Assume:

- orientation of S/C is known relative to inertial frame (no error)

- spin rate of body $\omega = \dot{\theta}$ is constant and known (from lightcurves)

- At time t_0 (or time of 1st image), the inertial frame, N, and chosen body frame, B, are aligned (different origins, same directions). Problem Statement

  $\theta(t_0) = \theta_0 = 0$

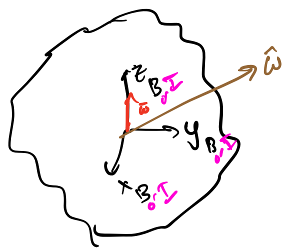
- The true pole axis is in the $\hat{z}$ direction of Inertial frame

- Initial incorrect pole axis estimate "given"

- uncertainty in pole within 1-10°

- Position of sun is known and direction of sun from body + S/C are equal

- Estimated prior shape model "given"



$$\Phi = \begin{array}{c} \begin{matrix} r & v & \alpha & \delta \end{matrix} \\ \begin{bmatrix} \cancel{3}1 & \neq 0 & & \cancel{\neq 0} \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{bmatrix} \end{array}$$

$$\vec{x} = [\vec{r} \;\; \vec{v} \;\; \alpha \;\; \delta]$$

Dynamics

$$\ddot{\vec{x}} = \frac{-\mu}{r^3}\vec{x}$$

$\mu \rightarrow \frac{m^3}{s^2}$ **NOT km**

$\vec{r} = [100^{km}, 0, 0]$

$\vec{v} = [0, 0, 0]$

OR

Kinematic Model

- we can compare models to known truth

Measurements

$$\hat{\omega}_c = T\,\hat{\omega}_I$$

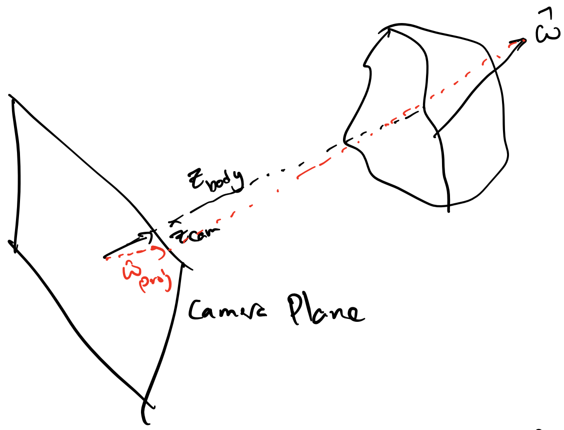
$$\hat{\omega}_c = \hat{\omega}_c + \Delta\vec{\omega} \rightarrow \begin{pmatrix} v_1 \\ v_2 \\ 0 \end{pmatrix}$$

$$\hat{\omega}_I = T^{-1}\,\hat{\omega}_c$$

$\hookrightarrow \alpha, \delta$

$- \dot{\alpha} = \dot{\delta} = 0$

$\dot{\vec{\omega}} = \vec{0}$



① We can project the assumed pole ($\hat{\omega}$) from 3D frame into the camera plane

$\hat{\omega}_C = T_I^C \hat{\omega}_I$ 3D pole in camera pole

$\hat{\omega}_{c,proj} : \hat{\omega}_c(1,2)$ 2D proj of 3D pole in camera.

Can reverse this without issue IF z_{body} is known

② We generate an image of bennu assuming this pole ($\hat{\omega}$)

Compare to pole extracted from a real image

This $\Delta\omega$ will be the MEASUREMENT

③ Correct $\hat{\omega}$ with $\Delta\omega$ and put into inertial frame

Get α, δ from $(\hat{\omega} + \Delta\omega)$

Tuesday 2:30 pm

image 1 to correct $\hat{\omega}$

$\hat{\omega}$ to predict for img 2

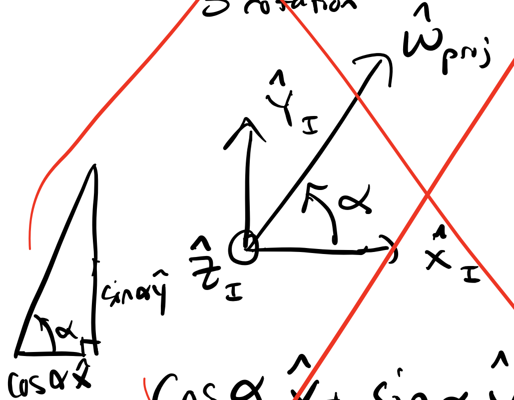
img 2 to correct new $\hat{\omega}$

3-α → RAAN

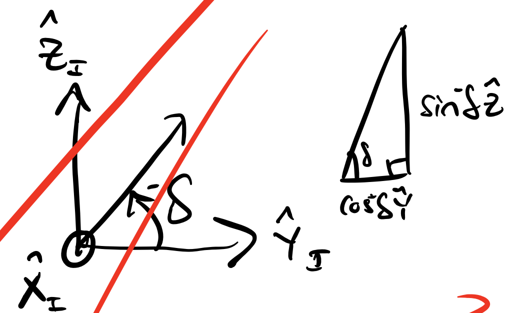
1-δ → Declination

} assumed from inertial

3 rotation



1 rotation (inclination)



$\cos\alpha\,\hat{x} + \sin\alpha\,\hat{Y}$

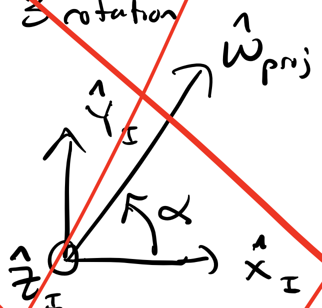
$\cos(-\delta)\,\hat{Y} + \sin(-\delta)\,\hat{z}$

$\cos\delta\,\hat{Y} - \sin\delta\,\hat{z}$

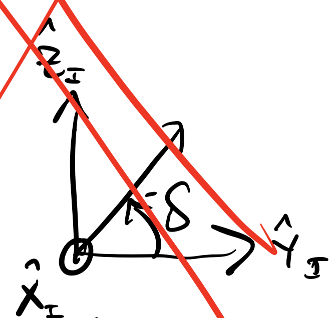
$$\vec{w} = \cos\alpha\,\hat{x} + (\sin\alpha + \cos\delta)\hat{Y} + (-\sin\delta)\hat{z}$$

$$\hat{\omega} = \frac{\vec{w}}{|\vec{w}|}$$

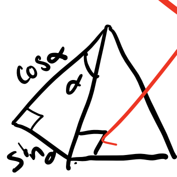
3 rotation

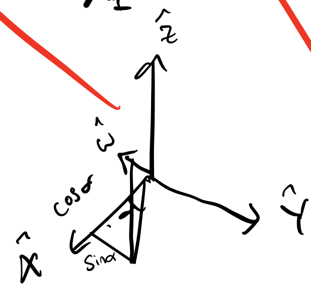


1 rotation (inclination



$\cos\alpha\,\hat{x} + \sin\alpha\,\hat{Y}$





Status Update

- Data has been developed, observed and predicted images are prepared from my PC in new run file 'run_ekf_polar'
  - observed images rescaled after being rotated so they match the scale of the predicted images (do I have pred vs. obs correct?)
- Initiated new EKF code using Dahlia's as framework.
  - still need to create ode45 function and H function
  - maskmatch.m used to find residual
  - kinda like a batch?

My unidays

Questions?

I – How to do above rotation. can I just use 3-1-3 rotation w last rotation $= 0$

$$\underset{\hookrightarrow \alpha}{3} - \overset{\hookrightarrow 90-\delta}{1} - \underset{\hookrightarrow 0^\circ}{3}$$

So $\hat{\omega} = T_{313}\,\hat{U}$?

– do I ever need this? or can I make up $\hat{\omega}_0$?

II – Random number seed in MATLAB is rng(1) sufficient?

III – A and H? Still confused

– they appear to be the same?

$$A_{n\times n} = A_{3\times 3} \overset{?}{=} H_{1\times m} = H_{3\times 3} \overset{?}{=} I_{3\times 3}$$

$\downarrow$ Seems like they are just the identity matrix to me

$f(\vec{x}, \vec{u}) =$ ✓

$h(\vec{x}, \vec{u}) =$ ✓

not same? ←

IV - How to compile time vector

- currently using angle instead
- for dynamics propagation need time element

$\dot{\vec{\omega}}$ look online and calculate

$$H = \begin{bmatrix} 0_{3\times3} & 0_{3\times3} & I_{3\times3} \end{bmatrix}$$

mc_poleerr → renders image