Forecasting Seasonal Time Series

Decomposition

Decomposition

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Decomposition is an alternative method of analysing/predicting time series that **explicitly** states how the components of the time series interact to produce the observed time series

It is an alternative time series method when **seasonal** and **trend** components are present

The explicit model for decomposition is of the form

$$Y_t = f(S_t, T_t, C_t, R_t)$$

The **form of the function** will be determined by the time series.

Additive model

The additive model takes the form;

$$Y_t = T_t + C_t + S_t + R_t$$

T=Trend, C=Cycle, S=Seasonality, R=Random

The components are added together to derive the time series (Y_t)

Time series needs to be **linear in trend and seasonality**

The seasonal component should be one where the **fluctuations due to seasonality are constant** over all seasonal cycles

Multiplicative model

The multiplicative model takes the form;

$$Y_t = T_t * C_t * S_t * R_t$$

T=Trend, C=Cycle, S=Seasonal, R=Random

The components are multiplied together to derive the time series (Y_t)

For the seasonal component, fluctuations due to seasonality will be **constant over all seasonal cycles only in relative magnitude**

Decomposition Steps

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1

- Removing seasonality and random or short term fluctuation from data using Centered Moving Average

2

- Averaging all values of the same quarter/monthly for the years eliminates randomness and generating seasonal indices and deseasonalisation

3

- To identify the trend component (T) in the deseasonalised time series via a trend equation (usually linear)

4

- Prediction with the decomposition model is really a re-composition of the projected values of the systematic components

Multiplicative Decomposition Example

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Time	Date	Takings $000's
1	Dec-12	668531.5
2	Mar-13	621398.8
3	Jun-13	553849.2
4	Sep-13	664512.2
5	Dec-13	759602.9
6	Mar-14	691864.2
7	Jun-14	627764.8
8	Sep-14	720862.9
9	Dec-14	749901.0
10	Mar-15	749365.4
11	Jun-15	674905.5
12	Sep-15	777192.4
13	Dec-15	809597.9
14	Mar-16	801350.5
15	Jun-16	732327.1
16	Sep-16	839228.6
17	Dec-16	924637.3
18	Mar-17	902566.8
19	Jun-17	795784.5
20	Sep-17	926676.5

Decomposition Steps

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1
- Season/ Random components identified and removed

2
- Seasonal Index estimation and deseasonalisation

3
- Cycle and Trend separation

4
- *Forecasting (Recomposing)*

Centred Moving Average

MA of **appropriate length** used to smooth the time series

Length of MA depends on Periodicity of Season

Quarterly: MA4

Monthly: MA12

Centred Moving Average: when **seasonal periodicity is even**

Forecasting Moving Average Vs Smoothing Moving Average

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Forecasting

Date	Dec-12	Mar-13	Jun-13	Sep-13	Dec-13	Mar-14	Jun-14	Sep-14
Takings $000's	668532	621399	553849	664512	759603	691864	627765	720863
					627073			

Date	Dec-12	Mar-13	Jun-13	Sep-13	Dec-13	Mar-14	Jun-14	Sep-14
Takings $000's	668532	621399	553849	664512	759603	691864	627765	720863
						649841		

Smoothing for Decomposition

Date	Dec-12	Mar-13	Jun-13	Sep-13	Dec-13	Mar-14	Jun-14	Sep-14
Takings $000's	668532	621399	553849	664512	759603	691864	627765	720863
		627073						

Date	Dec-12	Mar-13	Jun-13	Sep-13	Dec-13	Mar-14	Jun-14	Sep-14
Takings $000's	668532	621399	553849	664512	759603	691864	627765	720863
			649841					

Smoothing for Decomp MA Vs Forecasting MA

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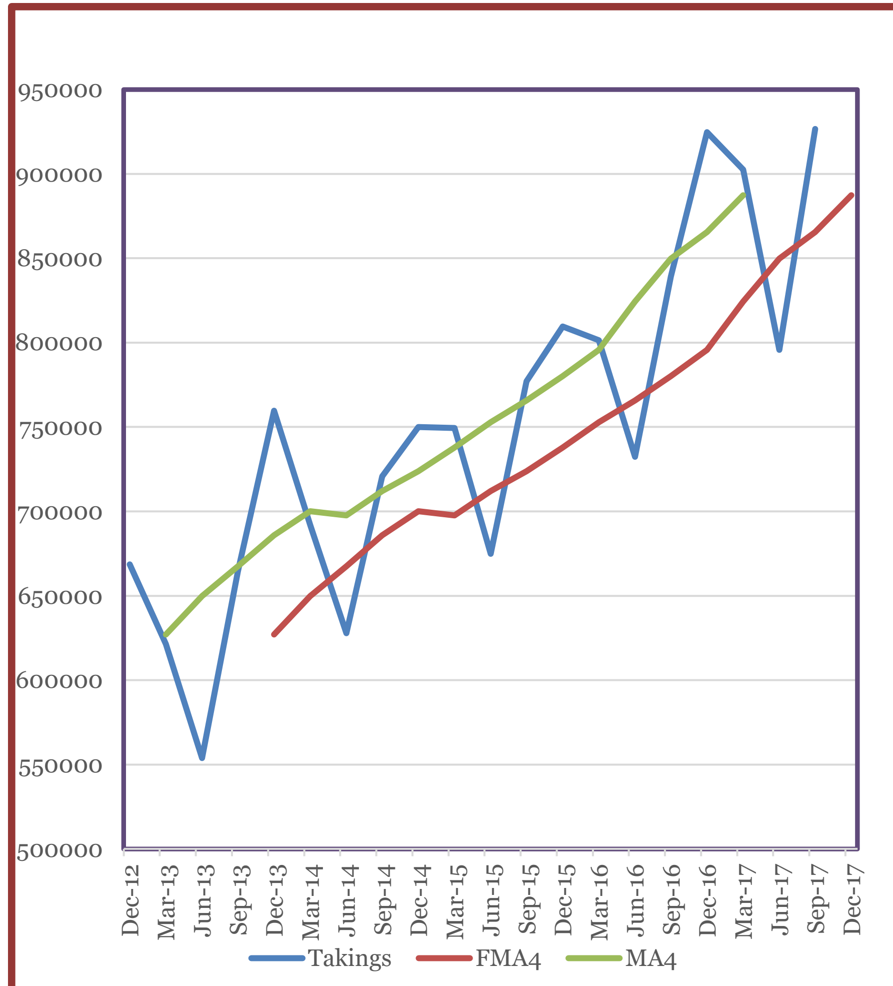
Date	Takings	FMA4	MA4
Dec-12	668532		
Mar-13	621399		627073
Jun-13	553849		649841
Sep-13	664512		667457
Dec-13	759603	627073	685936
Mar-14	691864	649841	700024
Jun-14	627765	667457	697598
Sep-14	720863	685936	711974
Dec-14	749901	700024	723759
Mar-15	749365	697598	737841
Jun-15	674906	711974	752765
Sep-15	777192	723759	765762
Dec-15	809598	737841	780117
Mar-16	801351	752765	795626
Jun-16	732327	765762	824386
Sep-16	839229	780117	849690
Dec-16	924637	795626	865554
Mar-17	902567	824386	887416
Jun-17	795785	849690	
Sep-17	926677	865554	
Dec-17		887416	



Note how FMA always understates forecast when trending

Step 1: Removing Seasonality & Random Variation

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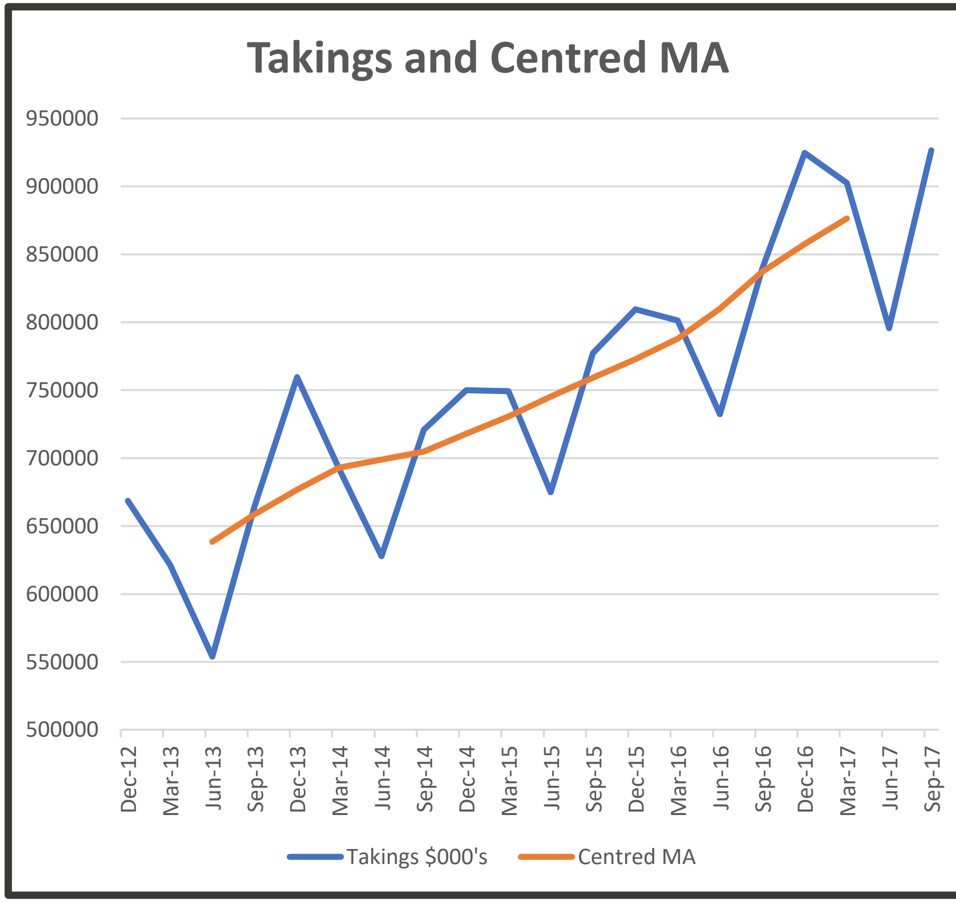
Time	Date	Takings $000's	MA4	Centred MA
1	Dec-12	668531.5		
2	Mar-13	621398.8	627073	
3	Jun-13	553849.2	649841	638457
4	Sep-13	664512.2	667457	658649
5	Dec-13	759602.9	685936	676697
6	Mar-14	691864.2	700024	692980
7	Jun-14	627764.8	697598	698811
8	Sep-14	720862.9	711974	704786
9	Dec-14	749901	723759	717866
10	Mar-15	749365.4	737841	730800
11	Jun-15	674905.5	752765	745303
12	Sep-15	777192.4	765762	759263
13	Dec-15	809597.9	780117	772939
14	Mar-16	801350.5	795626	787872
15	Jun-16	732327.1	824386	810006
16	Sep-16	839228.6	849690	837038
17	Dec-16	924637.3	865554	857622
18	Mar-17	902566.8	887416	876485
19	Jun-17	795784.5		
20	Sep-17	926676.5		



MA located in centre of period, not end: Used to smooth the time series, not to forecast

Step 2: Seasonal Index Estimation & Deseasonalisation

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1. Season/ Random components identified and removed
2. Seasonal Index estimation and deseasonalisation
3. Cycle and Trend separation
4. *Forecasting (Recomposing)*

Comparison of **original time series (Y)** with the **constructed time series (CMA = Centred Moving Average)** to evaluate **seasonal and random component**

Seasonal Relatives:

Additive: (Y-CMA)

Multiplicative: (Y/CMA)

Step 2: Seasonal Index Estimation & Deseasonalisation

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Time	Date	Takings $000's	MA4	Centred MA	Seasonal Relatives
1	Dec-12	668532			
2	Mar-13	621399	627073		
3	Jun-13	553849	649841	638457	0.867
4	Sep-13	664512	667457	658649	1.009
5	Dec-13	759603	685936	676697	1.123
6	Mar-14	691864	700024	692980	0.998
7	Jun-14	627765	697598	698811	0.898
8	Sep-14	720863	711974	704786	1.023
9	Dec-14	749901	723759	717866	1.045
10	Mar-15	749365	737841	730800	1.025
11	Jun-15	674906	752765	745303	0.906
12	Sep-15	777192	765762	759263	1.024
13	Dec-15	809598	780117	772939	1.047
14	Mar-16	801351	795626	787872	1.017
15	Jun-16	732327	824386	810006	0.904
16	Sep-16	839229	849690	837038	1.003
17	Dec-16	924637	865554	857622	1.078
18	Mar-17	902567	887416	876485	1.030
19	Jun-17	795785			
20	Sep-17	926677			

Seasonal Relatives:

= (Y/CMA)

= 553849/638456

= 0.867

If **Seasonal Relative >1** then seasonal relative is greater than yearly average

If **SR<1** then SR is less than yearly average.

Step 2: Seasonal Index Estimation & Deseasonalisation

1
- Season/ Random components identified and removed

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4
- *Forecasting (Recomposing)*

Seasonal Indexes estimated by:

Average of Seasonal Relatives

Sum of seasonal relatives should equal periodicity

ADJUSTMENT MAY BE NEEDED TO COVER ROUNDING ERRORS & DATA.

Step 2: Seasonal Index Estimation & Deseasonalisation

Time	Date	Takings $000's	MA4	Centred MA	Seasonal Relatives	Seasonal Index	Adjusted Seas Index
1	Dec-12	668532					
2	Mar-13	621399	627073			1.0177	1.0179
3	Jun-13	553849	649841	638457	0.867	0.8939	0.8940
4	Sep-13	664512	667457	658649	1.009	1.0145	1.0147
5	Dec-13	759603	685936	676697	1.123	1.0732	1.0734
6	Mar-14	691864	700024	692980	0.998		
7	Jun-14	627765	697598	698811	0.898		
8	Sep-14	720863	711974	704786	1.023		
9	Dec-14	749901	723759	717866	1.045		
10	Mar-15	749365	737841	730800	1.025		
11	Jun-15	674906	752765	745303	0.906		
12	Sep-15	777192	765762	759263	1.024		
13	Dec-15	809598	780117	772939	1.047		
14	Mar-16	801351	795626	787872	1.017		
15	Jun-16	732327	824386	810006	0.904		
16	Sep-16	839229	849690	837038	1.003		
17	Dec-16	924637	865554	857622	1.078		
18	Mar-17	902567	887416	876485	1.030		
19	Jun-17	795785					
20	Sep-17	926677					
21	**Dec-17**					**3.99919**	**4.00**

Seasonal Index (SI) derived as **mean** of relevant seasonal relatives

Adjusted SI is SI rescaled so that sum (ASI) = periodicity

Adjusted Seasonal Index

= (Seasonal Index / Sum of Indices) x Periodicity

e.g.
= (1.0732/3.99919) x 4 = 1.0734 for December

Note: this is only necessary if the sum of the indices is significantly different from the periodicity. Hence this is not done in the video.

Need for Adjusted Seasonal Relatives

	March	June	September	December	
2013		0.867	1.009	1.123	
2014	0.998	0.898	1.023	1.045	
2015	1.025	0.906	1.024	1.047	
2016	1.017	0.904	1.003	1.078	
2017	1.030				
Mean	***1.018***	***0.894***	***1.014***	***1.073***	**3.9992**

SR's should **sum to periodicity** (n_seasons)

Number of observations & **rounding error** may change calculation.

Step 2: Seasonal Index Estimation & Deseasonalisation

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1
- Season/ Random components identified and removed

2
- Seasonal Index estimation and deseasonalisation

3
- Cycle and Trend separation

4
- *Forecasting (Recomposing)*

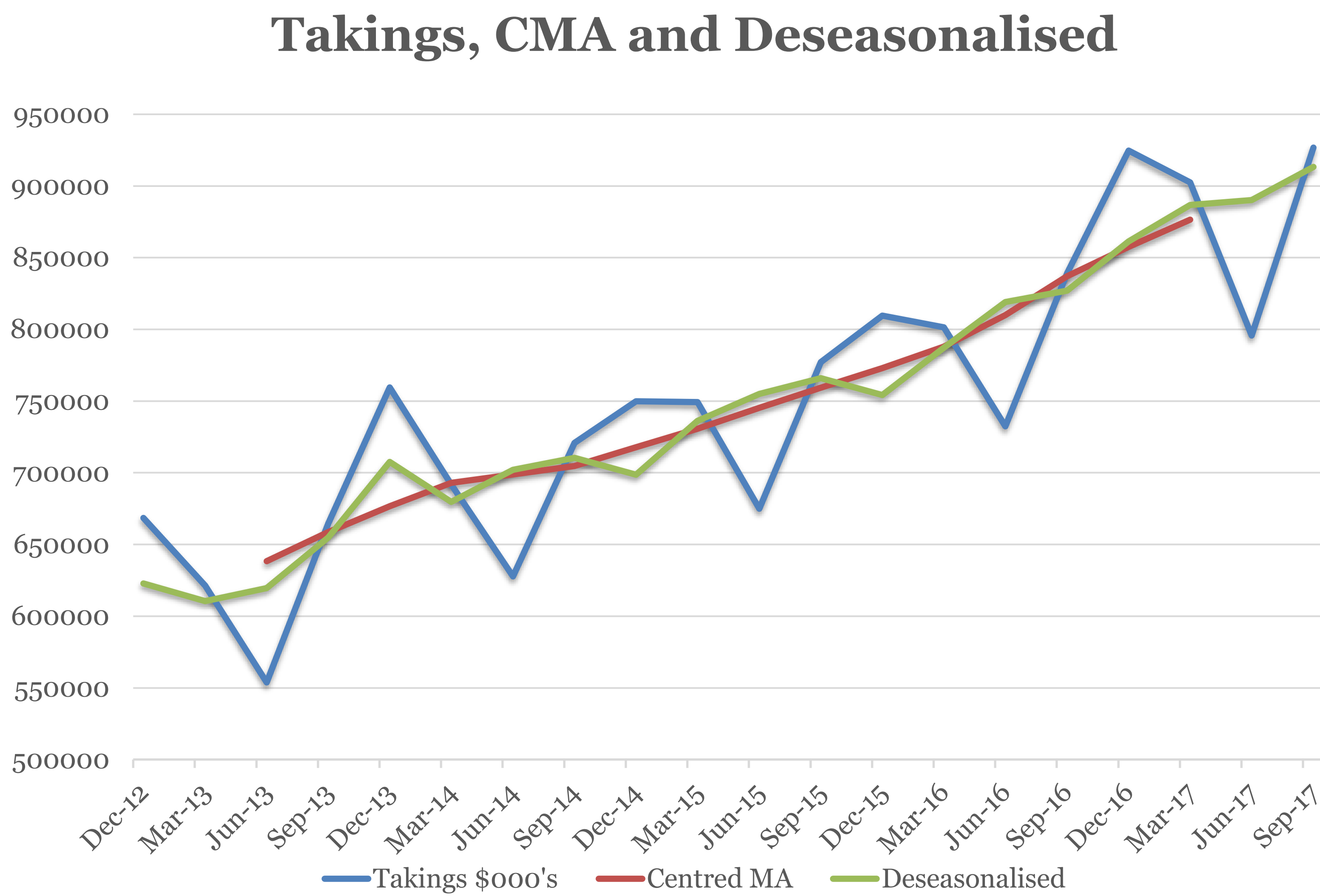
Deseasonalised data estimated by **ratio of original (actual) observation (Y_t)** and **adjusted seasonal index (SI)**

Deseasonalised data $= Y_t/SI$

Deseasonalised = Y_t/SI

Date	Takings $000's	MA4	Centred MA	Seasonal Relatives	Seasonal Index	Adjusted Seas Index	Deseasonalised
Dec-12	668532						622820
Mar-13	621399	627073			1.0177	1.0179	610489
Jun-13	553849	649841	638457	0.867	0.8939	0.8940	619612
Sep-13	664512	667457	658649	1.009	1.0145	1.0147	654891
Dec-13	759603	685936	676697	1.123	1.0732	1.0734	707665
Mar-14	691864	700024	692980	0.998			679718
Jun-14	627765	697598	698811	0.898			702162
Sep-14	720863	711974	704786	1.023			710426
Dec-14	749901	723759	717866	1.045			698626
Mar-15	749365	737841	730800	1.025			736209
Jun-15	674906	752765	745303	0.906			754890
Sep-15	777192	765762	759263	1.024			765940
Dec-15	809598	780117	772939	1.047			754241
Mar-16	801351	795626	787872	1.017			787282
Jun-16	732327	824386	810006	0.904			819116
Sep-16	839229	849690	837038	1.003			827078
Dec-16	924637	865554	857622	1.078			861415
Mar-17	902567	887416	876485	1.030			886721
Jun-17	795785						890094
Sep-17	926677						913260

Actual, Centred MA & Deseasonalised



Step 3: Cycle & Trend separation

1
- Season/ Random components identified and removed

2
- Seasonal Index estimation and deseasonalisation

3
- Cycle and Trend separation

4
- *Forecasting (Recomposing)*

Sometimes useful to separate the **trend** from **cycle**

Trend may be linear or some other form

Linear function:

$T_t = a + b^* t$

Where t is a time index

Regression of Deasonalised Data against t

Generate a trend line equation

$$\widehat{Yd}_t = a + bt$$

$$\widehat{Yd}_t = 587{,}764 + 15{,}464 * t$$

Inserting variable t for time

Time	Date	Takings $000's	MA4	Centred MA	Seasonal Relatives	Adjusted SI	Deseason'd	Time Trend
1	Dec-12	668532					622820	603228
2	Mar-13	621399	627073			1.0179	610489	618692
3	Jun-13	553849	649841	638457	0.867	0.894	619612	634156
4	Sep-13	664512	667457	658649	1.009	1.0147	654891	649620
5	Dec-13	759603	685936	676697	1.123	1.0734	707665	665084
6	Mar-14	691864	700024	692980	0.998		679718	680548
7	Jun-14	627765	697598	698811	0.898		702162	696012
8	Sep-14	720863	711974	704786	1.023		710426	711476
9	Dec-14	749901	723759	717866	1.045		698626	726940
10	Mar-15	749365	737841	730800	1.025		736209	742404
11	Jun-15	674906	752765	745303	0.906		754890	757868
12	Sep-15	777192	765762	759263	1.024		765940	773332
13	Dec-15	809598	780117	772939	1.047		754241	788796
14	Mar-16	801351	795626	787872	1.017		787282	804260
15	Jun-16	732327	824386	810006	0.904		819116	819724
16	Sep-16	839229	849690	837038	1.003		827078	835188
17	Dec-16	924637	865554	857622	1.078		861415	850652
18	Mar-17	902567	887416	876485	1.03		886721	866116
19	Jun-17	795785					890094	881580
20	Sep-17	926677					913260	897044
21	Dec-17							912508
22	Mar-18							927972
23	Jun-18							943436
24	Sep-18							958900

$$\widehat{Yd}_t = a + bt$$

$$\widehat{Yd}_t = 587{,}764 + 15{,}464 * t$$

Substitite t = 1, 2, 3,….24 into equation to obtain Time Trend values

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Inserting variable t for time

Time	Date	Takings $000's	MA4	Centred MA	Seasonal Relatives	Adjusted SI	Deseason'd	Time Trend
1	Dec-12	668532					622820	603228
2	Mar-13	621399	627073			1.0179	610489	618692
3	Jun-13	553849	649841	638457	0.867	0.894	619612	634156
4	Sep-13	664512	667457	658649	1.009	1.0147	654891	649620
5	Dec-13	759603	685936	676697	1.123	1.0734	707665	665084
6	Mar-14	691864	700024	692980	0.998		679718	680548
7	Jun-14	627765	697598	698811	0.898		702162	696012
8	Sep-14	720863	711974	704786	1.023		710426	711476
9	Dec-14	749901	723759	717866	1.045		698626	726940
10	Mar-15	749365	737841	730800	1.025		736209	742404
11	Jun-15	674906	752765	745303	0.906		754890	757868
12	Sep-15	777192	765762	759263	1.024		765940	773332
13	Dec-15	809598	780117	772939	1.047		754241	788796
14	Mar-16	801351	795626	787872	1.017		787282	804260
15	Jun-16	732327	824386	810006	0.904		819116	819724
16	Sep-16	839229	849690	837038	1.003		827078	835188
17	Dec-16	924637	865554	857622	1.078		861415	850652
18	Mar-17	902567	887416	876485	1.03		886721	866116
19	Jun-17	795785					890094	881580
20	Sep-17	926677					913260	897044
21	Dec-17						912508	912508
22	Mar-18						927972	927972
23	Jun-18						943436	943436
24	Sep-18						958900	958900

Time Trend values for periods 21-24 are forecasts for Deseasonalised data from Dec 17 to Sep-18

Step 4: Forecasting

1
- Season/ Random components identified and removed

2
- Seasonal Index estimation and deseasonalisation

3
- Cycle and Trend separation

4
- *Forecasting (Recomposing)*

Prediction with decomposition model is really a **re-composition of the projected values of the systematic components**

Original equation used to generate predictions

$$Y_f = T_f * C_f * S_f$$

Trend extrapolated from **trend equation**

Seasonality with **seasonal index**

Step 4: Forecasting

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1
- Season/ Random components identified and removed

2
- Seasonal Index estimation and deseasonalisation

3
- Cycle and Trend separation

4
- *Forecasting (Recomposing)*

$$Y_f = T_f * C_f * S_f$$

Trend extrapolated from **trend equation**

Seasonality with **seasonal index**

Cycle is **subjective or external prediction**

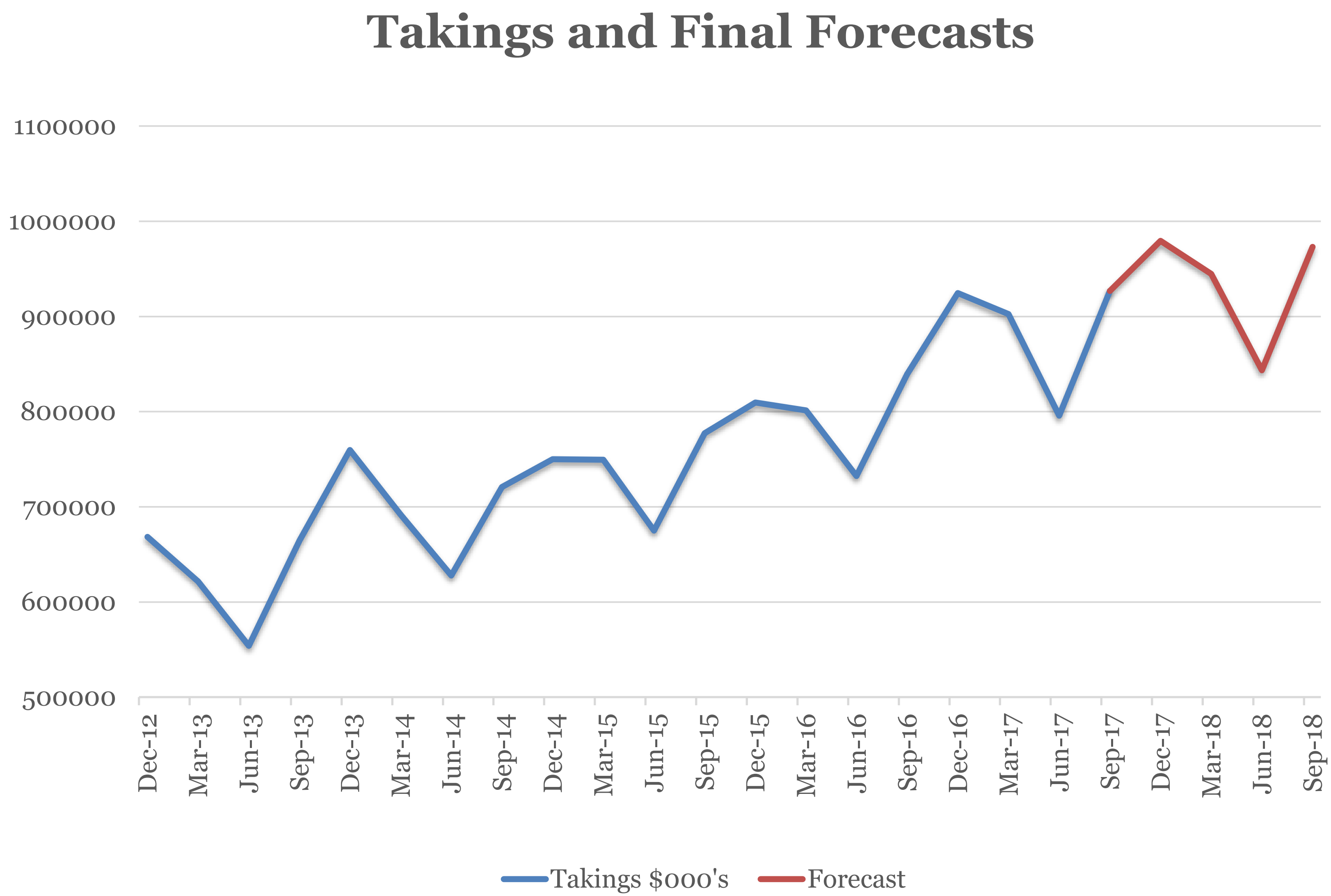
Forecast

Time	Date	Takings $000's	MA4	Centred MA	Seasonal Relatives	Seasonal Index	Adjusted SI	Deseason'd	Forecast
1	Dec-12	668532						622820	
2	Mar-13	621399	627073			1.0177	1.0179	610489	
3	Jun-13	553849	649841	638457	0.867	0.8939	0.8940	619612	
4	Sep-13	664512	667457	658649	1.009	1.0145	1.0147	654891	
5	Dec-13	759603	685936	676697	1.123	1.0732	1.0734	707665	
6	Mar-14	691864	700024	692980	0.998			679718	
7	Jun-14	627765	697598	698811	0.898			702162	
8	Sep-14	720863	711974	704786	1.023			710426	
9	Dec-14	749901	723759	717866	1.045			698626	
10	Mar-15	749365	737841	730800	1.025			736209	
11	Jun-15	674906	752765	745303	0.906			754890	
12	Sep-15	777192	765762	759263	1.024			765940	
13	Dec-15	809598	780117	772939	1.047			754241	
14	Mar-16	801351	795626	787872	1.017			787282	
15	Jun-16	732327	824386	810006	0.904			819116	
16	Sep-16	839229	849690	837038	1.003			827078	
17	Dec-16	924637	865554	857622	1.078			861415	
18	Mar-17	902567	887416	876485	1.030			886721	
19	Jun-17	795785						890094	
20	Sep-17	926677						913260	
21	Dec-17							**912502**	979474
22	Mar-18							**927966**	944549
23	Jun-18							**943429**	843469
24	Sep-18							**958893**	972980

Actual Forecasts = Trend Forecasts * ASI

Trend Forecasts from Trend Equation

Forecasts



Takings and Final Forecasts
1100000
1000000
900000
800000
700000
600000
500000
Dec-12
Mar-13
Jun-13
Sep-13
Dec-13
Mar-14
Jun-14
Sep-14
Dec-14
Mar-15
Jun-15
Sep-15
Dec-15
Mar-16
Jun-16
Sep-16
Dec-16
Mar-17
Jun-17
Sep-17
Dec-17
Mar-18
Jun-18
Sep-18
Takings $000's
Forecast

Additive Decomposition

If the data **(Y)** has additive seasonality, you decompose in a similar but not identical way to the multiplicative model

First steps are identical (generate suitable MA and centred MA)

Seasonal Relatives are then determined by **Y – CMA** (not Y/CMA)

Seasonal Indexes (SI) will be the average of similar season Seasonal Relatives (**adjust Seasonal Indexes so they add to "P" the seasonal period** >> P = 4 for quarterly data, P =12 for monthly data)

Deseasonalise original data by using **Y – SI** (not Y/SI)

To forecast, extrapolate Deseasonalised (D) data (using time trend) and then adjust for seasonality by using **D + relevant SI** (not D*SI)

Step 2: Seasonal Index Estimation & Deseasonalisation (Additive)

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Date	Takings $000 's	MA4	Centred MA	Seas Rels	Seas Index
Dec-2002	668531.5				
Mar-2003	621398.8	627072.9			14252.6
Jun-2003	553849.2	649840.8	638456.9	-84607.7	-75932.6
Sep-2003	664512.2	667457.1	658649.0	5863.3	10515.0
Dec-2003	759602.9	685936.0	676696.6	82906.3	54653.8
Mar-2004	691864.2	700023.7	692979.9	-1115.7	
Jun-2004	627764.8	697598.2	698811.0	-71046.2	
Sep-2004	720862.9	711973.5	704785.9	16077.0	
Dec-2004	749901.0	723758.7	717866.1	32034.9	
Mar-2005	749365.4	737841.1	730799.9	18565.5	
Jun-2005	674905.5	752765.3	745303.2	-70397.7	
Sep-2005	777192.4	765761.6	759263.4	17929.0	
Dec-2005	809597.9	780117.0	772939.3	36658.6	
Mar-2006	801350.5	795626.0	787871.5	13479.0	
Jun-2006	732327.1	824385.9	810006.0	-77678.9	
Sep-2006	839228.6	849690.0	837037.9	2190.7	
Dec-2006	924637.3	865554.3	857622.1	67015.2	
Mar-2007	902566.8	887416.3	876485.3	26081.5	Sum
Jun-2007	795784.5				3488.738
					872.184375

Seasonal Relatives:

= (Y- CMA)

= 553849 - 638456

= -84607.7

If **Seasonal Relative >0** then seasonal relative is greater than yearly average

If **SR<0** then SR is less than yearly average

Average of Seasonal Relatives used to create **Seasonal Indexes**

Additive Model

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Time	Date	Takings $000's	MA4	Centred MA	Seas Rels	Seas Index	Adjusted Seas Index	Deseasonal
1	Dec-12	668531.5						614749.9
2	Mar-13	621398.8	627072.9			14252.6	13380.4	608018.4
3	Jun-13	553849.2	649840.8	638456.9	-84607.7	-75932.6	-76804.8	630654
4	Sep-13	664512.2	667457.1	658649	5863.3	10515	9642.8	654869.4
5	Dec-13	759602.9	685936	676696.6	82906.3	54653.8	53781.6	705821.3
6	Mar-14	691864.2	700023.7	692979.9	-1115.7			678483.8
7	Jun-14	627764.8	697598.2	698811	-71046.2			704569.6
8	Sep-14	720862.9	711973.5	704785.9	16077			711220.1
9	Dec-14	749901	723758.7	717866.1	32034.9			696119.4
10	Mar-15	749365.4	737841.1	730799.9	18565.5			735985
11	Jun-15	674905.5	752765.3	745303.2	-70397.7			751710.3
12	Sep-15	777192.4	765761.6	759263.4	17929			767549.6
13	Dec-15	809597.9	780117	772939.3	36658.6			755816.3
14	Mar-16	801350.5	795626	787871.5	13479			787970.1
15	Jun-16	732327.1	824385.9	810006	-77678.9			809131.9
16	Sep-16	839228.6	849690	837037.9	2190.7			829585.8
17	Dec-16	924637.3	865554.3	857622.1	67015.2			870855.7
18	Mar-17	902566.8	887416.3	876485.3	26081.5	**Sum**	**Sum**	889186.4
19	Jun-17	795784.5				**3488.738**	**0**	872589.3
						872.184375		

SI adjusted so Sum SI = 0: (Take Sum SI and divide by 4. Subtract this amount from each SI)

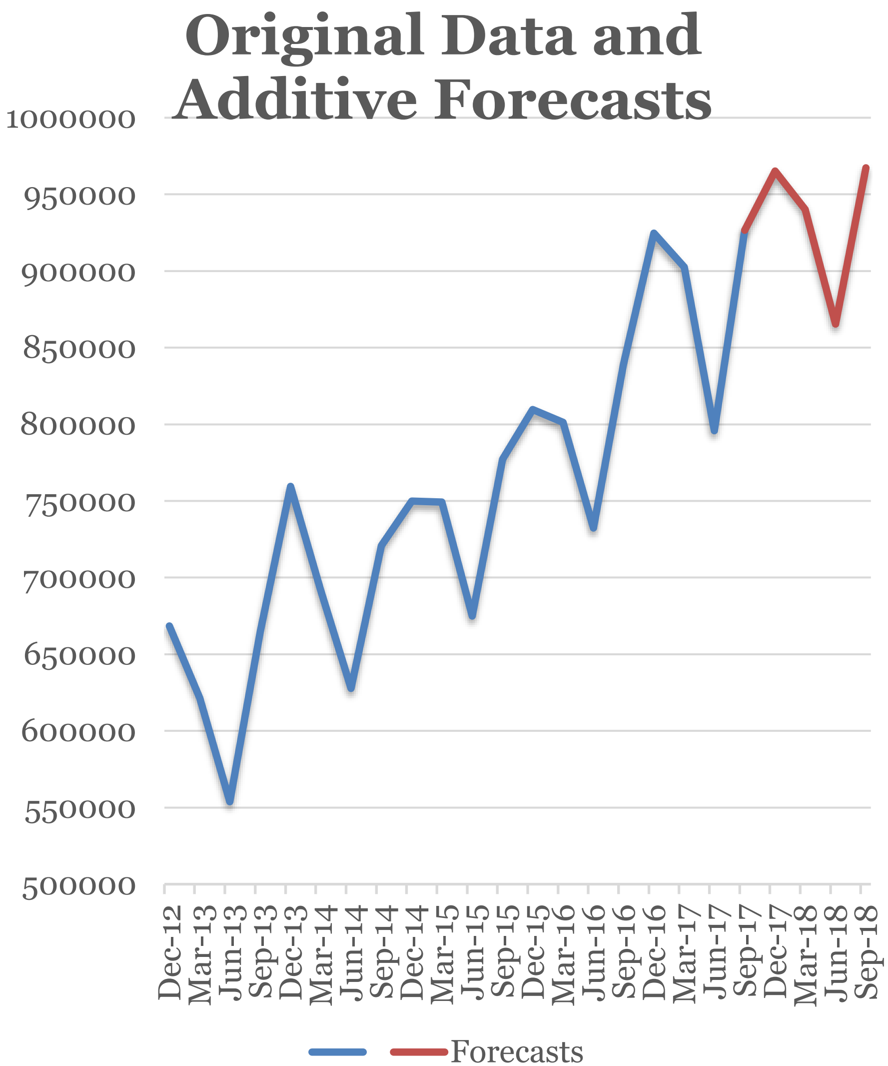
Decomposition-Additive

Time	Date	T, C, S, R Takings $000's	T, C Centred MA	S Adjusted Seas Index	T, C ,R Deseasonal	Tf, Sf Forecasts
1	Dec-2002	668531.5			614749.9313	
2	Mar-2003	621398.8		13380.40625	608018.3938	
3	Jun-2003	553849.2	638456.85	-76804.77188	630653.9719	
4	Sep-2003	664512.2	658648.95	9642.796875	654869.4031	
5	Dec-2003	759602.9	676696.575	53781.56875	705821.3313	
6	Mar-2004	691864.2	692979.863		678483.7938	
7	Jun-2004	627764.8	698810.963		704569.5719	
8	Sep-2004	720862.9	704785.875		711220.1031	
9	Dec-2004	749901	717866.113		696119.4313	
10	Mar-2005	749365.4	730799.888		735984.9938	
11	Jun-2005	674905.5	745303.188		751710.2719	
12	Sep-2005	777192.4	759263.438		767549.6031	
13	Dec-2005	809597.9	772939.275		755816.3313	
14	Mar-2006	801350.5	787871.5		787970.0938	
15	Jun-2006	732327.1	810005.95		809131.8719	
16	Sep-2006	839228.6	837037.913		829585.8031	
17	Dec-2006	924637.3	857622.125		870855.7313	
18	Mar-2007	902566.8	876485.288		889186.3938	
19	Jun-2007	795784.5			872589.2719	
20	Sep-2007	926676.5			917033.7031	
21	**Dec-2007**				**911445.5869**	**965227.2**
22	**Mar-2008**				**926859.8332**	**940240.2**
23	**Jun-2008**				**942274.0796**	**865469.3**

Actual Forecasts = Trend + AS I

Trend Forecasts from Trend Equation

Additive Model Forecasts



The forecasts look reasonable

Forecasts generated by using trend projection of Deseas. data and Seas. Indexes $(T_f + S_f)$

Cycle not included in forecasts **(assume neutral cycle, C=0)**

Can include Cycle by estimating Cycle index for forecast periods (C_f) and adding C_f to forecasts