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Key facts for luminous quasars

- Luminosity $L \sim 10^{39}$ W $(10^{13} L_{sun})$
- Size $R \leq 10^{13}$ m (from variability)
- stored energy in lobes $E_{st} \sim 10^{54}$ J
- Stable pointing $t \sim 10^8$ years

HOW IS THE ENERGY GENERATED ?

Suppose energy generation is by nuclear burning as in stars.
Burning mass m gives energy $E = \mu m c^2$
μ = "efficiency" of energy generation process; for H->He fusion $\mu_{nuc} = 0.007$

Mass required to produce stored energy E_{st} is $M = E_{st} / \mu c^2 \sim 10^{39}$ kg $(\sim 10^9 M_{sun})$
But all this mass is inside $R < 10^{13}$ m
Binding energy lost in collapse $> GM^2/R \sim 10^{55}$ J

In other words, if the object has to be that compact, MORE energy can be generated by collapse than by nuclear burning. But how can collapse energy actually be used ? Two routes have been suggested :

(1) Single collapse	- energy stored in rotation and magnetic field - radiated away slowly (LIKE GIANT PULSAR)
(2) Accretion onto existing large mass	- radiates newly generated energy as material dribbles in (LIKE GIANT X-RAY BINARY)

In either case, jet-axis = spin-axis ?

Most detailed work has been done on option (2). It is likely that any large compact object formed by route (1) would soon anyway collapse to a BLACK HOLE. Then the energy generated by new accretion would dominate. We will look more closely at accretion. However, there are ways to extract the rotational energy of a black hole.

ACCRETING BLACK HOLES

Consider object mass M radius R.
- small mass m falls to R : gains $E = GMm/R$
- efficiency $\mu = E/mc^2 = GM / c^2R$ (if it can be radiated ...)
- this is a maximum if object is a black hole $R = R_{Sch} = 2GM / c^2$

$$\Rightarrow \mu_{max} = \frac{GM}{c^2} \cdot \frac{c^2}{2GM} = \frac{1}{2}$$!!! cf nuclear 0.007 ....

Potentially then, this is the most efficient energy generation mechanism known.

ORBITING MATERIAL

- is there time to radiate before disappearing down the plughole ?
- incoming material almost certainly will have some angular momentum
- then problem is opposite - won't settle, only orbit ==> disc of material.

- need a source of viscosity :
- transfers momentum from fast moving layer to slow moving layer
==> ang.mom transported outwards, material can drift inwards slowly
- traditional molecular viscosity too slow
- unsolved problem : maybe turbulent viscosity, magnetic viscosity

ENERGY AVAILABLE IN DISC ACCRETION

However, as rotating material descends, only half the PE can ever be radiated....

Consider small mass m in orbit : PE : $V = \frac{-GMm}{r}$ KE : $T = \frac{GMm}{2r}$
Obeys Virial Theorem 2T + V = 0 ; different orbits have different total energy E=T+V

If mass descends by small amount Δr
PE lost is $\Delta V = \Delta r \cdot \frac{\partial V}{\partial r} = \Delta r . \frac{GMm}{r^2}$
KE of rotation gained is $\Delta T = \Delta r \cdot \frac{\partial T}{\partial r} = \Delta r . \frac{GMm}{2r^2}$

So $\Delta T = \Delta V/2$ i.e. half of ΔV ==> increased rotation
remaining half ==> heat ==> radiation.

EXPECTED ACCRETION LUMINOSITY

- can radiate until reaching R_{min} = last stable orbit = $3R_{Sch}$
- half lost PE available

==> $\mu = 1/12$

- if matter accretes at rate $\dot{M}$ (i.e. kg/sec, or solar masses/year) then luminosity is

$$\boxed{L = \mu \dot{M} c^2}$$

Detailed calculations : non-rotating BH μ = 6%, maximally rotating BH μ = 42%.

Doesn't depend on mass of black hole M_H ???
Can have luminosity as big as you like as long as you dump enough fuel ??

EDDINGTON LIMIT

Radiation pressure stops accretion above a critical luminosity...

- each photon carries momentum p=E/c
- collisions between outgoing photons and electrons, cross-section σ_T
- exchange momentum --> force given by dp/dt --> outward force is

  F_{out} = energy flux $m^{-2}/c \times$ cross-section per electron
- electrons drag protons with them - effective force on protons

• So at distance R from black hole of mass M_H and luminosity L

$$F_{out}(R) = \frac{L\sigma_T}{4\pi R^2 c}$$ outward force felt by a proton due to radiation

$$F_{in}(R) = \frac{GM_H m_p}{R^2}$$ inward force on proton due to gravity

no net force when F_{in}=F_{out} so accretion stops

==> maximum luminosity ("Eddington limit") $L_{max} = L_{Edd} = \dfrac{4\pi GM_H m_p c}{\sigma_T}$

EDDINGTON LIMITED ACCRETION

Putting some numbers in we find

$$\boxed{L_{Edd}(\text{Watts}) = 6.37 M_H(\text{kg}) = 1.26\times 10^{31}(M_H / M_{sun})}$$

Still true that luminosity due to accretion is $L_{acc} = \mu \dot{M} c^2$
So we get L_{Edd} when accretion rate is

$$\boxed{\dot{M}_{Edd} = \frac{L_{Edd}}{\mu c^2} = \frac{2.2\times 10^{-9}}{\mu}(M_H / M_{sun}) \quad \text{solar masses / year}}$$

• Assume μ~0.1 then L=10^{39} W can be explained by

M_H =10^8 M_{sun} accreting at Eddington limit, swallowing 2 solar masses/yr.

•Inner region ~ $10R_{Sch}$ = 20 GM_H/c^2 = 3×10^4 (M_H/M_{sun}) metres
==> a few light hours CONSISTENT WITH COMPACT VARIABILITY LIMITS

CHARACTERISTIC TEMPERATURES

(1) effective black body temp from L = $4\pi R^2\sigma T^4$
- assume L=L_{Edd}=6.37 M_H R=$3R_{Sch}$ = $6GM_H/c^2$

==> $\boxed{T_{eff} = 2.2\times 10^7 (M_H / M_{sun})^{-1/4}}$

For black hole 1 solar mass --> $T \sim 10^7$K ---> hard X-rays (cf X-ray binaries)
For black hole 10^8 solar mass --> $T \sim 10^5$K ---> UV-soft X-rays
about right for UV bump

(2) thermalised binding energy ("virial temperature")
- descending to radius R each proton gains $\Delta V = GM_H m_p/R$
- if virialised, $KE = -\Delta V/2$, and if thermalised $KE = 3kT/2$ gives proton temperature

==> expected proton temperature $T_p = GM_H m_p/3kR$
- falling to $R = 3 R_{Sch} = 6GM_H/c^2$ we get

$$T_p = \frac{m_p c^2}{18k} = 6 \times 10^{11}\ \mathrm{K}$$ regardless of black hole mass

- if protons and electrons reach equipartition, energy is shared
- so final electron temperature may be then factor two smaller

Note the energy per proton is 52 MeV - not large enough to create proton-anti-proton pairs, but easily large enough to produce electron-positron pairs ($m_e c^2 = 511$ keV). Given these very high energies, the material is in fact very unlikely to thermalise, and radiation will be dominated by high energy and/or non-thermal processes.

WHICH SITUATION APPLIES ?

If the material is optically thin, situation (2) may be closer to the truth. If the material is optically thick, then regardless of the raw processes which are hidden from view, the energy will get degraded on the way out and emerge at a temperature corresponding to situation (1).

COMPONENTS OF SPECTRAL ENERGY DISTRIBUTION (SED)

This is essentially an unsolved problem, but there are some reasonable guesses at various parts of the SED.

(1) UV bump. optically thick thermal emission ?
black hole size required for luminosity gives roughly correct Temp.
but spectrum broader than single temp
.... what do we get from whole accretion disc ?

(2) X-ray bump pair-plasma in very central part ?
hot corona above and below accretion disc ?
Note : varies more quickly as well as being much "hotter".

(3) IR bump heated dust at large distances from central object
broad SED from range of temps because of range of distances.
Note : does not vary, as expected from being much larger.

(4) radio lobes synchrotron emission from non-thermal electrons
... how is energy supplied to large distances to accelerate electrons ?

ACCRETION DISC : PREDICTED SPECTRUM

- mass m , descending dR ; half lost PE available for heat; so $dE = \frac{GM_H m}{2R^2} dR$
- to maintain steady state, $\dot{m}$ same through all radii
- assume that energy gained is turned into heat and radiated "on the spot"
- then emitted luminosity in annulus dR at R is

$$L(R)dR = \frac{GM_H \dot{m}}{2R^2} dR$$

- area of annulus $A(R)\, dR = 4\pi R\, dR$ (both sides !)
- if radiation is blackbody $L = area \times \sigma T^4$ ==> $T(R) = \left(GM_H \dot{m} / 8\pi R^3 \sigma\right)^{1/4}$

• max temp at $R_{min} \sim 3\, R_{sch}$ is $T_{\max} \approx 1.0 \times 10^7 \left({M_H}/{M_{sun}} \right)^{-1/4}$

(using $\dot{m}_{\max}$ and $\mu = 0.1$)

• but the overall observed spectrum is the sum of many blackbodies

at each radius :

emitting area	$A(R)\, dR = 2\pi R\, dR$	(only one side seen)
emitting temp	$T(R) = \left(GM_H \dot{m} / 8\pi R^3 \sigma\right)^{1/4}$	
spectrum for unit area	$I(\nu) = B[\nu, T(R)]$	
	$B_\nu = \frac{2\pi h \nu^3}{c^2\left(e^{h\nu/kT} - 1\right)}$	

(i.e. power emitted/Hz/m^2 in all outward directions by surface)

so the total disc luminosity is $$L_\nu = \frac{4\pi^2 h \nu^3}{c^2} \int_{R_{inner}}^{R_{outer}} \frac{R}{e^{h\nu/kT} - 1} \cdot dR$$

•$T_{max} = T(R_{in})$; peak at ν_{max} = const. × T_{max}, **Wien fall-off at higher frequencies**
•$T_{min} = T(R_{out})$; peak at ν_{min}= const. × T_{min}, **RJ fall-off at low frequencies** $L_\nu \propto \nu^2$

... what about intermediate frequencies ?

- can get approx shape by substituting $x = \frac{h\nu}{kT(R)}$ and noting that $T(R) \propto R^{-3/4}$

so $x \propto \nu R^{3/4} \Rightarrow R \propto x^{4/3}\nu^{-4/3}$ and $dR \propto x^{1/3}\nu^{-4/3}$

so the integrand above varies as $\int \frac{x^{5/3}}{e^x - 1} dx . \nu^{-8/3}$

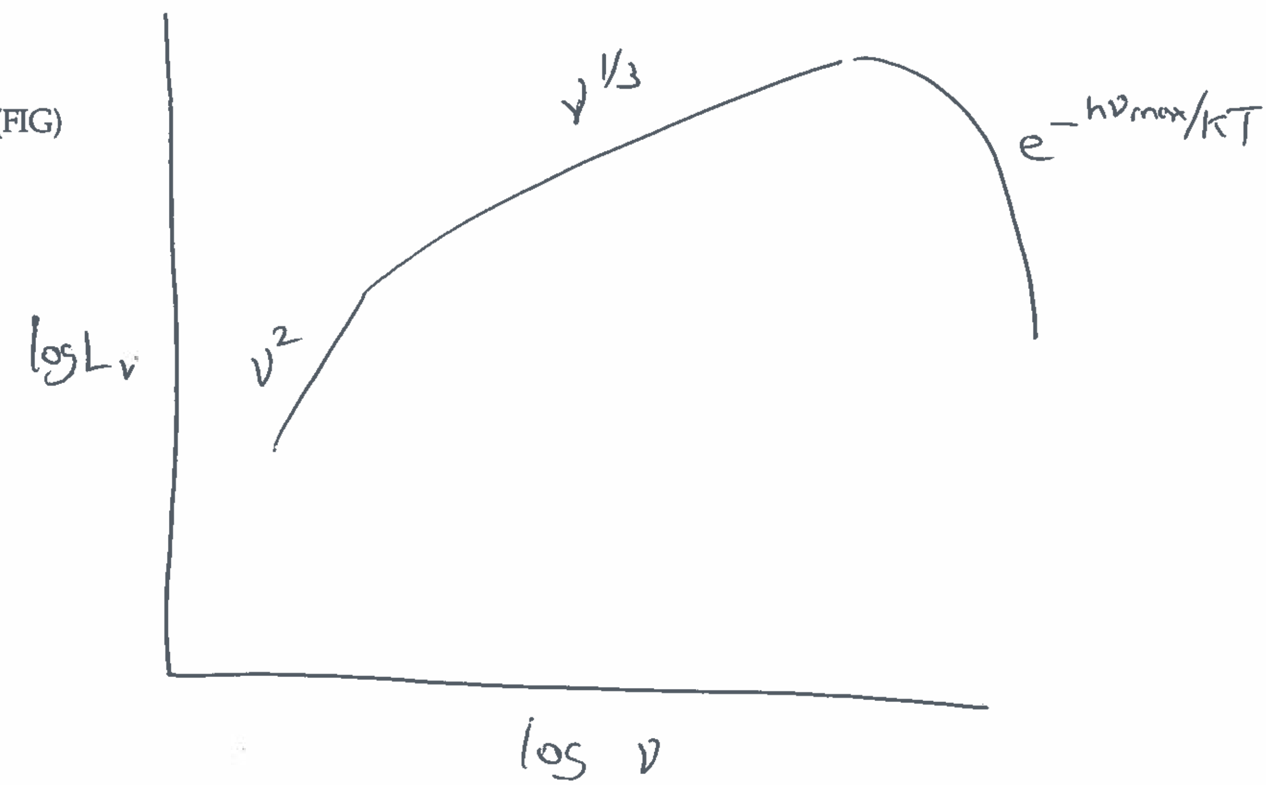
- the limits of the intergral in x correspond to those given by R_{in} and R_{out}

\- but as long as we considering frequencies in the range $\frac{kT(R_{out})}{h} << \nu << \frac{kT(R_{in})}{h}$ then the lower limit in x is <<1 and the upper limit is >>1 so that we get

$L_\nu \propto \nu^{1/3} \int_0^\infty \frac{x^{5/3}}{e^x - 1} dx$ but the integral is now just a number so ....

$$\boxed{L(\nu) \propto \nu^{1/3}}$$

(FIG)



IS THE UV BUMP AN ACCRETION DISC AROUND A BLACK HOLE ?

• big success is hitting the right spectral range : given black hole mass required to explain luminosity of quasars if radiating at Eddington limit, the expected BB temp is around 10^5K, giving a peak in the UV, as seen.

• spectral shape expected is of the right kind of width, but the predicted $\nu^{1/3}$ is not observed. Instead the typical optical-UV slope is in the range ν^0 to $\nu^{-0.5}$. Interestingly, cataclysmic variables (compact binary stars containing a white dwarf) do show a good fit to $\nu^{1/3}$, confirming that nature does produce simple accretion discs.

• but we know model is unrealistic in some ways :

- like stars, should include atmosphere rather than just BB radiation from hard surface ?
- assumption that PE is turned to heat and radiated on-the-spot could easily be wrong
- assumption that structure is geometrically thin may break down near the middle
  - ion-pressure supported "donut" models have been tried.

• rival model : optically thin fre-free emission from T~10^5 gas ? gives roughly ν^0 spectrum until cut-off frequency, which may be in unobserved XUV. This model probably doesn't work because if the source is small, as implied by variability, it must be optically thick. (See exercise sheet)

• interesting clues come from observations of optical/UV variability

- variability wavelength dependent - gets steadily larger through red to blue to UV
  - good news for accretion disc : should get hotter when brighter (for fixed mass)
  - bad news for free-free model : variations should be grey ?

- but all wavelengths vary simultaneously (to within observational limits)
  - bad news for accretion disc : different wavelengths come from different radii ==> should vary with time lag (see exercise sheet)

In summary :

UV bump is very probably multi-temperature optically thick thermal emission from around a black hole; but standard accretion disc model does not work.