



MACQUARIE
University

Forecasting Seasonal Time Series

Winters Exponential Smoothing



Seasonality

Previous smoothing models were appropriate for time series that were horizontal or had trend but not appropriate for time series with seasonal components

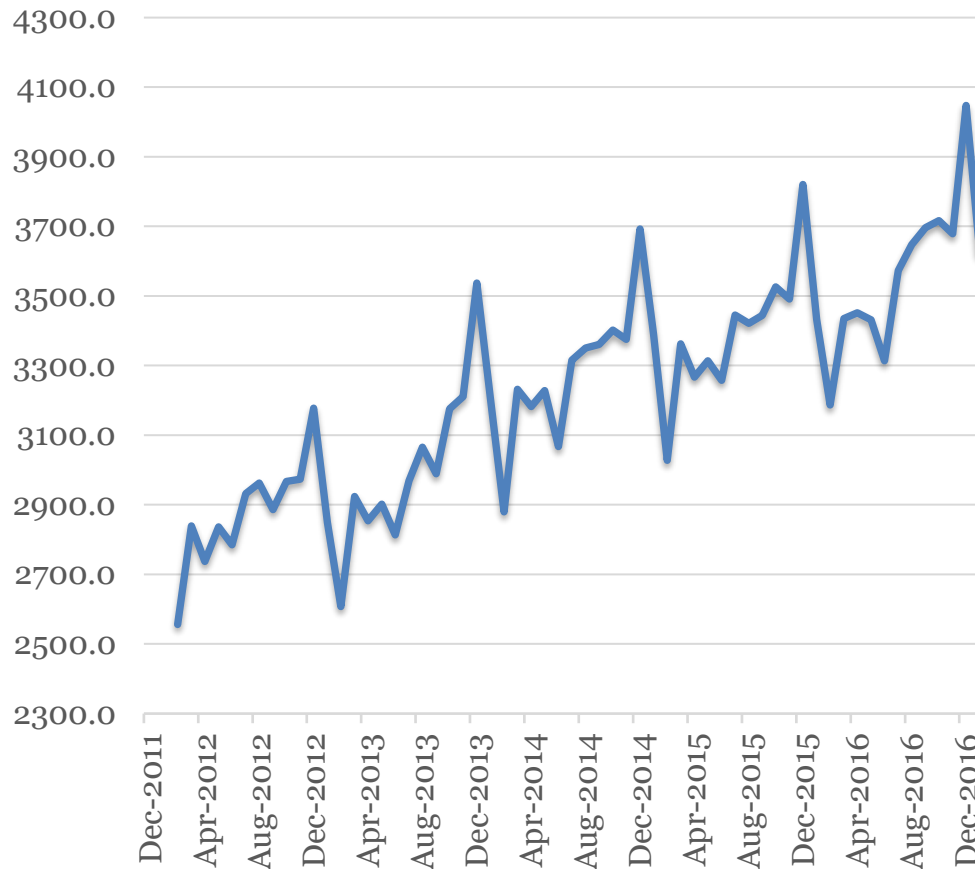
Time series may exhibit a seasonal component due to **weather, holiday periods, weekends** etc.

Seasonal components typically lead to **systematic fluctuations** of the level of the time series

The fluctuation pattern is typically **repeated for every seasonal cycle**

Seasonal Data Example

Cafe+Rest+TakeAway - Australia (\$m)



Monthly Café, Restaurant and Takeaway Sales (\$m) for Aust. Feb 12 – Jan 17

Seasonal fluctuation spiking in **December** each year

Due to **weather, holidays, Christmas**

Also **trend** and **random** component

Trend due to **inflation, population, market size**

Broad Types of Seasonality

Seasonality can be classified into two broad categories;
Additive and Multiplicative

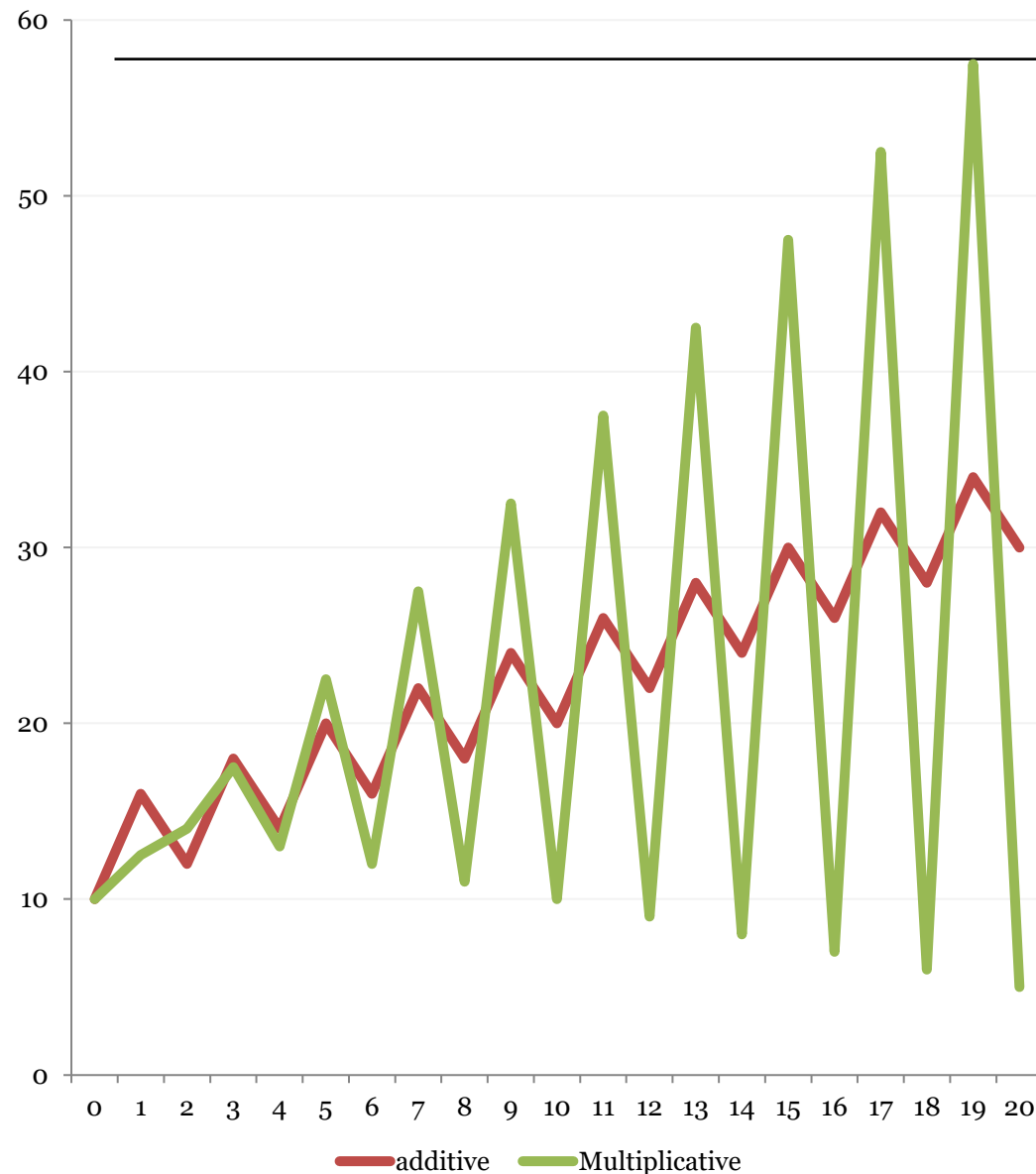
Additive is when seasonal fluctuations of the time series can be modelled by addition of a defined seasonal component

Multiplicative is when seasonal fluctuation of time series can be modelled by multiplication of a defined seasonal component

In Additive models, seasonal component size is absolute

In Multiplicative models, seasonal component is relative to the level of the time series

Additive Vs Multiplicative Seasonal Variation



Additive → Seasonal changes are fixed over the time series

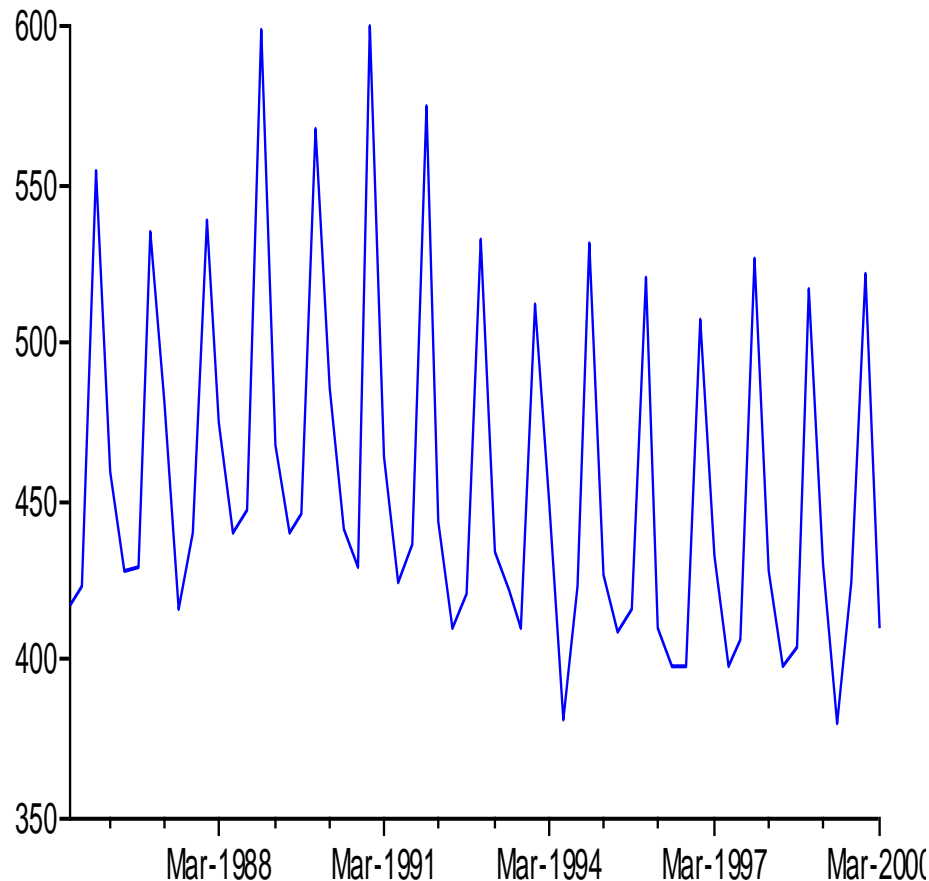
The magnitude of seasonal fluctuation does not vary with the level of the series)

Multiplicative → Seasonal changes are a fixed percentage of the time series

The magnitude of seasonal fluctuation varies with the level of the time series

Additive Seasonality

Original: Production: Beer (incl ale & stout, excl beer < 1.15% alcohol)
Megalitres



Quarterly Beer Production Aust.
(megalitres)

Data appears seasonal with
some trend (downward) and
possibly cycle

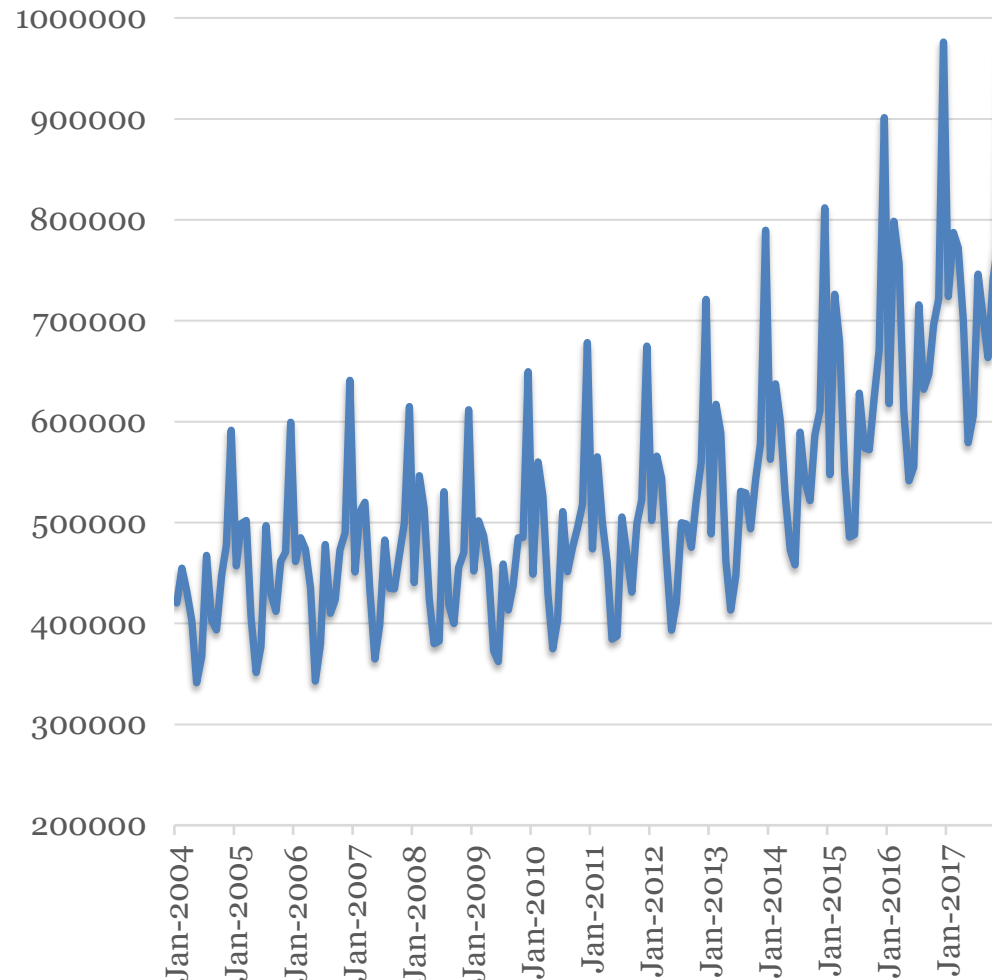
Due to **Christmas**, **weather**,
consumer taste changes

Seasonal fluctuation appears
constant and not dependent
on level of time series

Peaks & troughs **contained**
within a parallel band

Multiplicative Seasonality

Tourists



Monthly visitor arrivals Australia-number

Data appears seasonal with trend (upward) and cycle

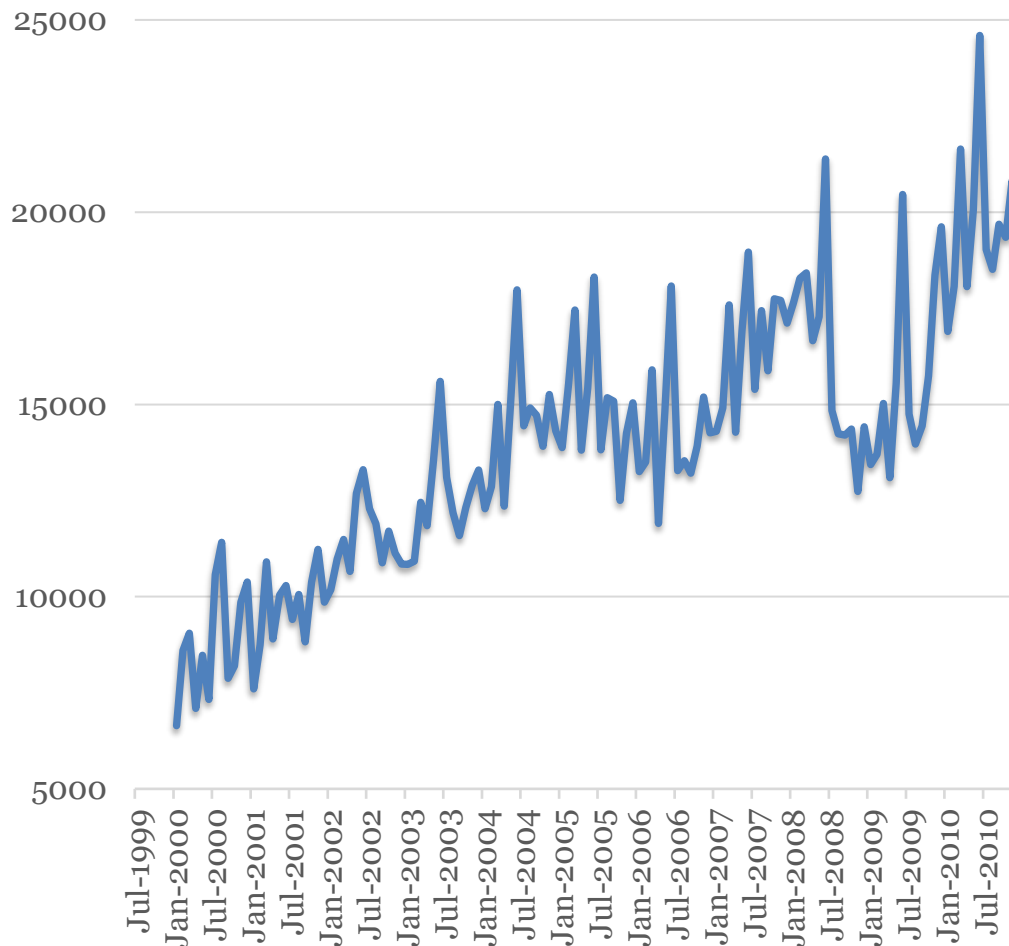
Due to: **Christmas**, weather, **globalisation**, tourism marketing, **Olympics**

Seasonal fluctuation appears relative and dependent on level of time series (but constant %)

Peaks, troughs not contained within a parallel band

Identifying Seasonality

SUV



It should be reasonably clear from a **time series graph** if seasonality is present

However it isn't always obvious eg. monthly SUV Sales (Jan 00 – Dec - 10)

Check the **ACF** and **PACF** for evidence of seasonality (**spikes** at **seasonal** values)

The time series may have to be **detrended** first for the seasonal spikes to appear clearly on **ACF** and **PACF**

Seasonal Models

If the time series has **seasonal components**, none of the previous models studied so far will be adequate

We will need to include a seasonal component in our models or adjust for seasonality when forecasting

An **extension of** Holt's method can accommodate seasonal effects

This model is the Winters Exponential Smoothing model

WES includes a **seasonality equation** and is a 3-parameter model with smoothing constants for **level: α** (alpha), **trend: β** (beta) and one for **seasonality: γ** (gamma)

WES (cont)

In both models, the first equation adjusts the actual value of the time series by a seasonal estimate

The second equation is a trend equation (as per Holt's)

The third equation provides an **updated** estimate of the seasonal estimate at each time period

The forecast equation allows for prediction “**m**” **periods** into the future.

The first part of the equation is a trend projection “m” periods ahead. The result is adjusted by an estimate of seasonality for that specific season (added or multiplied)

More on the WES Equations

The values of α, β, γ are all theoretically between 0 and 1 inclusive.

Choosing the values of the smoothing parameters is not easy. As a default, low values of α, β, γ (**0.2, 0.2, 0.2**) are used as a preliminary estimate.

The model also needs initialization values for L_1, T_1 and $S_1 - S_p$. Can use $S_1 - S_p = \mathbf{0}$ (**for Additive**) or **1** (**for Multiplicative**)

Changes to the smoothing parameters may **reduce error levels** and **improve accuracy** (usually based on an error criterion like MSE)

As with HES and SES, **SOLVER** can be used to derive the **“optimum” combination** of the smoothing parameters