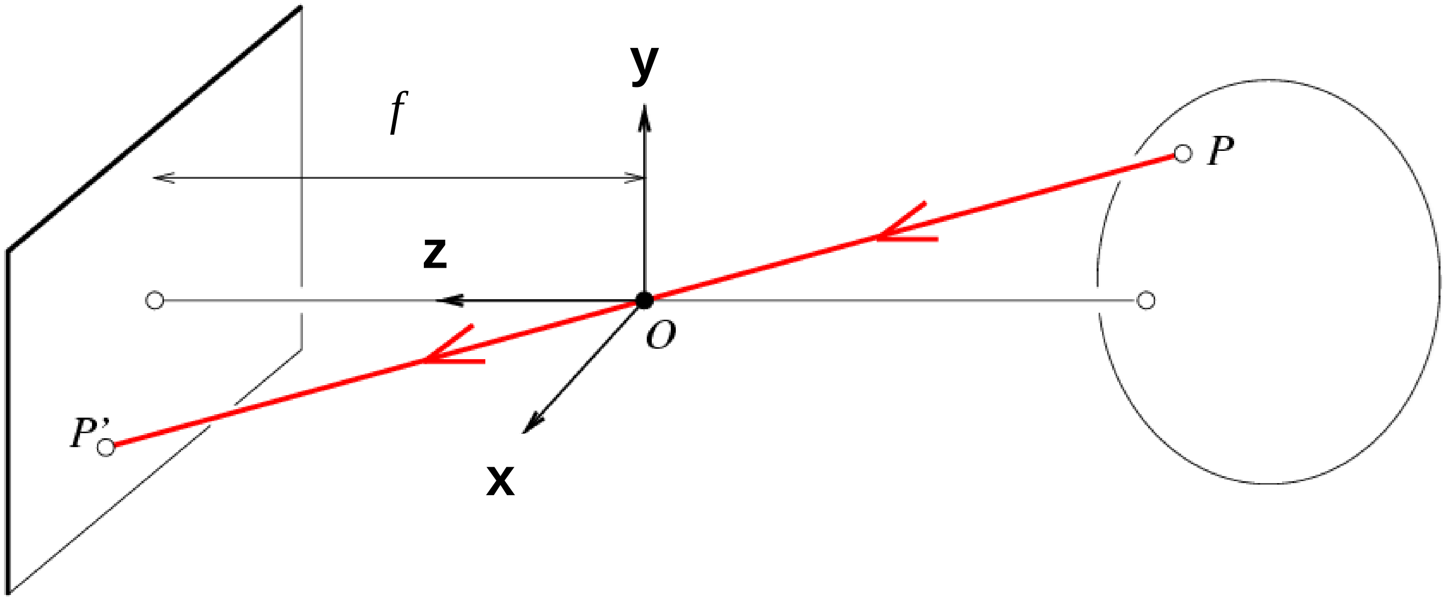
Slide by Steve Seitz

Modeling projection



- To compute the projection *P'* of a scene point *P*, form the **visual ray** connecting *P* to the camera center *O* and find where it intersects the image plane
  - All scene points that lie on this visual ray have the same projection in the image
  - Are there scene points for which this projection is undefined?

Modeling projection



The coordinate system

- The optical center (***O***) is at the origin
- The image plane is parallel to xy-plane (perpendicular to z axis)

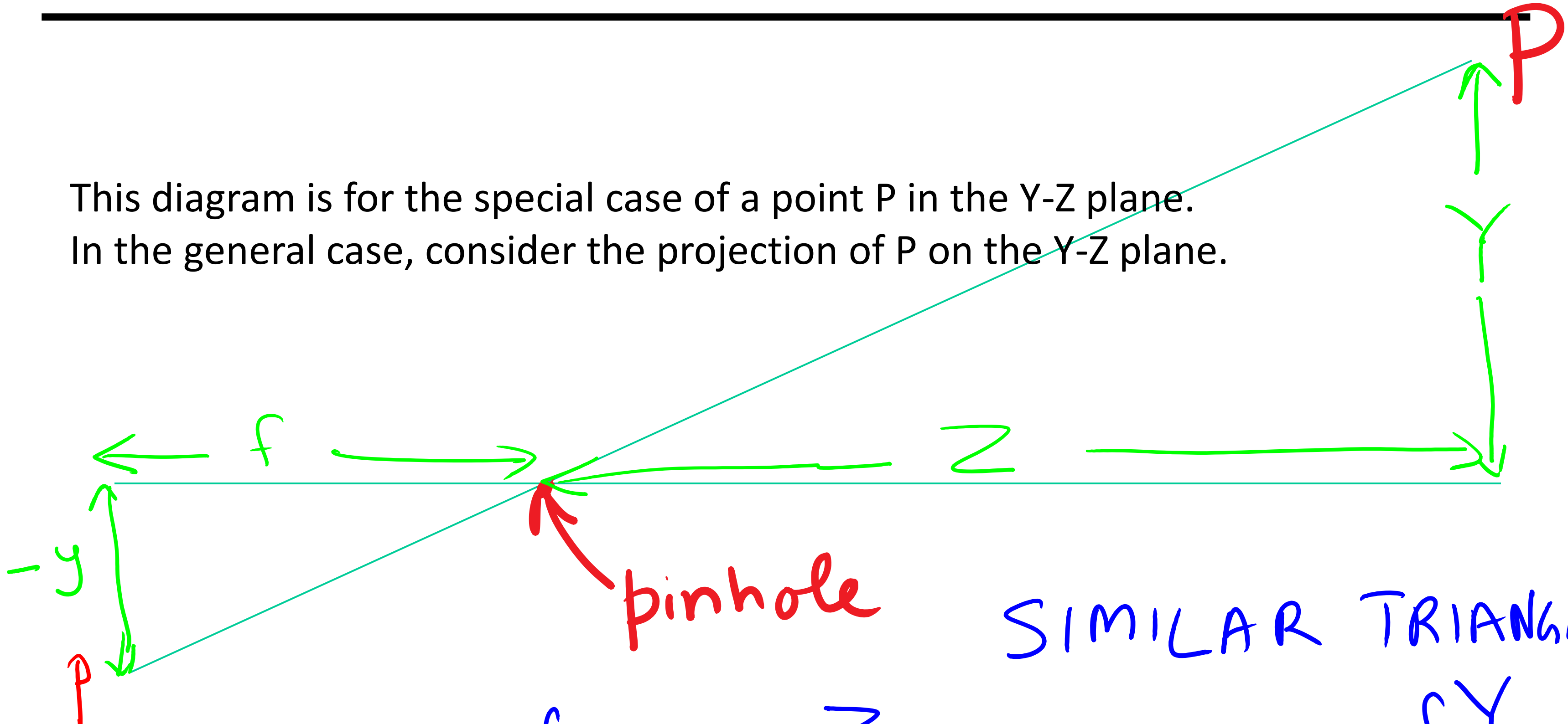
Projection equations

$$x = f\frac{X}{Z}, y = f\frac{Y}{Z}$$

Source: J. Ponce, S. Seitz

Let us prove this …

This diagram is for the special case of a point P in the Y-Z plane.
In the general case, consider the projection of P on the Y-Z plane.



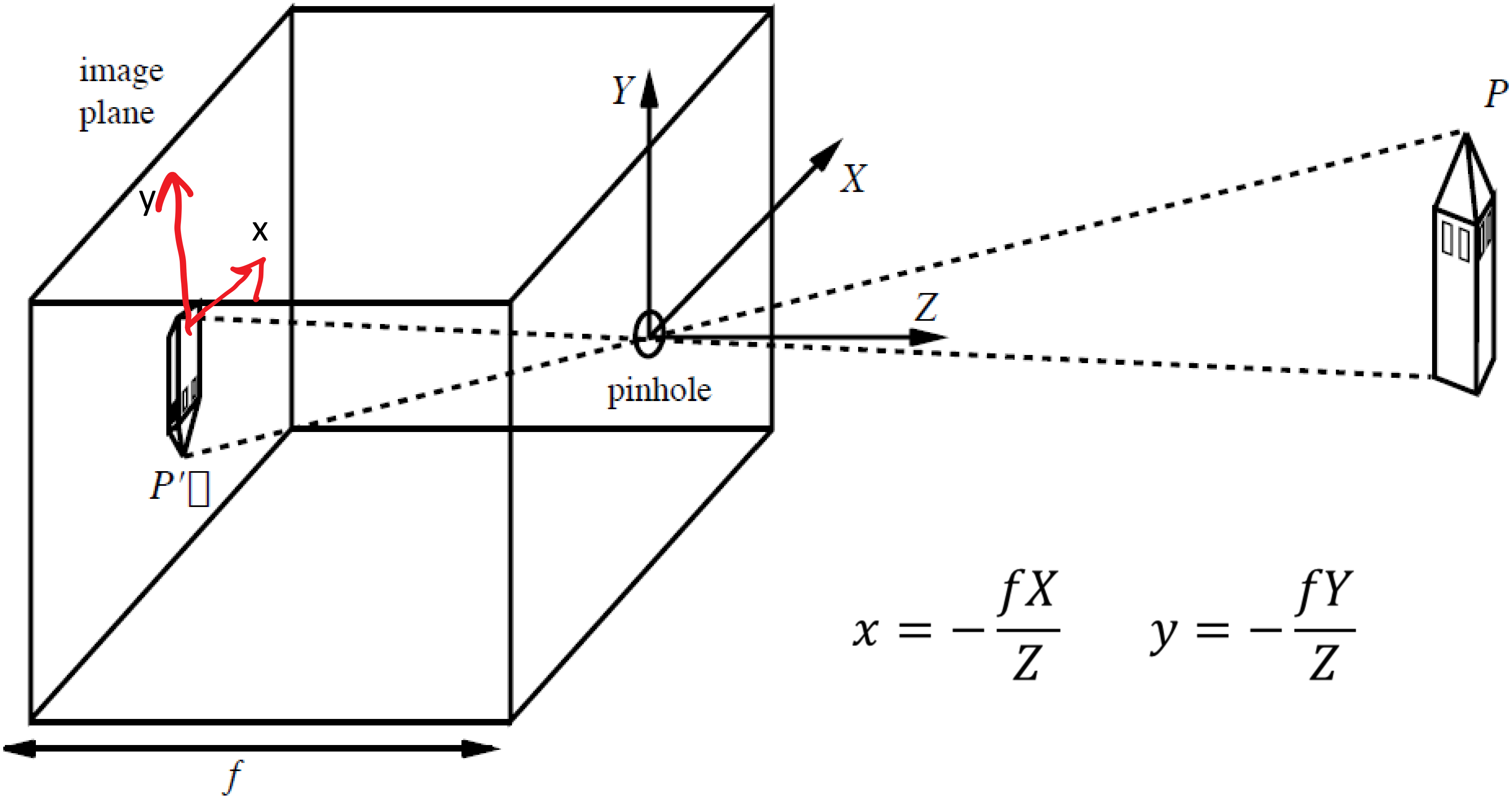
SIMILAR TRIANGLES

$$\frac{f}{-y} = \frac{Z}{Y} \Rightarrow y = \frac{-fY}{Z}$$

This is true even if the point P is not in the YZ plane.

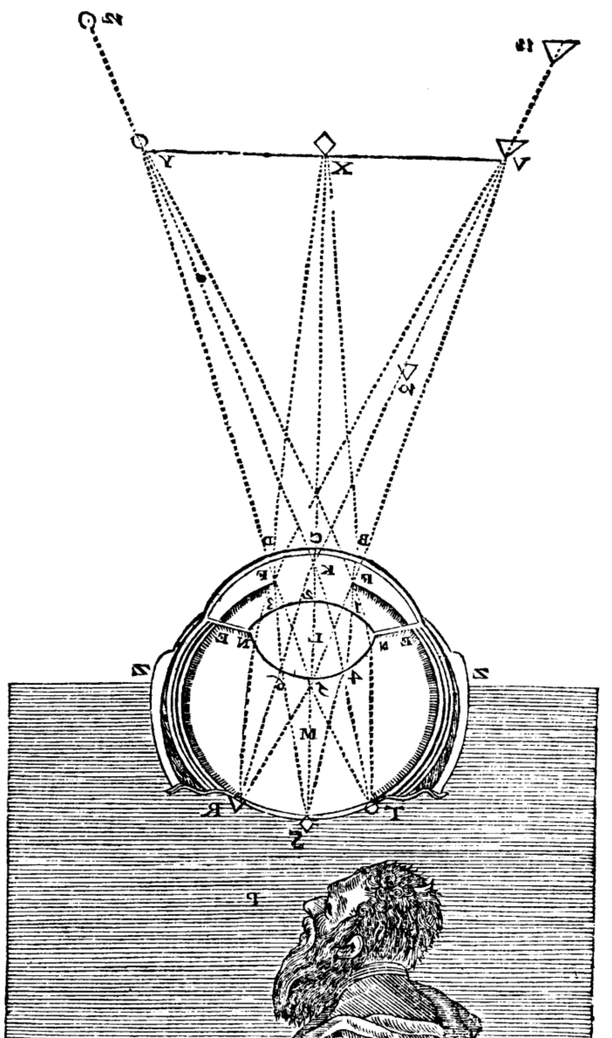
By similar reasoning $x = \frac{-fX}{Z}$

The Pinhole Camera



$$x = -\frac{fX}{Z} \qquad y = -\frac{fY}{Z}$$

The image is inverted



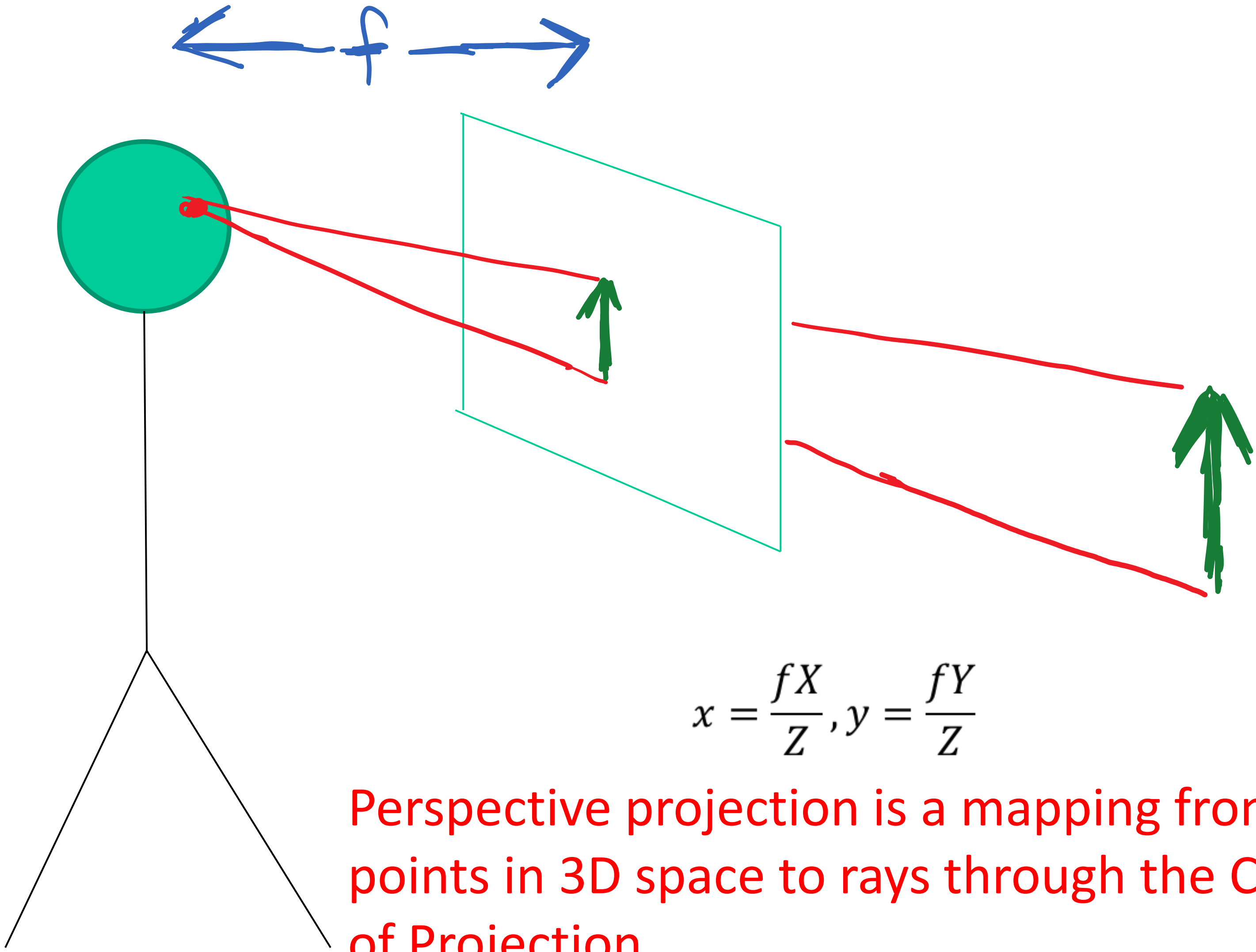
From Descartes(1637), La Dioptrique

This was pointed out by Kepler in 1604

But this is no big deal. The brain can interpret it the right way. And for a camera, software can simply flip the image top-down and right-left. After this trick, we get

$$x = \frac{fX}{Z} \qquad y = \frac{fY}{Z}$$

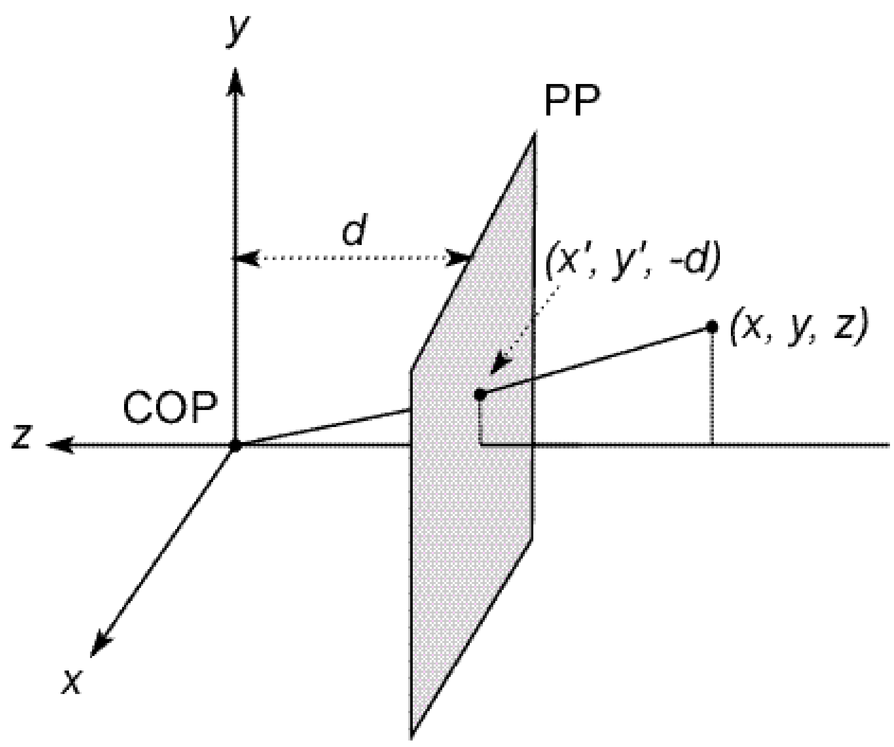
A projection model that avoids inversion



$$x = \frac{fX}{Z}, y = \frac{fY}{Z}$$

Perspective projection is a mapping from points in 3D space to rays through the Center of Projection

Simple trick to avoid inversion

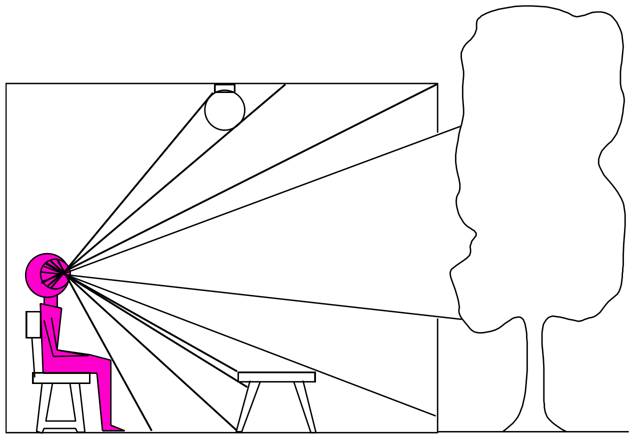


–

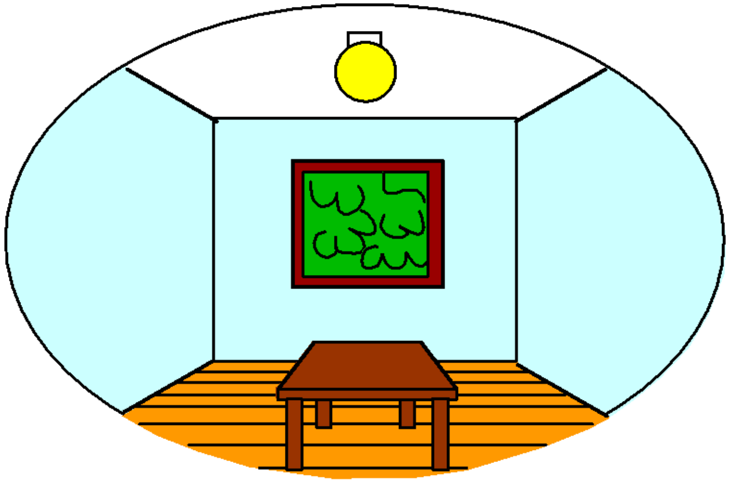
Dimensionality Reduction Machine (3D to 2D)

3D world

2D image



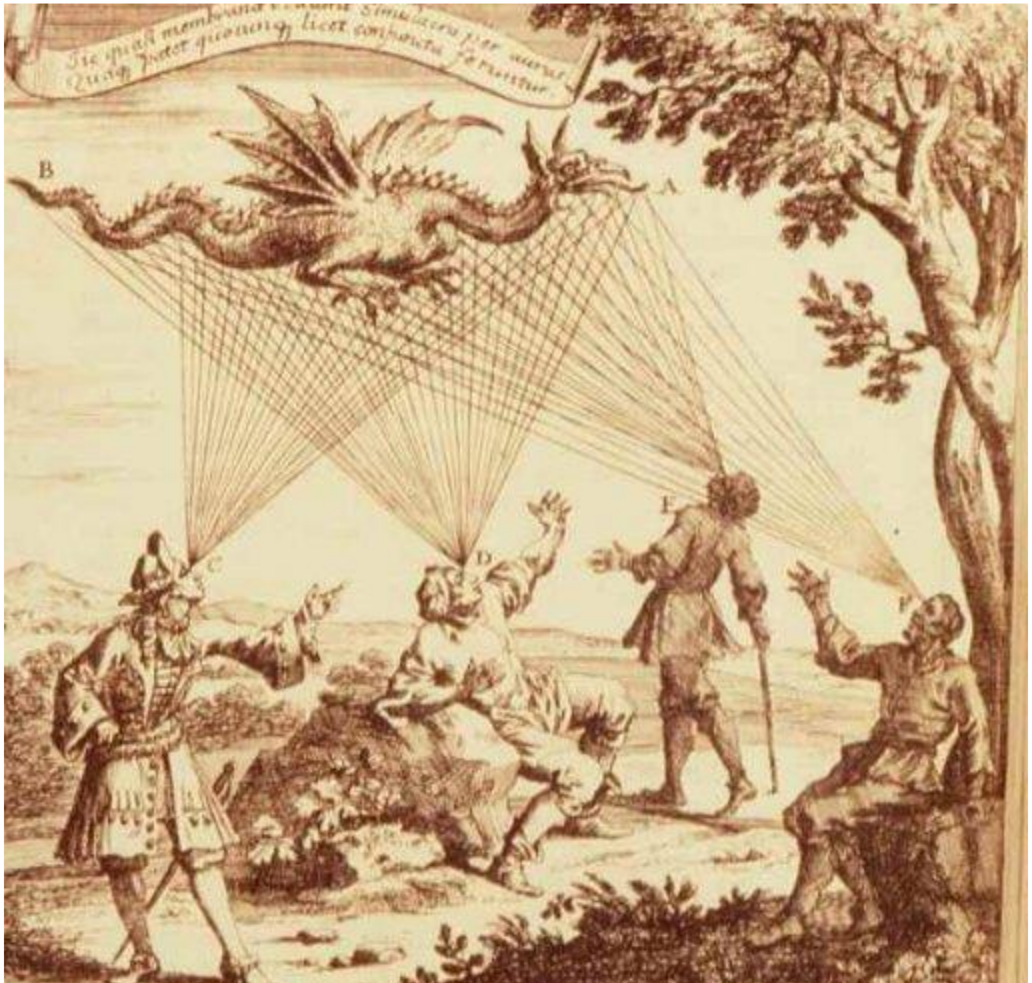
Point of observation



But there is a problem…

Emission Theory of Vision

"For every complex problem there is an answer that is clear, simple, and wrong."
-- H. L. Mencken



Eyes send out "feeling rays" into the world

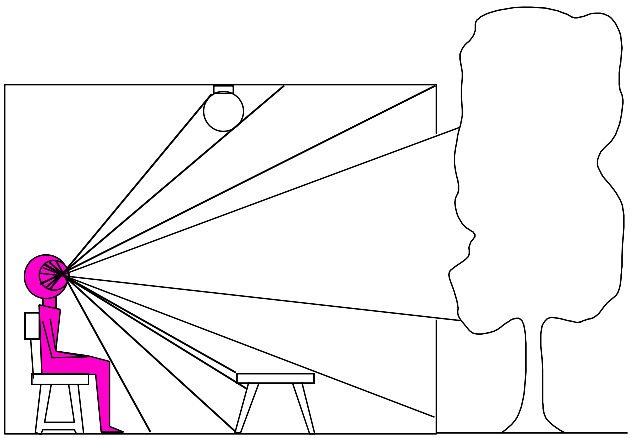
Supported by:

- Empedocles
- Plato
- Euclid (kinda)
- Ptolemy
- …
- 50% of US college students*

*http://www.ncbi.nlm.nih.gov/pubmed/12094435?dopt=Abstract

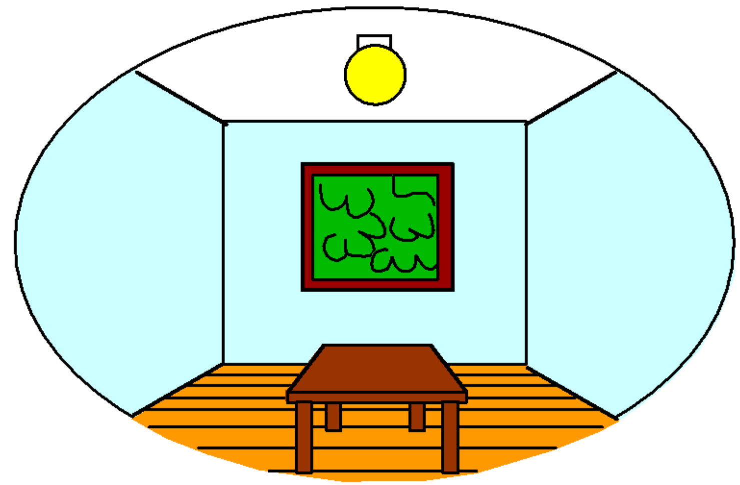
How we see the world

3D world



Point of observation

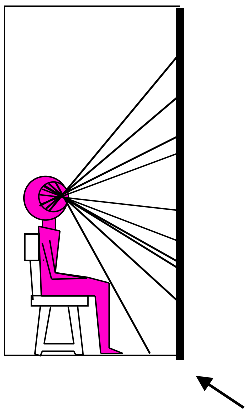
2D image



Figures © Stephen E. Palmer, 2002

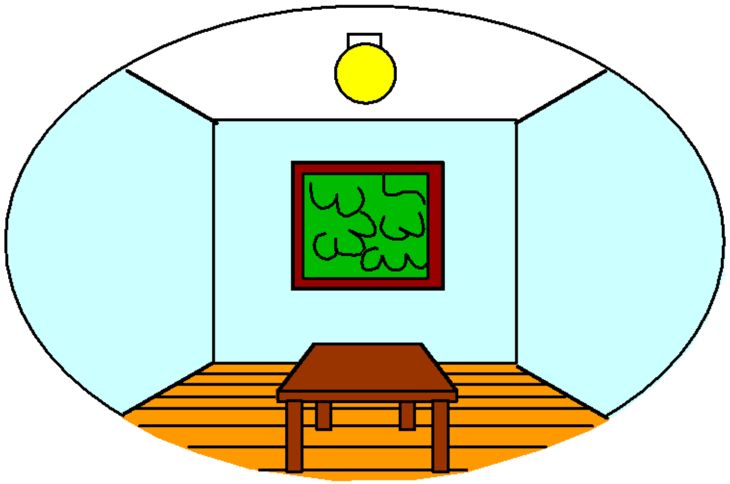
How we see the world

3D world



Painted
backdrop

2D image



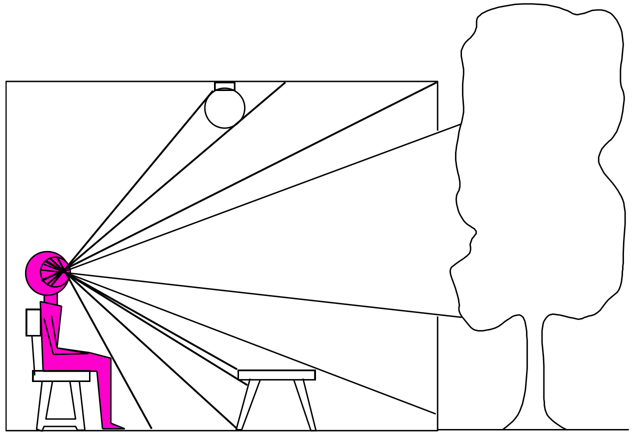
Fooling the eye

Fooling the eye

Making of 3D sidewalk art: http://www.youtube.com/watch?v=3SNYtd0Ayt0

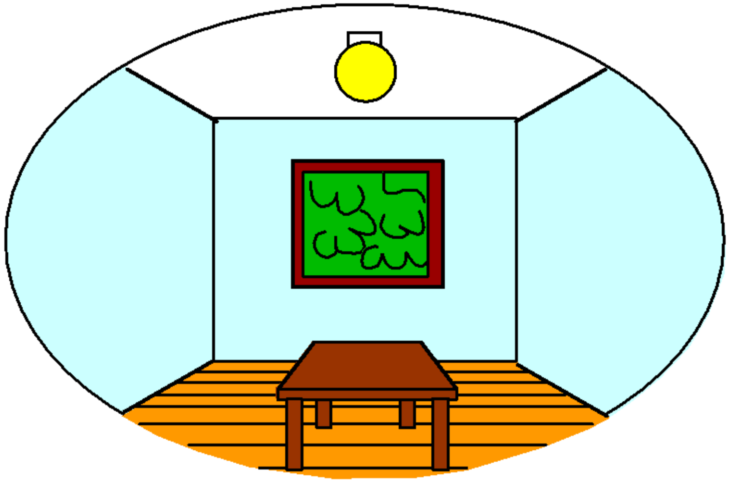
How we see the world

3D world



Point of observation

2D image

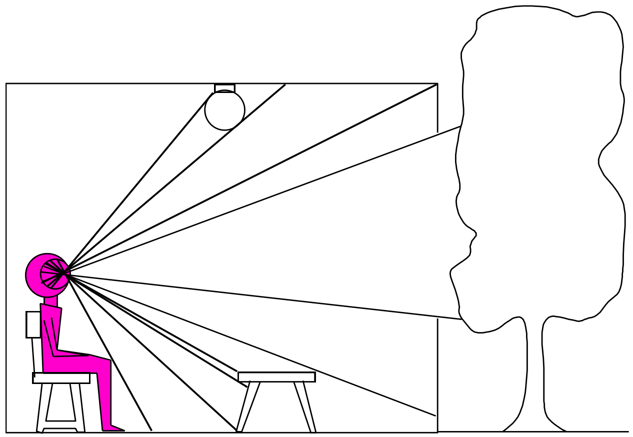


What is being lost?

- Distances (lengths)
- Angles

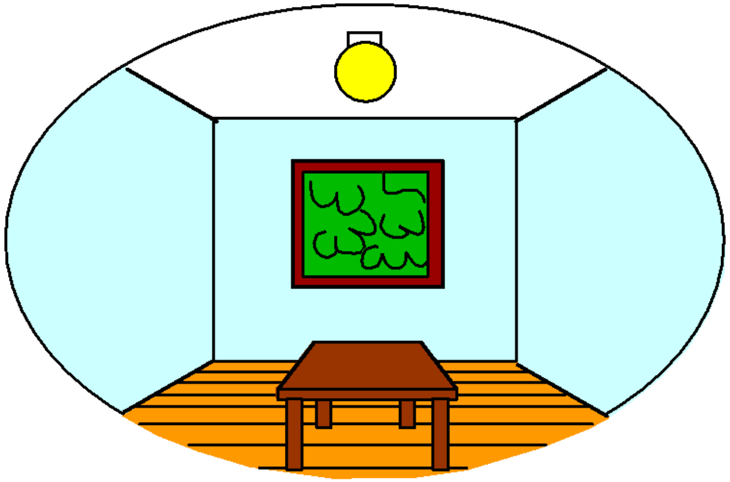
Dimensionality Reduction Machine (3D to 2D)

3D world



Point of observation

2D image



Why did evolution opt for such strange solution?

- Nice to have a passive, long-range sensor
- Can get 3D by moving around & experience

Funny things happen…

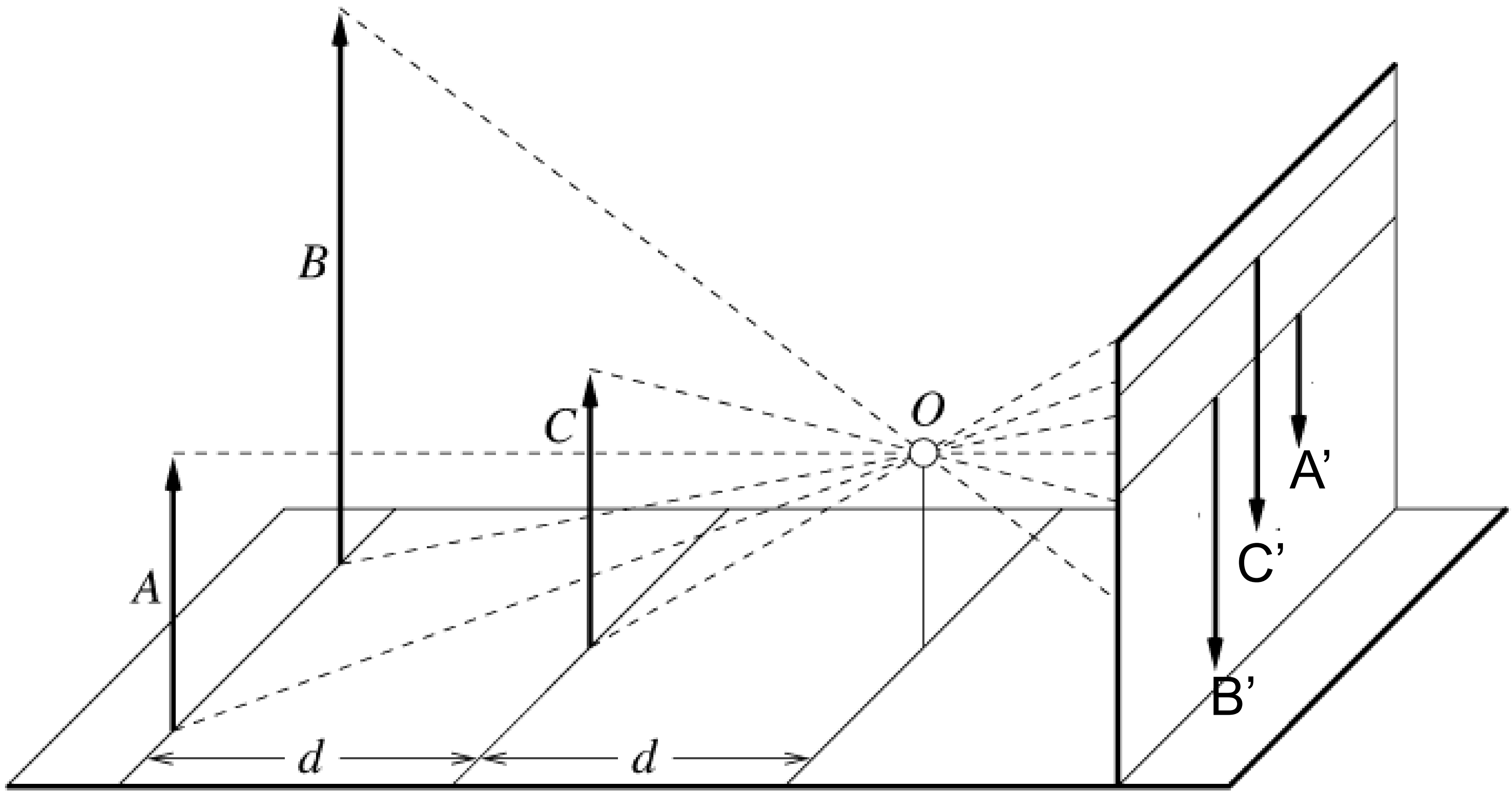
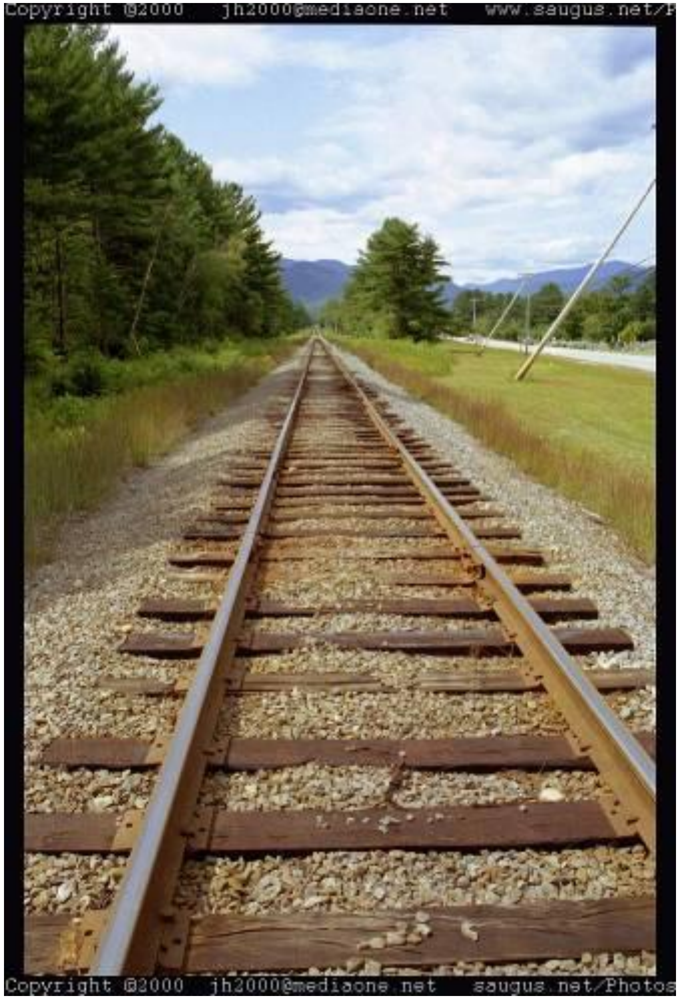


Figure by David Forsyth

Silly Euclid…



Parallel lines aren't…

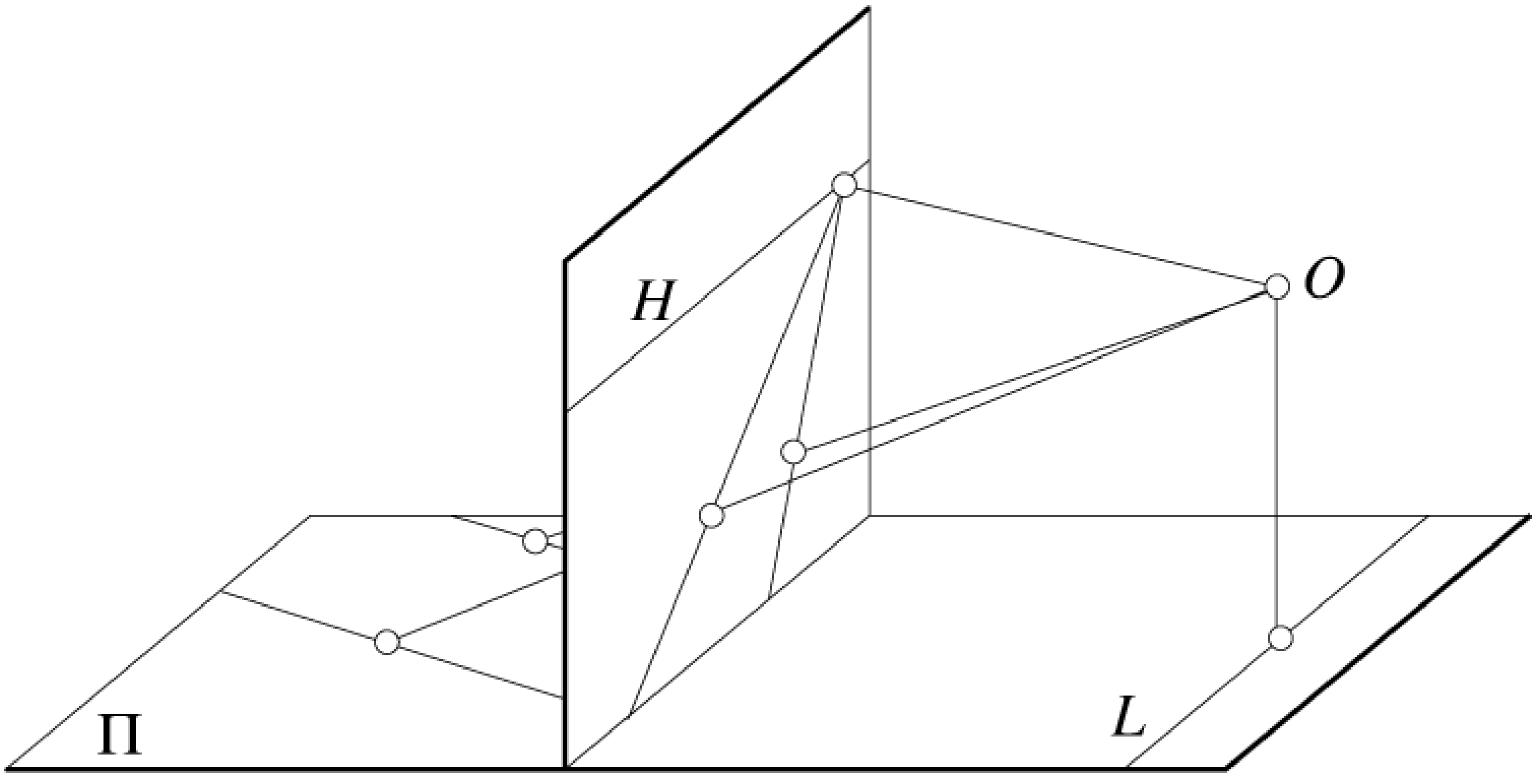
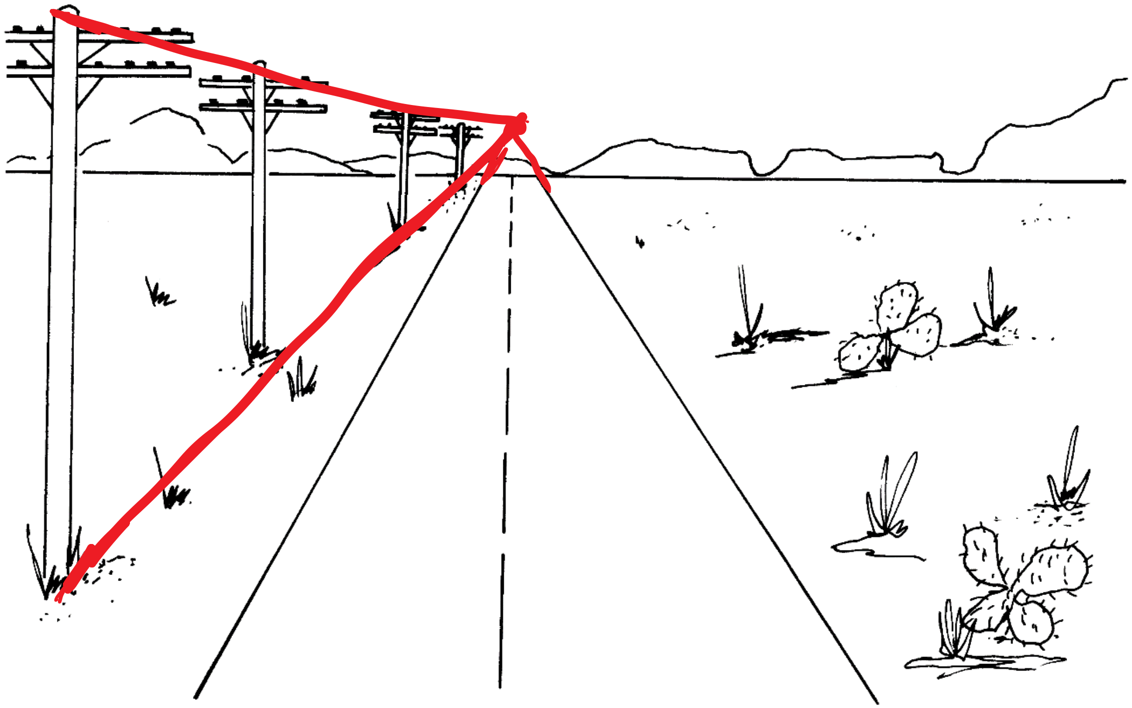


Figure by David Forsyth

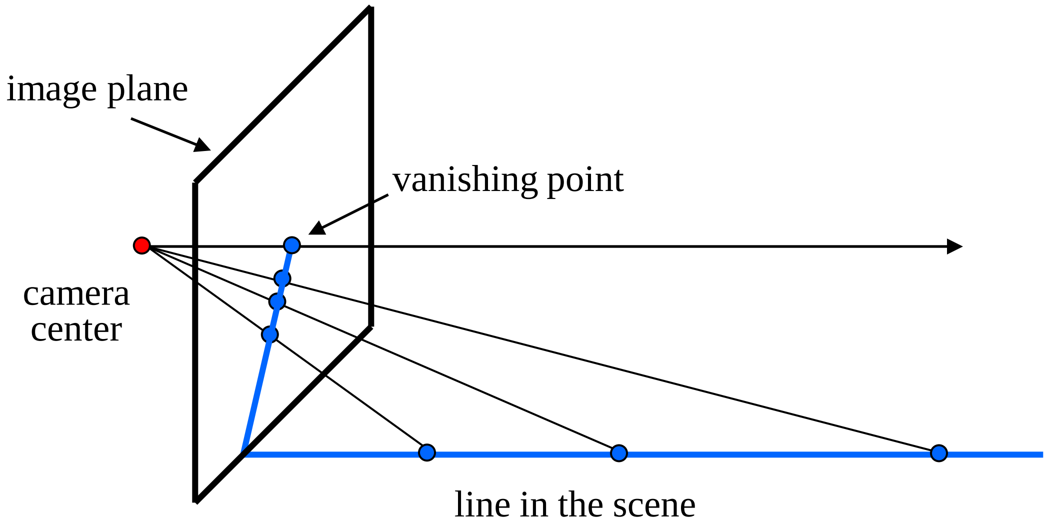
Parallel lines converge to a vanishing point



Proof

Let there be a point A and a direction vector D in three dimensional space.

Projection of a line



Vanishing point in vector notation

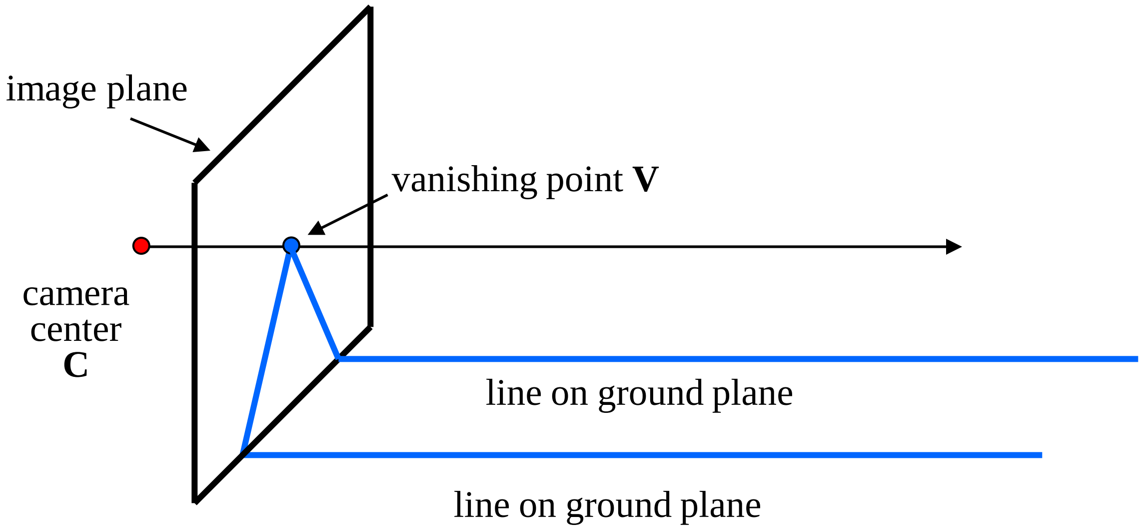
$$\mathbf{p} \;=\; f\frac{\mathbf{X}}{Z}$$

A line of points in 3D can be represented as $\mathbf{X} = \mathbf{A} + \lambda\mathbf{D}$, where $\mathbf{A}$ is a fixed point, $\mathbf{D}$ a unit vector parallel to the line, and λ a measure of distance along the line. As λ increases points are increasingly further away and in the limit:

$$\lim_{\lambda\to\infty} \mathbf{p} = f\frac{\mathbf{A} + \lambda\mathbf{D}}{A_Z + \lambda D_Z} = f\frac{\mathbf{D}}{D_Z}$$

i.e. the image of the line terminates in a *vanishing point* with coordinates $(fD_X/D_Z, fD_Y/D_Z)$, unless the line is parallel to the image plane ($D_Z = 0$). Note, the vanishing point is unaffected (invariant to) line position, $\mathbf{A}$, it only depends on line orientation, $\mathbf{D}$. Consequently, the family of lines parallel to $\mathbf{D}$ have the same vanishing point.

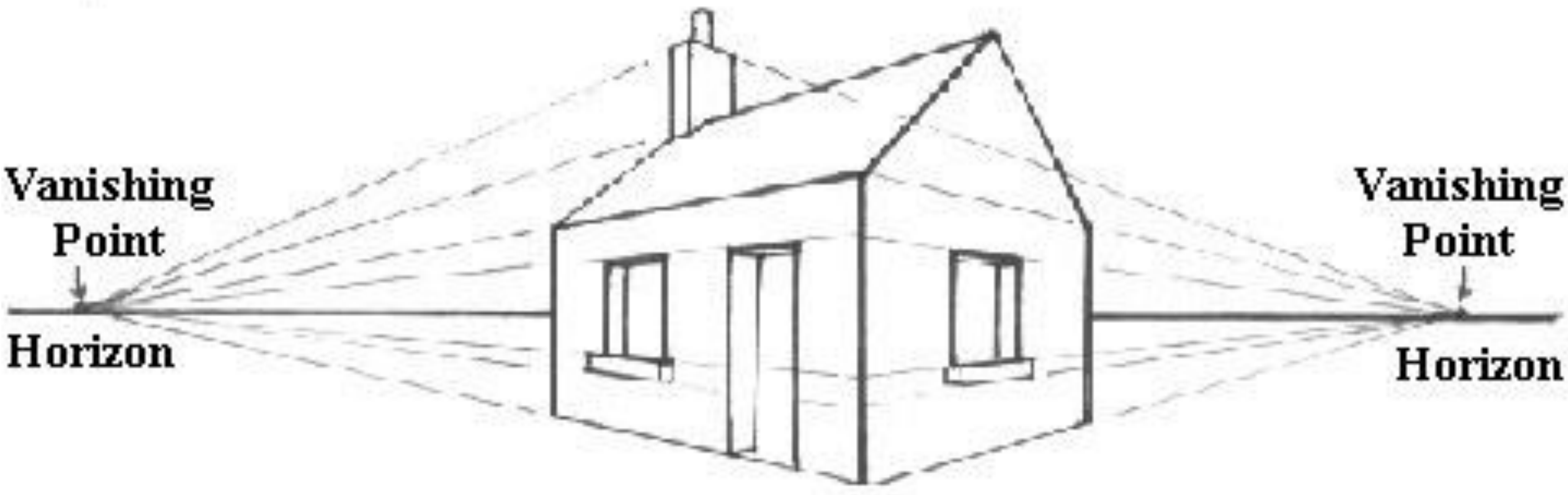
Vanishing points



Properties

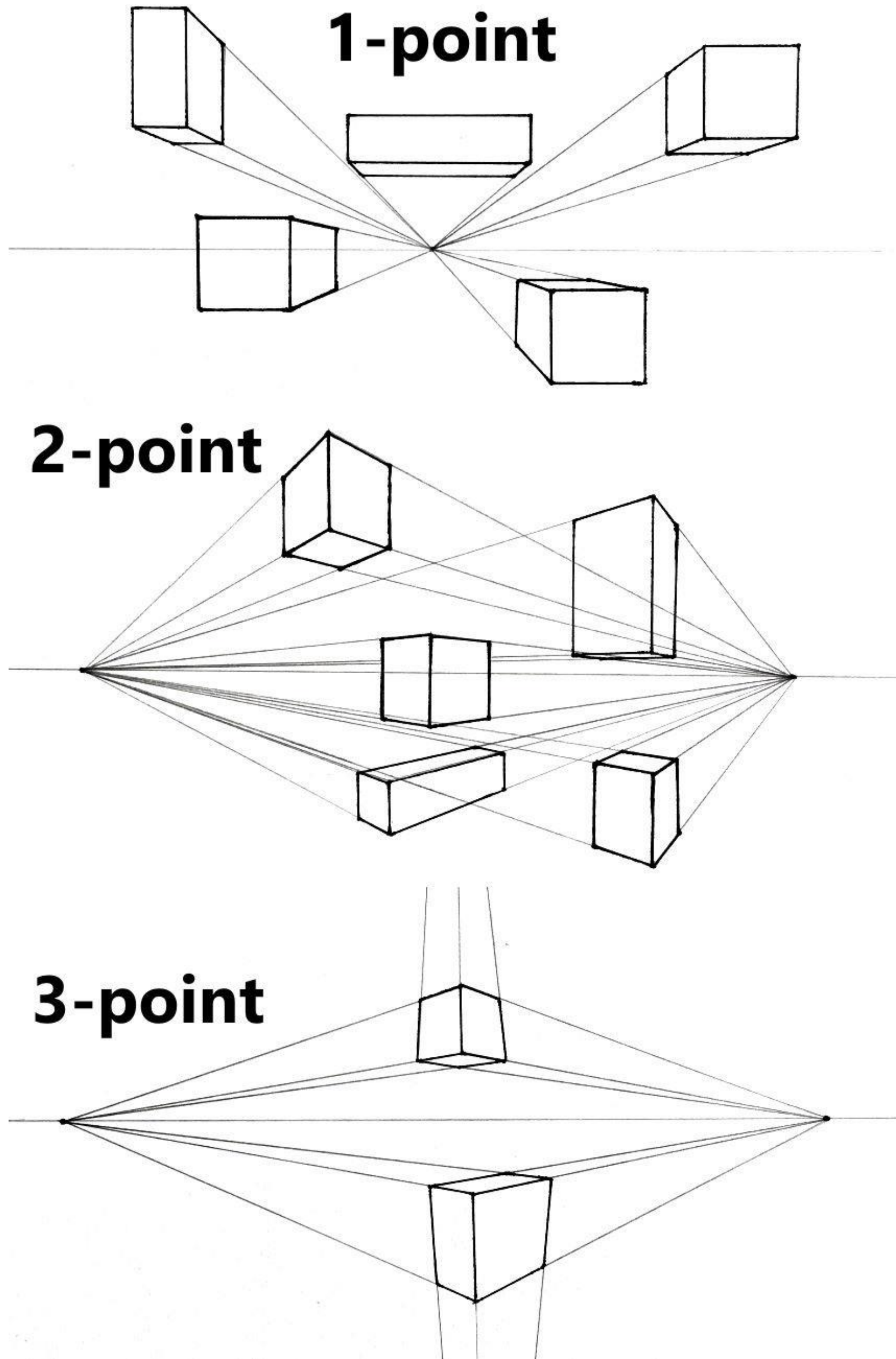
- Any two parallel lines have the same vanishing point **v**
- The ray from **C** through **v** is parallel to the lines
- An image may have more than one vanishing point

Each family of parallel lines has its own vanishing point

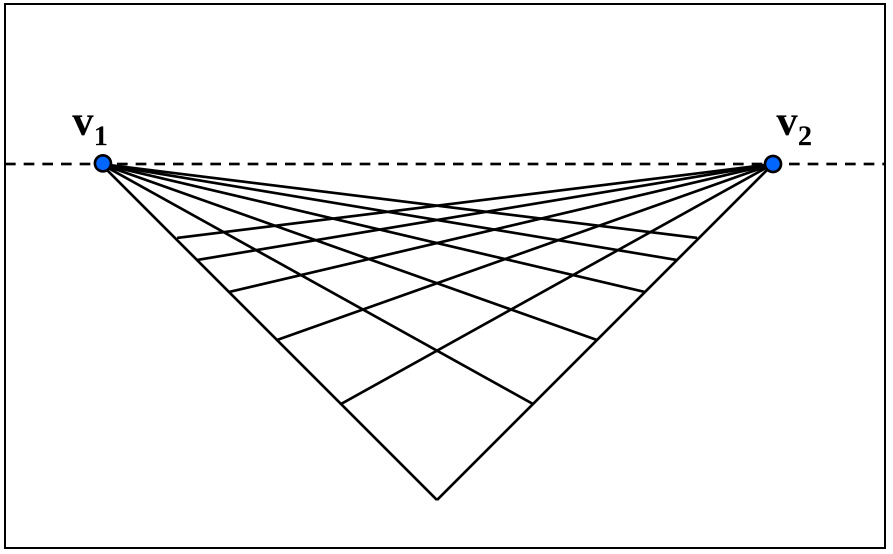


But this isn’t true of the vertical lines. They stay parallel. Why?

- How many vanishing points are there?



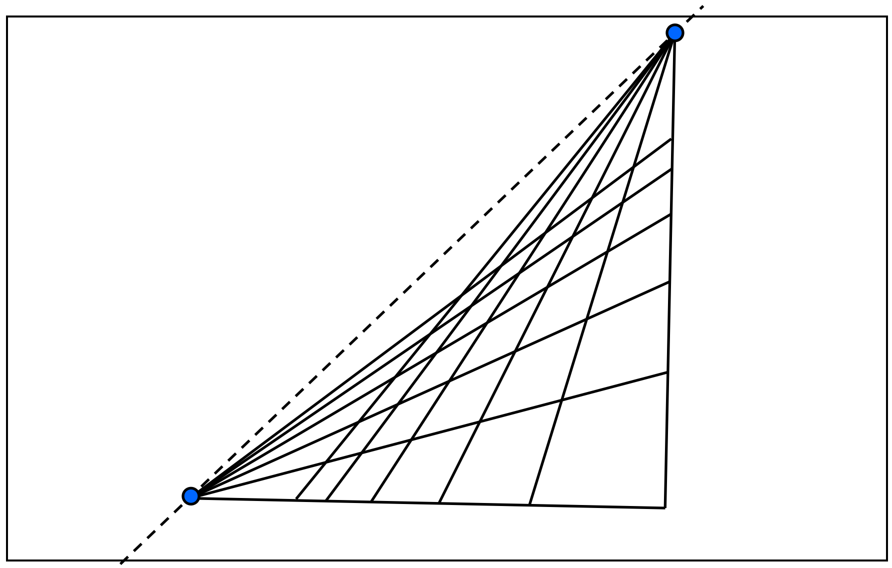
Vanishing lines



Multiple Vanishing Points

- Any set of parallel lines on the plane define a vanishing point
- The union of all of these vanishing points is the *vanishing line*
  - *horizon line* is a special case
- Note that different planes define different vanishing lines

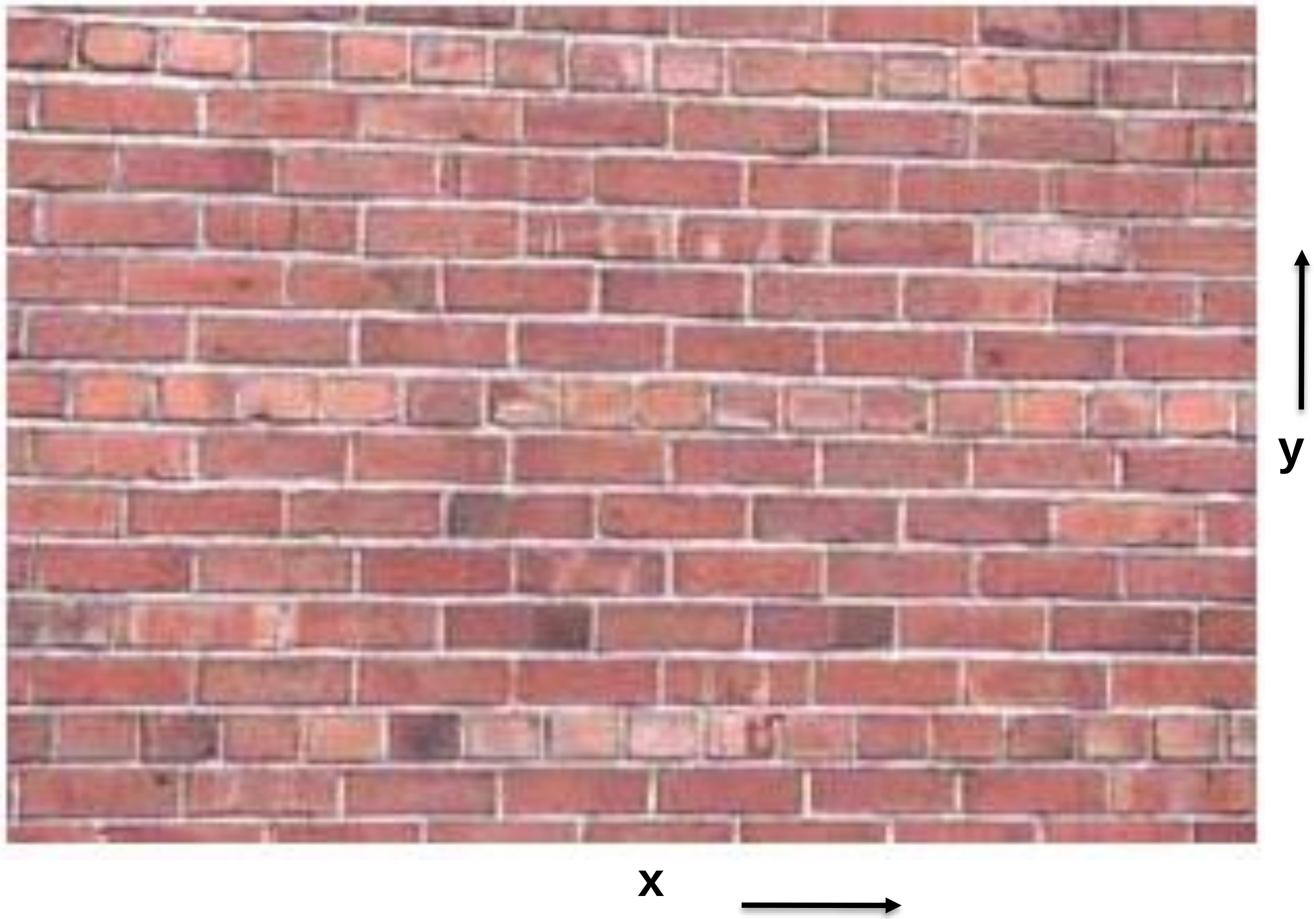
Vanishing lines



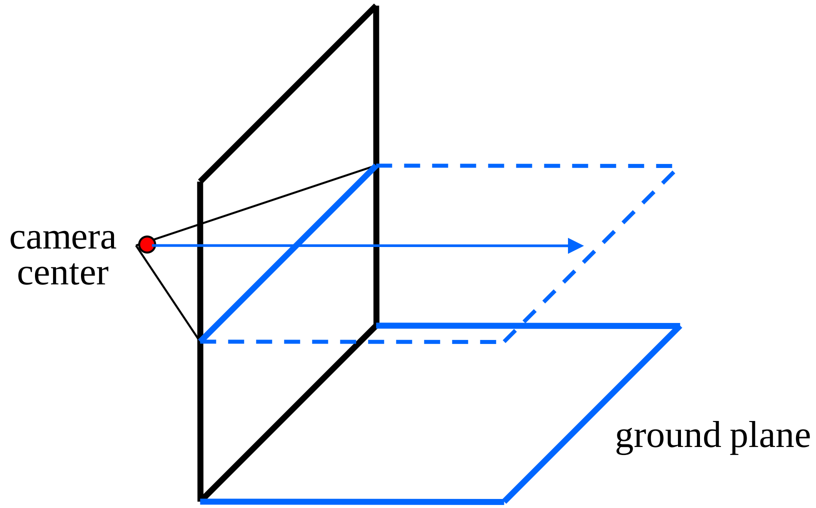
Multiple Vanishing Points

- Any set of parallel lines on the plane define a vanishing point
- The union of all of these vanishing points is the *vanishing line*
  - *horizon line* is a special case
- Note that different planes define different vanishing lines

What if you photograph a brick wall head-on?

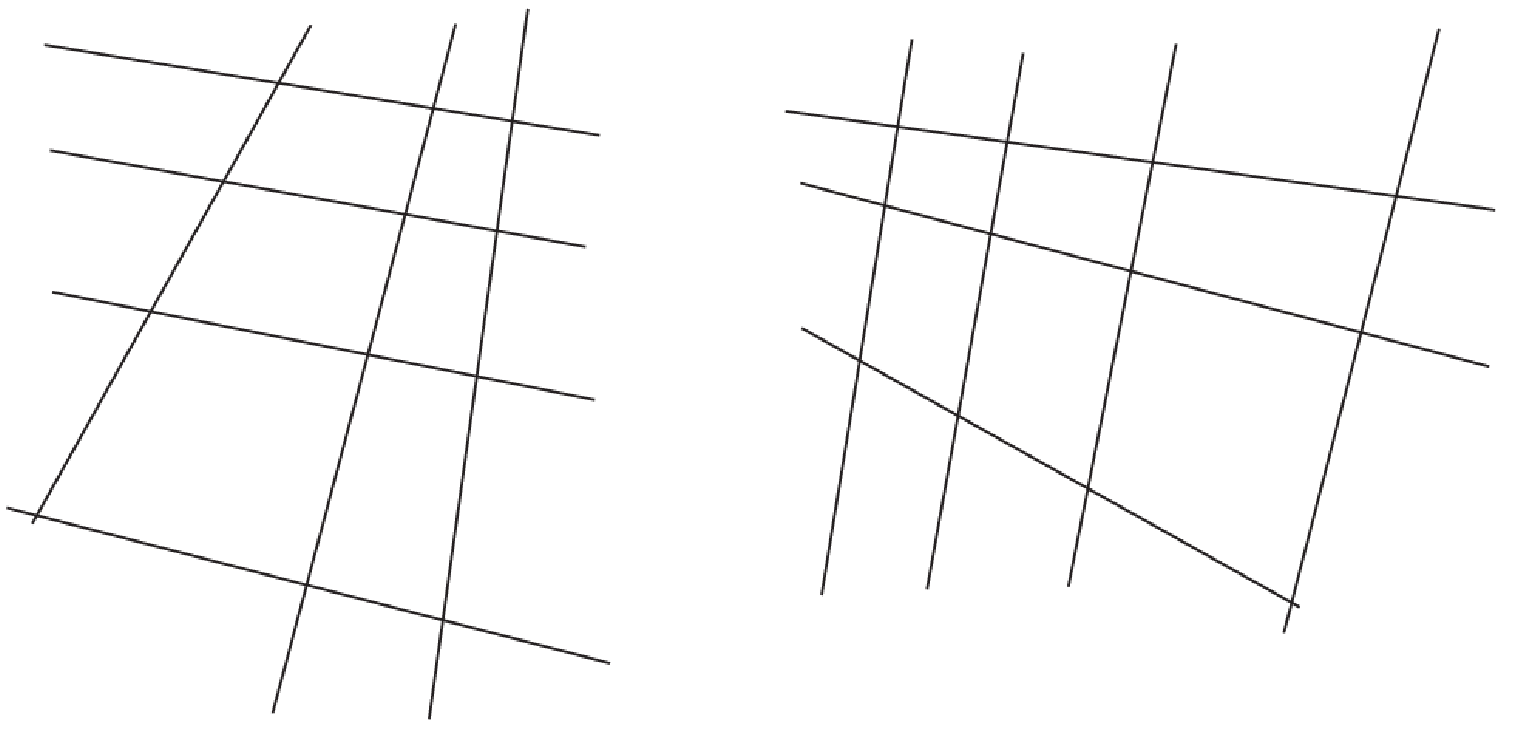


Special case: ground plane & horizon



- Vanishing line of the ground plane
  - All points at the same height as the camera project to the horizon
  - Points higher than the camera project above the horizon
  - Provides way of comparing height of objects

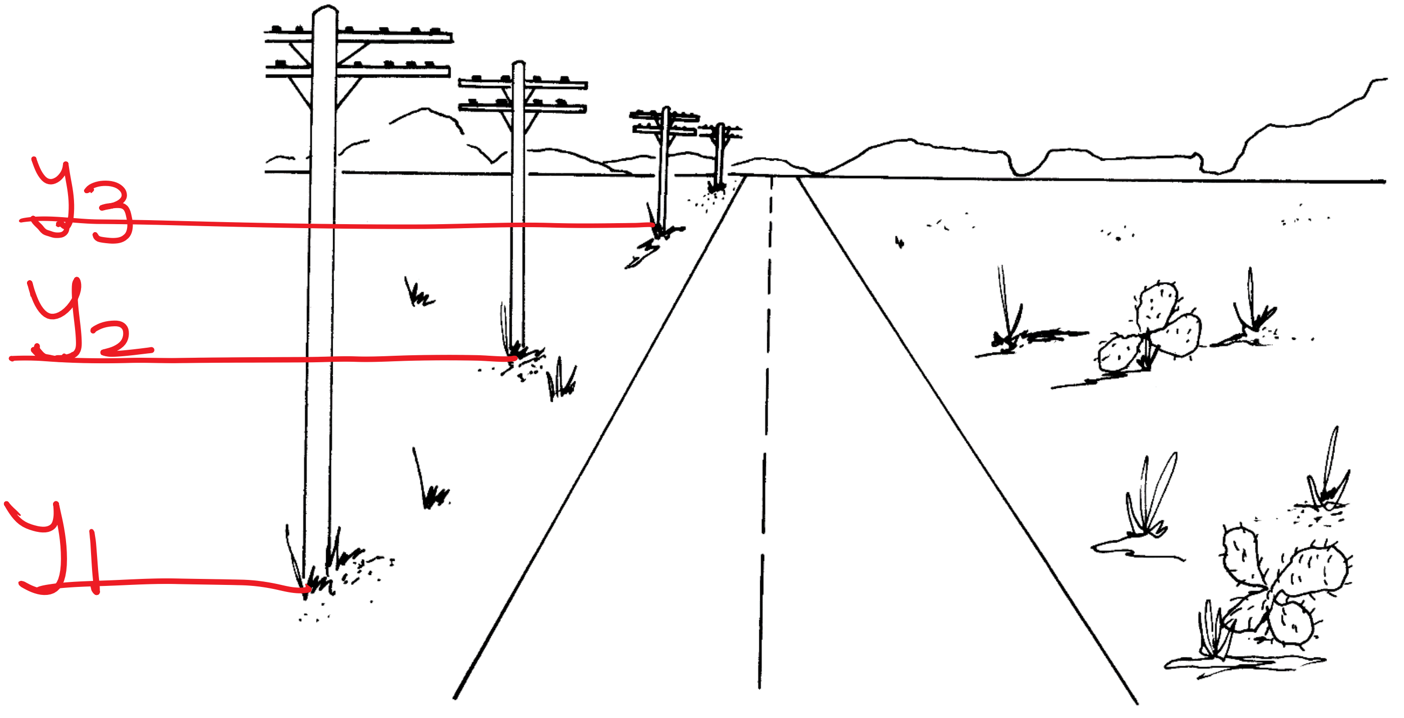
Vertical or Horizontal?



The horizon

Slide by Steve Seitz

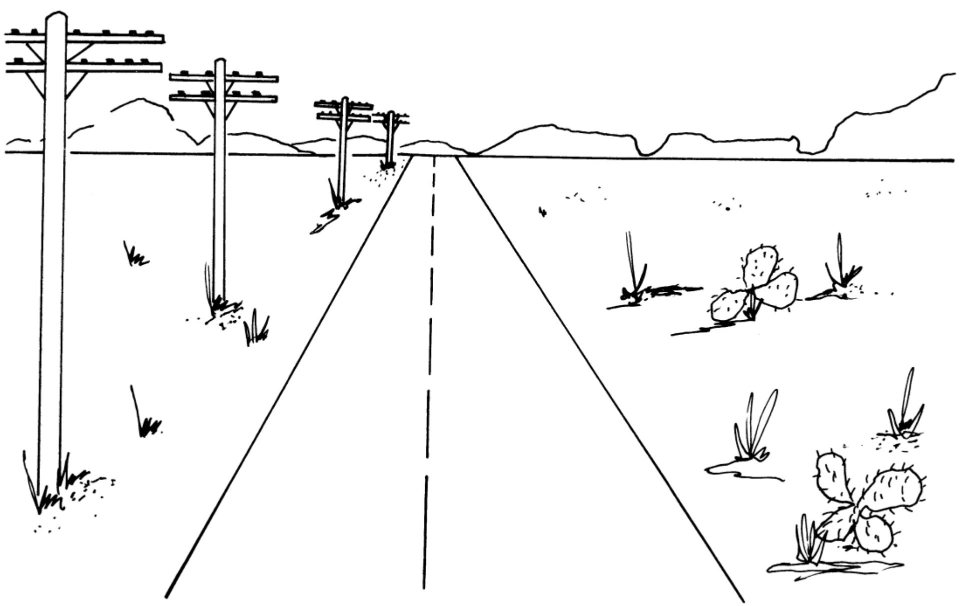
Nearer objects are lower in the image



Proof

The equation of the ground plane is $Y = -h$

A point on the ground plane will have y-coordinate $y = -fh/Z$



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Study: People Far Away From You Not Actually Smaller

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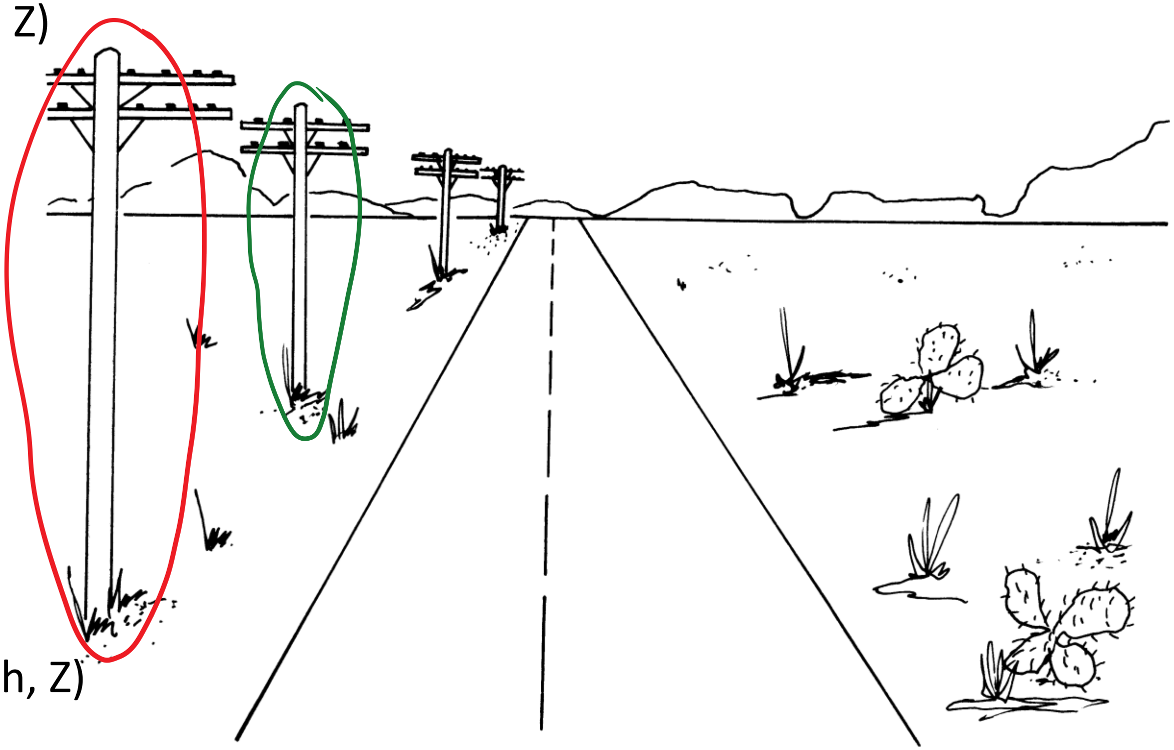
Researchers say that, contrary to prior assertions, the subject above stands at equal height at left and at right, and does not grow smaller as he walks away from the camera.

Nearer objects look bigger

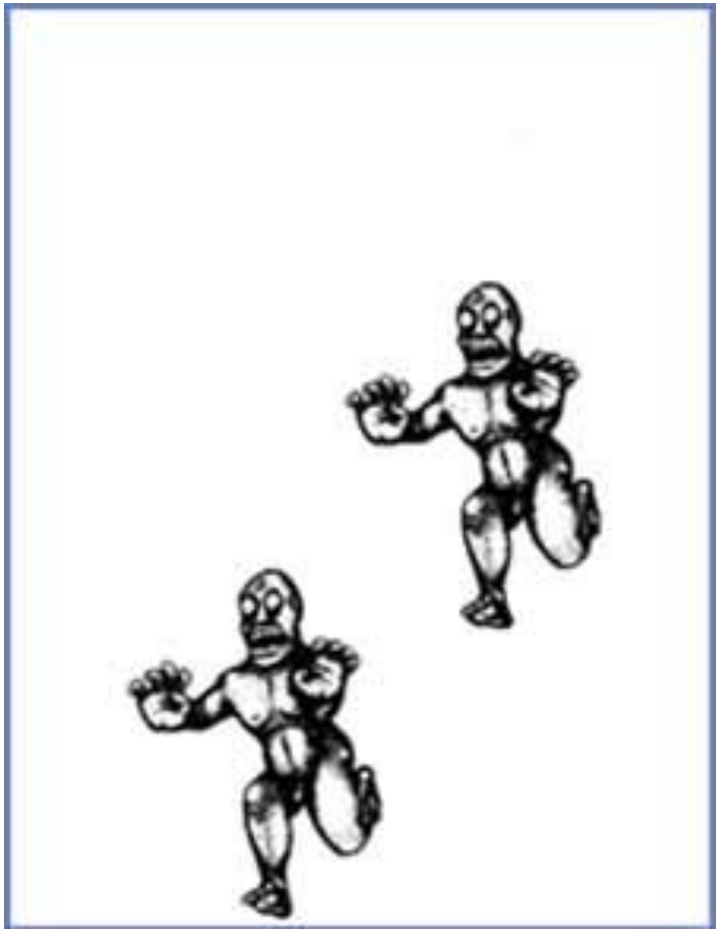
Top at $(X, \; L - h, Z)$

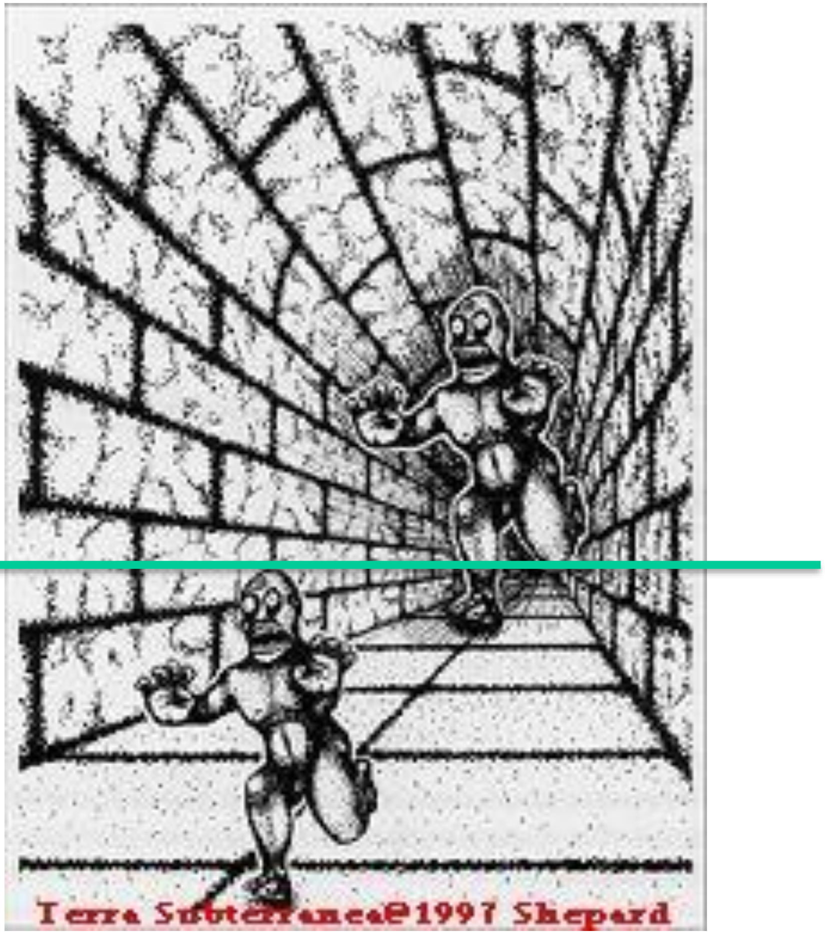


Bottom at $(X, \; - h, Z)$

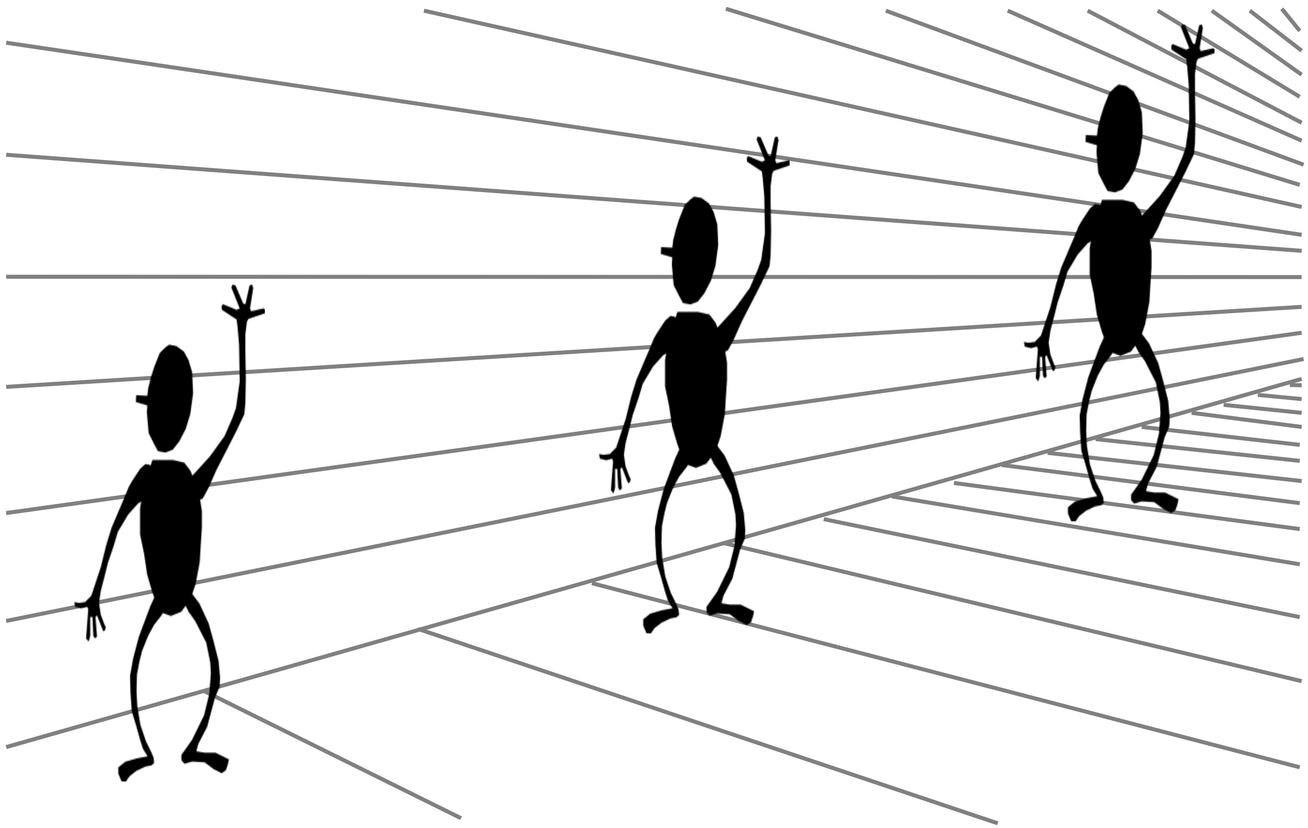
It is straightforward to calculate the projection of the top & bottom of the pole. The difference is the “apparent height”

Fun with vanishing points



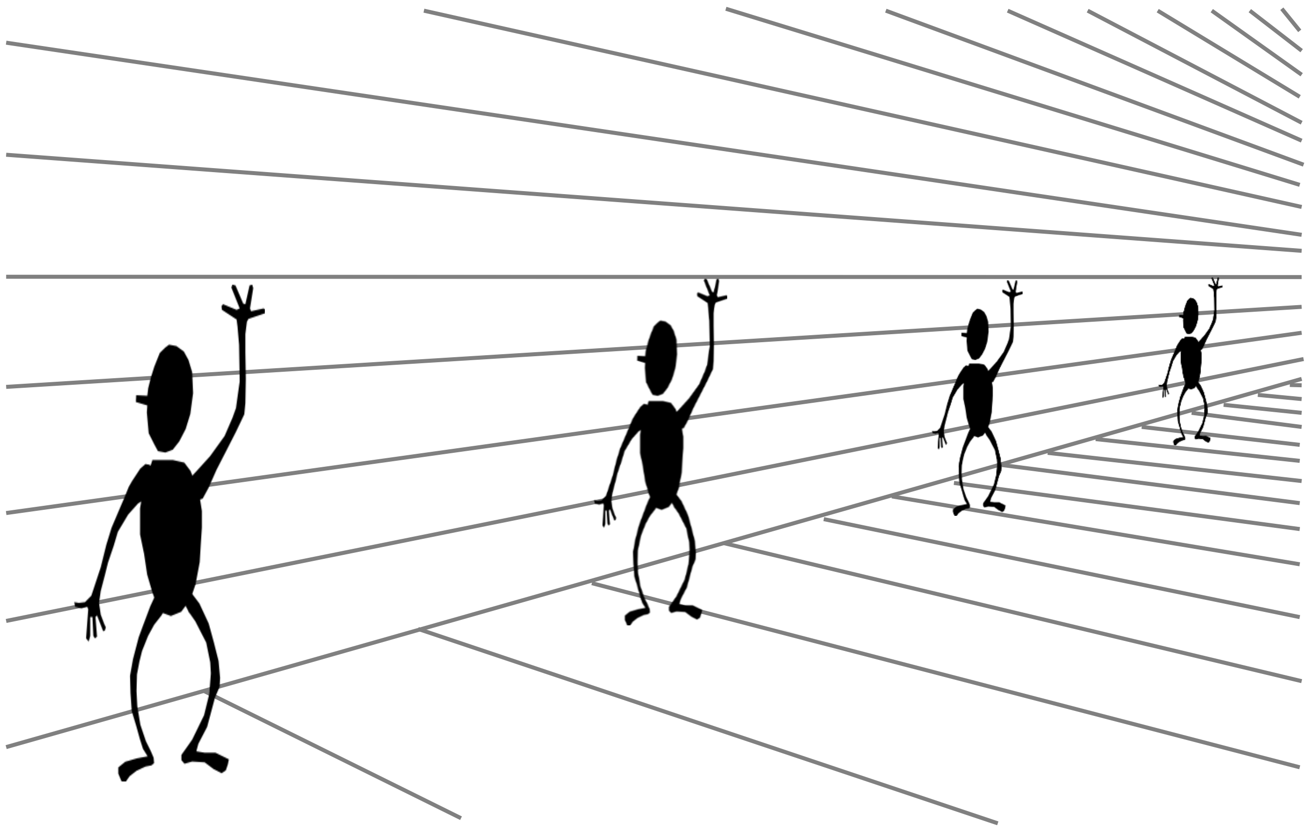


Vanishing points are perspective cues



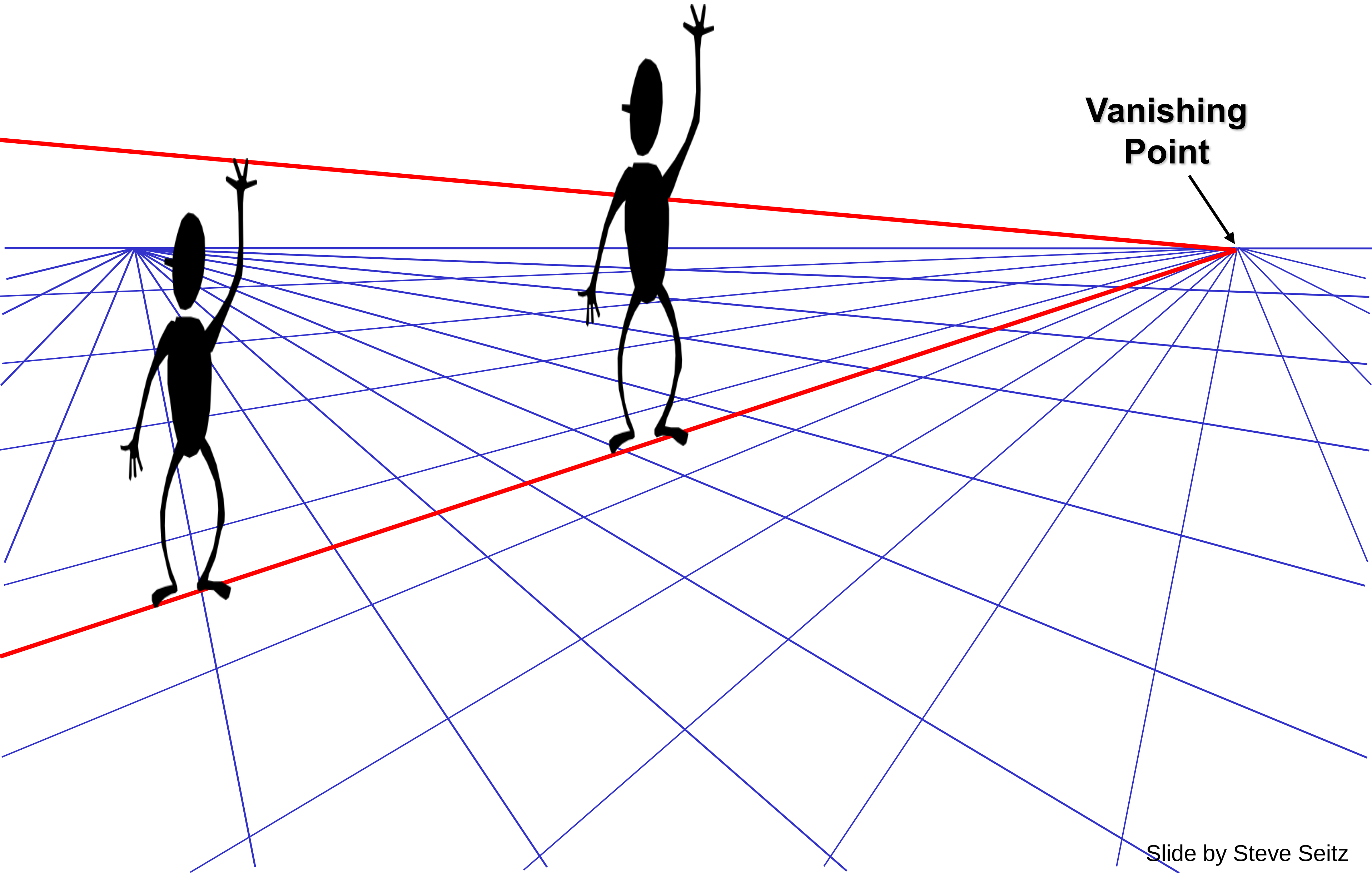
Slide by Steve Seitz

Vanishing points are perspective cues



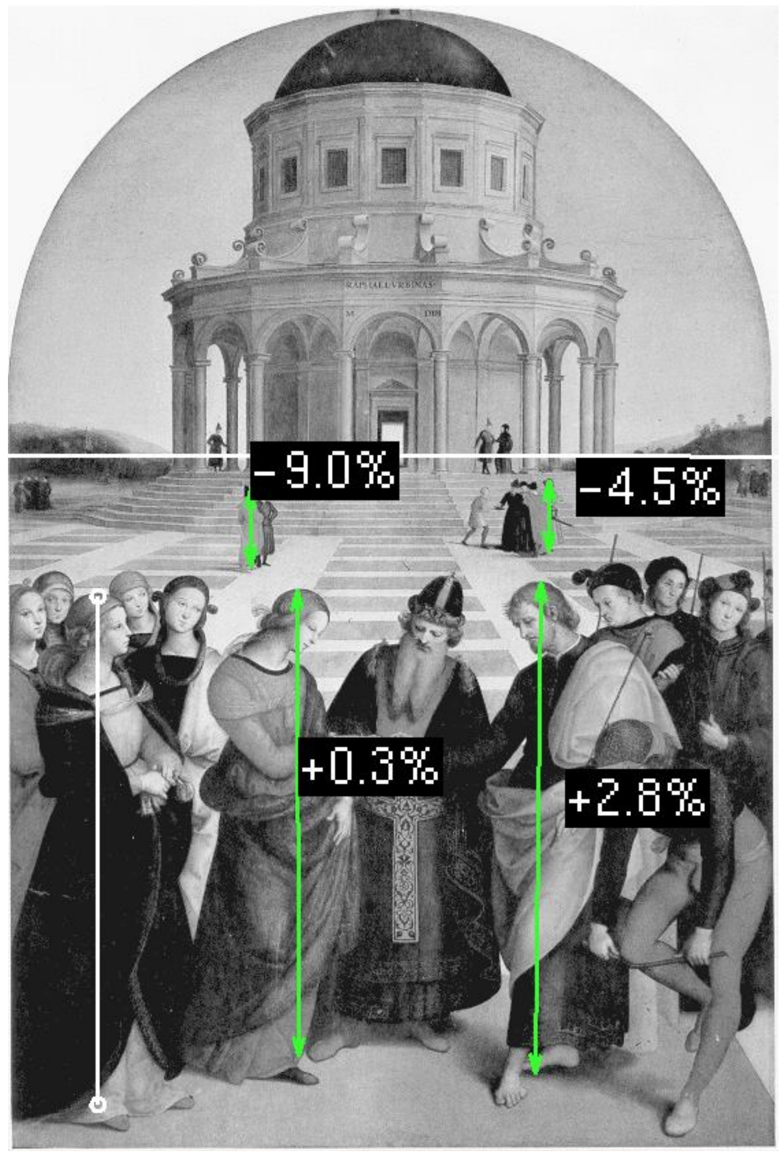
Slide by Steve Seitz

Comparing heights

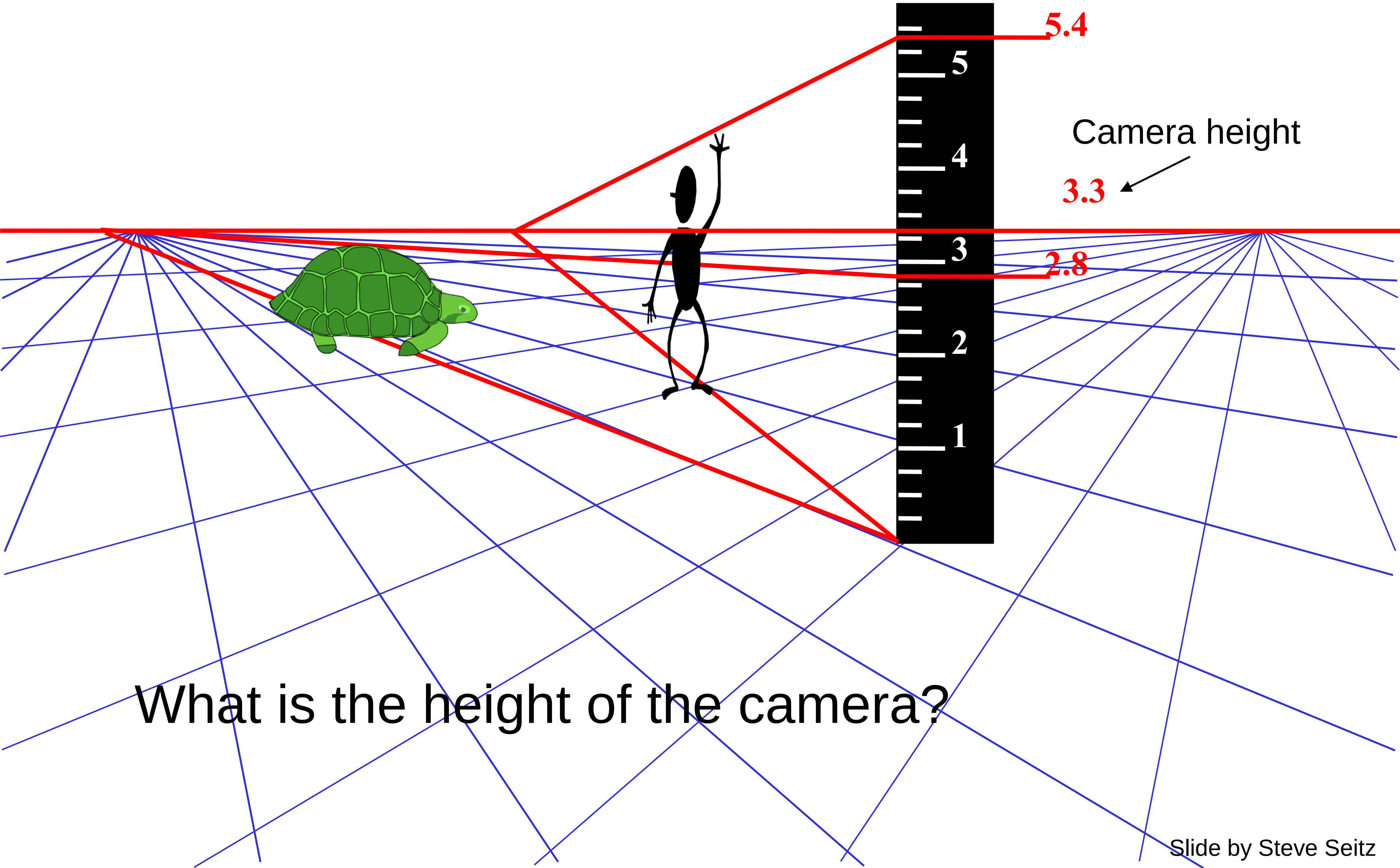


Slide by Steve Seitz

Too bad Rafael didn't know this…



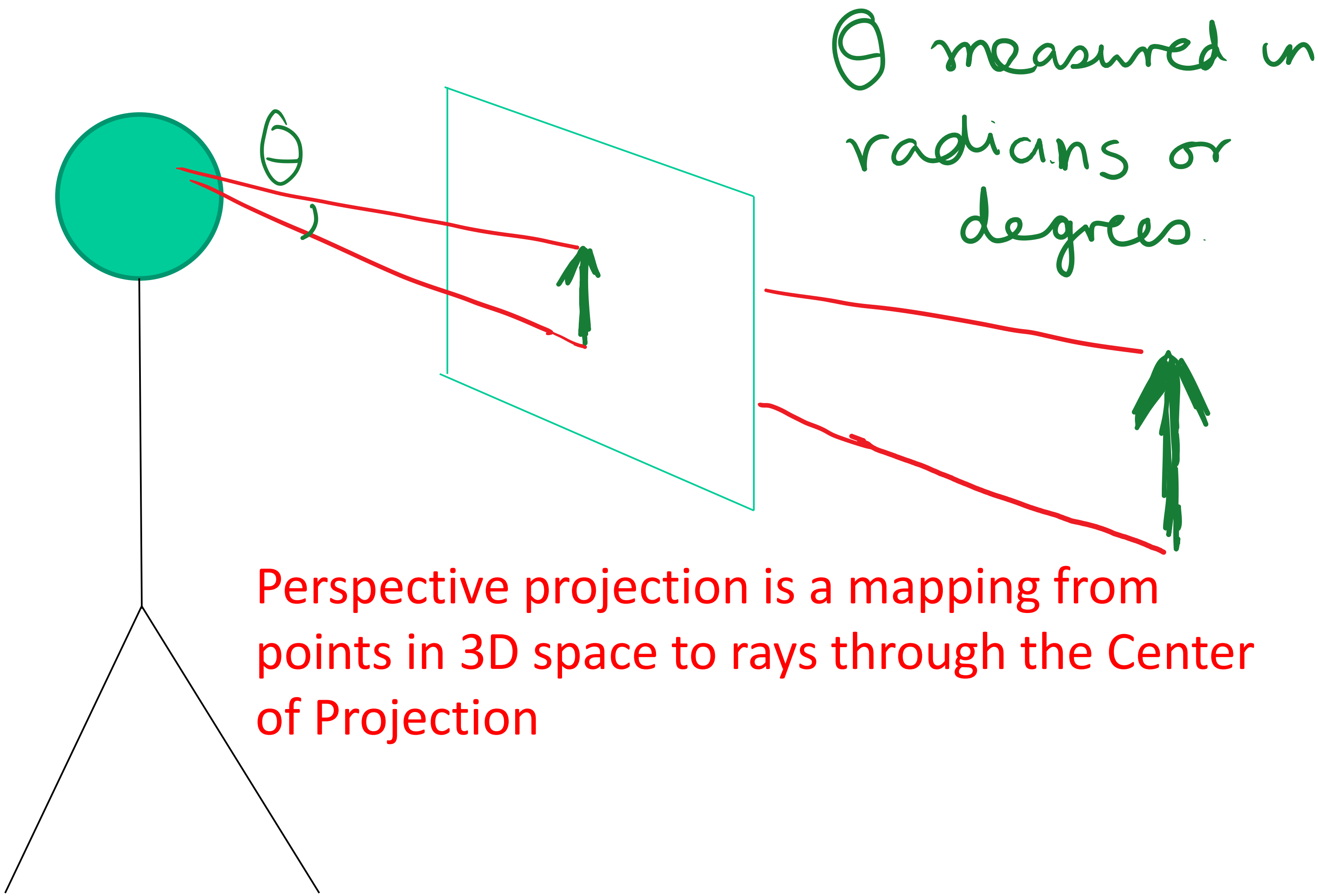
Measuring height



What is the height of the camera?

Slide by Steve Seitz

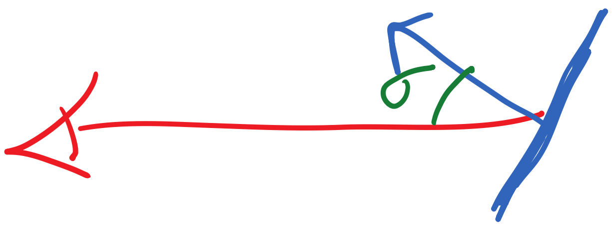
The natural measure of image size is visual angle



Perspective projection is a mapping from points in 3D space to rays through the Center of Projection

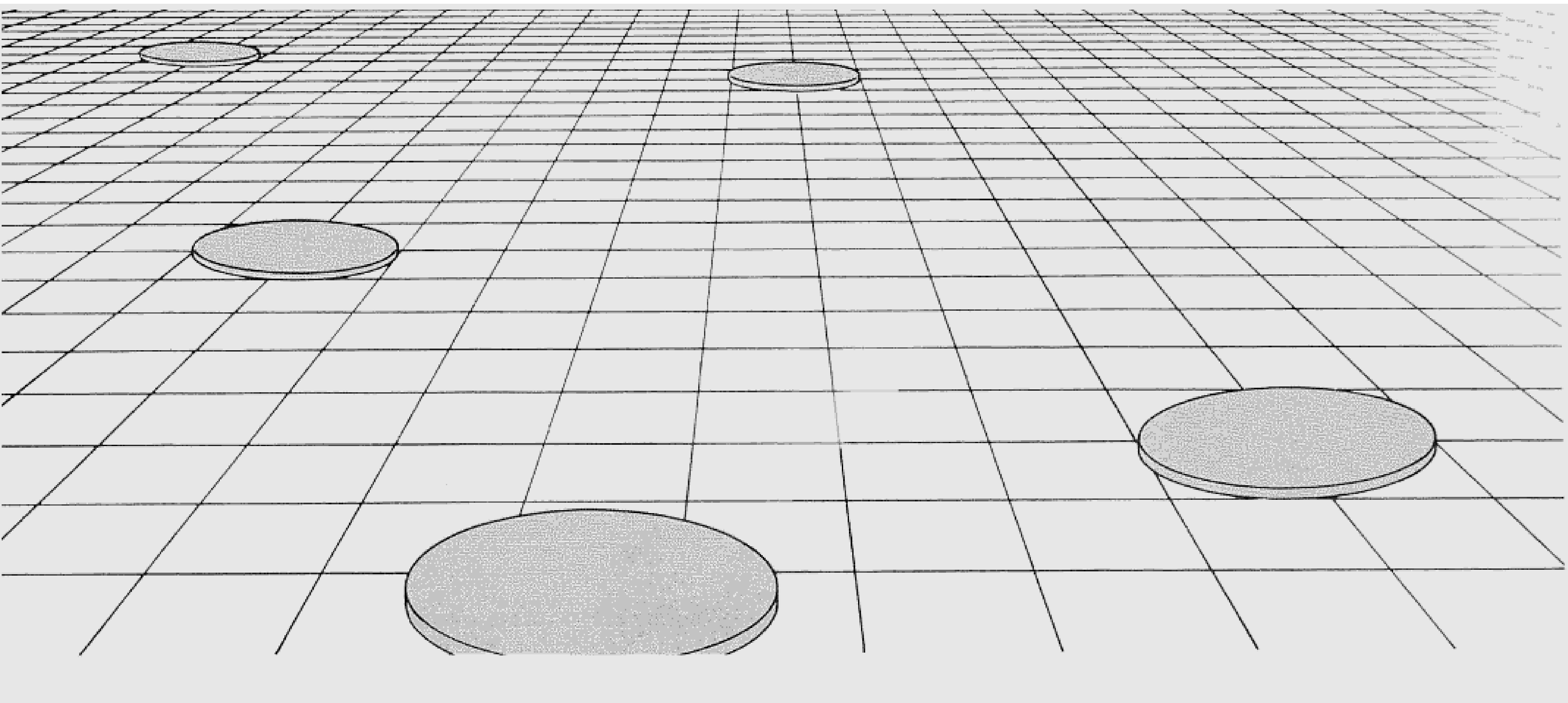
Two main effects of perspective projection

1. Distance – farther objects project to smaller sizes on the image plane. The scaling factor is 1/Z
2. Foreshortening – objects that are slanted with respect to the line of sight project to smaller sizes on the image plane. The scaling factor is cos σ



σ is the angle between the line of sight and the surface normal

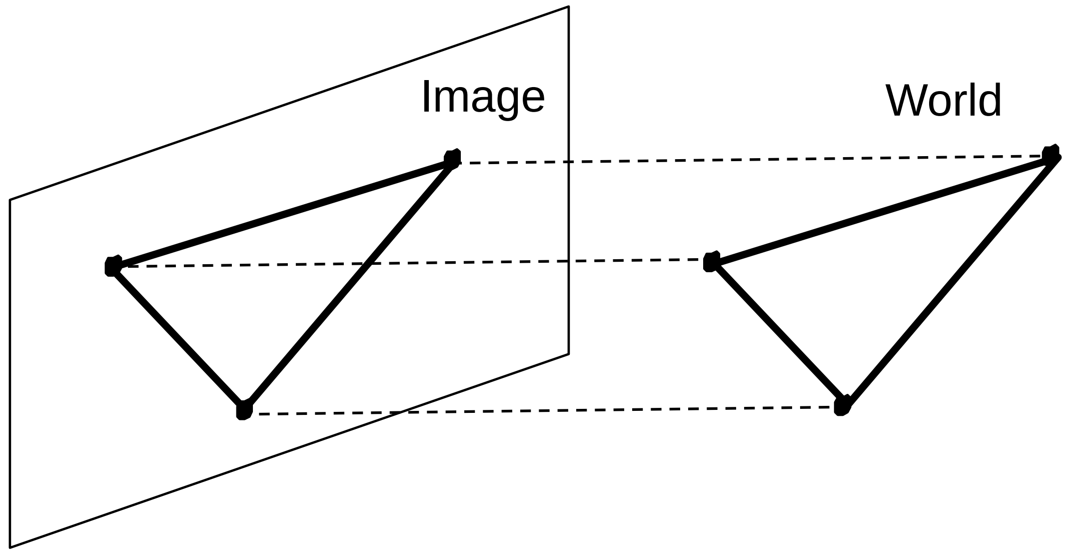
The slabs that are far away not only look smaller, but also more foreshortened



Other projections: Orthographic

Special case of perspective projection

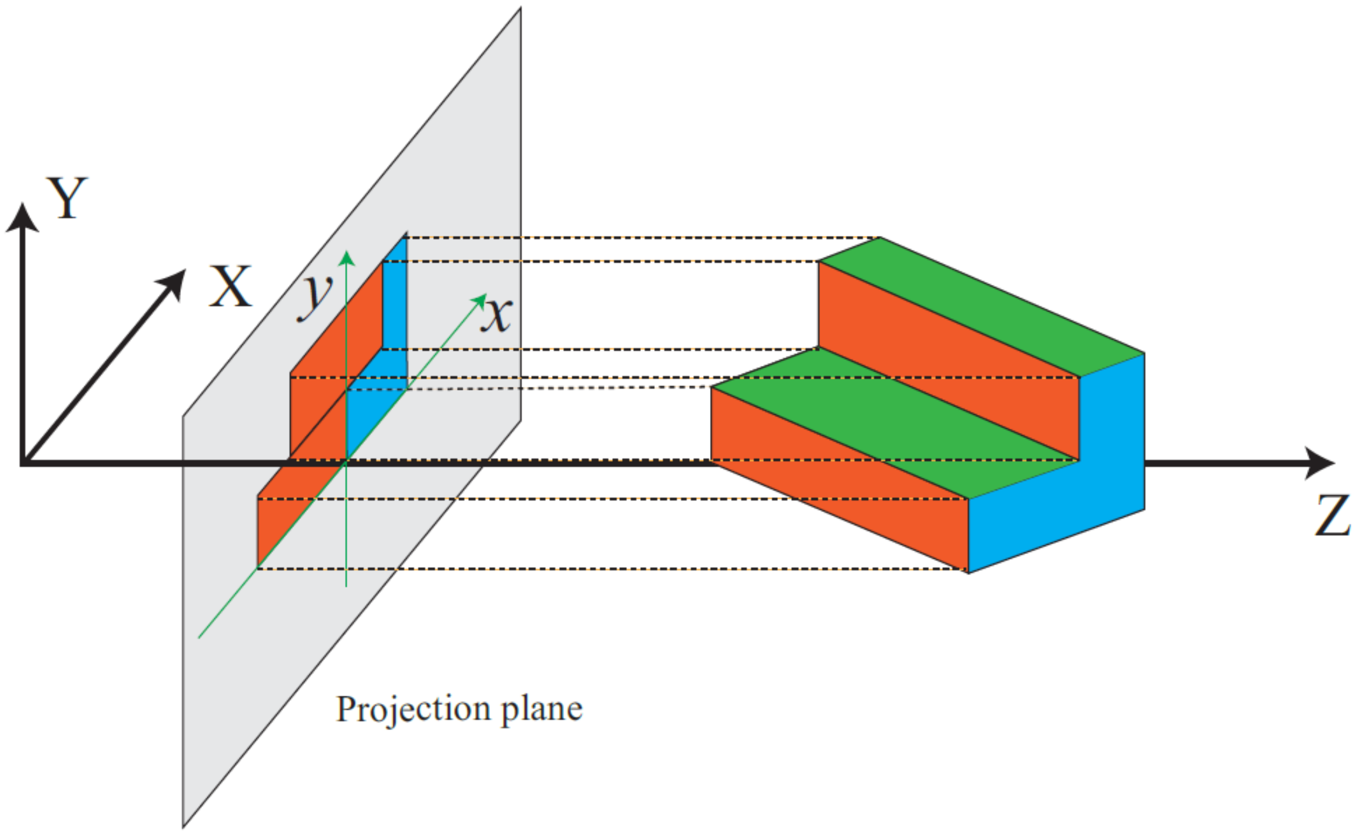
- Distance from the COP to the PP is infinite



- Also called “parallel projection”
- x’ = x
- y’ = y

Orthographic Projection

Figure 5.10: Orthographic projection. Projection is done by parallel rays orthogonal to the projection plane. In this example, we have $x = X$ and $y = Y$.

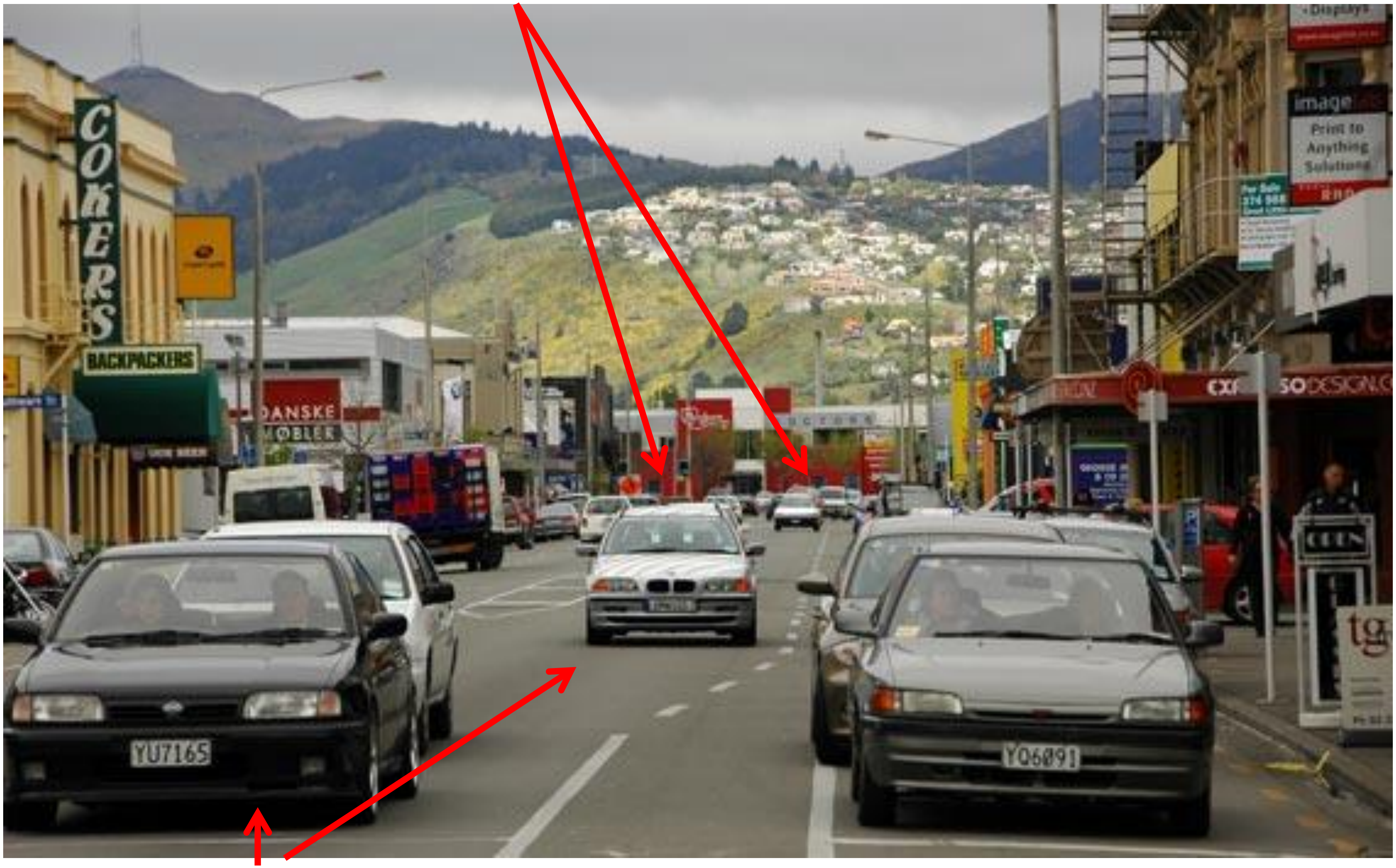


(a) (b) (c)

Figure 5.11: Straw camera example. (a) View through parallel straws. (b) The subject is a hand in sunlight. (c) The resulting image of the straw camera (using smaller straws than (a)). The image projection is orthographic.

Simpler way…

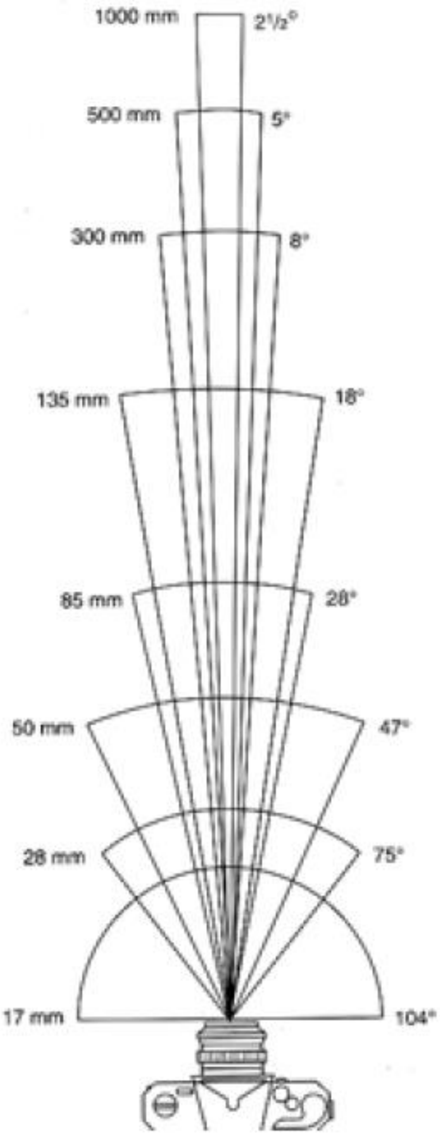
Far field: object appearance doesn't change as objects translate

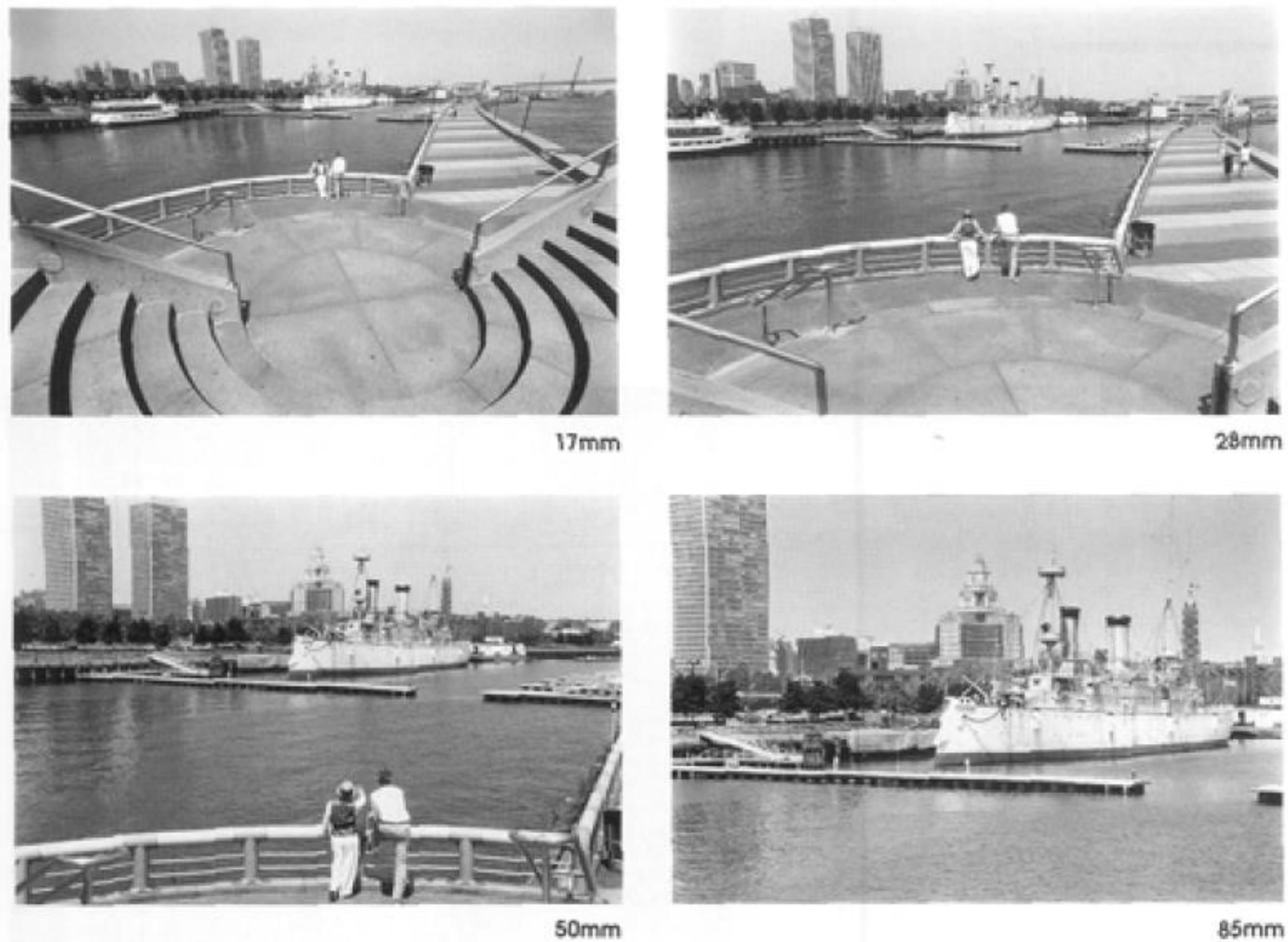


Near field: object appearance changes as objects translate

Focal Length, Zoom, Field of View

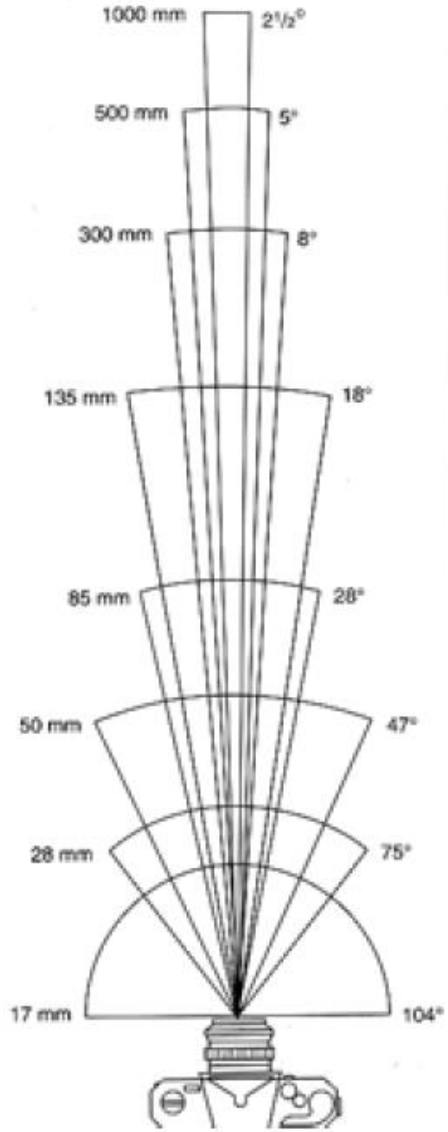
Field of View (Zoom)

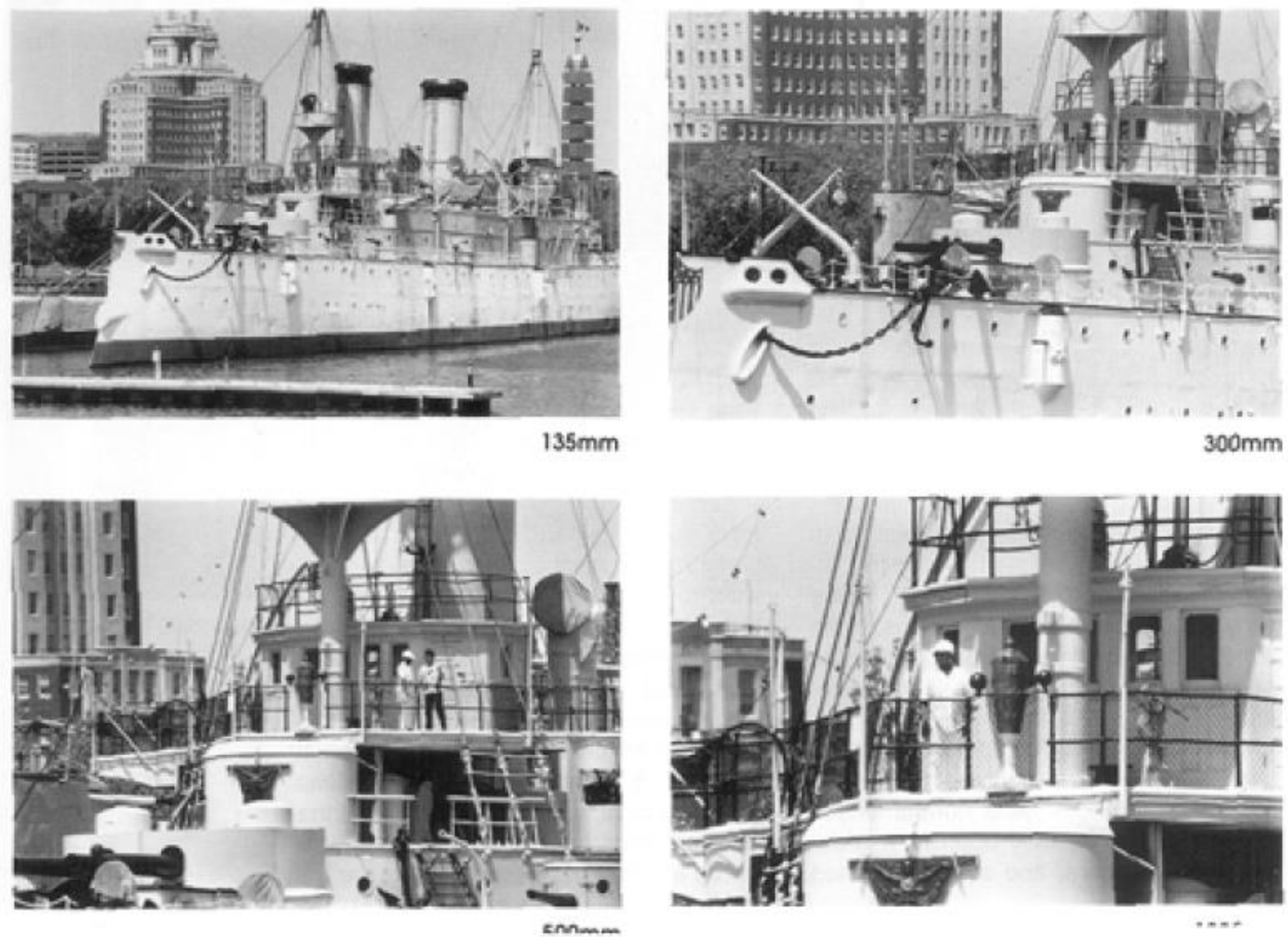




From London and Upton

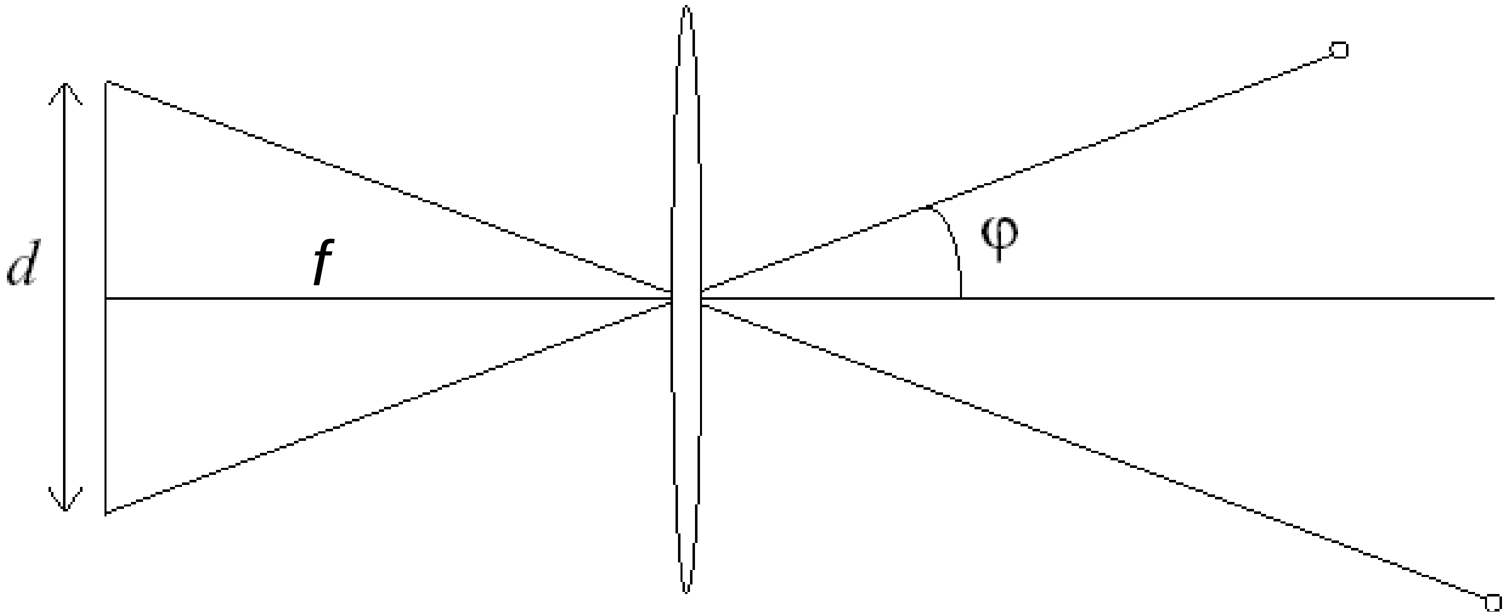
Field of View (Zoom) = Cropping





From London and Upton

FOV depends of Focal Length



Size of field of view governed by size of the camera retina:

$$\varphi = \tan^{-1}\left(\frac{d}{2f}\right)$$

Smaller FOV = larger Focal Length

Expensive toys…

Sigma 200-500mm F2.8 EX DG lens

What does 1600mm lens look like?

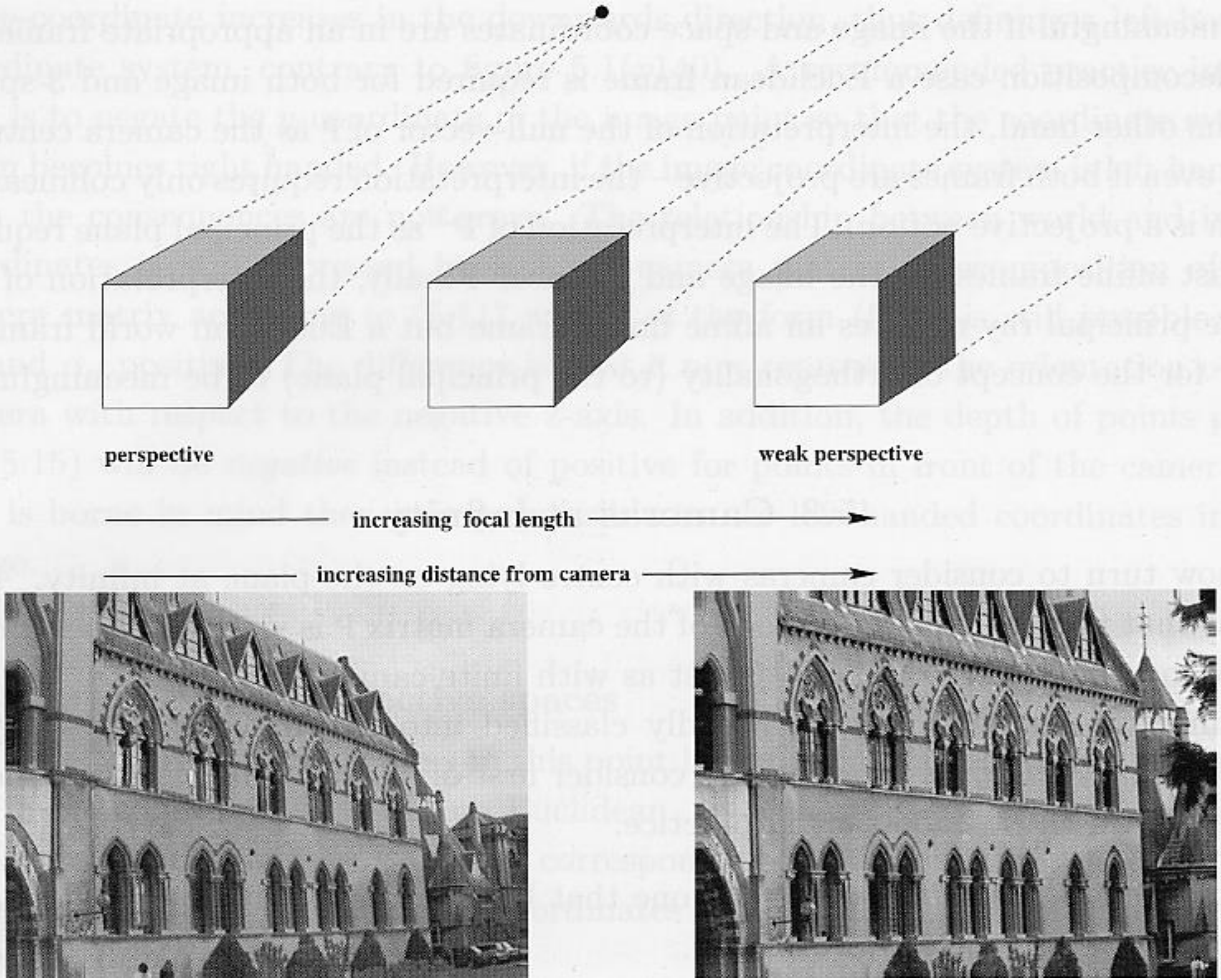
http://www.digitalpixels.net/varia/the-web/sigma-200-500mm-f28-ex-dg-lens-on-the-field/

http://dancarrphotography.com/blog/wp-content/uploads/2011/05/Canon_super_tele_comparison.jpg

Field of View / Focal Length

Large FOV / small f
+ Camera close to car

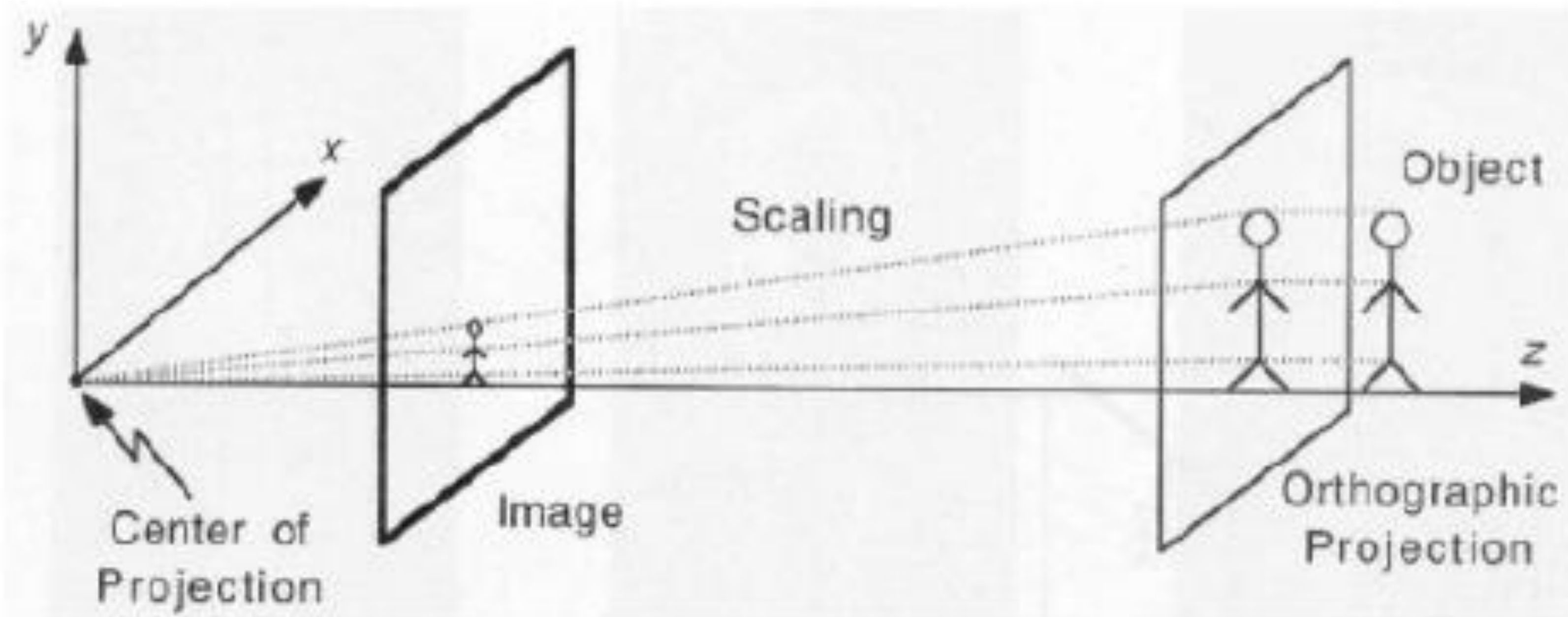
Small FOV / large f
+ Camera far from the car



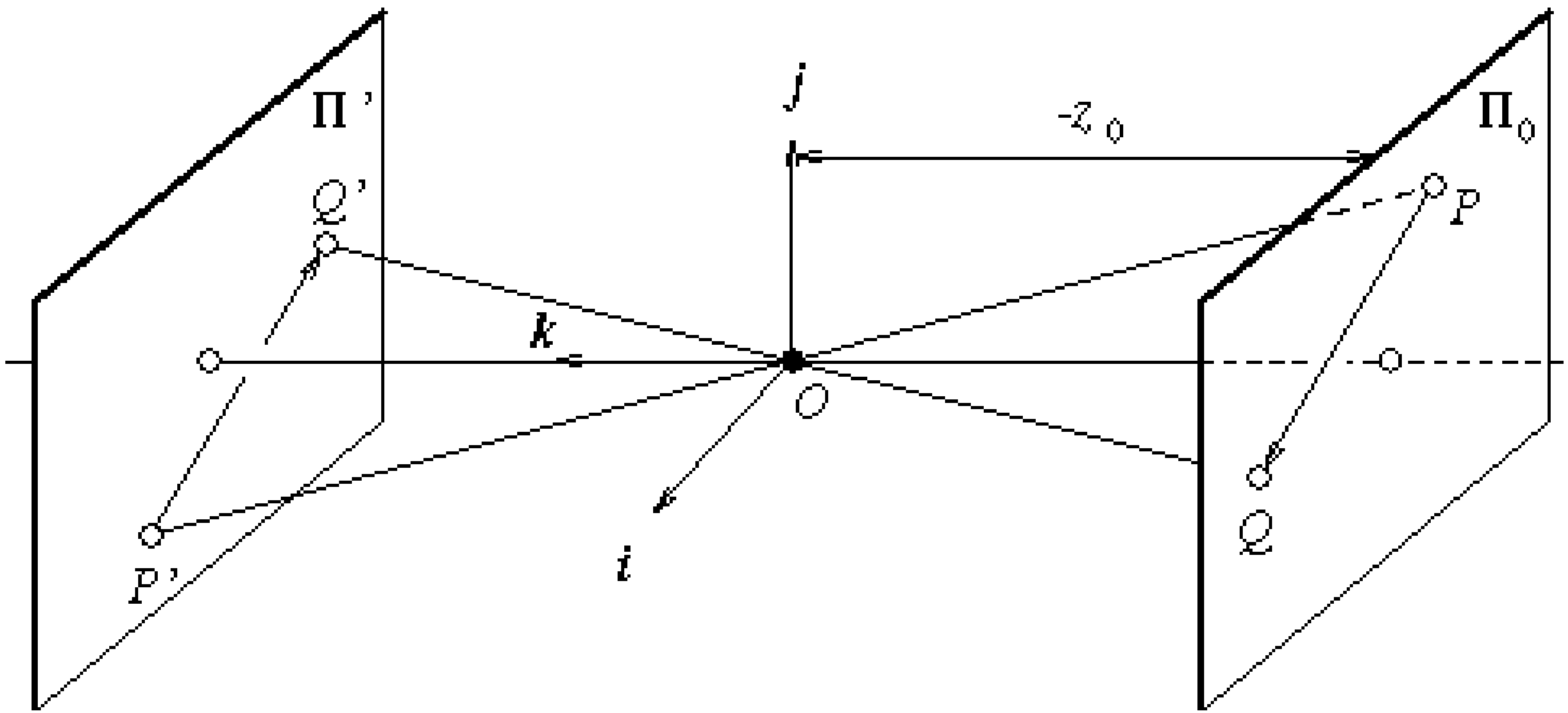
From Zisserman & Hartley

Focal length / distance in portraiture

Scaled Orthographic or “Weak Perspective”



Special Case: Scaled Orthographic

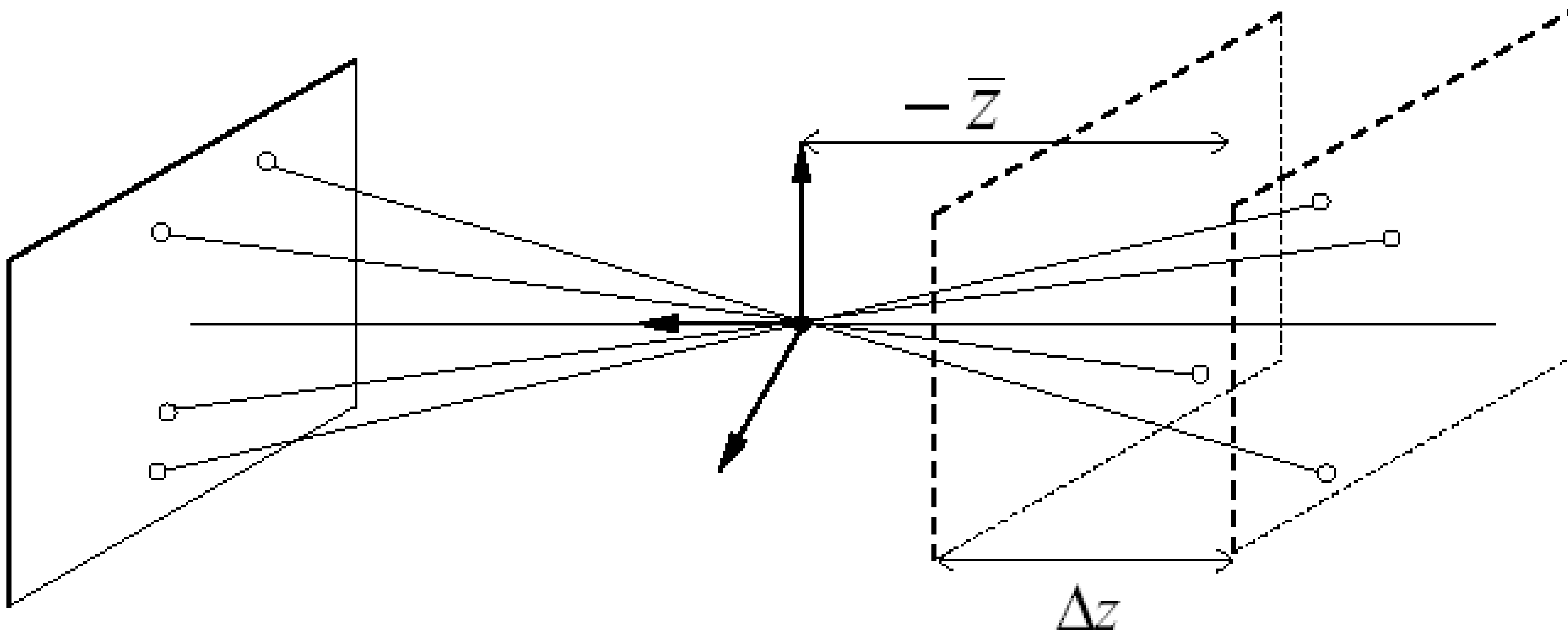


$$x' \approx -mx$$
$$y' \approx -my$$

$$m = -\frac{f'}{z_o}$$

If scene points are in a plane, projections are simply magnified by m

Special Case: Scaled Orthographic



If $\Delta z << -\bar{z}$: $\begin{matrix} x' \approx -mx \\ y' \approx -my \end{matrix}$ $\quad m = -\dfrac{f}{\bar{z}}$

Justified if scene depth is small relative to average distance from camera

Three camera projections

3-d point **2-d image position**

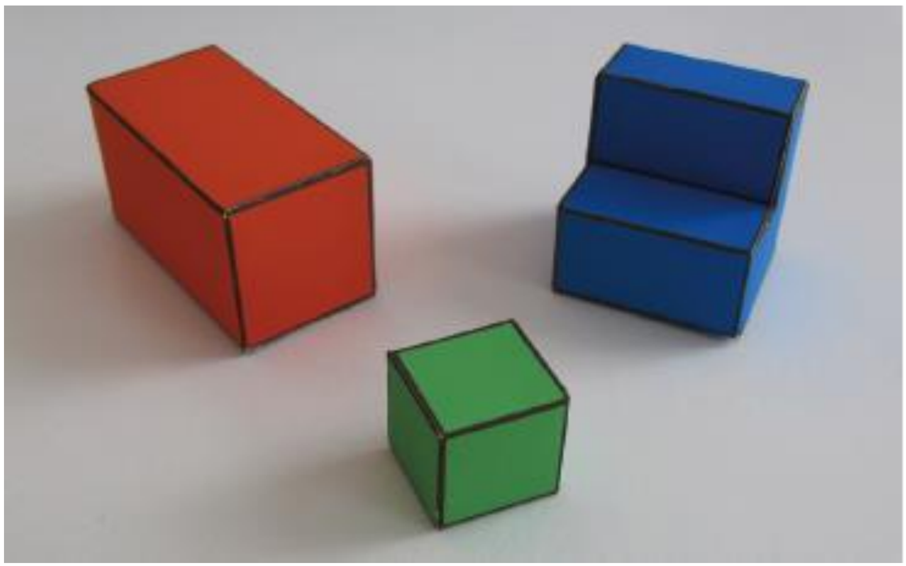
(1) Perspective: $(X, Y, Z) \rightarrow \left(\dfrac{fX}{Z}, \dfrac{fY}{Z}\right)$

(2) Weak perspective: $(X, Y, Z) \rightarrow \left(\dfrac{fX}{Z_0}, \dfrac{fY}{Z_0}\right)$

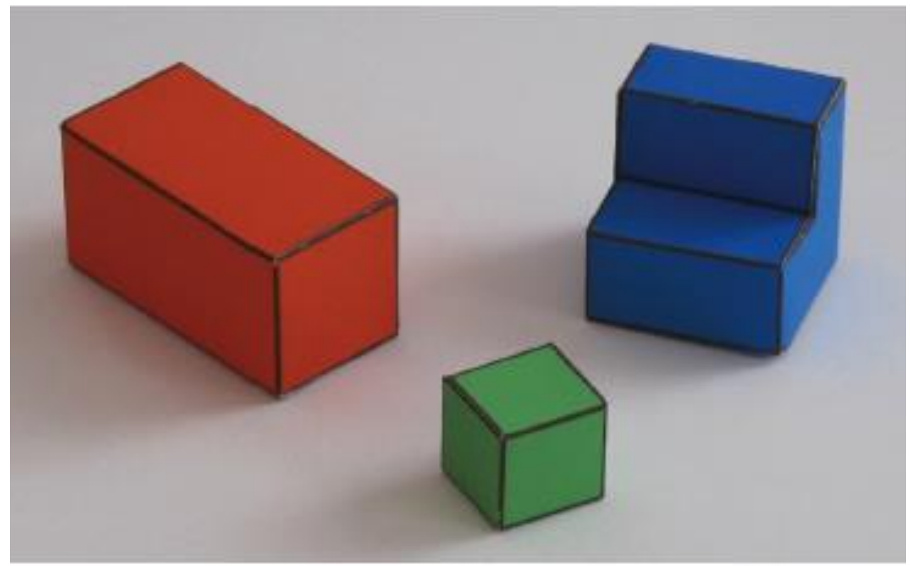
(3) Orthographic: $(X, Y, Z) \rightarrow (X, Y)$

which is perspective, which orthographic?

Perspective projection



Parallel (orthographic) projection



Matrix Time!

- (Scaled) Orthographic Projection

$$x = mX, y = mY$$

$$\begin{bmatrix} m & 0 & 0 \\ 0 & m & 0 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$$

- Perspective Projection

$$x = f\frac{X}{Z}, y = f\frac{Y}{Z}$$