Ear Search User Guide

Version 1.0

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Abstract

The Ear Search program implements isomorph-free generation of 2-connected graphs by ear augmentations. This document describes the interfaces used for customized searches, as well as describes three example searches: unique saturation, edge reconstruction, and extremal graphs with a fixed number of perfect matchings.

1 Introduction

The EarSearch library implements the generation algorithm of [7] to generate families of 2-connected graphs.

2 Data Management

3 Pruning

4 Canonical Deletion

5 Solutions

6 Example 0: 2-Connected Graphs

N	C_N	CPU time
5	10	0.01s
6	56	0.11s
7	468	0.26s
8	7123	10.15s
9	194066	5m 17.27s
10	9743542	7h 39m 28.47s
11	900969091	71d 22h 22m 49.12s

Table 1: Comparing C_N and the time to enumerate $\mathcal{C}_N$.

N	$E = 11$	$E = 12$	$E = 13$	$E = 14$	$E = 15$	$E = 16$	$E = 17$	$E = 18$	$E = 19$	$E = 20$
10	9	121	1034	5898	23370	69169	162593	317364	530308	774876
	0.01	0.16	1.73	12.99	65.88	167.12	472.68	972.62	2048.85	3631.71
11		11	189	2242	17491	94484	380528	1212002	3194294	7197026
		0.02	0.38	5.52	56.10	260.53	1212.89	4069.09	13104.24	32836.53
12			13	292	4544	46604	334005	1747793	7274750	24972998
			0.03	0.86	17.56	286.00	1226.71	6930.00	33066.80	125716.68
13				15	428	8618	113597	1031961	6945703	36734003
				0.05	1.83	44.64	469.02	5174.92	39018.15	227436.84
14					18	616	15588	257656	2925098	24532478
					0.08	3.82	90.51	1573.81	21402.18	183482.70
15						20	855	26967	519306	7654299
						0.12	7.56	198.84	4567.43	76728.79
16							23	1176	44992	1111684
							0.18	15.56	498.20	13176.05

Table 2: Comparing $C_{N,E}$ (above) and the time to enumerate $\mathcal{C}_{N,E}$ (below, in seconds).

7 Example 1: Unique Saturation

N	CPU time for $r = 4$	CPU time for $r = 5$	CPU time for $r = 6$
8	1.01s	7.90s	8.80s
9	31.51s	4m 12.75s	4m 14.90s
10	29m 31.46s	5h 24m 38.29s	8h 0m 47.43s
11	1d 8h 13m 59.16s	44d 20h 39m 34.66s	63d 13h 31m 24.30s
12	155d 7h 52m 36.51s		

Table 3: The time to search for uniquely K_r-saturated graphs with at most N vertices.

8 Example 2: Edge Reconstruction

N	$g(N)$	$\|\mathcal{R}_N\|$	Diff 1	Diff 2	Diff 3	CPU time
8	16	4804	145	177	187	8.01s
9	19	111255	6.19×10^3	5.72×10^3	4.77×10^3	5m 33.85s
10	22	3051859	7.13×10^5	6.00×10^5	4.21×10^5	6h 33m 40.59s
11	26	308400777	2.56×10^8	1.96×10^8	1.04×10^8	32d 19h 50m 51.37s
12	29	25615152888	4.86×10^9	4.25×10^9	1.96×10^9	8y 362d 0h 49m 8.35s

Table 4: Comparing $|\mathcal{R}_N|$ and the time to check $\mathcal{R}_N$. Here, $g(n) = 1 + \lfloor \log_2(N!) \rfloor$.

9 Example 3: p-Extremal Graphs

This problem is investigated in [8]. See [1] and [3] for background on this problem.

9.1 Perfect Matching Algorithms

Algorithm 1 CountPM(G, P)

Algorithm 2 is1Extendable(H)

References

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Algorithm 3 maximalSupergraphs(H)

p	N_p	c_p	Total CPU Time	p	N_p	c_p	Total CPU Time
5	8	2	0.02s	16	16	4	2h 07m 58.60s
6	10	3	0.04s	17	16	4	6h 46m 07.72s
7	10	3	0.18s	18	18	5	11h 45m 01.95s
8	12	3	0.72s	19	18	4	2d 17h 12m 31.85s
9	12	4	1.46s	20	18	5	4d 05h 28m 11.79s
10	12	4	5.95s	21	18	5	13d 17h 29m 12.45s
11	14	3	43.29s	22	20	5	42d 20h 40m 30.41s
12	14	5	44.01s	23	20	5	118d 07h 38m 36.84s
13	14	3	6m 39.80s	24	20	6	209d 10h 09m 54.98s
14	16	4	12m 10.40s	25	20	5	2y 187d 21h 48m 46.31s
15	16	6	12m 42.72s	26	20	5	7y 75d 13h 55m 10.27s
			.	27	22	6	10y 247d 21h 3m 13.94s

Table 5: Time analysis of the search for varying p values.