Sourdough notes (Ht?)

Use $\boxed{\text{plastic tub}}$; press on lid not clipped

$\boxed{\text{get therm.}}$ (Lakeland? Newgen formulaire?)
need 30°C

Reuse volume 300g ; prod^n 500 ; fed 975g
50g → leave for day

$\boxed{\text{Small tin}}$

Wheat leaven 440g → prod^n 480g → 1007g
160g

(1) permanent stock "mother" becomes acidic
feed & use some

(2) prod^n & holdback
constantly refreshed

Leppard 350/225
Pitta 300/200 slightly wetter
~67%

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(1) Body thru' fluid feels drag

$F_v = -K\eta v$ K = drag coeff of body
η = coeff of viscosity of fluid

sphere $K = 6\pi R$
[Stokes Law]

under gravity $ma = F_g - K\eta v$ $F_g = mg$

$\Rightarrow$ terminal velocity $v_L = \frac{F}{K\eta} = \frac{mg}{K\eta}$

but correct for buoyancy force
m' = displaced mass $v_L = \frac{(m-m')g}{K\eta}$

Air $\eta = 2\times10^{-4}$
Water $\eta = 2\times10^{-2}$
oil $\eta = 0.1$
Glycerine $\eta \approx 8$
$m^{-1}\,kg\,s^{-1}$

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(2) Fluid layer drag

$$\boxed{j_p = -\eta \frac{dv}{dx}}$$

j_p = momentum current density
– rate per unit area at which momentum is transferred across boundary

but force $= \frac{dp}{dt}$

so this is also F_v = force per unit area
[viscous stress]

Expressing as current makes analogy with diffusion

$j = -D\frac{dn}{dx}$
particle current, coeff of visc, concen^n gradient

or heat flow
$j_E = -\kappa\frac{dT}{dx}$

3) Molecular viscosity

gives $\eta = \frac{1}{3} n m \bar{v} \lambda$

$= \frac{1}{3} \rho \bar{v} \lambda$

λ = mfp $= \frac{1}{n \pi a^2}$

in robot model

$\boxed{\lambda = \frac{1}{n\sigma}}$

so expect $\eta \propto T^{1/2}$

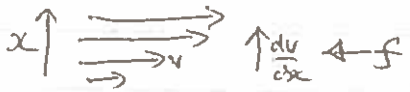
$\eta = \frac{1}{3} \frac{m^{1/2} (3kT)^{1/2}}{\sigma}$

4) Turbulent viscosity?

Does it still make sense to use η?

5) Effect on fluid

The force/unit area is perpendicular to the velocity gradient and also parallel to v



so slows down the fast layers

no net transfer of matter [check ?]

only of momentum

6) orbiting layers

the key difference is that if you slow down you can change orbit

-- but what really happens to two neighbour's layers?

Can't assume integrity of layers

think molecular

$\boxed{\text{outer} = \text{slower}}$

7) orbit reminders

$V^2 = \frac{GM}{R}$

$K = \frac{1}{2} m v^2 = \frac{GMm}{2R}$

$U = -\frac{GMm}{R}$

$\boxed{\text{Energy}}$ $K = \frac{GMm}{2R}$ $U = -\frac{GMm}{R}$

$\boxed{E = -\frac{GMm}{2R}}$

negative = bound

lower orbit, lower E

have to shed energy to descend

$\boxed{\text{AM}}$ $L = r \times p$ size $L = mrv$

$\boxed{L = \sqrt{GM}\, m R^{1/2}}$

lower orbit = lower AM

have to shed AM to descend

reminder: Torque $= r \times F$ $\frac{dL}{dt} = \tau$ cf $\frac{dp}{dt} = F$

8) Collisions

Before solving fluid, think of two bodies in same orbit

suppose ~~[illegible]~~ body A donates momentum & energy to body B.

A will move to lower orbit
B will move to higher orbit

must have both

$\Rightarrow$ a ring will spread

need to think of this in molecular terms

— particles diffuse

Also of course, AM change & E change don't match

— A gets faster on average ; K increases

— gas gets hotter

$\rightarrow$ note viscosity will then be a function of radius

9 Applying torque

Mass m orbits at R $\quad L = (GM)^{1/2} R^{1/2} m$

torque $\tau = \frac{dL}{dt}$ $\quad R = m^2 L^2 GM$

$$\frac{dL}{dR} = \frac{(GM)^{1/2} m R^{-1/2}}{2} \qquad \frac{dR}{dt} = \frac{dL}{dt} \cdot \frac{dR}{dL}$$

$$\frac{dR}{dt} = \frac{2\tau}{(GM)^{1/2} m} R^{1/2} \qquad \text{or } \frac{dL}{dt} \Big/ \frac{dL}{dR}$$

$$\frac{dm}{dt} = m \frac{dR}{dt} \qquad \dot{m} = \frac{2\tau R^{1/2}}{(GM)^{1/2} m}$$

10 Fluid cube $\quad$ Area A thickness ΔR

mass $\rho A \Delta R$

F on unit area $f = \eta \frac{dv}{dR}$ $\quad v = (GM)^{1/2} R^{-1/2}$

$\tau = fR$ $\quad \frac{dv}{dR} = \frac{1}{2} (GM)^{1/2} R^{-3/2}$

$$\dot{m} = \frac{2R^{1/2}}{(GM)^{1/2}} \cdot \frac{\eta \frac{1}{2} (GM)^{1/2} R^{-3/2} \cdot R}{\rho A \Delta R}$$

$$\dot{m} = \frac{\eta}{\rho A \Delta R} = \frac{\frac{1}{3} \rho \bar{v} \lambda}{\rho A \Delta R}$$

$A = 1$ $\quad \dot{m} \propto \frac{\bar{v} \lambda}{\Delta R}$

strain relative size change

stress force per unit area

11 Viscosity parameter

Accretion disc theorists tend to use

$\nu = \eta / \rho$ $\quad$ so with $f = -\eta \frac{dv}{dx}$

[not clear to me why!] $\quad$ and $\eta = \frac{1}{3} \rho \bar{v} \lambda$

$$f = -\frac{1}{3} \nu \rho^2 \lambda \bar{v}$$

12 Dissipation caused by viscosity

FKR p.64 for keplerian shear

$\nu \rightarrow$ viscous stress as a fⁿ of R

remembering work = force × distance

difference in stress across dR → net rate of work

part → rotation

part → dissipation

$$\Longrightarrow \quad D(R) = \frac{9}{8} \nu \Sigma \frac{GM}{R^3}$$

D = dissipated energy / unit area / sec at R

Σ = surface mass density $= \rho H$

so large dissipⁿ requires either large viscosity or large disc density

How does this compare with dissipⁿ given by an assumed $\dot{m}$?

(13) Dissipation vs $\dot{m}$

Descend by ΔR, mass m: $E = -\frac{GMm}{2R}$ $\quad \frac{dE}{dR} = -\frac{GMm}{2R^2}$

$\Rightarrow \Delta E = -\frac{GMm}{2R^2}\Delta R$ $\quad$ rate $-\frac{GM\dot{m}\Delta R}{2R^2}$

This is over annulus area $2\pi R \Delta R$

so $D(R) = \frac{GM\dot{m}}{4\pi R^3}$

cf $D(R) = \frac{9}{8}\eta H \frac{GM}{R^3}$

so viscosity produces $\dot{m} = \frac{9\pi}{2}\eta H$ kg s^{-1}

express H in units of R_s for $M_\odot$ $\quad = 2.24^{-22}\,\eta H\ M_\odot/yr^{-1}$

$$\boxed{\dot{m} = 6.62^{-11}\,\eta X M_8 \quad M_\odot yr^{-1}}$$

$X = H/R_s \quad M_8 = M/10^8 M_\odot$

impossible to produce enough $\dot{m}$/power even if disc really big ...
viscosity of air $\eta \sim 10^{-4}$ ...

(14) Hot sticky gas?

$\eta = \frac{1}{3}\frac{m^{1/2}(3kT)^{1/2}}{\sigma}$

use $\sigma_T = 6.65^{-29}$
$m_H = m_p = 1.67^{-27}$
$k = 1.38^{-23}$

$T = 10{,}000$ $\quad \eta = 1.31\times10^5$ wah?

(15) α prescription

$\nu = \alpha c_s H$ or $\alpha = \frac{\nu}{c_s H} = \frac{\eta}{c_s \rho H}$

c_s = sound speed $= \sqrt{\frac{\gamma P}{\rho}}$

ideal gas $\gamma = 5/3$ $\quad PV = NkT$ $\quad P = nkT$
$P = \frac{\rho kT}{m}$

$c_s = \sqrt{\frac{\gamma kT}{m}}$ $\quad$ cf molecular speed $\bar{v} = \sqrt{\frac{3kT}{m}}$

$\alpha = \frac{\frac{1}{3}\rho\bar{v}\lambda}{c_s\rho H}$ $\quad$ ie $\alpha \sim \lambda/H$ $\quad$ characteristic viscosity scale compared to disc thickness

$\boxed{\text{Efficient viscosity needs large scale blobs travel between collisions}}$

$\lambda \sim \alpha H$
$\eta \sim \frac{1}{3}\rho\bar{v}\alpha H$

(16) $\dot{m}$ vs α

so $\eta \sim \frac{1}{3}\rho\bar{v}\lambda \sim \frac{1}{3}\rho\bar{v}\alpha H$

$\dot{m} = 6.62^{-11}\,\frac{1}{3}\rho\bar{v}\alpha H \times M_8$ $\quad X = H/R_s$

$= \frac{6.62^{-11}}{3}\, n m_p \sqrt{\frac{3kT}{m_p}}\,\alpha \times \frac{2GM}{c^2} \times M_8$

$M = 10^8$ $\quad n = 10^{12} cm^{-3} = 10^{18} m^{-3}$

$\dot{m} = 1.7\times10^{-4}\, n_{18}\, T_{10K}^{1/2}\, X^2 M_8^2\, \alpha$

 but maybe $n \sim 10^{24}$? Anys, need $\alpha \sim 1$

(17) Radial drift speed / viscous timescale

$\dot{m}$ is of the order $M_\odot$/yr

how long to empty disc?

Disc mass is $\sim 10^3\, M_\odot$?

p82/112

spreading time

$t_{visc} = R^2/\nu \qquad V_R = \nu/R$

$$= \frac{R^2}{c_s H \alpha} \sim \frac{R^2}{c_s \lambda}$$

$$\text{or } \frac{R}{c_s} \cdot (R/H)$$

(18) Krolik timescales p. 164

$$t_{dyn} \sim \frac{1}{\Omega}$$

$$\text{thermal} \sim \frac{1}{\alpha \Omega}$$

$$t_{visc} \sim \frac{1}{\alpha} \frac{r^2}{h^2} \frac{1}{\Omega}$$

Accretion Disc reading

** FKR = the bible

Peterson ? no, fairly crude

Kembavi + Narlikar – brief, p112-121

** Krolik : ch 7

* Longair Vol 2 : Ch 16 short but clear as usual

→ 7.4 on timescales particularly important.

Alonso mini tutorial

① need to shed AM — look at E & L

② change AM with torque < local: friction lotic? / non-local: mag wind ?

$\tau = \frac{dL}{dt}$

(1) does viscosity work ?

③ standard viscous drag

$$f = -\eta \frac{dv}{dx}$$

④ molecular explanation — exchange of momentum

$$\eta = \frac{1}{3} \rho \bar{v} \lambda$$

mfp

particles = gas molecules, λ v. small

"particles" = gas eddies, λ large

[sometimes use $\nu = \eta / \rho$]

— biggest possible $\lambda \sim H$ disc thickness

⑤ Use $\nu = \alpha c_s H$ $\quad c_s \sim \bar{v}$

ie $\eta = \alpha \rho \bar{v} H$

cf $\eta = \frac{1}{3} \rho \bar{v} \lambda$ $\Big\} \; \alpha H = \frac{\lambda}{3}$

$\alpha \sim \lambda / H$

⑥ Two problems — luminosity and timescale

Keplerian $v(R)$
net torque
$\Rightarrow$ rate of doing work

FKR P69

$$D(R) = \frac{9}{8} \nu \Sigma \frac{GM}{R^3}$$

note $\Sigma = \rho H$

⑦ Compare to acc^n rate generated

$$D(R) = \frac{GM\dot{m}}{4\pi R^3} \qquad \dot{m} = \frac{9}{8} \nu \Sigma = \frac{9}{8} \eta H$$

luminosity $L = \mu \dot{m} c^2 = \frac{9}{8} \mu c^2 H^2 \rho \bar{v} \cdot \alpha$

eg $M = 10^3 M_\odot$ $\quad R_s = 2.95^4$
$H \sim 2.95^{10}$ say $\quad L_{Edd} \sim 10^{39}$

$\rho \sim 10^{21} m_p \sim 1.67^{-6}$ kg m^{-3}

$\mu = 0.1$

$v \sim c_s = \sqrt{\frac{3kT}{m_p}} \sim 1.6^4$ for $T = 10{,}000$

$\Rightarrow L = \cancel{2}1 \times 10^{35}$ still tricky if $\alpha \sim 1$
for tiny α, forget it!

⑧ What is molecular α ?

ionised gas $\rightarrow$ coulomb interactions

see Longair Vol I eq 10.4

$$t_c = 11.4^6 \frac{T^{3/2} A^{1/2}}{N Z^4 \ln \Lambda}$$

N = ptls m^{-3}

here can use $\ln \Lambda \sim 20$ $\quad Z = 1$, $A = 1$

$\lambda_c = V_s t_c$ $\quad T = 10{,}000$ $\quad N = 10^{21}$
$t_c = 5.5^{-10}$ sec $\quad \lambda = 4 \times 10^{-6}$ m

$\alpha \sim 10^{-15}$

9 Timescale

from FKR p82 $t_{visc} = R^2/\nu$

— & see the spreading graph!

[suitable revision ?]

from Kronk p164

$$t_{dyn} \sim \frac{1}{\Omega}$$

$$t_{therm} \sim \frac{1}{\alpha\Omega}$$

$$t_{visc} \sim \frac{1}{\alpha} \cdot \left(\frac{R}{H}\right)^2 \cdot \frac{1}{\Omega}$$

thick discs can change fast ?

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Outline Dec 9th
Full April 7th
Announce May 15th

5 year lifetime
long term legacy
diversity
new to RAS
leveraging

>1? yes

last time 90
1/3 non starters ---

["middle third"
not that good]

National Astronomical Research Institute of Thailand
(Public Organization)