1 Bondi Hoyle

Bondi & Hoyle (1944); The rate of accretion can be give as::

$$\dot{M} = \frac{4\pi G^2 M^2 \rho_\infty}{v^3} \tag{1}$$

2 System physics and model properties

STRAIGHT FROM Camilo Fontecilla, Zoltn Haiman, Jorge Cuadra, https://arxiv.org/abs/1810.02857v1. In this section, we first explain the relevant physics in our study, then we discuss the simplifications needed to implement a 1D simulation, and finally give details of our numerical code.

We model the inner– and circumbinary discs assuming their scale height h is much smaller than the distance to the centre r, so the system behaves as a standard, two-dimensional thin disc. Furthermore, we consider the primary black hole to be at the center of mass of the system, ignore the individual disc around the secondary, and assume that the secondary and both accretion discs follow circular, co-planar and prograde paths around the primary black hole. These assumptions allow us to model the system in one dimension, with all disc properties functions of the radial coordinate r only.

2.1 Viscosity, thickness and energetics

Viscosity acts as an angular momentum transport mechanism, which produces a torque between contiguous rings of the disc. The standard α-disc model (**?**) parametrizes the turbulent kinematic viscosity as a function of the sound speed c_s and the thickness of the disc h. While c_s depends on the total pressure, in this study we implement an alternative viscosity prescription called a β-disc model, which ensures thermal and viscous stability:

$$\nu = \alpha c_s h \beta, . \tag{2}$$

This allows us to put all the uncertainties in the constant $\alpha \leq 1$. Here, β is the ratio between the gas pressure $p_{\rm gas}$ and total pressure $p_{\rm tot} = p_{\rm gas} + p_{\rm rad}$ (with $p_{\rm rad}$ the radiation pressure). For simplicity, we consider the pressures to be determined by the mid-plane temperature of the disc T_c: $p_{\rm rad} = 4\sigma T_c^4/(3c)$ and $p_{\rm gas} = \rho k T_c/(\mu m_p)$, with k the Boltzmann constant, σ the Stefan-Boltzmann constant, and $\mu = 0.615$ the mean particle mass in units of the proton mass m_p for a plasma of solar metallicity.

The sound speed in the disc is given by $c_s^2 = p_{\rm tot}/\rho$ with $\rho = \Sigma/(2h)$ the volumetric density and Σ the surface density of the discs. Assuming hydrostatic equilibrium in the vertical direction, we can express the sound speed as a function of thickness h and angular velocity: $c_s = \Omega h$, with $\Omega = (GM(1+q)/r^3)^{1/2}$ the angular velocity of the material.

To close this system of equations, we relate the central temperature T_c to the surface density Σ, assuming photons are transported to the disc surface by vertical diffusion: $F = 8\sigma T_c^4/(3\Sigma\kappa)$ (where κ is the opacity due to electron scattering), and that viscous and tidal heating are dissipated in the form of radiation (**??**),

$$F = D_\nu + D_\Lambda = \frac{9}{8}\nu\Sigma\Omega^2 - \frac{1}{2}\Lambda\Sigma(\Omega - \Omega_s). \tag{3}$$

The tidal term D_Λ, as explained in **?**, comes from angular momentum conservation and from the assumption that orbits remain circular.

Using the above formulation leads to a system of equations, uniquely determining the disc properties at each radius and time:

$$\begin{aligned} T_c &= \left[\frac{3\kappa\Sigma^2\big(9\alpha c_s^2\Omega\beta - 4\Lambda(\Omega-\Omega_s)\big)}{64\sigma}\right]^{\frac{1}{4}}, \\ \beta &= \left[1 + \frac{8\sigma\mu m_p T_c^3 c_s}{3ck\Sigma\Omega}\right]^{-1}, \\ c_s &= \frac{8\sigma T_c^4}{3c\Omega\Sigma(1-\beta)}. \end{aligned} \tag{4}$$

It can be shown that there is only one real and positive solution for the central temperature. Solving the equations allows us to obtain the sound speed c_s, the thickness h of the disc, and finally the viscosity itself (**?**). Having the central temperature at each radius and assuming that the discs emit as a multi-temperature blackbody (**?**), we can obtain the spectral energy distribution (SED) and the bolometric luminosity ($L_{\rm bol}$) of the system as a function of time.

2.2 Surface Density evolution

At least two mechanisms will make the surface density of the discs evolve over time: viscosity, already explained in the previous subsection, and the tidal torque produced by the changing gravitational potential of the rotating binary. The latter is a 2D effect, and cannot be implemented directly in a 1D simulation. To bypass this, we adopt a commonly used recipe that models its effect on the accretion discs. Following the literature (**?**), [1] we define an orbit-averaged torque,

$$\Lambda = \begin{cases} -\dfrac{f}{2}q^2\Omega^2 r^2\left(\dfrac{r}{\Delta}\right)^4, & \text{if } r < a \\ \dfrac{f}{2}q^2\Omega^2 r^2\left(\dfrac{a}{\Delta}\right)^4, & \text{if } r \geq a \end{cases} \tag{5}$$

[1] See **????** for discussion about the tidal prescription.

whose functional form depends on whether we are in the inner ($r < a$) or circumbinary ($r \geq a$) discs. Here $f = 0.01$ is a dimensionless parameter that controls the strength of the torque, $\Delta =_{\max} \{R_h,\ h,\ |r-a|\}$ is a smoothing term, and $R_h = a(q/3)^{1/3}$ is the Hill radius of the secondary black hole. While this model was originally proposed for $q \ll 1$, it has been widely used for binaries up to $q = 0.3$ (**?????**).

While viscosity always transfers the angular momentum of the disc outwards, the tidal effect depends on the position. Inside the binary orbit ($r < a$) the tidal torque Λ adds angular momentum to the binary, producing an inward acceleration on the elements of the disc. On the other hand, for the circumbinary disc ($r \geq a$), this effect will add angular momentum to the gas, preventing it to cross the binary orbit. Here, the balance between the tidal effect and the viscous mechanism will produce a low density region called cavity or gap, which depends on the mass ratio of the binary.

As pointed out by **?**, for $q \geq 0.1$, eq. (5) gives an unphysically large tidal contribution on the discs outside the Lindblad resonances, which will alter the migration velocity of the binary and the surface density of the discs. Following their work we added an exponential cutoff for the tidal torque outside this region,

$$\Lambda = \begin{cases} -\dfrac{f}{2} q^2 \Omega^2 r^2 \left(\dfrac{r}{\Delta}\right)^4 \exp\left[-\left(\dfrac{r - r_{\rm IMLR}}{w_{\rm IMLR}}\right)^2\right] & \text{if } r \leq r_{\rm IMLR}, \\ \dfrac{f}{2} q^2 \Omega^2 r^2 \left(\dfrac{a}{\Delta}\right)^4 \exp\left[-\left(\dfrac{r - r_{\rm OMLR}}{w_{\rm OMLR}}\right)^2\right] & \text{if } r \geq r_{\rm OMLR}. \end{cases} \tag{6}$$

Here, $r_{\rm IMLR} = 0.63a$ and $r_{\rm OMLR} = 1.59a$ are the radius of the innermost and outermost Lindblad resonances, while $w_{\rm IMLR} = 370h$ and $w_{\rm OMLR} = 75h$ are the widths of the Gaussian smoothing. These values were used by **?** to reproduce the gap sizes of a $q = 0.11$ binary from **?**.

Following **?**, considering the mass continuity and angular momentum conservation equations in the discs, adding the viscous and tidal torques (eq. 5), the surface density evolution can be written as:

$$\frac{\partial \Sigma}{\partial t} = -\frac{1}{r}\frac{\partial}{\partial r}\left(-3r^{1/2}\frac{\partial}{\partial r}\left(\nu\Sigma r^{1/2}\right) + 2\frac{\Sigma\Lambda}{\Omega}\right), \tag{7}$$

where the first term at the right hand side is the effect of viscosity and the second one is produced by the tidal interaction between the binary and the discs.

3 What's the difference, ("If Any!!") between Proto-stellar disks & AGN disks ???!!!!

Table 1: What are the similarities and differences between Proto-stellar and AGN accretion disks?

	Proto-stellar disks	AGN disks
h/r	~ 0.1	
Adiabatic/isothermal?	Mainly adiabatic	
B-field strength	Interesting issue. Thought to be sensitive to MRI at later stages at least. However, there are some arguing that global magnetic fields may play a key in transporting angular momentum away.	
Mechamism(s) for turbulence generation	Self-gravity at early times, MRI later	
Dust chemistry	Certainly many people working on chemistry in these discs	
Dust opacity	Regarded as important for cooling	
Iron present?	Yes, and regarded as having, initially at least, an ISM composition.	

4 ADAFs, RIAFs, etcs.

5 ADAFs, RIAFs, etcs.

see reviews by Quataert [2001]; Narayan [2005]; early versions of RIAF models were called advection-dominated accretion flows [ADAFs] or ion tori;

Table 2

Temperature (cf. Virial temperature)	COLD	HOT
geometry (h/R)	thin, $\lesssim 0.1$	thick, ~ 0.5
gas opacity	optically thick	optically thin ($\tau < 1$)
$\dot{M}$	generally high	low(er)
radiation pressure	negligible	non-negligible
radiative cooling	generally efficient	generally inefficient
angular velocity	generally Keplerian	sub-Keplerian
Outflows? Jets?	Yes? No.	Yes, Yes
Feedback mode	"Radiative/Wind/Transition"	"Jet/Kinetic/Maintenances"
Named examples	"Slim"	advection dominated accretion flow (ADAF)
		(adiabatic inflow-outflow solution; ADIOS)
	Shakura-Sunyaev disk	(convection-dominated accretion flow; CDAF)
	(a.k.a "thin" + α)	radiatively inefficient accretion flow (RIAF)
		"luminous hot" (LHAF)
		SLE (Shaprio, Lightman, Eardley, 1976, ApJ, **204**, *187*)
Type of objects	quasars	Low-luminosity AGN (LLAGN)
		BHXBs in "Hard and Quiescent" state

$$T_{\rm vir} = \frac{\mu m_p}{2k_B} V_{\rm vir}^2 \simeq 3.6 \times 10^5 \left(\frac{V_{\rm vir}}{100\,{\rm km/s}} \right)^2 \tag{8}$$

$$T(R) = \left\{ \frac{3GM\dot{M}}{8\pi R^3 \sigma} \left[1 - \frac{R}{R_*} \right]^{1/2} \right\}^{1/4} \tag{9}$$

$$= \left(\frac{3GM\dot{M}}{8\pi \sigma R_*^3} \right)^{1/4} \tag{10}$$

$$\approx 5 \times 10^5 \left(\frac{M}{10^8 M_\odot} \right)^{1/4} \left(\frac{\dot{M}}{10^{23} \mathrm{kg\,s^{-1}}} \right)^{1/4} \left(\frac{r}{r_g} \right)^{-3/4} \mathrm{K} \tag{11}$$

$$\tag{12}$$

with $T = T_*(R/R_*)^{-3/4}$ for $R \gg R_*$.

$$T(R) = \left(\frac{3GM\dot{M}}{8\pi \sigma R_*^3} \right)^{1/4} \tag{13}$$

$$\approx 5 \times 10^5 \left(\frac{M}{10^8 M_\odot} \right)^{1/4} \left(\frac{\dot{M}}{10^{23} \mathrm{kg\,s^{-1}}} \right)^{1/4} \left(\frac{r}{r_g} \right)^{-3/4} \mathrm{K} \tag{14}$$

$$\tag{15}$$

$$T(R) = \left(\frac{3GM\dot{M}}{8\pi \sigma R_*^3} \right)^{1/4} \tag{16}$$

$$\approx 5 \times 10^5 \left(\frac{M}{10^8 M_\odot} \right)^{1/4} \left(\frac{\dot{M}}{10^{23} \mathrm{kg\,s^{-1}}} \right)^{1/4} \left(\frac{r}{r_g} \right)^{-3/4} \mathrm{K} \tag{17}$$

$$\approx 5 \times 10^5 \left(\frac{M}{10^8 M_\odot} \right)^{1/4} \left(\frac{\dot{M}}{0.6 M_\odot \mathrm{yr}^{-1}} \right)^{1/4} \left(\frac{r}{r_g} \right)^{-3/4} \mathrm{K} \tag{18}$$

$$\sim 8 \times 10^5 \left(\frac{M}{10^8 M_\odot} \right)^{1/4} \left(\frac{\dot{M}}{M_\odot \mathrm{yr}^{-1}} \right)^{1/4} \left(\frac{r}{r_g} \right)^{-3/4} \mathrm{K}$$

$$T(R) = \left(\frac{3GM\dot{M}}{8\pi \sigma R_*^3} \right)^{1/4} \approx 5{\times}10^5 \left(\frac{M}{10^8 M_\odot} \right)^{1/4} \left(\frac{\dot{M}}{0.6 M_\odot \mathrm{yr}^{-1}} \right)^{1/4} \left(\frac{r}{r_g} \right)^{-3/4} \mathrm{K}$$

6 This, THIS, THIS!!!!

http://www.scholarpedia.org/article/Accretion_discs

7 Astrophysics of accretion disks - Charles Gammie

"Disk's are at the heart of many of the most interesting problems in theoretical astrophysics." YouTube video.

7.1 Examples of Astro-disks

$$h/R \ll 1 \tag{19}$$

where h is the scale-height and R is the (local) radius. This is $\sim 10^{-3}$ for NGC 4258. This ratio is proportional to sound speed in the disk / rotational velocity, which is $\sim 1\text{kms}^{-1}/1000\text{kms}^{-1}$.

T is 1 billion degrees, and $n_\text{e} \sim 10^6$ means m.f.p. for Columbo scattering is very large. Introduce the "Knuddson number":

$$\kappa_\text{n} = \lambda_\text{mfp}/R \tag{20}$$

where R is the general 'scale of the system and κ_n for Sgr A* is around 10^5. Evolution of turbulence in collisionless plasmas.

Two more dimensionless numbers. Magnetic Reynolds number, which allows you to asses the non-ideal effects in the plasma around a young star

$$Re_m = \frac{c_s h}{\eta} \tag{21}$$

η is the electrical resistivity.

Toomre Q parameter, measures the importance of self-gravity in the disk.

$$Q = \frac{c_s \Omega}{\pi G \Sigma} \tag{22}$$

where Σ is the surface density of the disk. When $Q \lesssim 1$ then self-gravity becomes important in the disk.

7.2 Disk Evolution

All about ANGULAR MOMENTUM.

First, disk Equilibrium.

- Dynamical equilibrium:

$$\Omega = (GM/R^3)^(1/2) + O(H/R)^2 \qquad v_z = 0 \tag{23}$$

and $\Delta t \sim \Omega^{-1}$. *Nota bene* the scale height is proportional to the sound speed divided by the rotational frequency: $H = c_s/\Omega$.

In a PROTOPLANETARY disk, this is a GAS pressure dominated disk. Whereas in an AGN accretion disk, the pressure is dominated by the RADIATION pressure.

- Thermal equilibrium: $Q^+ = Q^-$

Q^+ is heating rate, Q^- is radiative cooling rate.

$$\Delta t \sim \Sigma c_s^2 / Q^+ \sim (\alpha\Omega)^{-1} \tag{24}$$

The heating in some disks is dominated by internal friction related to the dissipation of turbulence. And in an optically thick disk, $Q^+ \sim \sigma T^4$ (just like a star). α describes the intensity of turbulence inside the disk. So the $Q^+ \propto$ local dynamical time with constant of proportionality being α.

Typically, $\alpha \sim 1/10 - 1/100$ which is substantially longer than t for dynamical equilibrium.

- Viscous equilibrium: $\dot{M} = \text{const.}$

Time for which *inflow* equilibrium is reached.

$$\Delta t \sim M_{\text{disk}} / \dot{M} \sim (\alpha\Omega)^{-1} (R/H)^2. \tag{25}$$

References

Bondi H., Hoyle F., 1944, MNRAS, 104, 273