

Binomial Heaps

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Comp 401: Senior Seminar

3/05/2014

Outline

1. Binomial Trees
2. Binomial Heaps
3. Implementation Structure
4. Standard Functions
5. Uses of Binomial Heaps

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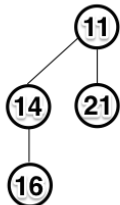
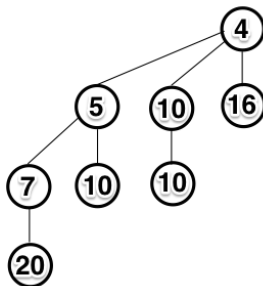
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Binomial Trees

- A **Binomial Tree** is a specific type of tree that includes the following specifications:
 1. The **order** or **rank** of the binomial tree is the number of children of the root node.
 2. A Binomial Tree of order 0 is a single node.
 3. A Binomial Tree of order k has k child nodes, all of which are the roots of binomial trees of orders $k - 1, k - 2, \dots, 2, 1, 0$ from left to right.

Binomial Trees: Examples

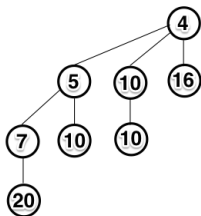
- If a binomial tree has order k , the orders of the k child nodes decrease from left to right from $k - 1$ to 0.

 $k = 2$  $k = 0$  $k = 3$  $k = 1$ 

Binomial Trees

- If a Binomial Tree has an order k :
 1. The tree has 2^k nodes.
 2. The height of the tree is k .
 3. There are $\binom{k}{d}$ nodes at depth d .
- $\binom{k}{d} = \frac{k!}{d!(k-d)!}$ is known as the Binomial Coefficient.

Example: $k = 3, d = 2$



$$\binom{k}{d} = \binom{3}{2} = \frac{3!}{2!(3-2)!} = \frac{6}{2 * 1} = \frac{6}{2} = 3$$

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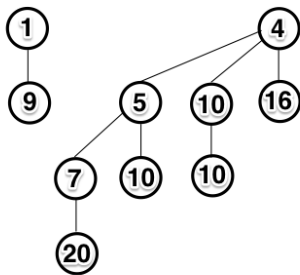
Binomial Heaps

- A **Binomial Heap** is a collection of binomial trees that satisfy the following two binomial heap properties:
 1. The key of any node is greater than or equal to the key of its parent (minimum-heap property).
 2. There cannot be two binomial trees of the same order.

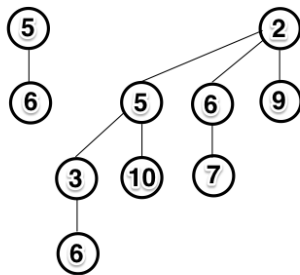
Binomial Heap: Property #1 (minimum-heap)

- The first property (minimum-heap) ensures that the root is the smallest key in each binomial tree.
- The smallest key of the entire heap is one of the roots.

Min-Heap Property ✓



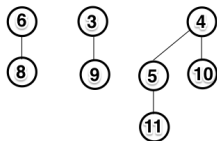
Min-Heap Property ✗



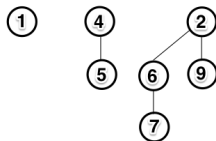
Binomial Heap: Property #2

- The order of each binomial tree must be unique.

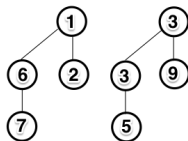
Property #2 ×



Property #2 ✓

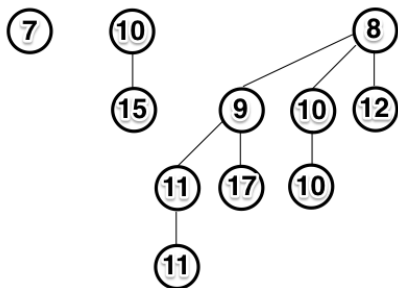


Property #2 ×



Binomial Heap: Property #2, Cont.

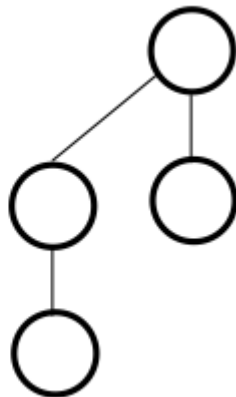
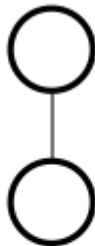
- The second property ensures that if a binomial heap has n nodes, then it will have at most $\lfloor \log n \rfloor + 1$ binomial trees.
- The total number of nodes can also be thought of as a binary string, where each binomial tree represents a bit.



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Implementation Structure



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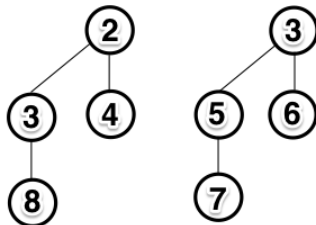
Standard Functions: Merge

- Two k ordered binomial trees are merged into one $k + 1$ ordered binomial tree.

Merge

Input: Node* root_A, Node* root_B

```
if root_A.key ≤ root_B.key then
    root_B.right_sibling ← root_A.child
    root_A.child ← root_B
    root_B.parent ← root_A
    return root_A
else
    root_A.right_sibling ← root_B.child
    root_B.child ← root_A
    root_A.parent ← root_B
    return root_B
end if
```



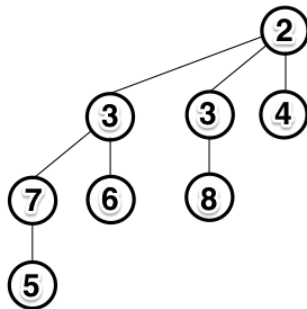
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end if
```



Standard Functions: Join



Standard Functions: Insert



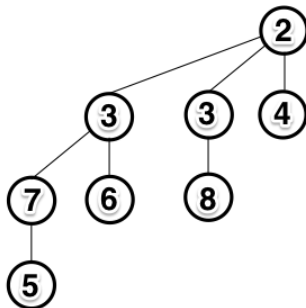
Standard Functions: DeleteMinimum



BinomialHeap : DeleteMinimum

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    return root_B
end if
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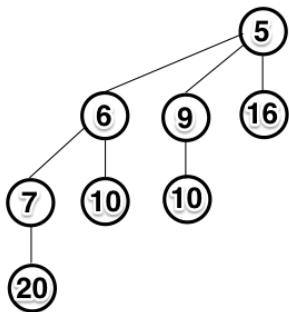


Standard Functions: DecreaseKey

- Decreases the key of a given node in the Binomial Heap

BinomialHeap : DecreaseKey

Input: Node* node, int new_key
decrease_node.key \leftarrow new_key
Node* temp \leftarrow decrease_node
while temp.parent \neq NULL **do**
 if temp.key < temp.parent.key
 then
 swap(temp.key, temp.parent.key)
 else
 break
 end if
 temp \leftarrow temp.parent
end while

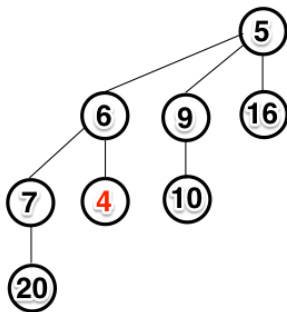


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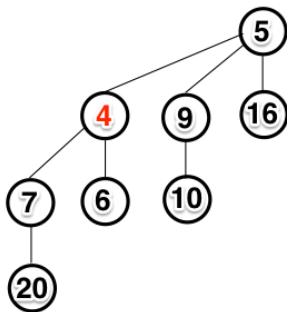


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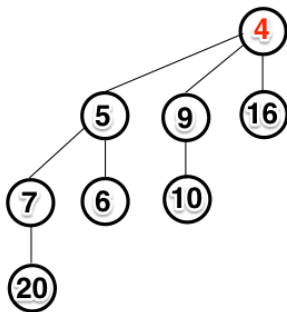


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Standard Functions: Delete

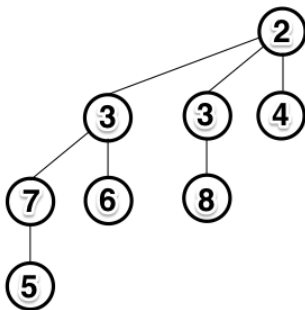
- Deletes a given node from the Binomial Heap

BinomialHeap : Delete

Input: Node* delete_node

BinomialHeap.DecreaseKey(delete_node, -9999)

BinomialHeap.DeleteMinimum()



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References

- ▶ KEVIN WAYNE.
Priority Queues.
<http://www.cs.princeton.edu/wayne/kleinberg-tardos/pdf/BinomialHeaps.pdf>,
2013.
- ▶ UNKNOWN.
Binomial Heap.
http://en.wikipedia.org/wiki/Binomial_heap, 2015.