20 Advanced Time Complexity Problems

Problem 1: Triple Nested with Division

```
count ← 0
for i ← 1 to n do
    j ← n
    while j > 0 do
        for k ← 1 to j do
            count ← count + 1
        j ← j/2
```

Problem 2: Logarithmic Multiplication Chain

```
count ← 0
i ← 1
while i ≤ n do
    j ← 1
    while j ≤ n do
        k ← 1
        while k ≤ n do
            count ← count + 1
            k ← k × 3
        j ← j × 2
    i ← i × 2
```

Problem 3: Nested Square Roots

```
count ← 0
i ← n
while i ≥ 1 do
    j ← n
    while j ≥ 1 do
        count ← count + 1
        j ← √j
    i ← √i
```

Problem 4: Fibonacci-style Iteration

```
count ← 0
i ← 1; j ← 1
while i ≤ n do
    for k ← 1 to i do
        count ← count + 1
    temp ← i + j
    i ← j
    j ← temp
```

Problem 5: Dependent Nested Loops

```
count ← 0
for i ← 1 to n do
    for j ← 1 to i×i do
        for k ← 1 to j do
            count ← count + 1
```

Problem 6: Mixed Division Pattern

```
count ← 0
for i ← n to 1 (decrement by i/2) do
    for j ← 1 to n do
        count ← count + 1
```

Problem 7: Exponential Inner Loop

```
count ← 0
for i ← 1 to n do
    j ← 1
    while j ≤ n do
        count ← count + 1
        j ← j × i
```

Problem 8: Harmonic Series Pattern

```
count ← 0
for i ← 1 to n do
    for j ← i to n (increment by i) do
        count ← count + 1
```

Problem 9: Complex Division Chain

```
count ← 0
i ← n
while i > 0 do
    j ← i
    while j > 0 do
        k ← n
        while k > 0 do
            count ← count + 1
            k ← k/3
        j ← j/2
    i ← i/4
```

Problem 10: Polynomial with Logarithm

```
count ← 0
for i ← 1 to n do
    for j ← 1 to i×i do
        k ← n
        while k > 0 do
            count ← count + 1
            k ← k/2
```

Problem 11: Reverse Geometric Series

```
count ← 0
i ← n
while i ≥ 1 do
    for j ← 1 to n do
        for k ← 1 to i do
            count ← count + 1
    i ← i/2
```

Problem 12: Nested Logarithmic Square

```
count ← 0
i ← 2
while i ≤ n do
```

```
    j ← 2
    while j ≤ i×i do
        count ← count + 1
        j ← j × 2
    i ← i × 2
```

Problem 13: Triple Division Different Bases

```
count ← 0
for i ← n down to 1 (divide by 2 each time) do
    for j ← n down to 1 (divide by 3 each time) do
        for k ← n down to 1 (divide by 5 each time) do
            count ← count + 1
```

Problem 14: Cube Root Pattern

```
count ← 0
i ← n
while i ≥ 1 do
    j ← i
    while j ≥ 1 do
        count ← count + 1
        j ← j^(2/3)  // cube root of j squared
    i ← i^(1/2)  // square root
```

Problem 15: Conditional Nested Growth

```
count ← 0
for i ← 1 to n do
    if i is a power of 2 then
        for j ← 1 to n×n do
            count ← count + 1
    else
        for j ← 1 to n do
            count ← count + 1
```

Problem 16: Logarithm of Factorial Pattern

```
count ← 0
for i ← 1 to n do
    j ← i
    while j > 0 do
        for k ← 1 to i do
            count ← count + 1
        j ← j/2
```

Problem 17: Mixed Multiplication and Division

```
count ← 0
i ← 1
while i ≤ n do
    j ← n
    while j ≥ i do
        count ← count + 1
        j ← j - i
    i ← i × 2
```

Problem 18: Exponential Base Change

```
count ← 0
```

```
for i ← 1 to log₂(n) do
    for j ← 1 to 2^i do
        for k ← 1 to n do
            count ← count + 1
```

Problem 19: Nested Square with Division

```
count ← 0
i ← n
while i > 0 do
    for j ← 1 to i do
        for k ← 1 to j do
            count ← count + 1
    i ← √i
```

Problem 20: Complex Harmonic Pattern

```
count ← 0
for i ← 1 to n do
    j ← i
    while j ≤ n do
        k ← j
        while k > 0 do
            count ← count + 1
            k ← k/2
        j ← j + i
```

Tips for Solving:

1. **Identify loop types:** Linear, logarithmic, exponential
2. **Check dependencies:** Is the inner loop dependent on the outer loop variable?
3. **Look for patterns:** Geometric series, harmonic series, arithmetic series
4. **Use summation notation:** Write out what you're summing
5. **Simplify step by step:** Don't try to solve everything at once
6. **Consider dominant terms:** In sums, the largest term usually determines complexity

Good luck! These problems cover various techniques including:

- Geometric series summation
- Harmonic series patterns
- Multiple base divisions
- Dependent vs independent loops
- Polynomial combinations with logarithms
- Square root and cube root iterations