A Guide to Bayes

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Abstract

This is a simple document that discusses the basis and basics of Bayes.

Contents

1 Bayes' Theorem

Bayes' theorem is:

$$P(A|B) = \frac{P(B|A)\,P(A)}{P(B)} \tag{1}$$

where

$P(A)$ and $P(B)$ are the probabilities of observing A and B without regard to each other.

$P(A|B)$, a conditional probability, *is the probability of observing event A given that B is true.*

$P(B|A)$ is the probability *of observing event B given that A is true.*

2 Bayesian inference

Bayesian inference derives the posterior probability as a consequence of two antecedents, a prior probability and a "likelihood function" derived from a statistical model for the observed data. Bayesian inference computes the posterior probability according to Bayes' theorem:

$$P(H|E) = \frac{P(E|H)\,P(H)}{P(E)} \tag{2}$$

where:

| denotes a conditional probability; more specifically, it means "given".

H stands for any hypothesis whose probability may be affected by data (called evidence below). Often there are competing hypotheses, from which one chooses the most probable.

the evidence E corresponds to new data that were not used in computing the prior probability.

$P(H)$, the prior probability, is the probability of H before E is observed. This indicates one's previous estimate of the probability that a hypothesis is true, before gaining the current evidence.

$P(H|E)$ the posterior probability, is the probability of H given E, i.e., after E is observed. This tells us what we want to know: the probability of a hypothesis given the observed evidence.

$P(E|H)$ is the probability of observing E given H. As a function of E with H fixed, this is the likelihood. The likelihood function should **not** be confused with $P(H|E)$ as a function of H rather than of E. It indicates the compatibility of the evidence with the given hypothesis.

$P(E)$ is sometimes termed the marginal likelihood or "model evidence". This factor is the same for all possible hypotheses being considered. (This can be seen by the fact that the hypothesis H does not appear anywhere in the symbol, unlike for all the other factors.) This means that this factor does not enter into determining the relative probabilities of different hypotheses.

Note that Bayes' rule can also be written as follows:

$$P(H \mid E) = \frac{P(E \mid H)}{P(E)} \cdot P(H) \tag{3}$$

where the factor $\frac{P(E|H)}{P(E)}$ represents the impact of E on the probability of H.

3 SED FITTING APPROACH

Ricci et al. (2016)

4 References

https://en.wikipedia.org/wiki/Bayesian_inference
https://en.wikipedia.org/wiki/Bayes%27_theorem

Croom et al. (2004)

References

Croom S. M., Smith R. J., Boyle B. J., Shanks T., Miller L., Outram P. J., Loaring N. S., 2004, MNRAS, 349, 1397

Ricci C., et al., 2016, ArXiv e-prints