MACQUARIE University

Forecasting Trend Time Series

Incorporating Steps and Trends

Trend Fitting

Where the time series has a trend, a **trend fitting and extrapolation** approach may be used for prediction

Trend fitting is a method where the time series is linked to **some function of a time index**

$$Y_t = f\,(\text{time})$$

A **linear trend** equation is typically assumed although it depends on the trend observed

Prediction is based on extrapolation by **substitution of the appropriate value for the time index**

Trend Fitting (cont)

Assuming a **linear** trend the equation is

$$Y_t = a + b * t$$

where t = a time index and α and β are constants

The values of α and β typically **estimated by regression** of the time series (Y) against time index (t)

EXCEL has **numerous alternative ways of estimating the above equation** and/or trend fitting/extrapolation including the **regression routine**

Holts Exponential Smoothing (HES)

The two general methods already studied (MA, SES) are useful when the time series is **predominantly horizontal** but will **not be good predictors** when the **time series has other systematic components**

If the time series has a trend then MA and SES will be poor predictors

A simple **extension of the SES model (Holt's Model)** which incorporates a **trend component** can be used for better prediction

Like SES, **Holts Exponential Smoothing (HES)** uses a smoothing algorithm to **remove random influences** from the time series revealing the underlying systematic components.

HES Equations

HES is characterised by three equations;

$$1.\ L_t = \alpha Y_t + (1 - \alpha)(L_{t-1} + T_{t-1})$$

$$2.\ T_t = \beta(L_t - L_{t-1}) + (1 - \beta)(T_{t-1})$$

$$3.\ F_{t+m} = L_t + mT_t$$

The first equation is for **level**, the second for **trend** and the third is the **forecasting equation** **for "m" periods into the future**

HES Equations (cont)

L_t= Smoothed level at period (t)

Y_t= Actual time series value at period t

α = Smoothing constant for level

T_t= Trend estimate at period t

β= Smoothing constant for the trend
($0 <= \beta <= 1$)

m = Number of periods ahead to be forecast

F_{t+m} = Holt's forecast value for period t + m

HES (cont.)

The values of α, β are **arbitrarily determined**

Typically **between 0 and 1 inclusive** although some programs (eg MINITAB) ignore this restriction

Try different α, β to determine the "optimum" combination (as assessed by error criteria **(MSE, MAE, MAPE)**

SOLVER in EXCEL can also be used to find the optimum by minimising a chosen error criterion

Initialisation of the model requires initial estimates for L_t and T_t.
L_t is usually the initial time series value (Y_1)
T_t is usually the average of the increase/decrease in the first few periods (use either zero or (Y_2-Y_1) or ((Y_3-Y_1)/2))