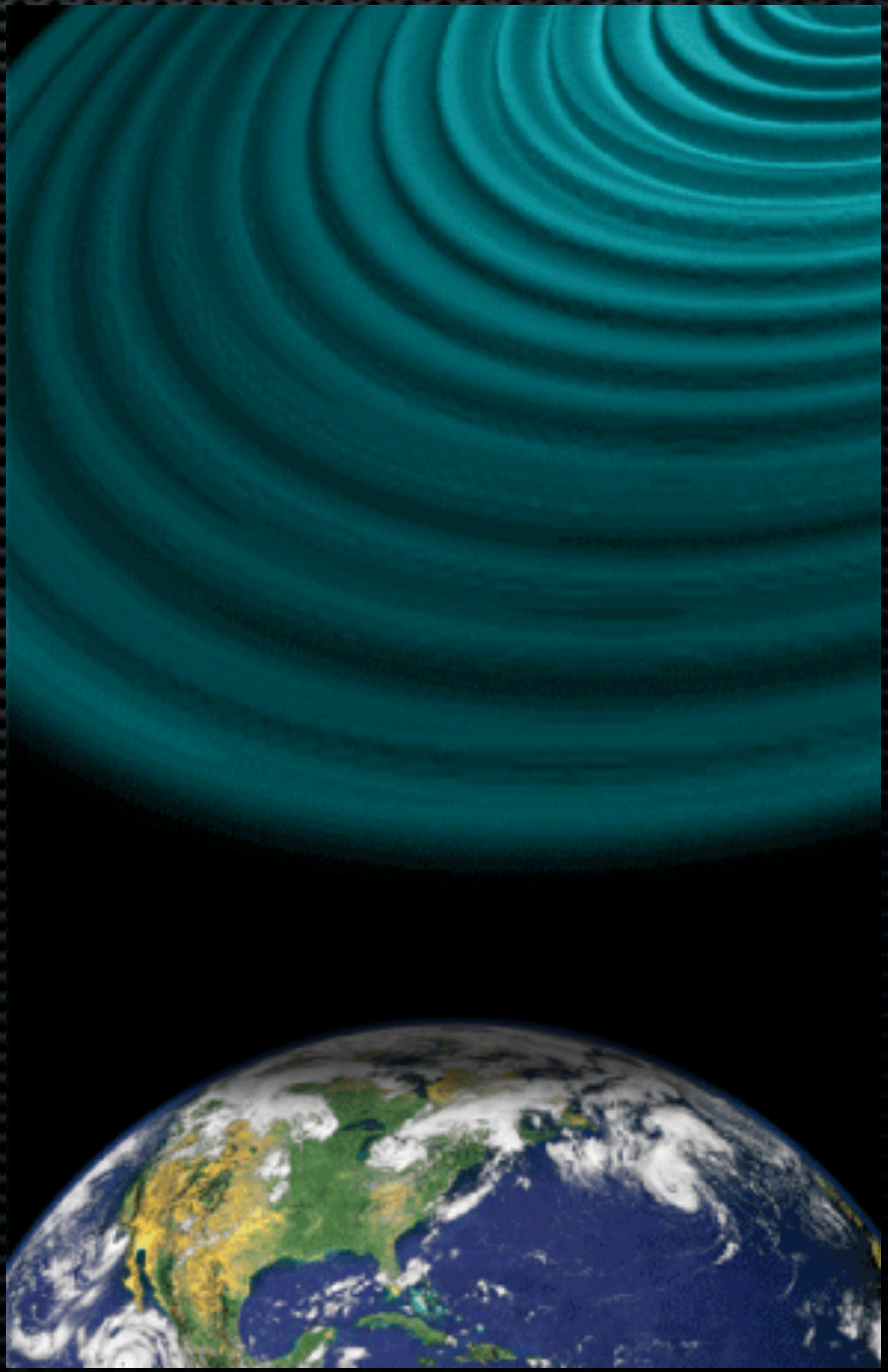
Gravitational wave cosmology



Daniel Holz

The University of Chicago

Thunder and lightning

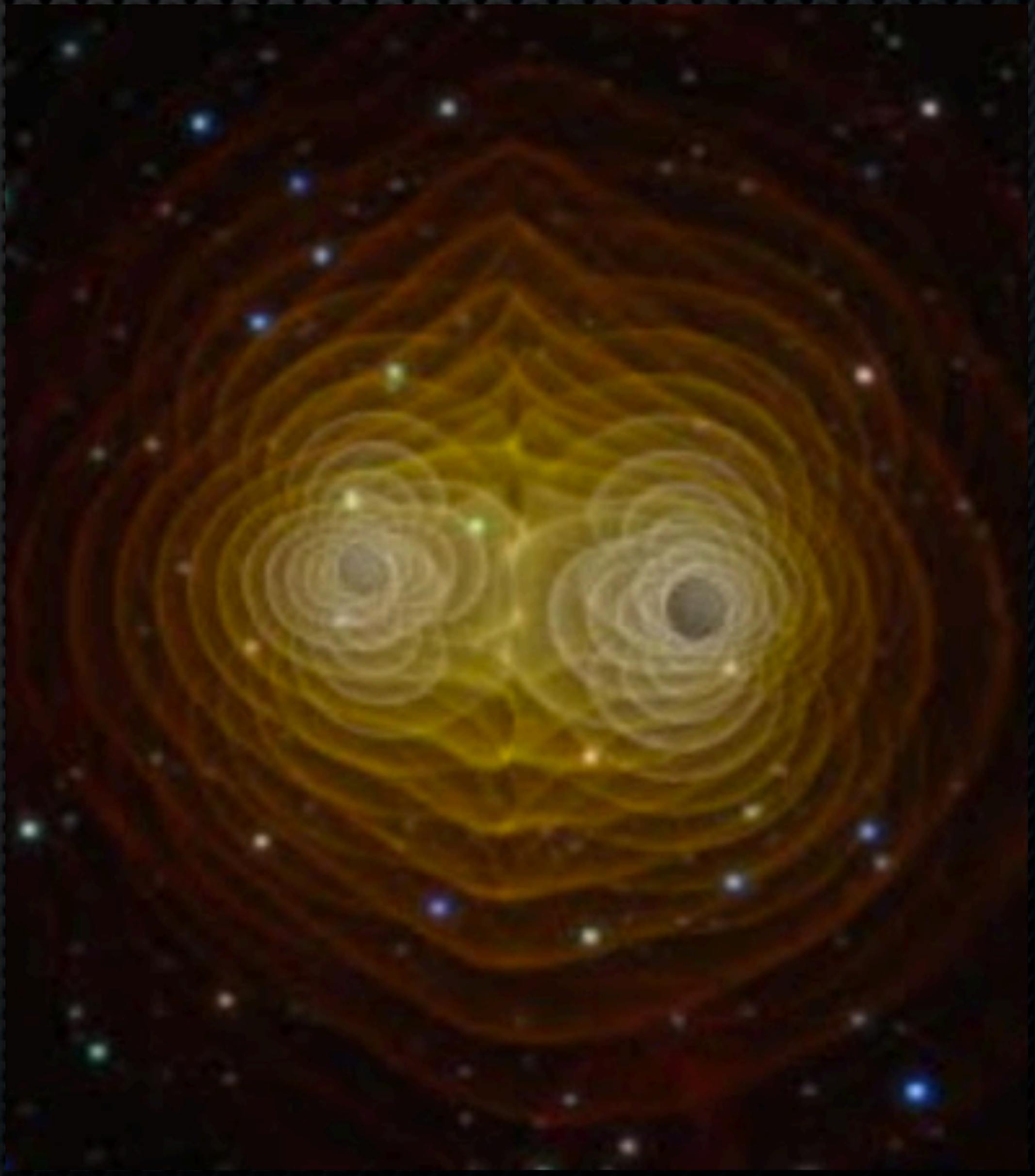
Thus far we’ve only seen the Universe (and 95% of it is dark: dark matter and dark energy).

In the the next few years we will finally be able to listen to the Universe.

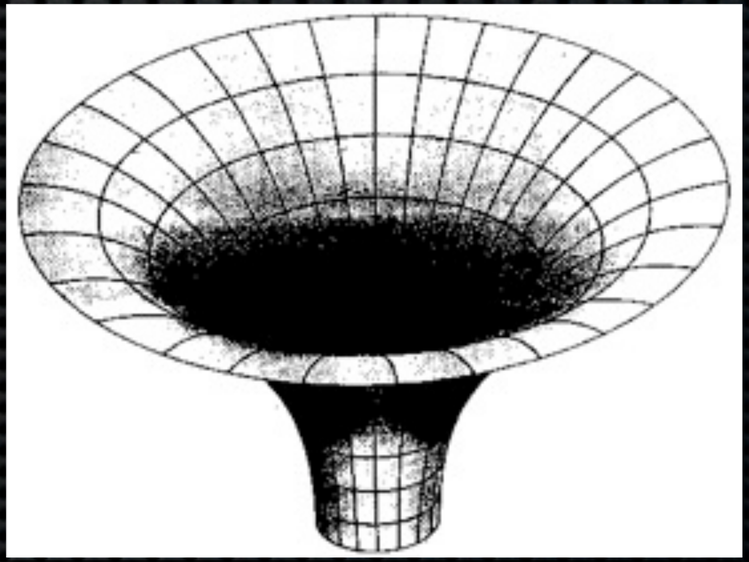
This will be revolutionary!

Outline

- Lecture 1: introduction to gravitational waves
- Lecture 2: how to detect gravitational waves
- Lecture 3: what we might learn from gravitational waves

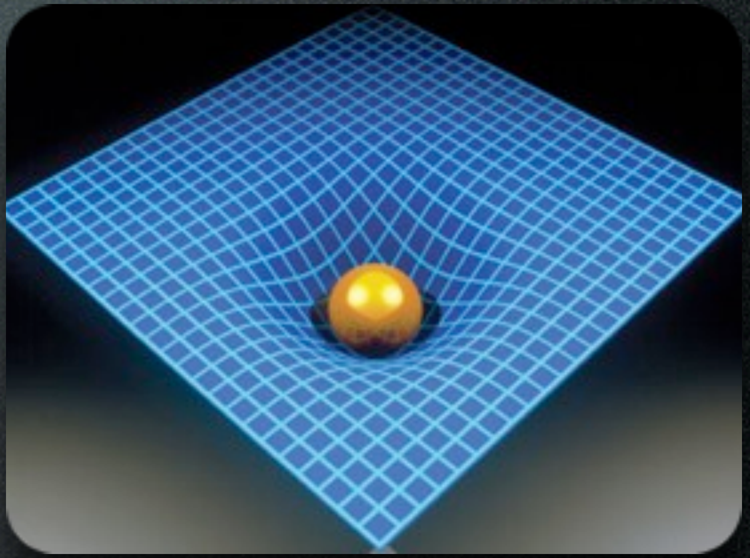


Why should you care about detecting gravitational waves?



- Confirm Einstein
    - predicted a Century ago
- Probe strong gravity
- Teach us about astrophysics and cosmology
- A revolutionary new and completely different way to study the Universe
- Imminent!!

General Relativity

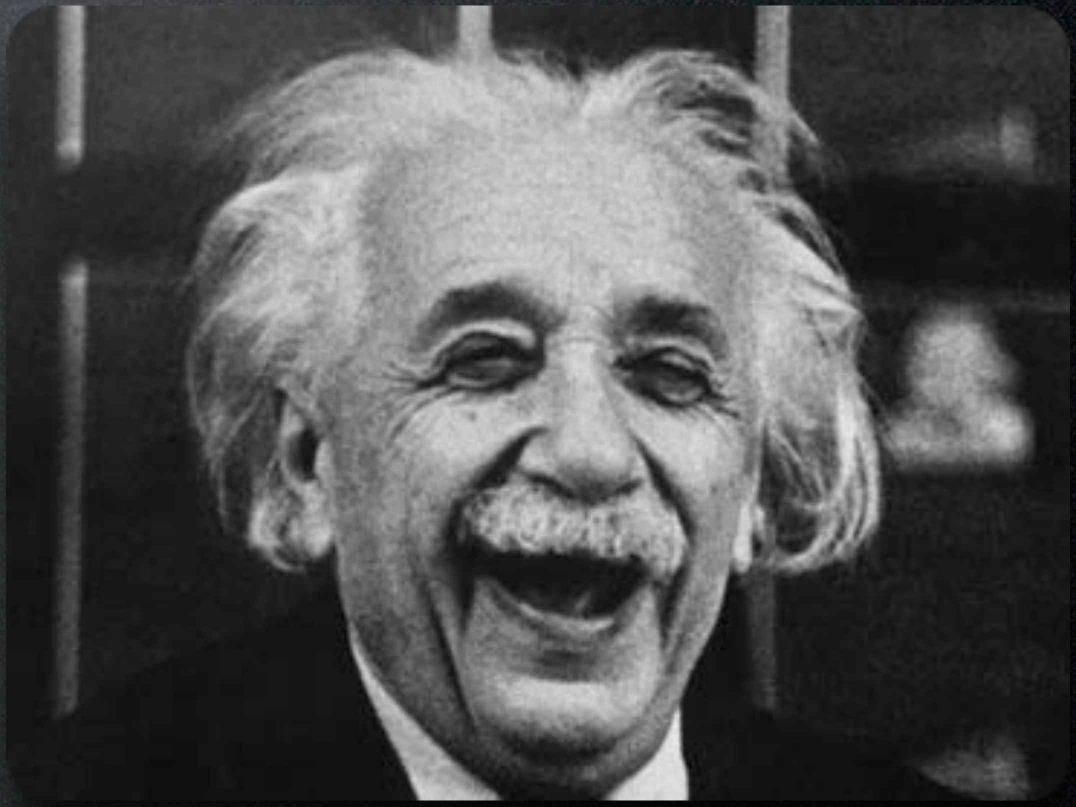


- Theory of gravity
- Beautiful and compelling:
  one of the triumphs of modern physics
- Space and time are intertwined:
  spacetime
- Spacetime tells matter how to move, and matter tells spacetime how to curve

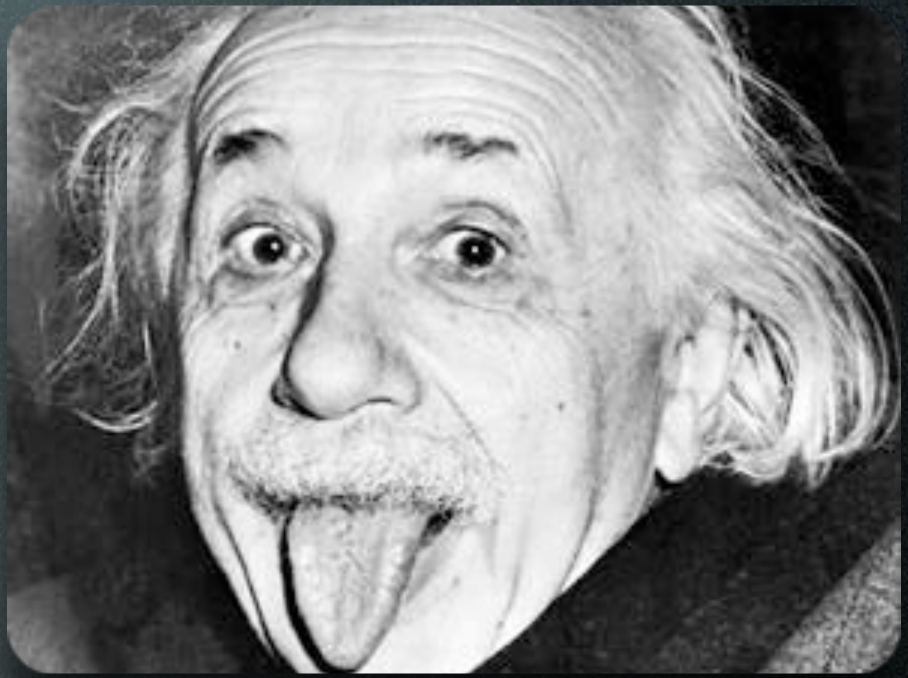
General relativity

Key predictions:

- perihelion precession of Mercury
- the Universe is dynamic
- bending of light
- black holes
- gravitational waves



Gravitational waves
Sordid history
Theory



- 1916: Einstein predicts gravitational waves
- 1922: Eddington: "Gravitational waves travel at the speed of thought"
- 1936: Einstein writes a paper showing that gravitational waves don't exist
- 1950–1960: Theoretical arguments about existence of gravitational waves
- 1970s: Consensus that gravitational waves exist

Sordid history

Experiment

- 1916: Einstein predicts gravitational waves
- 1960: Joe Weber starts building detectors
- 1969: Joe Weber announces first detection of gravitational waves

Sordid history

Experiment

EVIDENCE FOR DISCOVERY OF GRAVITATIONAL RADIATION*

J. Weber
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(Received 29 April 1969)

Coincidences have been observed on gravitational-radiation detectors over a base line of about 1000 km at Argonne National Laboratory and at the University of Maryland. The probability that all of these coincidences were accidental is incredibly small. Experiments imply that electromagnetic and seismic effects can be ruled out with a high level of confidence. These data are consistent with the conclusion that the detectors are being excited by gravitational radiation.

Some years ago an antenna for gravitational radiation was proposed.[1] This consists of an elastic body which may become deformed by the dynamic derivatives of the gravitational potentials, and its normal modes excited. Such an antenna measures, precisely, the Fourier transform of certain components of the Riemann curvature tensor, averaged over its volume. The theory has been developed rigorously, starting with Einstein's field equations to deduce[2] equations of motion. Neither the linear approximation nor the energy-flux relations are needed to describe these experiments, but their use enables discussion in terms of more familiar quantities. All array is a new set of windows for studying the universe.

Search for gravitational radiation in the vicinity of 1660 Hz.—A frequency in the vicinity of 1660 Hz was selected because the dimensions are convenient for a modest effort and because this frequency is swept through during emission in a supernova collapse. It was expected that once the technology was refined, detectors could be designed for search for radiation from sources with radio or optical emission, such as the pulsars. A knowledge of the expected frequency and Q of a source enormously increases the probability of successful search.

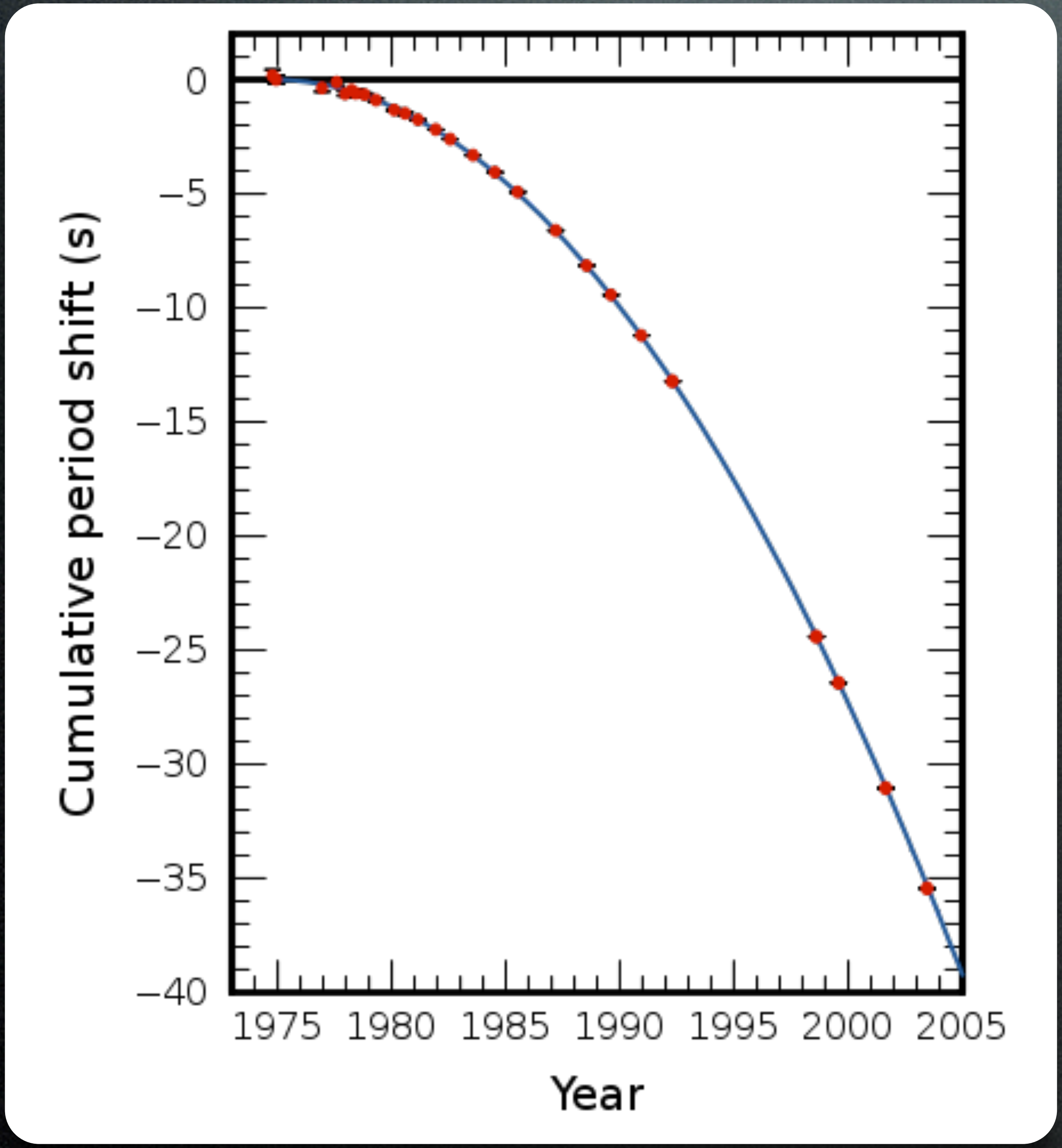
Sordid history

Experiment

- 1916: Einstein predicts gravitational waves
- 1960: Joe Weber starts building detectors
- 1969: Joe Weber announces first detection of gravitational waves
- 1970–: Weber's waves are never reproduced
- 1974–1979: Hulse-Taylor binary pulsar

Sordid history

Experiment



Sordid history

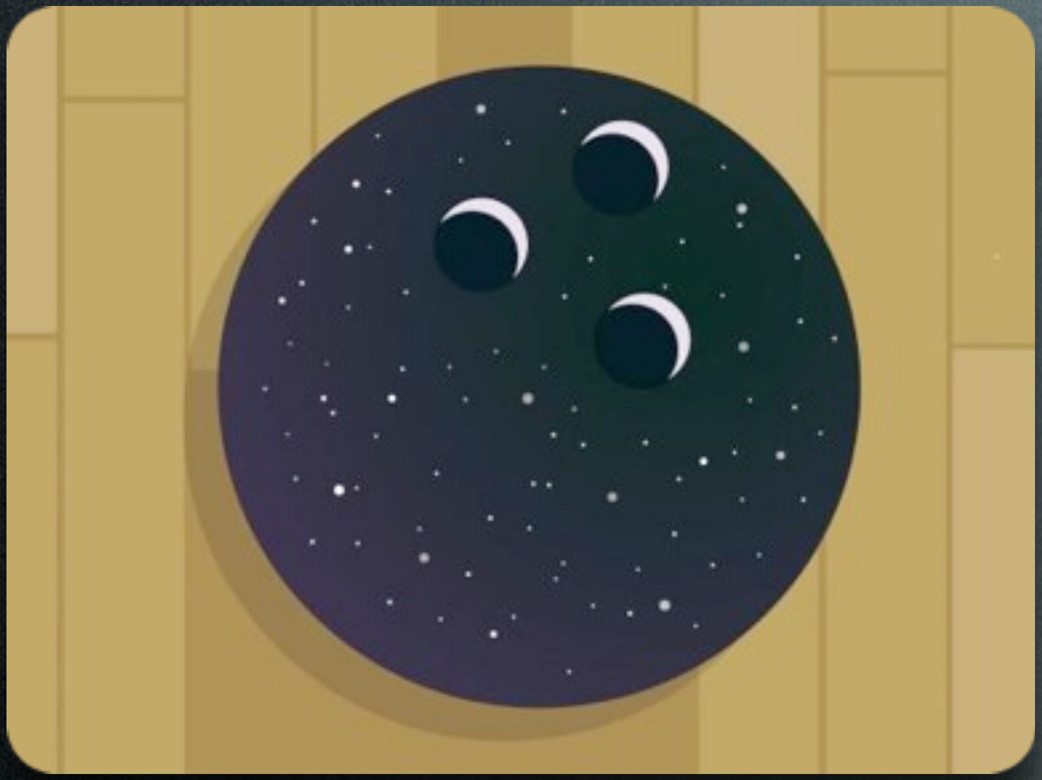
Experiment

- 1916: Einstein predicts gravitational waves
- 1960: Joe Weber starts building detectors
- 1969: Joe Weber announces first detection of gravitational waves
- 1970–: Weber's waves are never reproduced
- 1974–1979: Hulse-Taylor binary pulsar
- 1993: Nobel prize to Hulse and Taylor
- 1992–today: Laser Interferometer Gravitational-Wave Observatory (LIGO)

What are gravitational waves?

Heuristic arguments

- Electromagnetism:
    - If you shake an electron, you get light
- General relativity:
    - If you shake a massive object, you get gravitational waves

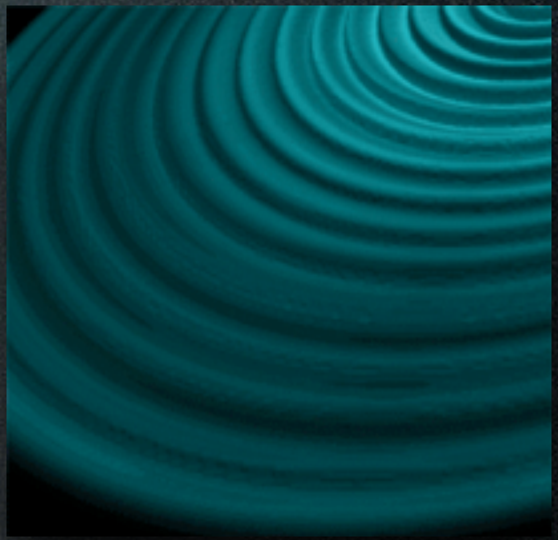


What are gravitational waves?

Heuristic arguments

- Nothing travels faster than the speed of light. Not even information.
- If a massive object moves, how do we find out?
    - Something must travel from the object to us, to let us know. This is a gravitational wave.

GWs from GR



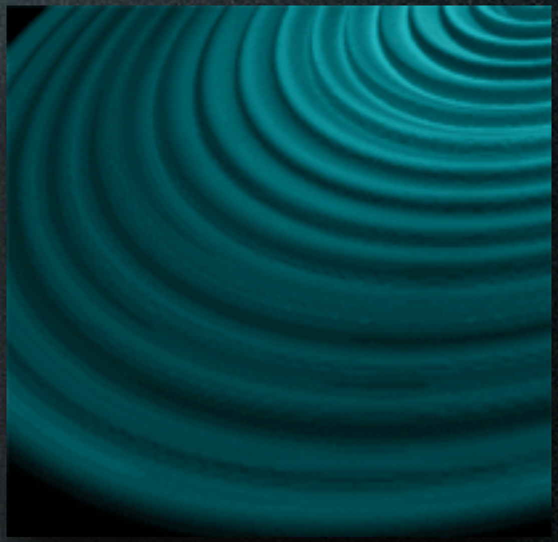
- Start with a metric:

$$g_{\alpha\beta}(x) = \eta_{\alpha\beta} + h_{\alpha\beta}(x)$$

- $\eta_{\alpha\beta}$ is flat Minkowski spacetime
- $h_{\alpha\beta}(x)$ is a small perturbation
- GR gives a solution:

$$h_{\alpha\beta}(x) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} f(t-z)$$

GWs from GR



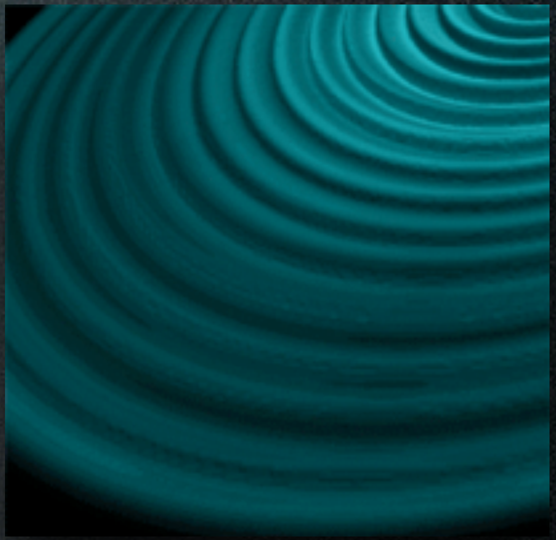
- Writing everything out:

$$ds^2 = -dt^2 + [1 + f(t-z)]\, dx^2 + [1 - f(t-z)]\, dy^2 + dz^2$$

- Plug in:

$$f(t-z) = a \sin [w(t-z)]$$

- This is a gravitational wave of amplitude a and frequency ω traveling in the z direction



GWs from GR

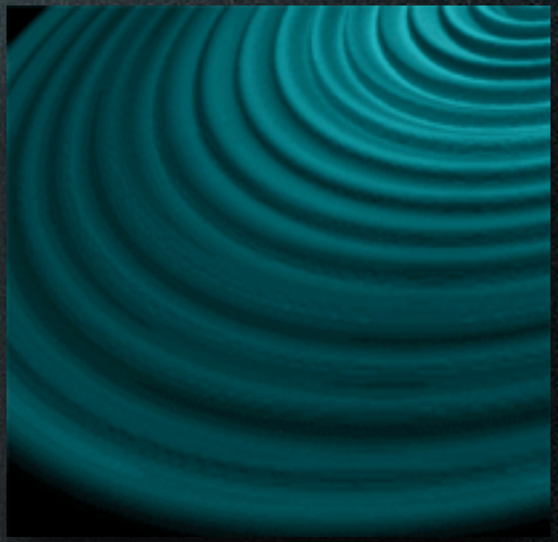
- There is another polarization:

$$h_{\alpha\beta}(x) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} f(t-z)$$

- Most general form for linearized gravitational wave propagating in the z direction:

$$h_{\alpha\beta}(t,z) = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & f_+(t-z) & f_\times(t-z) & 0 \\ 0 & f_\times(t-z) & -f_+(t-z) & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

Energy in GWs?

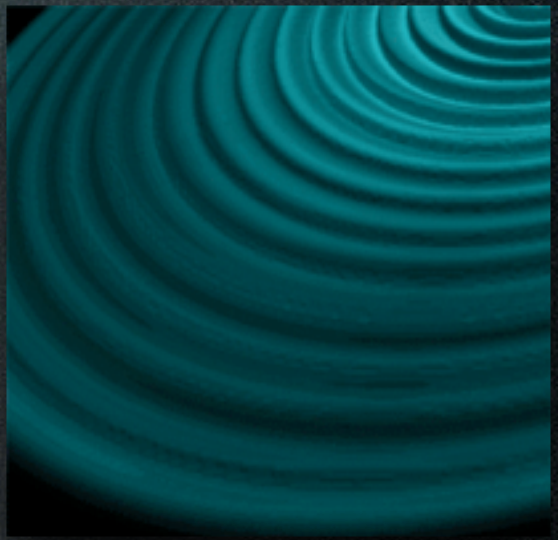


- In Newtonian gravity, the energy density is given by:

$$\epsilon_{\mathrm{Newton}}(\vec{x}) = -\frac{1}{8\pi G}[\vec{\nabla}\Phi(\vec{x})]^2 = -\frac{1}{8\pi G}[\vec{g}(\vec{x})]^2$$

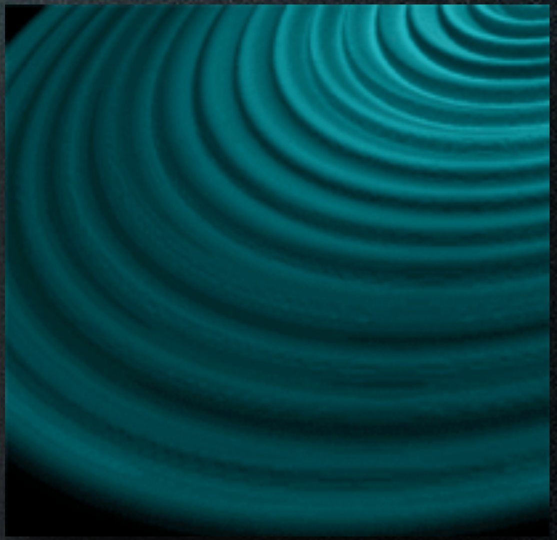
- There is no analog in general relativity!
- Can always find a frame where there is no gravity (first derivatives of metric vanish in local inertial frame)

Energy in GWs?



- Can find expression in short-wavelength approximation
- Hand-wavy argument:
    - Energy must depend on wavelength and amplitude, since $\omega = 0$ and $a = 0$ should not carry energy
    - Energy must go as even power of wavelength and amplitude (otherwise negative energy for negative frequency/ amplitude)

Energy in GWs?



- Comparing to waves on a string or E&M waves, expect to go as square
- Putting in units, must have:

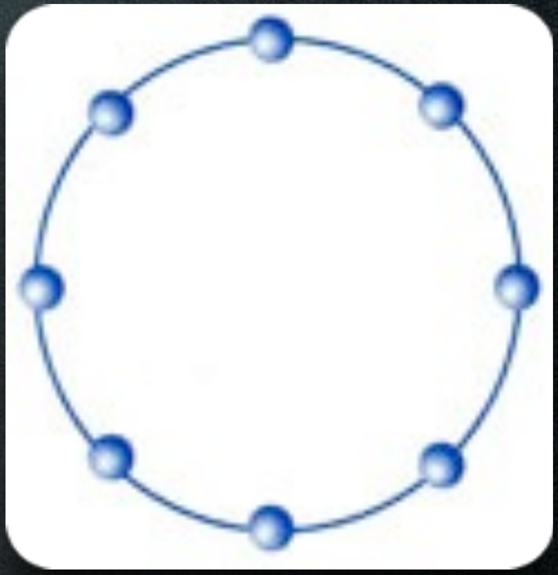
$$F \propto \frac{c^3}{G} f^2 a^2$$

- Exact solution:

$$F = \frac{\pi c^3}{4G} f^2 a^2$$

- GWs carry energy!

What do GWs do?

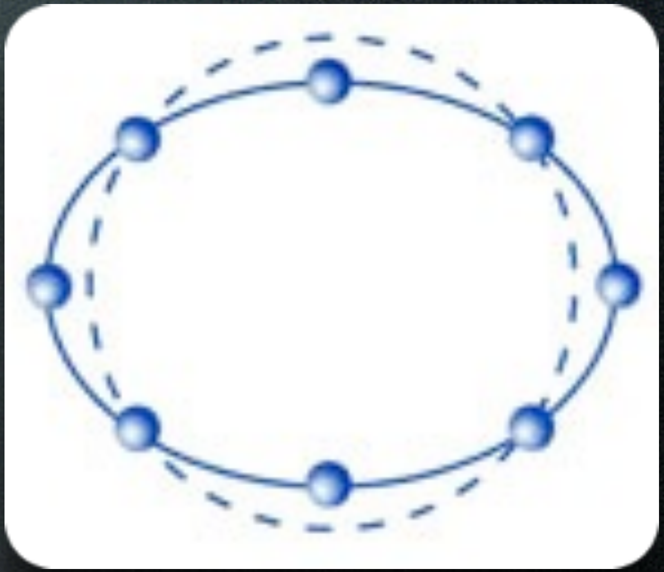


- Consider a wave propagating in z direction
- Have two free particles, particle A at the origin, and particle B at coordinate position (x_B, y_B, z_B)
- Particles have initial four-velocities:

$$u^\alpha_{\text{particle A}} = u^\alpha_{\text{particle B}} = (1, 0, 0, 0)$$

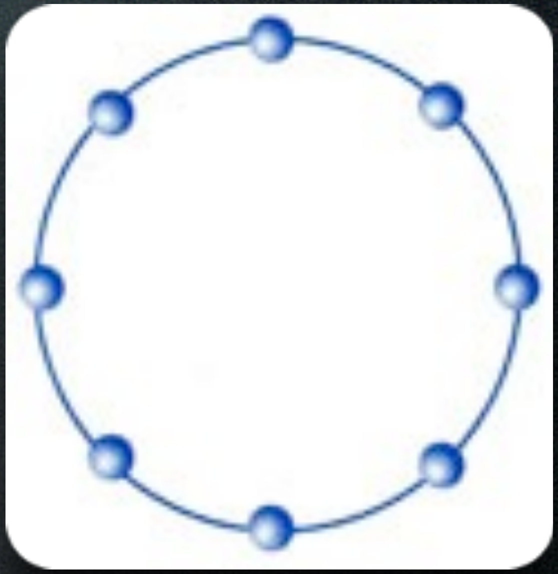
- Before wave passes particles are in flat space, so particles remain at rest

What do GWs do?



- Solve geodesic equation to first order to find out what happens while GW passes: $$\frac{d^2x^i}{d\tau^2} = -\Gamma^i_{\alpha\beta}\frac{dx^\alpha}{d\tau}\frac{dx^\beta}{d\tau}$$
- Christoffel symbols vanish in background (flat) metric, so to first order have:
$$\frac{d^2\delta x^i}{d\tau^2} = -\delta\Gamma^i_{\alpha\beta}u^\alpha u^\beta = -\delta\Gamma^i_{tt}$$
- time-time Christoffel symbol vanishes, so perturbations vanish. Particles stay at fixed coordinate position!

What do GWs do?



- The proper distance between the particles varies with time:

$$L(t) = \int_0^{L_*} \left[1 + h_{xx}(t,0)\right]^{1/2}$$

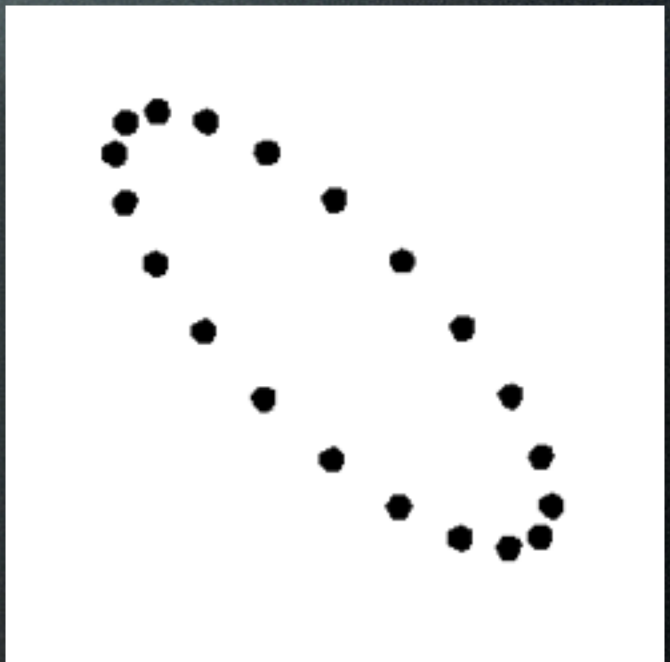
- Waves are very weak, so have:

$$L(t) \approx L_* \left[1 + h_{xx}(t,0)/2\right]$$

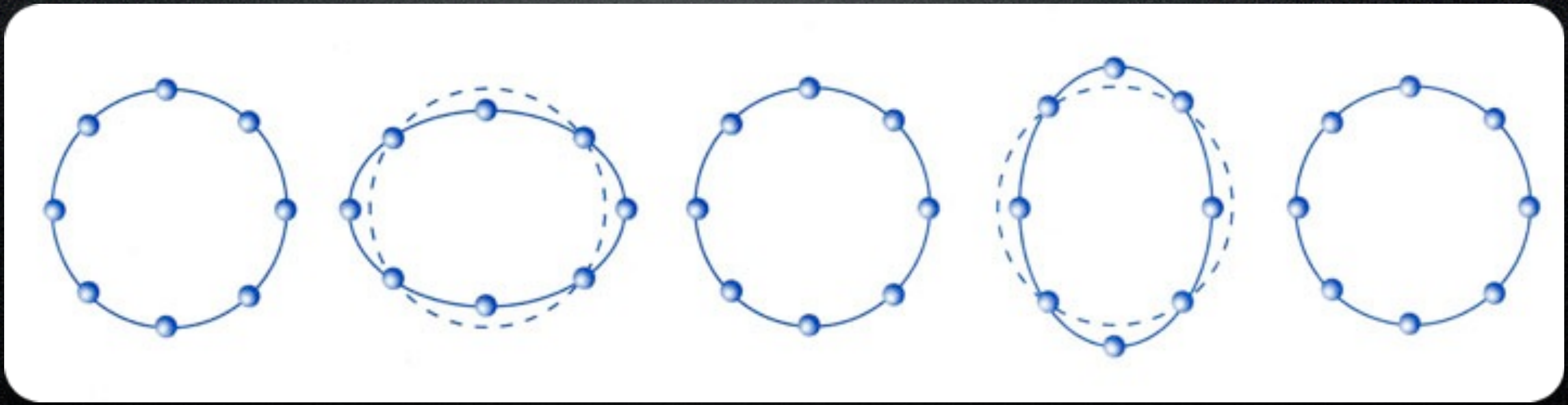
- Fractional change in distance (strain) given by:

$$\frac{\delta L(t)}{L_*} = \frac{1}{2} h_{xx}(t,0) = \frac{1}{2} a \sin(\omega t)$$

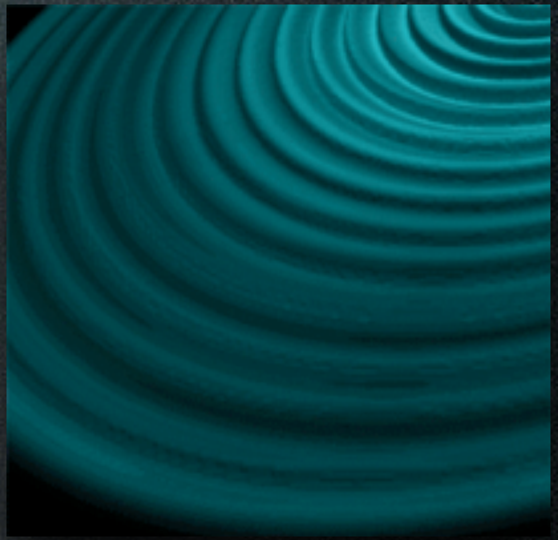
What do gravitational waves do?



- Alternatively stretch and shrink the distance between two points

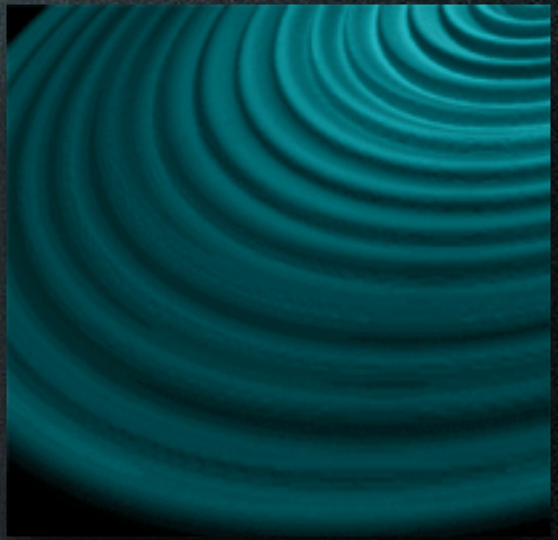


Properties of GWs



- Propagate at the speed of light
- Transverse ($h_{zz} = 0$)
- Two polarizations (+ and x)
- Carry energy
- Affect the relative separation of test particles

GW emission



- Quadrupole formula gives the total power radiated in gravitational waves:

$$L_{\mathrm{GW}} = \frac{G}{5c^5}\left\langle \dddot{I}_{ij}\,\dddot{I}^{ij} \right\rangle$$

where $\dddot{I}_{ij}$ is the quadrupole moment tensor

- Luminosity of GW sources:

$$L_{\mathrm{GW}} \sim \frac{c^5}{G} \sim 10^{59}\,\frac{\mathrm{erg}}{\mathrm{s}}$$

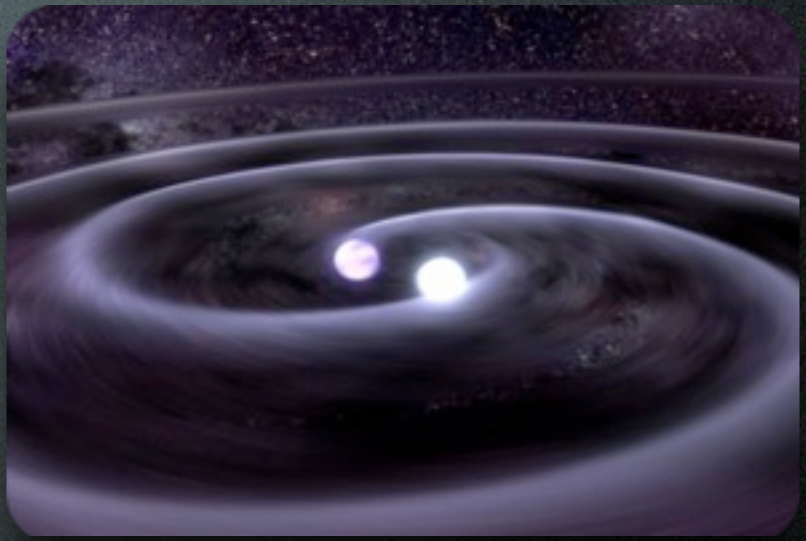
Where do gravitational waves come from?

- Essentially everything generates gravitational waves
- Essentially all sources of gravitational waves are staggeringly weak
- To produce strong GWs need large masses (e.g., the mass of the Sun) moving very fast (e.g., near the speed of light)

Strong sources of gravitational waves

- Two black holes (or neutron stars) crash into each other
- A star falls into a big black hole
- A supernova explodes (asymmetrically)
- Big bang/inflation (maybe)

GWs from binary black holes



- Two black holes each of mass M, separated by $2R$
- Solve quadrupole formula:

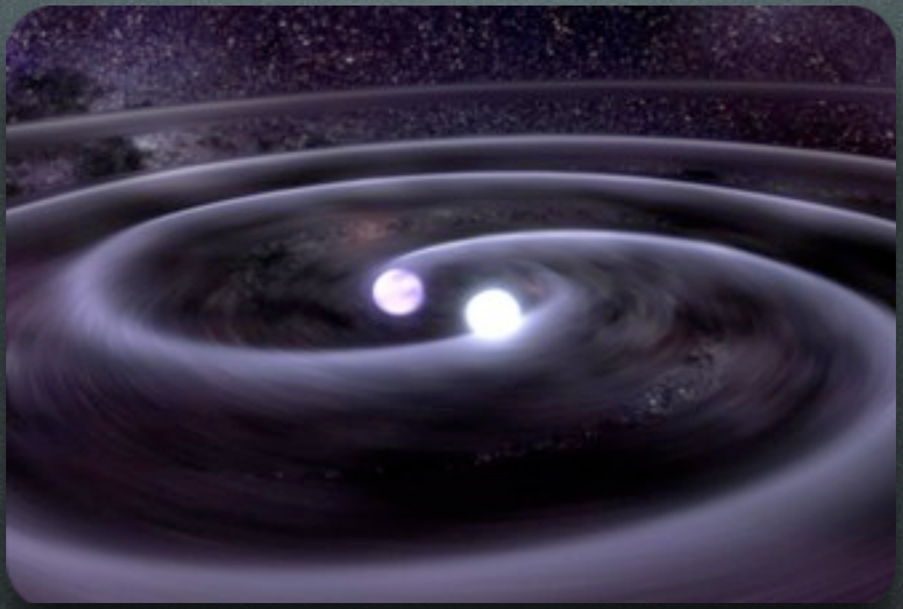
$$L_{\mathrm{GW}} = \frac{128G}{5c^5} M^2 R^4 \Omega^6$$

- Plug in Kepler's law:

$$L_{\mathrm{GW}} = \frac{128}{5} 4^{1/3} \frac{c^5}{G} \left(\frac{\pi G M}{c^3 P} \right)^{10/3}$$

$$= 1.9 \times 10^{33} \left(\frac{M}{M_{\odot}} \frac{1\,\mathrm{hour}}{P} \right)^{10/3} \frac{\mathrm{erg}}{\mathrm{s}}$$

GWs from binary black holes



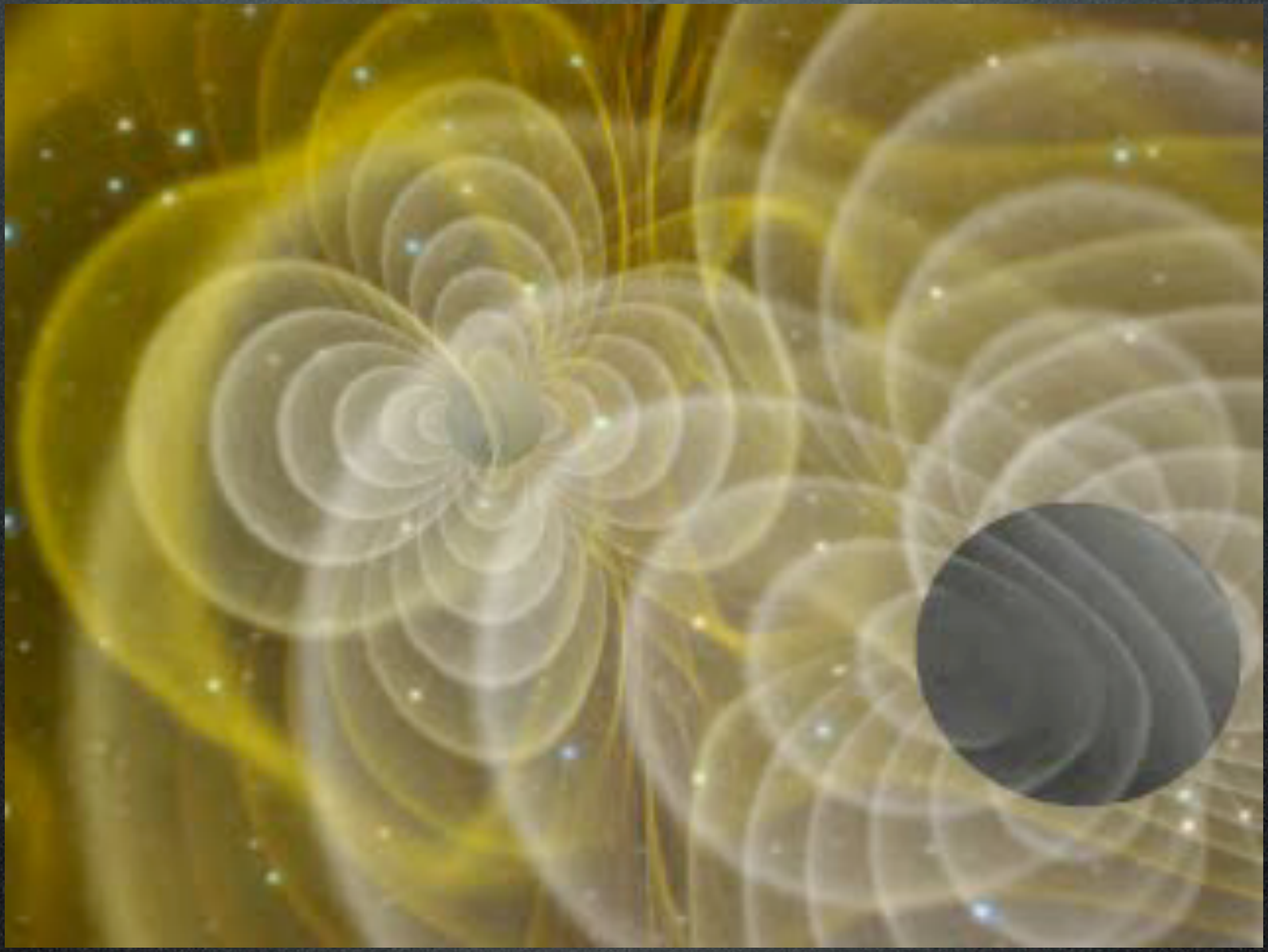
- Assume GW emission plus Kepler's laws:

$$\frac{dP}{dt} = -\frac{96}{5}\pi 4^{1/3}\left(\frac{2\pi M}{P}\right)^{5/3}$$

$$= -3.4 \times 10^{-12}\left(\frac{M}{M_\odot}\frac{1\,\mathrm{hour}}{P}\right)^{5/3}$$

- This gives the full time evolution/waveform

GWs from binary black holes



NASA