INSTRUCTIONS

- You have 2 hours to complete the exam individually.

- The exam is closed book, closed notes, closed computer, and closed calculator, except for one hand-written 8.5" × 11" crib sheet of your own creation.

- Mark your answers **on the exam itself**. We will *not* grade answers written on scratch paper.

Last (Family) Name	
First (Given) Name	
Student ID Number	
Berkeley Email	
Teaching Assistant	◯ Alex Stennet ◯ Christina Zhang ◯ Jennifer Tsui ◯ Alex Wang ◯ Derek Wan ◯ Jenny Wang ◯ Cameron Malloy ◯ Erica Kong ◯ Kevin Li ◯ Chae Park ◯ Griffin Prechter ◯ Nancy Shaw ◯ Chris Allsman ◯ Jemin Desai
Exam Room and Seat	
Name of the person to your left	
Name of the person to your right	
All the work on this exam is my own. **(please sign)**	

POLICIES & CLARIFICATIONS

- You may use built-in Python functions that do not require import, such as `min`, `max`, `pow`, and `abs`.

- For fill-in-the blank coding problems, we will only grade work written in the provided blanks. You may only write one Python statement per blank line, and it must be indented to the level that the blank is indented.

- Unless otherwise specified, you are allowed to reference functions defined in previous parts of the same question.

1. (11 points)   No Capes!

For each of the expressions in the table below, write the output displayed by the interactive Python interpreter when the expression is evaluated. The output may have multiple lines. Each expression has at least one line of output.

- If an error occurs, write **ERROR**, but include all output displayed before the error.
- To display a function value, write **FUNCTION**.
- If an expression would take forever to evaluate, write **FOREVER**.

The interactive interpreter displays the value of a successfully evaluated expression, unless it is `None`.

Assume that you have started `python3` (not `ipython` or other variants) and executed the code shown on the left first, then you evaluate each expression on the right in the order shown. Expressions evaluated by the interpreter have a cumulative effect.

```
1   mr, incredible = 13, 21
2
3   def el(ast, i, girl):
4       if ast > i or i / girl:
5           print('stretch')
6       return incredible
7
8   zen = 7
9
10  def fro(zone):
11      def where(is_my):
12          print('supersuit')
13          return is_my + 2
14      print(zone + zen)
15      return where
16
17  dash = 4
18
19  def edna(mo, de=4):
20      jack = 5
21      if de // 3 < 2:
22          return zen
23      return mo(jack) + dash
24
25  jack = lambda jack: edna(jack)
26
27  def vi(o, let):
28      if let % 3:
29          return o(3)
30      print('hidden')
31      return vi(print, let + 2)
```

Expression	Interactive Output
print(4, 5) + 1	4 5 **ERROR**
incredible / dash	5.25
el(mr, 4, 0)	stretch 21
sam = fro(zen)	14
edna(sam, 7)	supersuit 11
vi(dash, incredible)	hidden 3
vi(jack, dash)	7

2. (10 points)   Dog Days

Fill in the environment diagram that results from executing the code below until the entire program is finished, an error occurs, or all frames are filled. *You may not need to use all of the spaces or frames.*

A complete answer will:

- Add all missing names and parent annotations to all local frames.
- Add all missing values created or referenced during execution.
- Show the return value for each local frame.

```
Global frame
                    pet | "dog"
                    bug | 3
                     to | ————————————→  func to(do) [parent=Global]
                   walk | ————————————→  func walk(time) [parent=Global]

f1:   walk      [parent: Global]
                   time | ————————————→  func λ() [parent=Global]
                    pet | "cat"
                    bed | ——————
           Return Value | "itch"

f2:   λ         [parent: Global]
                        | ——————————————→ func λ(bye) [parent=f3]
                        | ——————
                        | ——————
           Return Value | ——————

f3:   to        [parent: Global]
                     do | "dog"
                    bug | 5
           Return Value | ——————

f4:   λ         [parent: f3]
                    bye | 3
                        | ——————
           Return Value | -2

f5:   λ         [parent: f1]
                        | ——————
                        | ——————
           Return Value | "itch"
```

```
1   pet = "dog"
2   bug = 3
3
4   def to(do):
5       bug = 5
6       if do == "sleep":
7           return "zzz"
8       return lambda bye: bye - bug
9
10  def walk(time):
11      pet = "cat"
12      bed = time()
13      if not bed(bug):
14          return to("sleep")
15      else:
16          return (lambda: "itch")()
17
18  walk(lambda: (to(pet) or not 2*pet))
```
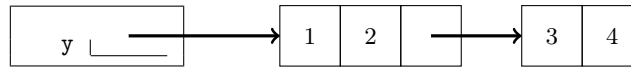
3. (7 points)   List-Man and the Box

Draw box-and-pointer diagrams for the state of the lists after executing each block of code below. You don't need to write index numbers or the word "list". Please erase or cross out any boxes or pointers that are not part of a final diagram.
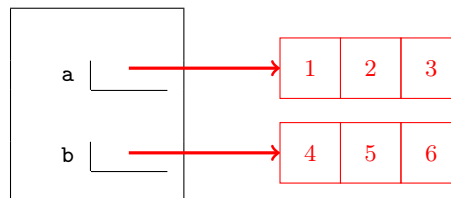
An example is given below:
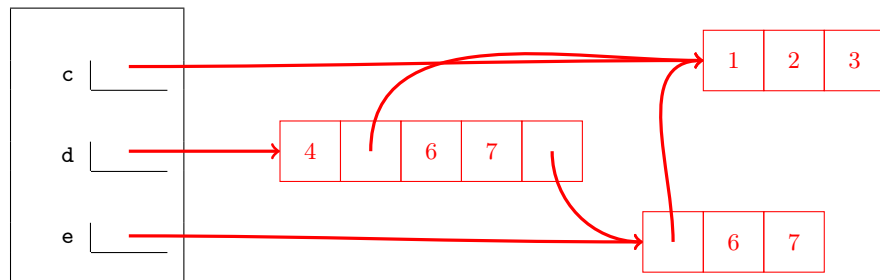
```
1  y = [1, 2, [3, 4]]
```


(a) (2 pt)

```
1  a = [1, 2, 3]
2  b = [4, 5, 6]
3  a.insert(a[0], b)
4  b.pop(a.pop(1))
```

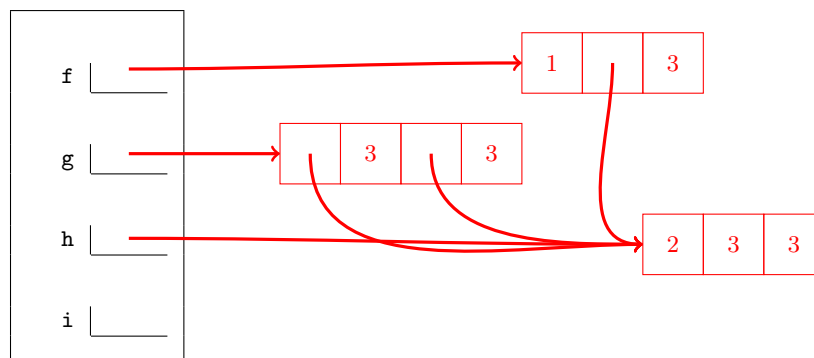The code errors after removing from **a** but before removing from **b**.

(b) (2 pt)

```
1  c = [1, 2, 3]
2  d = [4, c, 6, 7]
3  e = [d[x] for x in c]
4  d.append(e)
```


(c) (3 pt)

```
1  f, h = [1], [2]
2  f.extend([h, 3])
3  g = f[1:] + f[1:]
4  i = 0
5  while i < len(f):
6      if i % 2 == 0:
7          h.append(g[f[i]])
8      i += 1
```

4. (6 points)   Book Club

(a) **(2 pt)** A function is a higher order function if it has at least one of two particular properties. What are those two properties?

    1. Return a function

    2. Take in a function as input

(b) **(2 pt)** Give one reason why you shouldn't violate abstraction barriers. Limit your answer to 15 words or less.

When you violate abstraction barriers, changing the implementation is no longer possible
OR
Violating abstraction barriers makes your code less readable

(c) **(2 pt)** What's a difference between tuples and lists besides syntax? Limit your answer to 10 words or less.

List are mutable; tuples are immutable

5. (6 points)   Won't You Be My Neighbor?

(a) **(4 pt)** Write `repeat_digits`, which takes a positive integer `n` and returns another integer that is identical to `n` but with each digit repeated.

```
def repeat_digits(n):
    """Given a positive integer N, returns a number with each digit repeated.
    >>> repeat_digits(1234)
    11223344
    """
    last, rest = n % 10, n // 10
    if rest == 0:
        return last * 11 # or last * 10 + last, or equivalent
    return repeat_digits(rest) * 100 + last * 11

    OR

    last, rest = n % 10, n // 10
    if n == 0:
        return 0
    return repeat_digits(rest) * 100 + last * 10 + last
```

(b) **(2 pt)** Let d be the number of digits in `n`. What is the runtime of `repeat_digits` with respect to d?

    ○ $\Theta(1)$ ○ $\Theta(\log d)$ ○ $\Theta(\sqrt{d})$ ● $\Theta(d)$ ○ $\Theta(d^2)$ ○ $\Theta(2^d)$

6. (6 points) Ocean's Eight

Write `eight_path`, which takes in a tree `t` and returns a list of labels following a path from the top of the tree (the root) to a leaf whose sum is divisible by 8. If there are multiple such paths, return the leftmost one. If there is no such path, return None.

The tree data abstraction is provided here for your reference. Do not violate the abstraction barrier!

```
def tree(label, branches=[]):              def is_leaf(t);
    return [label] + list(branches)            return not branches(t)


def label(t):                              def branches(t):
    return t[0]                                return t[1:]
```

```
def eight_path(t):
    """Returns a path_so_far of the labels from the root to a leaf whose sum is a multiple of eight,
    or return None if no path exists.

    >>> t1 = tree(5, [tree(2),
                      tree(1, [tree(3),
                              tree(2)])
                  ])
    >>> eight_path(t1)
    [5, 1, 2]
    >>> t2 = tree(9, [t1])
    >>> eight_path(t2)
    [9, 5, 2]
    """
    def helper(t, path_so_far):
        path_so_far = path_so_far + [label(t)]
        if is_leaf(t) and sum(path_so_far) % 8 == 0:
            return path_so_far
        for b in branches(t):
            result = helper(b, path_so_far)
            if result:
                return result
    return helper(t, [])
```
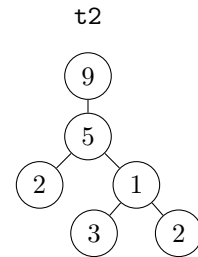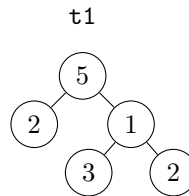
7. (6 points) Never Tell Me The Odds

Han and Lando are playing a game of Sabacc. The rules are as follows:

- Two players take turns drawing from a deck of cards
- Each turn, players can choose to draw either 1 or 2 cards
- A player wins when there are no cards left at the start of their turn.

Han and Lando are both very good at the game, so they play *optimally.* That is, if there is a move they can take that will allow them to win (assuming their opponent also plays optimally), they will take it.

Write `sabacc_winner`, which takes a number of cards and two players, and returns the winner if both players play optimally and player 0 goes first.

You *must* use recursion to solve this problem. Writing the closed-form solution will receive no credit.

```
def sabacc_winner(cards, player0, player1):
    """Returns the winner of a game of Sabacc if players can take 1 or 2 cards
    per turn and both players play optimally. Assume that it is player0's turn.
    >>> sabacc_winner(0, 'Han', 'Lando')
    'Han'
    >>> sabacc_winner(1, 'Han', 'Lando')
    'Lando'
    >>> sabacc_winner(2, 'Han', 'Lando')
    'Han'
    >>> sabacc_winner(3, 'Han', 'Lando')
    'Han'
    >>> sabacc_winner(4, 'Han', 'Lando')
    'Lando'
    """
    if cards == 0:
        return player0
    if cards == 1:
        return player1
    take_one = sabacc_winner(cards - 1, player1, player0)
    take_two = sabacc_winner(cards - 2, player1, player0)
    if take_one == player0 or take_two == player0:
        return player0
    return player1
```

8. (8 points)   Mr. Stark, I Don't Feel So Good

A *messenger function* is a function that takes a single word and returns another messenger function, until a period is provided as input, in which case a sentence containing the words provided is returned. At least one word must be provided before the period.

As an example, here's a simple messenger function that returns a sentence with all of the words that have been provided.

```
>>> simple_messenger("Avengers")("assemble")(".")
'Avengers assemble.'
>>> simple_messenger("Get")("this")("man")("a")("shield")(".")
'Get this man a shield.'
```

Write `thanos_messenger`, which is a messenger function that discards every other word that's provided. The first word should be included in the final sentence, the second word should be discarded, and so on.

```
def thanos_messenger(word):
    """A messenger function that discards every other word.

    >>> thanos_messenger("I")("don't")("feel")("so")("good")(".")
    'I feel good.'
    >>> thanos_messenger("Thanos")("always")("kills")("half")(".")
    'Thanos kills.'
    """
    assert word != '.', 'No words provided!'
    def make_new_messenger(message, skip_next):
        def new_messenger(word):
            if word == '.':
                return message + '.'
            if skip_next:
                return make_new_messenger(message, False)
            return make_new_messenger(message + " " + word, True)
        return new_messenger
    return make_new_messenger(word, True)
```

9. (0 points)   Your Mission, Should You Choose To Accept It

In this extra credit problem, you may choose one of two options.

- Mark the choice to "Betray" and write a positive integer in the blank below. The one student who writes the *smallest, unique positive integer* will receive *two* (2) extra credit points but only if fewer than 90% of students choose the next option.
- Mark the choice to "Work Together". If at least 90% of students choose this option, all students who chose this option will receive *one* (1) extra credit point and those who marked the choice to "Betray" will receive zero (0) extra credit points.

Will you *work together*? Or will you *betray* your fellow students? It is up to you.

○ Betray _____

● Work Together

10. (0 points)   I've Been Waiting For You

If you finish early, please stay in your seat until time is up and we are ready to move on to the group section. After you've reviewed your work, if you have extra time, you can try this challenge problem while you wait (not for credit).

Work on this problem quietly until the individual section is over. We will not accept questions on this problem.

(a) (0 pt) Implement `lcs`, which finds the length of the longest common subsequence between two words. Note that the ordering of the characters must be preserved.

```
def lcs(word1, word2):
    """ Finds the length of the longest common subsequence between two words.
    >>> lcs("water", "loo")
    0
    >>> lcs("mamma", "mia")  # ma
    2
    >>> lcs("super", "trouper")  # uper
    4
    """
    if word1 == "" or word2 == "":
        return 0
    elif word1[0] == word2[0]:
        return 1 + lcs_len(word1[1:], word2[1:])
    else:
        return max(lcs_len(word1, word2[1:]), lcs_len(word1[1:], word2))
```

(b) (0 pt) What is the runtime (in the worst case) of `lcs`? By *worst case*, we mean we want you to consider inputs that would be particularly time consuming for `lcs`.

$\Theta(n \cdot m \cdot 2^{min(n,m)})$

(c) (0 pt) Now, implement `lcs_fast` which does the same thing as `lcs`, but has a time complexity of $\Theta(m * n)$, where m is the number of characters in `word1` and n is the number of characters in `word2`.

```python
def lcs_fast(word1, word2):
  cache = {}

  def helper(i, j):
      if i >= len(word1) or j >= len(word2):
          return 0
      elif (i, j) in cache:
          return cache[(i, j)]
      elif word1[i] == word2[j]:
          cache[(i, j)] = 1 + helper(i + 1, j + 1)
      else:
          cache[(i, j)] = max(helper(i, j + 1), helper(i + 1, j))
      return cache[(i, j)]

  return helper(0, 0)
```

(d) (0 pt) Provide a high-level proof as to why the runtime of your implementation of `lcs_fast` is $\Theta(m * n)$.

This solution is very similar to the regular solution except we now keep track of the index of the character we're checking instead of cutting down our word. We know that both indices will increment from 0 to the length of their respective strings. We also know that we are checking every combination of `i` and `j`. There are at most `m*n` combinations of `i` and `j`. In our first solution we had multiple recursive calls that covered duplicate combinations. However because we are saving unique `(i, j)` combinations in a dictionary, we won't be doing any duplicate work and so we will have at most `m*n` recursive calls. Our runtime would then be $\Theta(m * n)$.

11. (0 points)   Terminal

Done? Still need something to do?

(a) (0 pt) Can you name the movie coming out / already released this summer that each problem title is referencing?

1. The Incredibles 2
2. Dog Days
3. Ant-Man and the Wasp
4. Book Club
5. Won't You Be My Neighbor?
6. Oceans's 8
7. Solo: A Star Wars Story
8. Avengers: Infinity War
9. Mission: Impossible - Fallout
10. Mamma Mia! Here We Go Again
11. Terminal

(b) (0 pt) Any feedback for us on how this exam went / how the course is going so far?