Bayesian classifier based on maximum-likelihood estimation

According to Bayesian decision theory, the discriminant function should be:

$$g_i(x) = \ln p\,(x|w_i) + \ln p(w_i) \tag{1}$$

We assume that data approximately satisfy Gaussian distribution. Thus, according to the multi-dimension Gaussian distribution:

$$\mathrm{p(x)} = \frac{1}{(2\pi^{\frac{d}{2}}|\Sigma|^{\frac{1}{2}})} \exp[-\frac{1}{2}(x-\mu)^t \Sigma^{-1}(x-\mu)] \tag{2}$$

We have to obtain the two parameters- mean (μ) and covariance (Σ).

The maximum-likelihood estimation of mean and covariance for multivariate Gaussian function is:

$$\hat{\mu} = \frac{1}{n}\sum_{k=1}^{n} x_k \tag{3}$$

$$\hat{\Sigma} = \frac{1}{n}\sum_{k=1}^{n}(x_k - \hat{\mu})\,(x_k - \hat{\mu})^t \tag{4}$$

From the four formulas above, finally we get the discriminant function with Gaussian distribution:

$$g_i(x) = x^t W_i x + {w_i}^t x + w_{i0} \tag{5}$$

$$W_i = -\frac{1}{2}{\mu_i}^t {\Sigma_i}^{-1} \tag{6}$$

$$w_i = {\Sigma_i}^{-1}\mu_i \tag{7}$$

$$w_{i0} = -\frac{1}{2}{\mu_i}^t {\Sigma_i}^{-1}\mu_i - \frac{1}{2}\ln|\Sigma_i| + \ln p(w_i) \tag{8}$$