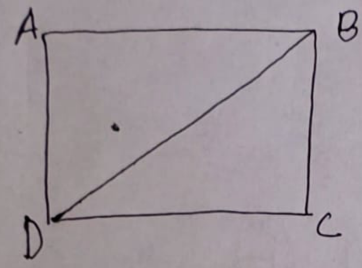
Q1.

a.

W4D1 Solution

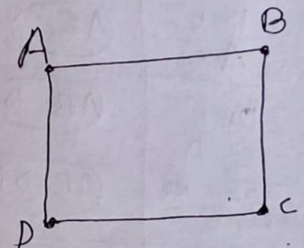
a.



Adjacency matrix

	A	B	C	D
A	0	1	0	1
B	1	0	1	1
C	0	1	0	1
D	1	1	1	0

Hamiltonian cycle

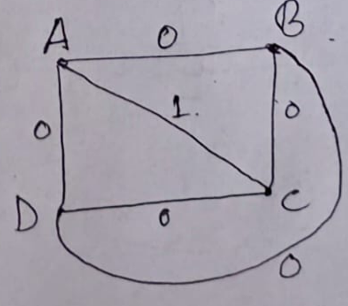


A-B-C-D

Reducing to TSP

	A	B	C	D
A	0	0	1	0
B	0	0	0	0
C	1	0	0	0
D	0	0	0	0

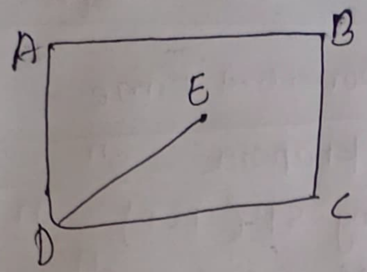
TSP



- TSP visit with cos K=0

b.

b.



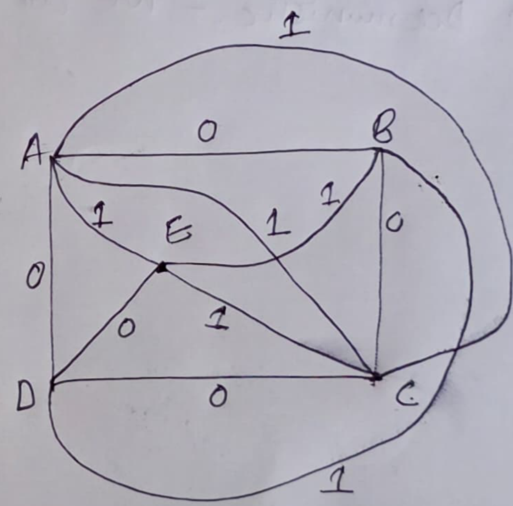
Adjacency matrix

	A	B	C	D	E
A	0	1	0	1	0
B	1	0	1	0	0
C	0	1	0	1	0
D	1	0	1	0	1
E	0	0	0	1	0

No Hamiltonian cycle

Reducing to TSP

	A	B	C	D	E
A	0	0	1	0	1
B	0	0	0	1	1
C	1	0	0	0	1
D	0	1	0	0	0
E	1	1	1	0	0



TSP can not visit with
cost K=0

Q2.

Problems.

- **HamiltonianCycle (HC):** Given an (undirected) graph $G = (V, E)$, does Gcontain a Hamiltonian cycle?
- **TSP (decision version):** Given a complete weighted graph $K = (V, E')$with integer edge weights $w(\cdot,\cdot)$and a bound B, is there a Hamiltonian cycle of total weight $\leq B$?

(We use the usual fact that TSP is in NP because a proposed tour can be checked in polynomial time.)

Reduction $f$from HC to TSP

Input to HC: graph $G = (V, E)$with $|V| = n$.

Construct a complete graph $K = (V, E')$on the same vertex set and define weights

$$w(u, v) = \begin{cases} 1 & \text{if } \{u, v\} \in E \text{ (edge of } G\text{)}, \\ 2 & \text{if } \{u, v\} \notin E \text{ (non-edge of } G\text{)}. \end{cases}$$

Set the TSP bound $B = n$. Output the TSP instance (K, w, B).

This transformation is clearly computable in polynomial time (just examine each pair of vertices and set weight 1 or 2).

Correctness

1. **If Ghas a Hamiltonian cycle.**
   Then that cycle uses only edges of G. In Kthose edges each have weight 1, so the corresponding Hamiltonian cycle in Khas total weight n. Thus $(K, w, B)$is a YES instance of TSP (weight $\leq B$).
2. **If $(K, w, B)$is a YES instance of TSP (there is a tour of weight $\leq n$).**
   Any Hamiltonian cycle in Khas exactly $n$edges. If the tour has weight $\leq n$, every edge in it must have weight 1(because if even one edge had weight $2$the sum $\geq (n-1) \cdot 1 + 2 = n + 1 > n$). Hence all edges of the tour are edges of G, so that tour is a Hamiltonian cycle of G. Therefore Ghas a Hamiltonian cycle.

So Ghas a Hamiltonian cycle iff the constructed TSP instance has a tour of weight $\leq n$. That proves HC $\leq_p$TSP.

Conclusion

- TSP is in NP (certificate = tour, verify sum in polynomial time).
- HC is NP-complete by assumption and we have shown a polynomial-time reduction HC $\leq_p$TSP, so TSP is NP-hard.
- Therefore TSP is NP-complete.