Binomial Heap

Introduction and Exercises

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Binomial Trees

- A Binomial Tree is a specific type of tree that includes the following specifications:
  1. The order or rank of the binomial tree is the number of children of the root node.
  2. A Binomial Tree of order 0 is a single node.
  3. A Binomial Tree of order k has k child nodes, all of which are the roots of binomial trees of orders $k-1$, $k-2$, ..., 2, 1, 0 from left to right.

Excercise: Binomial Trees

- Determine the order of the following binomial trees (note how if the tree has order k, the orders of the child nodes decrease from left to right from $k-1$ to 0).

Figure 1

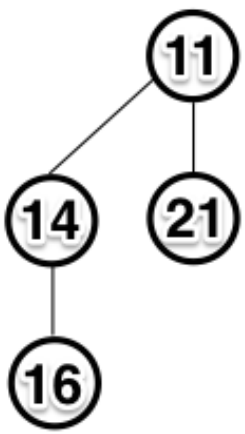


Figure 2

Figure 3

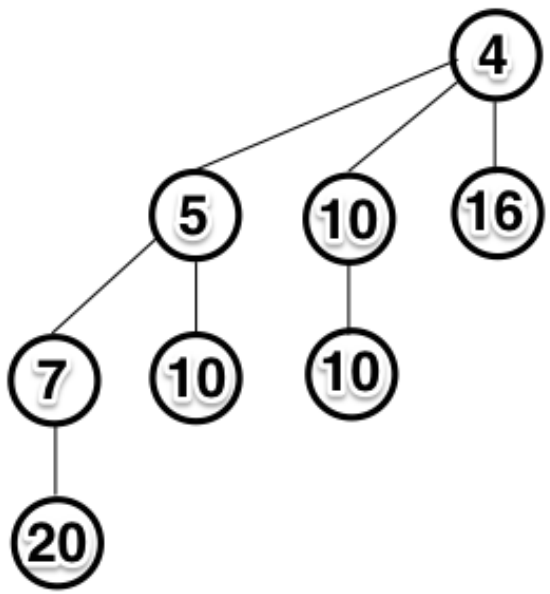


Figure 4

Binomial Heaps

- A Binomial Heap is a collection of binomial trees that satisfy the following two binomial heap properties:
  1. The key of any node is greater than or equal to the key of its parent (minimum-heap property).
  2. There cannot be two binomial trees of the same order.
- The first property (minimum-heap) ensures that the root is the smallest key in each binomial tree. Similarly, the smallest key of the entire heap is one of the roots.
- The second property ensures that the if a binomial heap has n nodes, then it will have at most $\lfloor log\ n \rfloor + 1$ binomial trees.
- Binomial heaps are used to implement priority queues.

Excercise: Binomial Heap Property #1

- Determine if the following binomial trees satisfy the minimum-heap property.

Figure 1

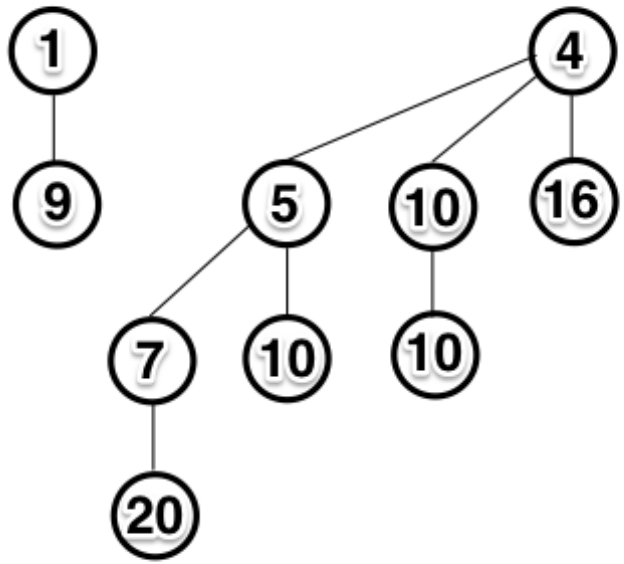
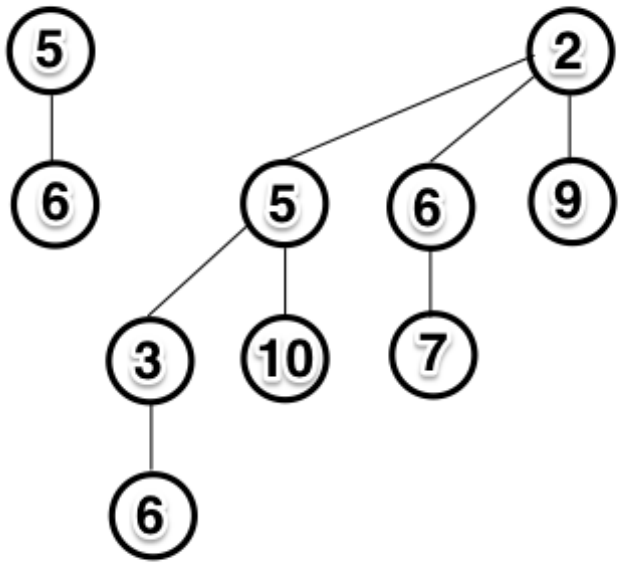


Figure 2



Excercise: Binomial Heap Property #2

- Determine if the following structures are valid Binomial Heaps.

Figure 1

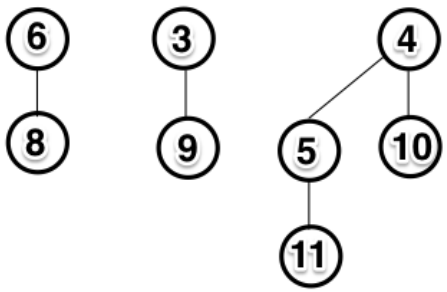


Figure 2

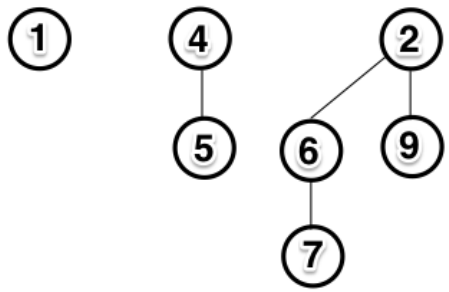
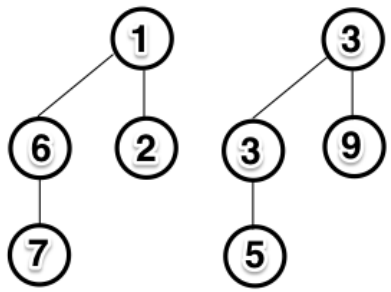


Figure 3



Excercise: Binomial Heap Property #2, Cont.

Figure 4