

## STELLAR ATMOSPHERES. I: STELLAR OPACITY

### 5.1 Introduction and Definitions

The light we receive from a star originates in its atmosphere, the layers of gas overlying the opaque interior. The photons produced there carry away the gravitational energy released when the star forms from a collapsing gas cloud, the energy released by the thermonuclear reactions taking place in the core of the star during its lifetime and, once thermonuclear reactions stop altogether, the energy released by the cooling of the star's interior, as is the case in white dwarfs. The photons collected by our telescopes carry with them information about the temperature, density, and chemical composition of the atmospheric layers from which they have escaped (see Figure 5.1). In order to decode this information we must understand how light travels through the gas which makes up a star.

It is useful at this point to clarify what we mean by some of the quantities we are going to use. We define the specific intensity (or simply the

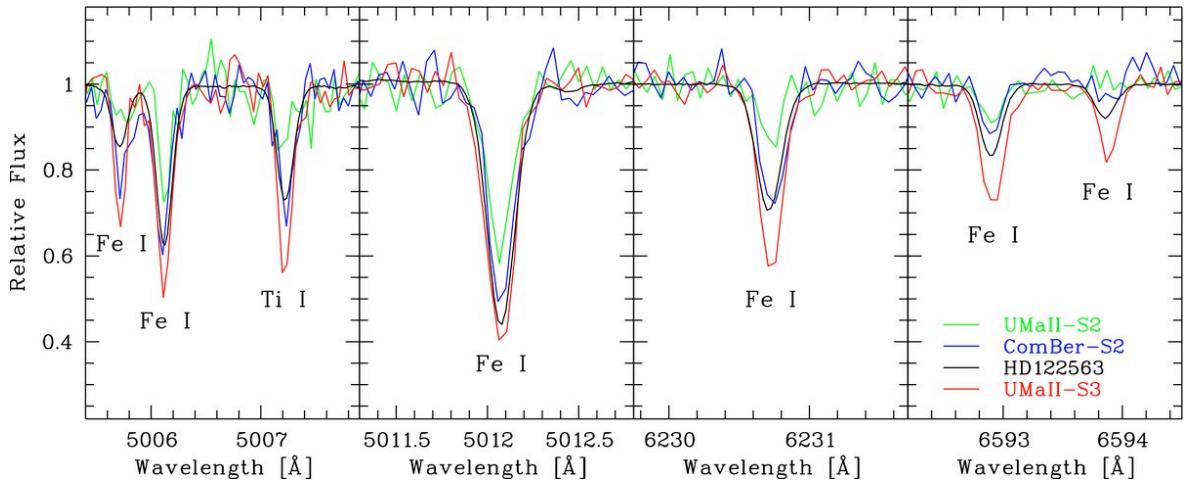


Figure 5.1: Absorption lines in four stars of similar effective temperatures by different Fe abundance, as follows: UMa II-S2 (green),  $[\text{Fe}/\text{H}] = -3.2$ ; ComBer-S2 (blue),  $[\text{Fe}/\text{H}] = -2.9$ ; HD 122563 (black),  $[\text{Fe}/\text{H}] = -2.8$ ; and UMa II-S3 (red),  $[\text{Fe}/\text{H}] = -2.3$ . The iron abundance is measured on a log scale relative to its value in the Sun; thus a star with  $[\text{Fe}/\text{H}] = -3$  has only 1/1000 of the solar Fe/H ratio (by number), one with  $[\text{Fe}/\text{H}] = -2$  has 1/100 the solar ratio and so on.

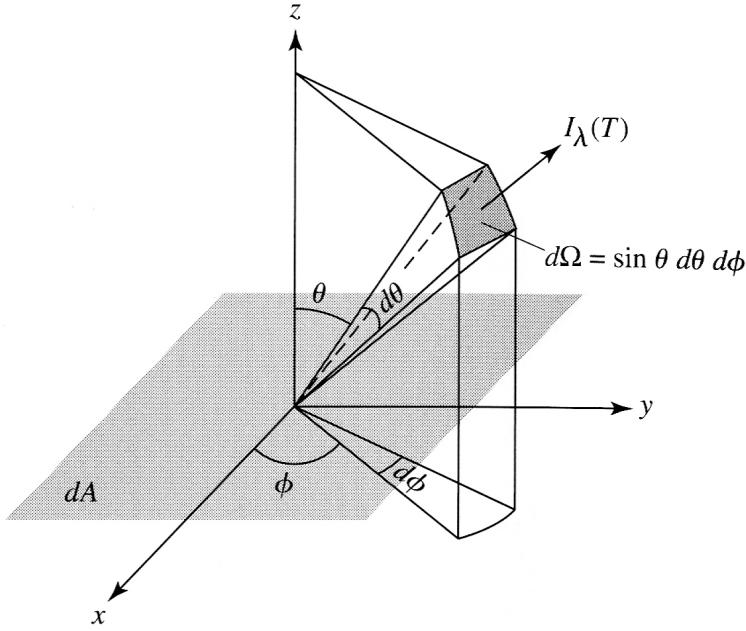


Figure 5.2: Nomenclature for the definition of specific intensity  $I_\lambda$ . (Reproduced from Carroll & Ostlie's *Modern Astrophysics*).

intensity)  $I_\lambda$ ,

$$E_\lambda d\lambda = I_\lambda d\lambda dt dA \cos \theta \sin \theta d\theta d\phi \quad (5.1)$$

as the amount of electromagnetic radiation energy with wavelength between  $\lambda$  and  $\lambda + d\lambda$  that passes in time  $dt$  through the star surface area  $dA$  into a solid angle  $d\Omega = \sin \theta d\theta d\phi$  (see Figure 5.2). Thus the units of intensity are (in the cgs systems used by astronomers)  $\text{erg s}^{-1} \text{cm}^{-2} \text{\AA}^{-1} \text{sr}^{-1}$ .

The mean intensity  $J_\lambda$  is obtained by integrating the intensity over all directions and dividing by the solid angle enclosed by a sphere, which is  $4\pi \text{ sr}$ . Thus:

$$J_\lambda = \frac{1}{4\pi} \int I_\lambda d\Omega = \frac{1}{4\pi} \int_{\phi=0}^{2\pi} \int_{\theta=0}^{\pi} I_\lambda \sin \theta d\theta d\phi \quad (5.2)$$

For an isotropic radiation field,  $J_\lambda = I_\lambda$ . A blackbody radiates isotropically by definition; thus  $J_\lambda = B_\lambda$ .

The specific energy density  $u_\lambda$  measures how much energy is contained within the radiation field, that is the energy per unit volume having a wavelength between  $\lambda$  and  $\lambda + d\lambda$ :

$$u_\lambda d\lambda = \frac{1}{c} \int I_\lambda d\lambda d\Omega = \frac{4\pi}{c} J_\lambda d\lambda \quad (5.3)$$

Thus, for a blackbody radiation field,

$$u_\lambda d\lambda = \frac{4\pi}{c} B_\lambda d\lambda = \frac{8\pi hc/\lambda^5}{e^{hc/\lambda kT} - 1} d\lambda \quad (5.4)$$

and in frequency, rather than wavelength, units:

$$u_\nu d\nu = \frac{4\pi}{c} B_\nu d\nu = \frac{8\pi h\nu^3/c^3}{e^{h\nu/kT} - 1} d\nu. \quad (5.5)$$

The total energy density is found by integrating over all wavelengths (or frequencies):

$$u = \int_0^\infty u_\lambda d\lambda. \quad (5.6)$$

For a blackbody:

$$u = \frac{4\pi}{c} \int_0^\infty B_\lambda(T) d\lambda = \frac{4\sigma}{c} T^4 = aT^4 \quad (5.7)$$

where  $a \equiv 4\sigma/c$  is the radiation constant.

## 5.2 Local Thermodynamic Equilibrium

We can define the temperature of a star in many different ways:

1. *The effective temperature*, defined in terms of the luminosity of the star (and its radius) according to eq. 2.13;
2. *The excitation temperature*, defined by the relative populations of different excited levels of an atom or ion according to Boltzmann equation (eq. 3.2);
3. *The ionisation temperature*, defined by the relative populations of different ionisation stages of an atom according to Saha equation (eq. 3.4);
4. *The kinetic temperature*, defined by the Maxwell-Boltzmann distribution:

$$n_v dv = n \left( \frac{m}{2\pi kT} \right)^{3/2} e^{-mv^2/2kT} 4\pi v^2 dv \quad (5.8)$$

where  $n_v$  is the number of particles per unit volume with speeds between  $v$  and  $v + dv$ ,  $n$  is the volume density (of all particles),  $m$  is the mass and the other symbols have their usual meanings;

5. *The colour temperature*, being the temperature of the blackbody whose spectral energy distribution resembles most closely that of the star.

Except for the effective temperature, which is an important global descriptor of the star, all of the other temperatures apply to any location within the stellar interior and vary according to the physical conditions of the gas. Of course, all of them become the same in the ideal situation of the particles and radiation being in equilibrium. In such a steady-state condition, there is no net flow of energy in nor out of a volume element, nor any transfer of energy between matter and radiation. Every process, such as the absorption of a photon, occurs at the same rate as its inverse process, such as the emission of a photon. Such an idealised condition is referred to as thermodynamic equilibrium. A blackbody is by definition in thermodynamic equilibrium.

Clearly this ideal condition does not apply to stars. There is a net outward flow of energy through a star. The temperature varies from millions of degrees in the core to thousands of degrees in the atmosphere. As the gas particles collide with one another and interact with the radiation field by absorbing and emitting photons, the description of the processes of excitation and ionisation becomes quite complex. Despite all of this complications, we can still apply the the approximation of Local Thermodynamic Equilibrium (LTE), *provided the typical distance travelled by particles and photons between collisions—their mean free path—is small compared to the scale over which the temperature changes significantly*. This situation applies to most of the stellar interior, where density and temperature are high, so that the mean distance between collisions is small. You can think of this situation as the particles and photons being confined to a limited volume of nearly constant temperature.

### 5.3 Opacity

As a beam of light of intensity  $I_\lambda$  travels through a gas, some of the photons will be removed through scattering off and absorption by ions, atoms and molecules in the gas. We can write:

$$dI_\lambda = -\kappa_\lambda \rho I_\lambda ds, \quad (5.9)$$

where  $ds$  is the distance travelled,  $\rho$  is the density of the gas and  $\kappa_\lambda$  is the absorption coefficient, or opacity. In terms of the photon mean free path:

$$\mu = \frac{1}{\kappa_\lambda \rho} = \frac{1}{\sigma_\lambda n}$$

where  $\sigma$  is the cross-section for interaction and  $n$  is the number density of particles. With these definitions,  $\kappa$  has units  $\text{cm}^2 \text{ g}^{-1}$ ,  $\rho$  has units  $\text{g cm}^{-3}$ ,  $\sigma$  is in  $\text{cm}^2$ , and  $n$  is in  $\text{cm}^{-3}$ . Thus, both products on the denominators of the above equation can be thought of as the fraction of photons scattered off the beam per cm travelled.

We define the optical depth as:

$$\tau_\lambda = \int_0^s \kappa_\lambda \rho ds \quad (5.10)$$

In the case of a photon travelling from the stellar interior to the surface,  $s = 0$  at the starting point, and  $\tau_\lambda = 0$  at the surface of the star. Thus, we can think of the optical depth as the number of mean free paths for the

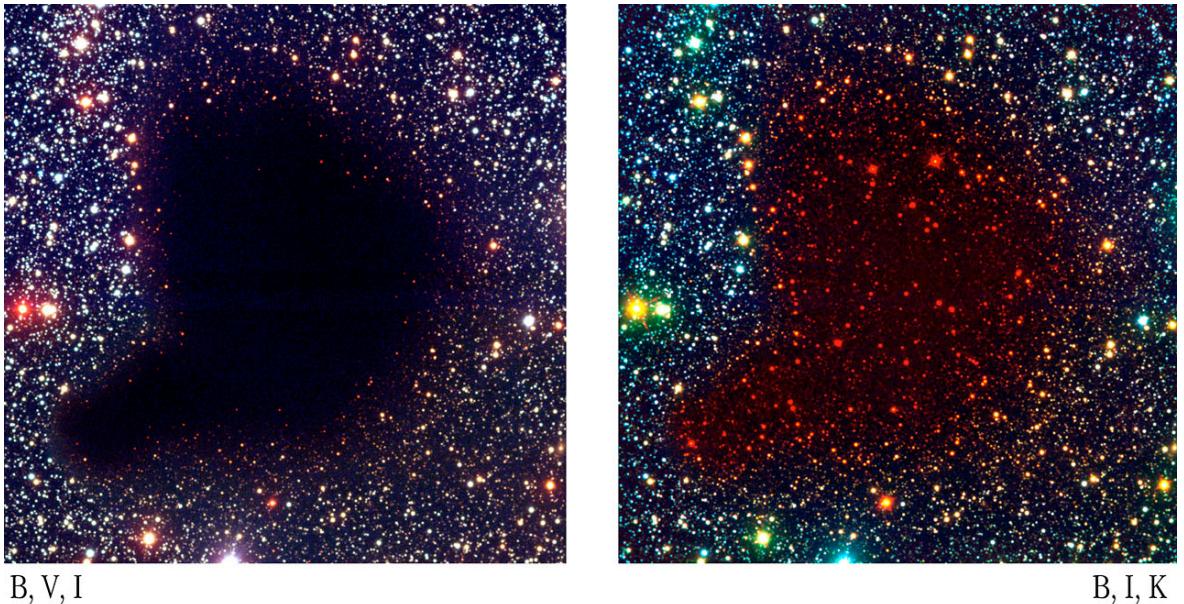


Figure 5.3: Colour composites of ESO images of the Dark Cloud Barnard 68 taken through different optical and near-IR filters. Dust in the cloud hides stars located behind it; because dust scatters infrared light less efficiently than optical light, the opacity of the cloud is reduced in the colour composite that includes the near-IR  $K$ -band filter. The cloud is located at a distance of 160 pc and is seen against the background of a rich star field in the Milky Way. Curiously, there are no foreground stars. Barnard 68 seems to be a molecular cloud in the earliest phase of collapse to form new stars; for this reason it is the subject of many studies at a variety of wavelengths.

photon, from a given location in the star's interior to the surface. Note that in all the above equations we have explicitly indicated that  $\kappa_\lambda$ ,  $\sigma_\lambda$  and  $\tau_\lambda$  are all functions of wavelength. Figure 5.3 is a vivid demonstration of the wavelength dependence of the opacity, in this case of interstellar dust.

Integrating eq. 5.9 and with the definition of eq. 5.10, we have:

$$I_\lambda = I_{\lambda,0} e^{-\tau_\lambda} \quad (5.11)$$

where  $I_{\lambda,0}$  is the intensity at wavelength  $\lambda$  that would be measured in the absence of absorption/scattering. Gas with  $\tau_\lambda \gg 1$  is said to be optically thick; conversely, if  $\tau_\lambda \ll 1$  the gas is optically thin. From the above, it can be appreciated that we typically do not see deeper into a stellar atmosphere (at a given wavelength) than unit optical depth, i.e.  $\tau_\lambda \approx 1$ .

## 5.4 Sources of Opacity

When dealing with stellar atmospheres, the opacity can be due to one of four main physical processes (or a combination of them):

1. Bound-bound transitions. These are the familiar transitions between different energy levels which cause absorption (or emission) lines at discrete wavelengths. Thus  $\kappa_{\lambda,bb}$  is small or zero at all wavelengths except those which correspond to the energy difference between two atomic levels.

It is of interest to consider what happens to the photon after it is absorbed. If the electron spontaneously returns to the same energy level from which the upward transition took place, a photon with the same energy is re-emitted but in a random direction. So, the net effect is one of scattering. On the other hand, if the electron returns to its original state via two or more transitions to intermediate energy levels, then the net effect is to degrade the average energy of the photons in the radiation field.

2. Bound-free absorption. This is the familiar process of photoionisation which will occur for all photon energies  $h\nu > \chi_n$ , where  $\chi_n$  is the ionisation potential of a given atomic energy level. The difference  $\Delta E = h\nu - \chi_n$  is the kinetic energy of the free electron. Thus,  $\kappa_{\lambda,bf}$

is one source of continuum opacity (as opposed to opacity at discrete wavelengths as is the case for  $\kappa_{\lambda,bb}$ ).

3. Free-free absorption. This is the inverse process of free-free emission (*bremssstrahlung*) in which a free electron is decelerated by the electric potential of an ion and, as a result, radiates. Thus, in free-free absorption, a photon is absorbed by a free electron and an ion, which share the photon's momentum and energy.  $\kappa_{\lambda,ff}$  also contributes to the continuum opacity.
4. Electron scattering. This is the scattering of photons by free electrons without change of photon energy (Thompson scattering). The cross-section for Thompson scattering is independent of wavelength, so that  $\kappa_{es}$  is also a source of continuum opacity. Its value is very small,  $\sigma_T = 6.7 \times 10^{-25} \text{ cm}^2$ , some seven orders of magnitude smaller, for example, than the cross-section for photoionisation of neutral hydrogen from the ground state,  $\sigma_0 = 6.3 \times 10^{-18} \text{ cm}^2$ . Thus, Thomson scattering is important only when the density of free electrons is very high, that is when the gas is fully ionised (or nearly so), as is the case in the atmospheres of the hottest stars.

#### 5.4.1 H<sup>-</sup> and Molecules

The temperatures of stars later than spectral type F0 ( $T_{\text{eff}} \lesssim 7300 \text{ K}$ ) are sufficiently low for non-negligible concentrations of H<sup>-</sup> to be present. With a binding energy of only 0.754 eV ( $\sim 1/20$  the binding energy of the electron in neutral hydrogen), corresponding to a wavelength of 16.4  $\mu\text{m}$ , H<sup>-</sup> is an important source of continuum opacity. Even at longer wavelengths, the H<sup>-</sup> ion adds to the continuum opacity through free-free absorption. In A and B stars, photoionisation of H and free-free absorption are the main sources of continuum opacity. In O stars, He photoionisation and Thomson scattering also contribute. In the coolest stars, significant concentrations of molecules, as opposed to atoms and ions, can survive, providing additional sources of opacity through bound-bound (many molecular bands), bound-free, and photodissociation (the absorption of a photon breaks the weak molecular bond) transitions (see Figure 5.4).

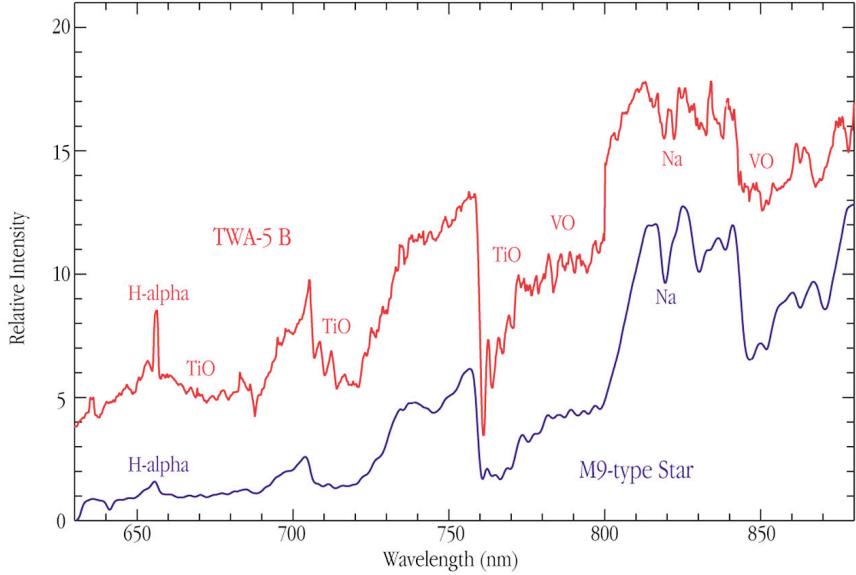


Figure 5.4: At optical and infrared wavelengths the spectra of cool stars are dominated by strong molecular bands.

## 5.5 Mean Opacities

It can be appreciated that the full calculation of stellar opacities, which depend on the chemical composition, pressure and temperature of the gas, as well as the wavelength of the incident light, is a complex endeavour. The problem can be simplified by using a mean opacity averaged over all wavelengths, so that only the dependence on the gas physical properties remains. The most commonly used is the Rosseland mean opacity, defined as:

$$\frac{1}{\langle \kappa \rangle} = \frac{\int_0^\infty \frac{1}{\kappa_\nu} \frac{\partial B_\nu(T)}{\partial T} d\nu}{\int_0^\infty \frac{\partial B_\nu(T)}{\partial T} d\nu}. \quad (5.12)$$

Note that this is an harmonic mean, in which the greatest contribution comes from the lowest values of opacity, weighted by a function that depends on the rate at which the blackbody spectrum varies with temperature.

Returning to the different sources of opacity considered earlier, let us consider their Rosseland means. There is no simple analytical expression that describes the multitudes of absorption lines resulting from bound-bound transitions. However, we can do somewhat better for the bound-free and free-free transitions, whose mean opacities can be approximated

by Kramers opacity law:

$$\langle \kappa_{bf} \rangle = \kappa_{0,bf} \rho T^{-3.5} \quad (5.13)$$

and

$$\langle \kappa_{ff} \rangle = \kappa_{0,ff} \rho T^{-3.5} \quad (5.14)$$

where  $\kappa_{0,bf}$  and  $\kappa_{0,ff}$  are constants that depend on the composition of the gas.

For electron scattering, which has no wavelength, density, nor temperature dependence, the Rosseland mean takes a particularly simple form:

$$\kappa_{es} = \kappa_{0,es} \frac{1}{\mu_e} \quad (5.15)$$

where  $1/\mu_e$  is the number of electrons per nucleon and  $\kappa_{0,es}$  is the opacity of a fully ionised, pure hydrogen gas (in which case  $1/\mu_e = 1$ , clearly). Recalling (lecture 1) that we use the symbols  $X$ ,  $Y$ , and  $Z$  to denote the mass fractions of, respectively, H, He and everything else (metals),

$$\frac{1}{\mu_e} = X + \frac{1}{2}Y + \frac{1}{2}(1 - X - Y) \simeq \frac{1}{2}(1 + X)$$

since helium and the most abundant heavier elements are (nearly) fully ionised in the interior of stars, so that the average number of electrons per nucleon is  $\approx 1/2$ .

The mean opacity due to  $H^-$  does not have a simple analytical approximation.

The full calculation of stellar opacities is a major endeavour. For this reason, the *Opacity Project* (OP) was set-up in 1984 as an international collaboration to calculate the extensive atomic data required to estimate stellar envelope opacities and to compute Rosseland mean opacities and other related quantities. It brought together the work of research groups from France, Germany, the United Kingdom, the United States and Venezuela, under the leadership of M. J. Seaton at University College London. The project involved, among other aspects, the computation by *ab initio* methods of accurate atomic properties such as energy levels, oscillator strengths and photoionization cross sections. Similar work conducted at the Lawrence Livermore National Laboratory, the OPAL project, gave results generally in good agreement with those of the OP.

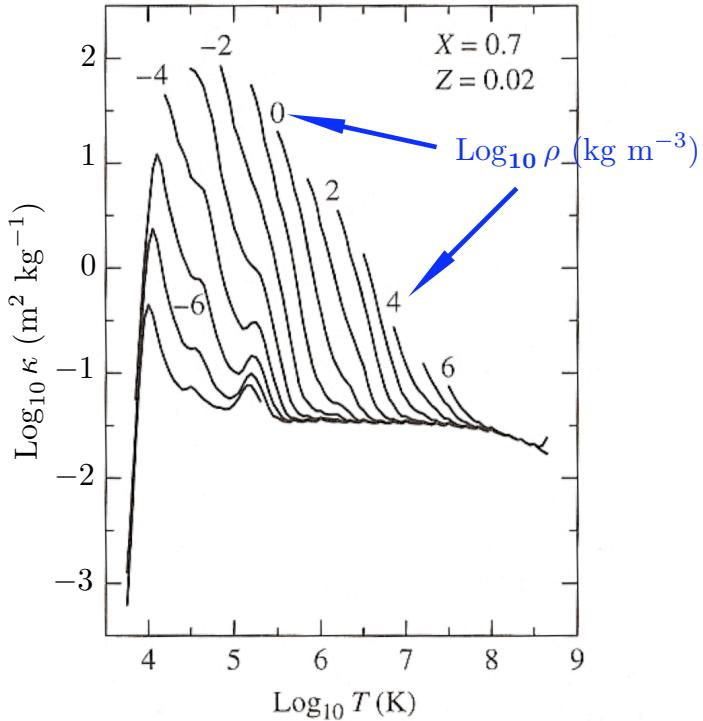


Figure 5.5: Rosseland mean opacities calculated by the OPAL project for ‘standard’ solar abundances (Iglesias & Rogers 1996).

Figure 5.5 shows the OPAL calculations of the Rossland mean opacity as a function of density and temperature for a gas of solar composition. It is worthwhile considering the behaviour of  $\langle \kappa \rangle$  even though we cannot derive the curves ourselves. The first thing to notice is that at a fixed temperature the opacity increases with density (as one may expect). Focussing on one of the curves, note that  $\langle \kappa \rangle$  at first increases rapidly with increasing  $T$ , reflecting the increase in the number of free electrons produced by ionisation of H and He. The decline with increasing  $T$  beyond the peak value of  $\langle \kappa \rangle$  follows a Kramers law,  $\langle \kappa \rangle \propto T^{-3.5}$ , and is due mostly to bound-free and free-free absorption. All the curves come together at very high temperatures, when the gas is fully ionised and the main source of opacity is electron scattering (eq. 5.15). The small bumps superimposed on some of the curves occur at the temperatures where He and higher mass elements (mostly Fe) are ionised, releasing additional electrons into the plasma.

Opacities are an essential ingredient of model stellar atmospheres.

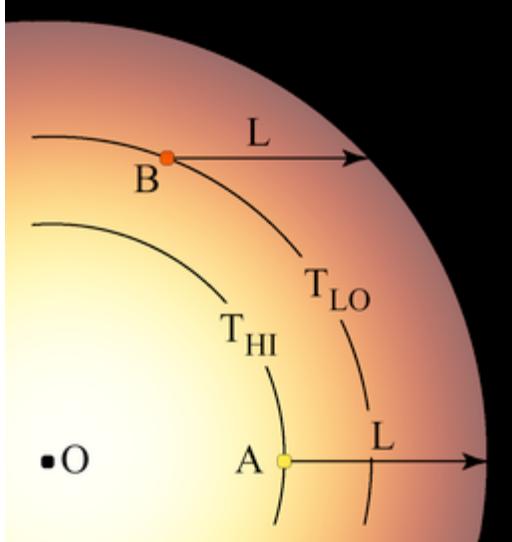


Figure 5.6: A given linear distance  $L$  probes to different depths into a stellar atmosphere depending on the angle of the line of sight to the radial coordinate.

## 5.6 Limb Darkening

It was stated earlier that we typically see no further into a star than unity optical depth. A more careful treatment actually shows that the level within a stellar atmosphere from which most of the photons of wavelength  $\lambda$  escape is at optical depth  $\tau_\lambda \simeq 2/3$ . Indeed, the condition  $\tau_\lambda \simeq 2/3$  defines the stellar photosphere—the layer of a star’s atmosphere from which the light we see originates.

There are two consequences of this realisation. First, the condition applies to all viewing angles; therefore, the distance  $ds$  corresponding to the condition  $\tau_\lambda = 2/3$  will probe further into the star’s interior at the centre of a stellar disk than at its edges (see Figure 5.6). Second, recalling the definition of optical depth,

$$\tau_\lambda = \int_0^s \kappa_\lambda \rho \, ds \quad (5.10)$$

it is obvious that if the opacity  $\kappa_\lambda$  increases at some wavelength, then  $ds$  must be smaller to satisfy the condition  $\tau_\lambda \simeq 2/3$ . Thus, we see further into a star in its continuum light than at the wavelengths of discrete absorption lines.

These two effects explain a phenomenon known as ‘limb darkening’, first recognised in the Sun, whereby the light emitted in successive annuli from the centre decreases in intensity and becomes progressively redder, as

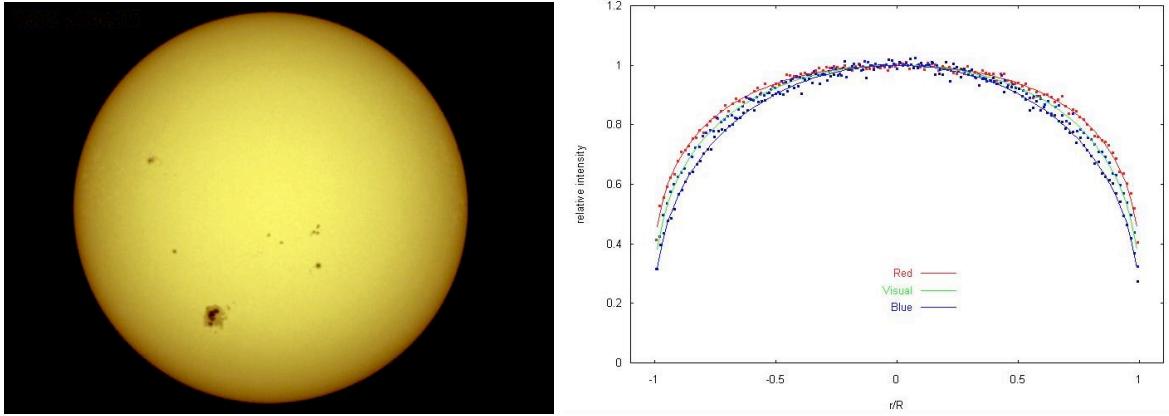


Figure 5.7: Limb darkening in the Sun  $r/R$  is the fractional distance from the centre of the solar disk. Note that the effect is more pronounced at shorter wavelengths: near the edge of the disk, the Sun is not only dimmer but also redder.

shown in Figure 5.7. Sightlines near the limb do not penetrate as deeply into the Sun’s atmosphere by the time  $\tau_\lambda \simeq 2/3$  is reached; since the Sun’s temperature decreases outwards from the centre, such sightlines see light from cooler regions of the Sun’s atmosphere.

While most easily seen in the Sun because of its proximity to Earth, limb darkening has now been seen in a handful of nearby stars, and its effects have been recognised in the light curves of eclipsing binaries and of microlensing events (when one star passes in front of a second, more distant, star whose brightness can be greatly amplified as light rays are bent towards us by gravitational lensing).

## 5.7 Summary of Definitions

$I_\lambda$ : Intensity ( $\text{erg s}^{-1} \text{ cm}^{-2} \text{ \AA}^{-1} \text{ sr}^{-1}$ )

$B_\lambda$ : Mean intensity of blackbody radiation ( $\text{erg s}^{-1} \text{ cm}^{-2} \text{ \AA}^{-1} \text{ sr}^{-1}$ )

$u$ : energy density ( $\text{erg cm}^{-3}$ )

$\kappa_\lambda$ : opacity ( $\text{cm}^2 \text{ g}^{-1}$ )

$\tau_\lambda$ : optical depth

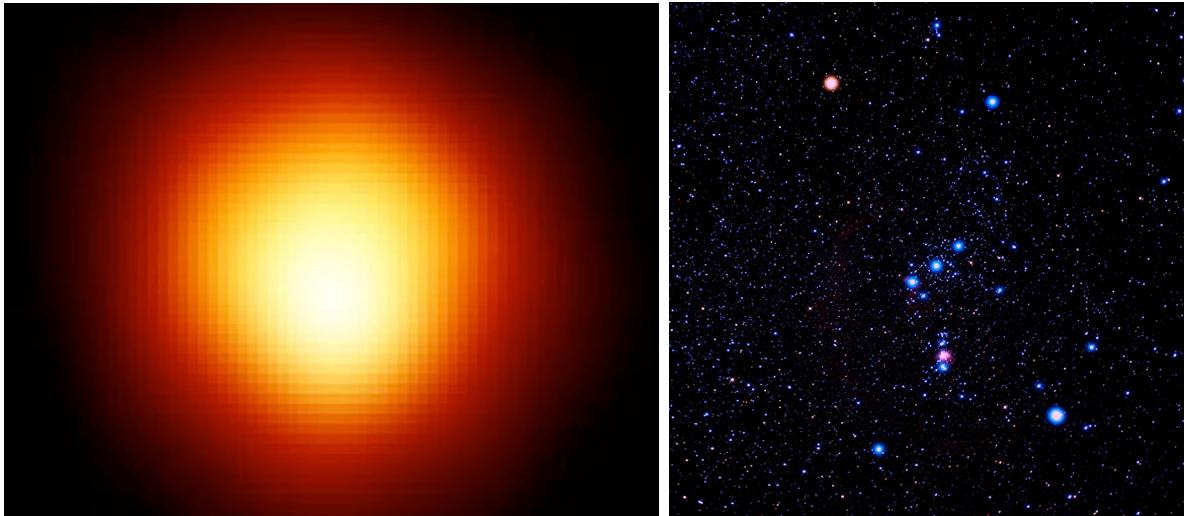


Figure 5.8: This is the first direct image of the surface of a star other than the Sun. The star is Betelgeuse ( $\alpha$  Ori), a red supergiant (M2 Iab) at a distance of  $\sim 150$  pc; among the ten brightest stars in the night sky, it is easily recognised in the constellation of Orion (right). The image on the left was obtained with the *Hubble Space Telescope* at near-ultraviolet wavelengths. The diameter of the star at  $2500\text{ \AA}$  is 2.2 times larger than at optical wavelengths, as a result of the higher UV opacity. Limb-darkening is evident in the *HST* image. The diameter of  $\alpha$  Ori is  $\sim 1000$  times larger than that of the Sun.