Parzen window

Parzen-window is a classic and performance-driven non-parameter method for estimating densities. The basic idea is to use the average density of points within a hypercube to interpret the overall density function. General speaking, if we assume x is an arbitrary point in the space, N is the total sample number, to predict the $p_N(x)$, which is the density at the location of x, we made a hypercube with length of h, and the volume is $V_N = h_N^d$, we can compute the samples that fall into the hypercube and then get the predicted density $p_N(x)$. To be more specific, the steps are as following:

1. Choose window function

In this case, we chose the Gaussian function as our kernel function, which could be interpreted as:

$$\varphi(u) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2} u^2) \tag{1}$$

And the window function could be interpreted as:

$$\varphi(\frac{x - x_i}{h_N}) = \frac{1}{\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left(\frac{x - x_i}{h_N}\right)^2\right] \tag{2}$$

We choose Gaussian function cause it could make the predict function more smooth, and only have one parameter.

2. Choose width of the window

Different windows result in different classification results, and the width of the Gaussian window function and the number of training samples are determinant factors to the classification. While the number of sample is only one, they the density function will only have one value, and while n is approaching ∞, the predicted p(x) will be close to the true value. In this experiment, we adopted one-leave-out on the training set to get choose a proper h.

3. Calculate conditional probability

$$p_n\left(x \mid w_i\right) = \frac{1}{n_i} \sum_{j=1}^{n_i} \frac{1}{V_n} \varphi\left(\frac{x - x_i}{h}\right) \tag{3}$$

4. Classify according to Bayes criterion

Classify the test sample according to Bayes criterion, and put x to the class which could make the $p_n(\omega_i \mid x)$ biggest.

$$p_n(\omega_i \mid x) = \frac{P(x \mid \omega_{\mathrm{i}}) \cdot P(\omega_i)}{\sum_{i=1}^{n} P(x \mid \omega_i) \cdot P(\omega_i)} \tag{4}$$