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Computability of the Cosmos

In mathematics, there are certain fundamental laws that are the building blocks of the most complex problems. Mathematical logic can be broken down into specific and simple axioms that can be used to form extremely complicated patterns. Simple laws lead to more abstract ideas that go beyond mathematics, as it is also a common pattern in physics, biology, and computing. Understanding mathematical logic requires further exploration into whether such rules and schemas are fundamental to how the entire universe is governed. If there are a finite number of such rules and schemas, then there must be a way in which the entire universe can be computable. If the universe is computable, then arguably everything known is programmed in a computer—perhaps a computer much more advanced than the average desktop. This paper will begin by delving into the thesis that the universe is a digital computer, a cellular automaton (CA). It will discuss uncomputability, and how in the CA framework, it is theoretically possible to compute any problem that can be computed algorithmically.

The concept that governance of the universe can be programmed into a computer was postulated by Konrad Zuse, the inventor of the first fully functional programmable computer and first ever programming language.¹ In his 1969 book, *Space Computes*, Zuse argues that “all information can be broken up into yes-no values (bits).”² Zuse uses an “automaton theoretical way of thinking” which means the universe is viewed as “a lapse of states,” which follow from predetermined rules.³ This suggests that our universe is governed by a finite number of simple mathematical transition rules and processes.

Zuse further posits that the physical universe runs on a certain type of digital computer—similar to CA. CA is defined as “discrete, abstract computational systems.”⁴ To be discrete means that CA “are composed of a finite or denumerable set of homogeneous, simple units, the atoms or cells.”⁵ These cells are abstract in that they can be defined mathematically and implemented physically. Because CA are computational systems, they can compute algebraic problems. Zuse then applies this concept to the entire universe. He argues that if the universe is seen as CA, then the cosmos is made of single cells that represent a finite automata. He writes, “individual cells can accept a limited number of states and have therefore only a limited

¹ “Konrad Zuse.” *Encyclopædia Britannica*, Encyclopædia Britannica, inc., www.britannica.com/biography/Konrad-Zuse. Accessed 6 Oct. 2023.

² Zenil, Hector, and Roger Penrose. *A Computable Universe: Understanding and Exploring Nature as Computation*. World Scientific, 2013. <https://philpapers.org/archive/ZUSRR.pdf> (Space Computes)

³ *Id.*

⁴ Berto, Francesco and Jacopo Tagliabue, “Cellular Automata”, *The Stanford Encyclopedia of Philosophy* (Spring 2022 Edition), Edward N. Zalta (ed.), URL = <https://plato.stanford.edu/archives/spr2022/entries/cellular-automata/>. (Berto)

⁵ *Id.*

information content. This is equally true for the entire cosmos, if we make suitable assumptions about its limits.”⁶ Zuse’s point undermines the concept that individual cells, as components of a computational system, have inherent limitations in the number of states they can assume, consequently simplifying the entire system into a discrete— and thus computable— process.

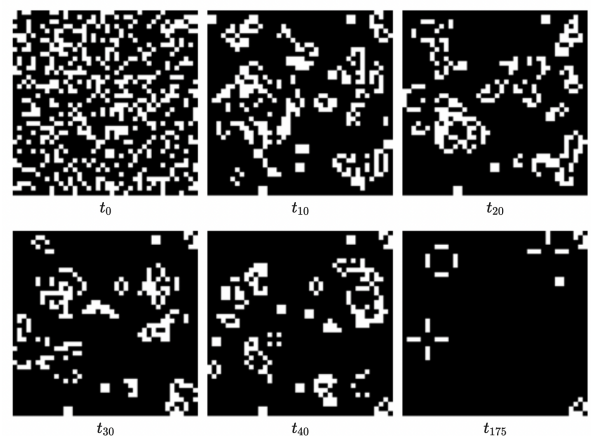
If Zuse is correct in his thesis, then all physical phenomena with which we are familiar are large-scale patterns from the evolution of computation operating everywhere in the universe at the smallest scale. This idea can be explored with a simple illustration of cellular automata as seen in John Conway’s game aptly titled *Game of Life*.

“Conway’s genetic laws are delightfully simple. First note that each cell of the checkerboard (assumed to be an infinite plane) has eight neighboring cells, four adjacent orthogonally, four adjacent diagonally. The rules are:

1. Survivals. Every counter with two or three neighboring counters survives for the next generation.
2. Deaths. Each counter with four or more neighbors dies (is removed) from overpopulation. Every counter with one neighbor or none dies from isolation.
3. Births. Each empty cell adjacent to exactly three neighbors--no more, no fewer--is a birth cell. A counter is placed on it at the next move.

It is important to understand that all births and deaths occur *simultaneously*. Together they constitute a single generation or, as we shall call it, a ‘move’ in the complete ‘life history’ of the initial configuration.”⁷

The rules of the *Game of Life* are simple, but designed specifically “to make the behavior of the population unpredictable.”⁸ In the game, the decisions and patterns come from the player. In the computer versions of the game, these decisions are made by algorithms.⁹ As time passes, as seen in computer models of the game, complex and seemingly non-random patterns emerge. The *Game of Life* shows how complex patterns on a large scale may emerge from simple computational rules on a small scale.¹⁰ Even in such an elementary setting, “periodic structures, stable blocks and complex moving patterns come into existence.”¹¹ When assessing



⁶ See: Zuse

⁷ Gardner, Martin. “MATHEMATICAL GAMES.” *Scientific American*, vol. 223, no. 4, 1970, pp. 120–23. JSTOR, <http://www.jstor.org/stable/24927642>. Accessed 6 Oct. 2023.

⁸ *Id.*

⁹ Caballero, Lorena et al. “Conway’s “Game of Life” and the Epigenetic Principle.” *Frontiers in cellular and infection microbiology* vol. 6 57. 14 Jun. 2016, doi:10.3389/fcimb.2016.00057 (Caballero)

¹⁰ Sprevak, Mark, et al. “Zuse’s Thesis, Gandy’s Thesis, and Penrose’s Thesis.” *Physical Perspectives on Computation, Computational Perspectives on Physics*, edited by Michael E. Cuffaro and Samuel C. Fletcher, Cambridge University Press, Cambridge, 2018, pp. 39–59. (Sprevak)

¹¹ See: Berto

his game, Conway remarks that in a large enough space and starting in a random state, “after a long time, intelligent, self-reproducing animals will emerge and populate some parts of the space.”¹² Conway's contemplation of the potential emergence of intelligent, self-reproducing entities within the *Game of Life* reflects the tantalizing possibility that within vast computational spaces, complex life-like behaviors might spontaneously evolve, mirroring the universe's capacity to foster diverse forms of life. The structures in the *Game of Life* are thought-provoking because, like atoms, cells, and people, “they maintain cohesion, move, reproduce, interact with each other. They are governed by their own rules,” much like the patterns that are noticeable in the cosmos.¹³ The game signifies the notion that both microcosmic and macrocosmic phenomena are governed by intrinsic rules. As environments become more complex, the development becomes unpredictable. When observing patterns emerging in the *Game of Life*, “there must surely arise true living ‘life-forms,’ perhaps themselves evolving into more complex, possibly sentient, ‘organisms.’”¹⁴ The *Game of Life* highlights the idea that simplicity can be the precursor to unforeseen intricacy. The potential emergence of true living ‘life-forms’ and even sentient ‘organisms’ within an infinite lattice shows the remarkable capacity of computational systems to give rise to lifelike entities, reflecting the diversity of life in the universe.

The universe is different than the *Game of Life*, but there are noticeable parallels. While “these are not the rules of our universe... perhaps other transition rules are.”¹⁵ There may be transitional rules that guide our universe in the way that certain rules guide the *Game of Life*. Philosopher T’ Hooft notes, “I think Conway’s *Game of Life* is the perfect example of a toy universe. I like to think that the universe we are in is something like this.”¹⁶ The observation that, despite the stark differences between our universe and Conway’s *Game of Life*, there are discernible similarities that invite contemplation about present cosmic reality. In the last 50 years, several scientists have joined Zuse to postulate “that the physical universe *is*, fundamentally, a discrete computational structure. Everything in our world—quarks, trees, human beings, remote galaxies—is just a pattern in a CA, much like a glider in *Life*.”¹⁷ The perpetuality of the *Game of Life* produces unexpected outcomes, even when the rules are known. This emphasizes the challenges of comprehending the fundamental rules governing reality.

Although the theory is fascinating, Zuse’s thesis is not without critique. Arguably, the cosmos cannot be considered a computer when there are instances of uncomputability. This line of reasoning begs the question: *is the universe even computable?* Alan Turing’s 1936 paper *On Computable Numbers* defines “computable” as being carried out in a finite number of steps.¹⁸ It follows that if everything in the universe can be carried out in a set of steps, then the cosmos

¹² Ilachinski, Andrew, 2001, *Cellular Automata*, Singapore: World Scientific Publishing. (Ilachinski)

¹³ See: Sprevak

¹⁴ See: Ilachinski

¹⁵ Deutsch, D. 2003. ‘It from qubit’. In J. Barrow, P. Davies, and C. Harper, eds, *Science and Ultimate Reality*. Cambridge University Press, 90–102.

¹⁶ ’t Hooft, G. 2002. ‘Looking at life with Gerardus ’t Hooft’. *Plus Magazine* (January). <http://plus.maths.org/issue18/features/thooft/>

¹⁷ *Id.*

¹⁸ Turing, Alan M., 1936, “On Computable Numbers with an Application to the Entscheidungsproblem”, *Proceeding of the London Mathematical Society*, 42: 230–265. doi:10.1112/plms/s2-42.1.230

itself is indeed computable. However, Turing further outlines well-defined mathematical problems the Turing machine cannot solve, including the Hilbertian Entscheidungs problem.¹⁹ Generally, it is clear that there *must* be real numbers that cannot be computed by any Turing machine, because there are *more* real numbers than there are Turing-machine programs.²⁰ As Georg Cantor proved in 1874, there are vastly more real numbers than whole numbers.²¹ Moreover, mathematician Kurt Gödel's Incompleteness Theorems further illuminates constraints within formal axiomatic theories, revealing inherent limits to what can be proven. Gödel showed there are limits to the probability in formal axiomatic theories.²² This work inspired mathematician Roger Penrose, which he "regard[s] as providing a strong case for human understanding being something essentially non-computable"—understanding being "one manifestation of human consciousness."²³ When there are clear instances of uncomputable mathematic problems in existence, arguably, it is difficult to believe that the universe is a computer. The presence of uncomputable phenomena raises a fundamental question regarding the computability of the universe. If there are aspects of the cosmos that defy computation, it prompts one to contemplate whether the universe, in its entirety, adheres to computable principles.

Turing also believed that all the numbers that can be computed can be done with a computer (of sorts). This is represented in the Church-Turing thesis, which claims that "that every effective computation can be carried out by a Turing machine."²⁴ The Church-Turing thesis forms the foundational premise for the computability of various problems and processes. In fact, computer scientist Stephen Wolfram extended the claim to produce a physical form of the Church-Turing hypothesis, which says that the universal Turing machine can simulate any physical system.²⁵ He concludes that "universal computers are as powerful in their computational capacities as any physically realizable system can be, so that they can simulate any physical system."²⁶ By "physical," Wolfram means the actual laws of nature, including "not only actually existing systems but also idealized physical systems [...] and physically possible systems that do not actually exist, but that could exist, or did exist (e.g. in the universe's first moments), or will exist."²⁷ Wolfram's extension of the Church-Turing hypothesis into a physical realm suggests

¹⁹ *Id.*

²⁰ Copeland, B. Jack, "The Church-Turing Thesis", *The Stanford Encyclopedia of Philosophy* (Summer 2020 Edition), Edward N. Zalta (ed.), URL = <<https://plato.stanford.edu/archives/sum2020/entries/church-turing/>>. (Copeland)

²¹ Cantor, G., 1874, "Ueber eine Eigenschaft des Inbegriffs aller reellen algebraischen Zahlen", *Journal für die reine und angewandte Mathematik*, 77: 258–262.

²² Raatikainen, Panu, "Gödel's Incompleteness Theorems", *The Stanford Encyclopedia of Philosophy* (Spring 2022 Edition), Edward N. Zalta (ed.), URL = <<https://plato.stanford.edu/archives/spr2022/entries/goedel-incompleteness/>>.

²³ Zenil, Hector. "A computable universe: Understanding and exploring nature as computation." *Forward by Roger Penrose*, 2013, <https://doi.org/10.1142/8306>.

²⁴ See: Copeland

²⁵ See: Sprevak

²⁶ Wolfram, S. Undecidability and intractability in theoretical physics. *Physical Review Letters* 54, 8 (Feb. 1985), 735–738.

²⁷ See: Sprevak

that universal computers, exemplified by Turing machines, possess the computational capability to emulate any conceivable physical system.

This notion encompasses not only currently existing systems, but also hypothetical and idealized ones, including those that may have existed in the universe's early stages or those that could potentially emerge. Research and modelling shows “that *Life* can compute everything a universal Turing machine can and therefore, taking on board Turing's Thesis, function as a general purpose computer.”²⁸ The assertion that CA, such as Conway's *Game of Life*, have the computational capacity to perform tasks equivalent to a universal Turing machine reinforces the idea that the Church-Turing thesis encompasses a wide array of computational scenarios. CA, when configured with appropriate rules, can simulate a universal Turing machine, suggesting its capability to compute diverse problems, aligning with the Church-Turing thesis. That means “CA with suitable rules can emulate a universal Turing machine and therefore compute, and given the Church-Turing thesis, compute anything.”²⁹ This alignment with the Church-Turing thesis suggests that within the CA framework, it becomes theoretically possible to compute any problem or process that can be computed algorithmically.

This idea extends to determinist physical forms, too. Robin Gandy, a student of Turing, created a thesis to argue that “every discrete deterministic physical assembly is computable (assuming that there is an upper bound on the speed of propagation of effects and signals, and a lower bound on the dimensions of an assembly's components).”³⁰ Gandy proposes that any discrete and deterministic physical system can be computed, provided certain conditions are met. These conditions involve constraints on signal propagation, speed, and the dimensions of the system's components, thereby reinforcing the idea of computational universality in physical systems. These conclusions have led some people to believe that uncomputable problems do not discount the concept that the universe could still be computable, a theory known as *ontic pancomputationalism*.³¹ The proposition that uncomputable problems do not necessarily invalidate the concept of a computable universe has given rise to the theory of ontic pancomputationalism. This perspective suggests that even in the face of certain inherently uncomputable problems, the universe could still fundamentally operate according to computable principles, echoing the profound implications of computational theory in our understanding of reality.

One such approach follows along the lines of physicist Edward Fredkin's “Finite Nature Hypothesis.” The hypothesis explains that “ultimately every quantity of physics, including space and time, will turn out to be discrete and finite; that the amount of information in any small volume of space-time will be finite and equal to one of a small number of possibilities.”³² The

²⁸ See: Berto

²⁹ *Id.*

³⁰ See: Sprevak

³¹ Piccinini, Gualtiero and Corey Maley, “Computation in Physical Systems”, *The Stanford Encyclopedia of Philosophy* (Summer 2021 Edition), Edward N. Zalta (ed.), URL = <<https://plato.stanford.edu/archives/sum2021/entries/computation-physicalsystems/>>.

³² Fredkin, Edward, 1993, “A New Cosmogony”, in *PhysComp '92: Proceedings of the Workshop on Physics and Computation*, IEEE Computer Society Press, pp. 116–121. doi:10.1109/PHYCMP.1992.615507

hypothesis theorizes that the foundational elements of the physical world, such as space and time, are fundamentally discrete and finite. This perspective suggests that even in small volumes of space-time, the amount of information present is finite and can assume only a limited number of states. A finite nature implies that physics operates in a manner similar to CA.³³ So, perhaps even with recognition of uncomputability, the universe could still be computable. Furthermore, it proposes that the underlying framework of physics shares similarities with cellular automata, specialized computational models. So if a cellular automaton is a model that satisfies this hypothesis, then “underneath the laws of physics as we know them today it could be that there lies a simple program from which all the known laws (...) emerge.”³⁴ If cellular automata indeed align with Fredkin’s hypothesis, it raises the intriguing possibility that beneath our current understanding of the laws governing the universe, there might exist a fundamental and uncomplicated program. This program could serve as the basis from which all the established laws of physics originate, ushering in a transformative perspective on the nature of the physical world.

Zuse’s thesis that the universe is a CA computer and the implications of the *Game of Life* open questions into what is known about the universe. While there are problems that are uncomputable, it is possible to compute any problem that can be computed algorithmically. With that understanding, the universe can be viewed as a finite system of cells. If finite, the cosmos is computable, much like the mathematical systems that govern logic. Perhaps the universe itself is a grand computer program.

³³ *Id.*

³⁴ Wolfram, S., 1983, “Statistical Mechanics of Cellular Automata”, *Reviews of Modern Physics*, 55(3): 601–644. doi:10.1103/RevModPhys.55.601