INSTRUCTIONS

- You have 2 hours to complete the exam.

- The exam is closed book, closed notes, closed computer, closed calculator, except one hand-written 8.5" × 11" crib sheet of your own creation and the 2 official 61A midterm study guides attached to the back of this exam.

- Mark your answers ON THE EXAM ITSELF. If you are not sure of your answer you may wish to provide a *brief* explanation.

Last name	
First name	
SID	
Login	
TA & section time	
Name of the person to your left	
Name of the person to your right	
All the work on this exam is my own. (**please sign**)	

For staff use only

Q. 1	Q. 2	Q. 3	Q. 4	Q. 5	Total
/12	/14	/8	/8	/8	/50

Blank Page

1. (12 points) Class Hierarchy

For each row below, write the output displayed by the interactive Python interpreter when the expression is evaluated. Expressions are evaluated in order, and **expressions may affect later expressions**.

Whenever the interpreter would report an error, write ERROR. You *should* include any lines displayed before an error. *Reminder*: The interactive interpreter displays the `repr` string of the value of a successfully evaluated expression, unless it is `None`. Assume that you have started Python 3 and executed the following:

```python
class Worker:
    greeting = 'Sir'
    def __init__(self):
        self.elf = Worker
    def work(self):
        return self.greeting + ', I work'
    def __repr__(self):
        return Bourgeoisie.greeting
class Bourgeoisie(Worker):
    greeting = 'Peon'
    def work(self):
        print(Worker.work(self))
        return 'My job is to gather wealth'
class Proletariat(Worker):
    greeting = 'Comrade'
    def work(self, other):
        other.greeting = self.greeting + ' ' + other.greeting
        other.work() # for revolution
        return other
jack = Worker()
john = Bourgeoisie()
jack.greeting = 'Maam'
```
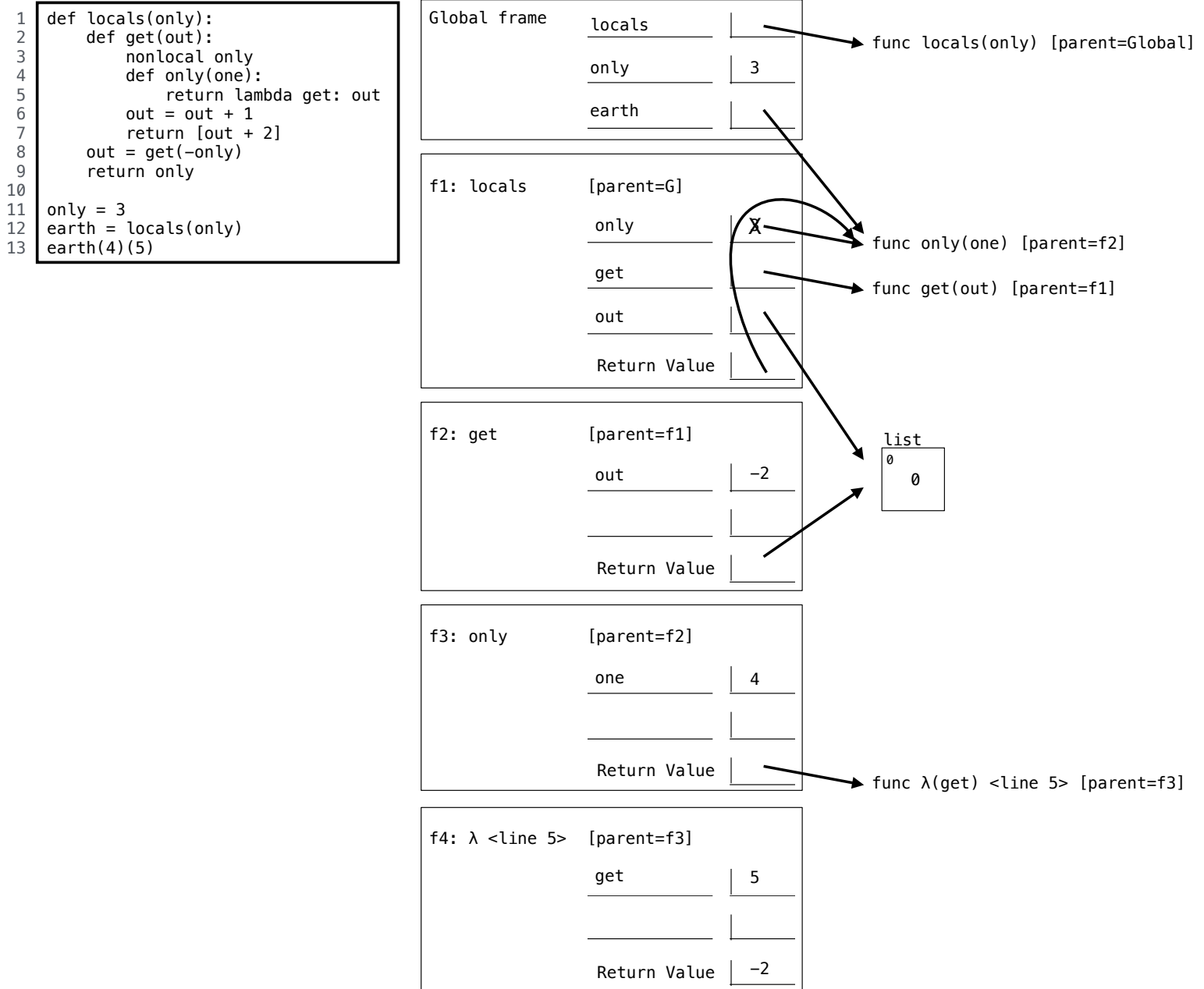
Expression	Interactive Output
5*5	25
1/0	ERROR
Worker().work()	'Sir, I work'
jack	Peon
jack.work()	'Maam, I work'

Expression	Interactive Output
john.work()[10:]	Peon, I work 'to gather wealth'
Proletariat().work(john)	Comrade Peon, I work Peon
john.elf.work(john)	'Comrade Peon, I work'

2. (14 points)   Space

(a) **(8 pt)** Fill in the environment diagram that results from executing the code below until the entire program is finished, an error occurs, or all frames are filled. *You may not need to use all of the spaces or frames.*

A complete answer will:

- Add all missing names and parent annotations to all local frames.
- Add all missing values created during execution.
- Show the return value for each local frame.

```
1  def locals(only):
2      def get(out):
3          nonlocal only
4          def only(one):
5              return lambda get: out
6          out = out + 1
7          return [out + 2]
8      out = get(-only)
9      return only
10
11 only = 3
12 earth = locals(only)
13 earth(4)(5)
```

Global frame

locals	
only	3
earth	

func locals(only) [parent=Global]

f1: locals   [parent=G]

only	X
get	
out	
Return Value	

func only(one) [parent=f2]

func get(out) [parent=f1]

f2: get   [parent=f1]

out	−2
Return Value	

list
0
0

f3: only   [parent=f2]

one	4
Return Value	

func λ(get) <line 5> [parent=f3]

f4: λ <line 5>   [parent=f3]

get	5
Return Value	−2

(b) (6 pt) Fill in the blanks with the shortest possible expressions that complete the code in a way that results in the environment diagram shown. You can use only brackets, commas, colons, and the names `luke`, `spock`, and `yoda`. **You *cannot* use integer literals, such as 0, in your answer!** You also cannot call any built-in functions or invoke any methods by name.
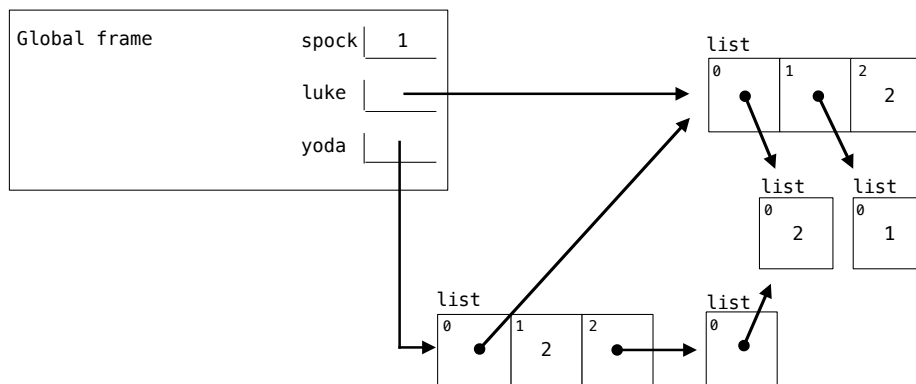
```
spock, yoda = 1, 2

luke = [[yoda], [spock], yoda                                               ]

yoda = 0

yoda = [luke, luke[yoda][yoda]                                              ]

yoda.append(luke[:spock]                                                    )
```

3. (8 points) This One Goes to Eleven

(a) **(4 pt)** Fill in the blanks of the implementation of `sixty_ones` below, a function that takes a `Link` instance representing a sequence of integers and returns the number of times that 6 and 1 appear consecutively.

```
def sixty_ones(s):
    """Return the number of times that 1 follows 6 in linked list s.

    >>> once = Link(4, Link(6, Link(1, Link(6, Link(0, Link(1))))))
    >>> twice = Link(1, Link(6, Link(1, once)))
    >>> thrice = Link(6, twice)
    >>> apply_to_all(sixty_ones, [Link.empty, once, twice, thrice])
    [0, 1, 2, 3]
    """

    if s is Link.empty or s.rest is Link.empty:

        return 0

    elif s.first == 6 and s.rest.first == 1:

        return 1 + sixty_ones(s.rest.rest)

    else:

        return sixty_ones(s.rest)
```

(b) **(4 pt)** Fill in the blanks of the implementation of `no_eleven` below, a function that returns a list of all distinct length-n lists of ones and sixes in which 1 and 1 do not appear consecutively.

```
def no_eleven(n):
    """Return a list of lists of 1's and 6's that do not contain 1 after 1.

    >>> no_eleven(2)
    [[6, 6], [6, 1], [1, 6]]
    >>> no_eleven(3)
    [[6, 6, 6], [6, 6, 1], [6, 1, 6], [1, 6, 6], [1, 6, 1]]
    >>> no_eleven(4)[:4] # first half
    [[6, 6, 6, 6], [6, 6, 6, 1], [6, 6, 1, 6], [6, 1, 6, 6]]
    >>> no_eleven(4)[4:] # second half
    [[6, 1, 6, 1], [1, 6, 6, 6], [1, 6, 6, 1], [1, 6, 1, 6]]
    """

    if n == 0:

        return [[]]

    elif n == 1:

        return [[6], [1]]

    else:

        a, b = no_eleven(n-1), no_eleven(n-2)

        return [[6] + s for s in a] + [[1, 6] + s for s in b]
```

4. (8 points)   Tree Time

(a) **(4 pt)** A `GrootTree` *g* is a binary tree that has an attribute `parent`. Its parent is the `GrootTree` in which *g* is a branch. If a `GrootTree` instance is not a branch of any other `GrootTree` instance, then its `parent` is `BinaryTree.empty`.

BinaryTree.empty should not have a `parent` attribute. Assume that every `GrootTree` instance is a branch of at most one other `GrootTree` instance and not a branch of any other kind of tree.

Fill in the blanks below so that the `parent` attribute is set correctly. You may not need to use all of the lines. Indentation is allowed. You *should not* include any `assert` statements. Using your solution, the doctests for `fib_groot` should pass. The `BinaryTree` class appears on your study guide.

Hint: A picture of `fib_groot(3)` appears on the next page.

```
class GrootTree(BinaryTree):
    """A binary tree with a parent."""
    def __init__(self, entry, left=BinaryTree.empty, right=BinaryTree.empty):

        BinaryTree.__init__(self, entry, left, right)

        self.parent = BinaryTree.Empty

        for b in [left, right]:

            if b is not BinaryTree.empty:

                b.parent = self

def fib_groot(n):
    """Return a Fibonacci GrootTree.

    >>> t = fib_groot(3)
    >>> t.entry
    2
    >>> t.parent.is_empty
    True
    >>> t.left.parent.entry
    2
    >>> t.right.left.parent.entry
    1
    >>> t.right.left.parent.right.parent.entry
    1
    """
    if n == 0 or n == 1:
        return GrootTree(n)
    else:
        left, right = fib_groot(n-2), fib_groot(n-1)
        return GrootTree(left.entry + right.entry, left, right)
```
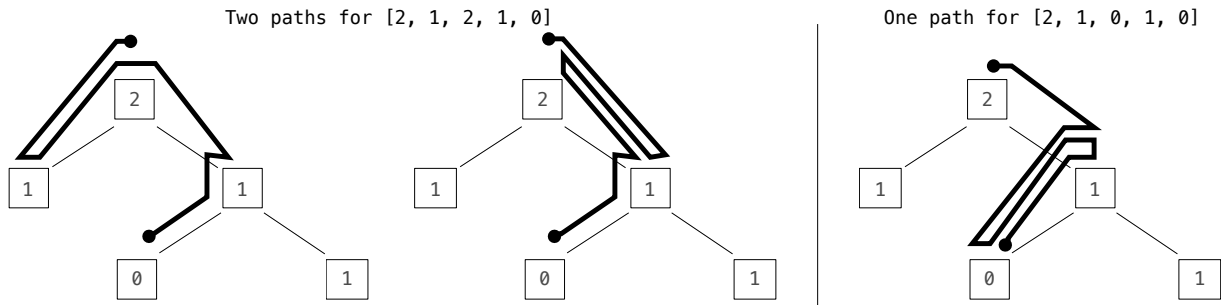
(b) (4 pt) Fill in the blanks of the implementation of `paths`, a function that takes two arguments: a `GrootTree` instance `g` and a list `s`. It returns the number of paths through `g` whose entries are the elements of `s`. A path through a `GrootTree` can extend either to a branch or its `parent`.

You may assume that the `GrootTree` class is implemented correctly and that the list `s` is non-empty.

The two paths that have entries `[2, 1, 2, 1, 0]` in `fib_groot(3)` are shown below (left). The one path that has entries `[2, 1, 0, 1, 0]` is shown below (right).

Two paths for [2, 1, 2, 1, 0]   One path for [2, 1, 0, 1, 0]

```
def paths(g, s):
    """The number of paths through g with entries s.

    >>> t = fib_groot(3)
    >>> paths(t, [1])
    0
    >>> paths(t, [2])
    1
    >>> paths(t, [2, 1, 2, 1, 0])
    2
    >>> paths(t, [2, 1, 0, 1, 0])
    1
    >>> paths(t, [2, 1, 2, 1, 2, 1])
    8
    """

    if g is BinaryTree.empty or s == [] or g.entry != s[0]:


        return 0


    elif len(s) == 1 and g.entry == s[0]:


        return 1


    else:


        extensions = [g.left, g.right, g.parent]


        return sum(paths(x, s[1:]) for x in extensions)
```

5. **(8 points) Abstraction and Growth**

(a) **(6 pt)** Your project partner has invented an abstract representation of a sequence called a `slinky`, which uses a `transition` function to compute each element from the previous element. A `slinky` explicitly stores only those elements that cannot be computed by calling `transition`, using a `starts` dictionary. Each entry in `starts` is a pair of an index key and an element value. See the doctests for examples.

Help your partner fix this implementation by crossing out as many lines as possible, but leaving a program that passes the doctests. Do not change the doctests. The program continues onto the following page.

```python
def length(slinky):
    return slinky[0]
def starts(slinky):
    return slinky[1]
def transition(slinky):
    return slinky[2]

def slinky(elements, transition):
    """Return a slinky containing elements using transition.

    >>> s = slinky(range(3, 10), lambda x: x+1)
    >>> length(s)
    7
    >>> starts(s)
    {0: 3}
    >>> get(s, 2)
    5
    >>> t = slinky([2, 4, 10, 20, 40], lambda x: 2*x)
    >>> starts(t)
    {0: 2, 2: 10}
    >>> get(t, 3)
    20
    >>> slinky([], abs)
    [0, {}, <built-in function abs>]
    >>> slinky([5, 4, 3], abs)
    [3, {0: 5, 1: 4, 2: 3}, <built-in function abs>]
    """
    starts = {}
    for index in range(len(elements)):
        if index == 0 or elements[index] != transition(elements[index-1]):
            starts[index] = elements[index]
    return [len(elements), starts, transition]
```

```
def get(slinky, index):
    """Return the element at index of slinky."""
    start = index
    while start not in starts(slinky):
        start = start - 1
    value = starts(slinky)[start]
    while start < index:
        value = transition(slinky)(value)
        start = start + 1
    return value
```

(b) (2 pt) Circle the Θ expression below that describes the number of operations required to compute `slinky(elements, transition)`, assuming that

- n is the initial length of `elements`,
- d is the final length of the `starts` dictionary created,
- the `transition` function requires constant time,
- the `pop` method of a list requires constant time,
- the `len` function applied to a `list` requires linear time,
- the `len` function applied to a `range` requires constant time,
- adding or updating an entry in a dictionary requires constant time,
- getting an element from a list by its index requires constant time,
- creating a list requires time that is proportional to the length of the list.

$\Theta(1)$ $\qquad$ $\Theta(n)$ $\qquad$ $\Theta(d)$ $\qquad$ $\Theta(n^2)$ $\qquad$ $\Theta(d^2)$ $\qquad$ $\Theta(n \cdot d)$