

The Sieve of Eratosthenes using MPI

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Outline

1. Parallel Programming and MPI
2. The Sequential Algorithm
3. The Parallel Algorithm
4. Sequential vs. Parallel Comparison

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Introduction

- **Parallel Computing** is the use of multiple computers or processors to reduce the time needed to solve a single computational problem.
- A **task** is a single program including local memory and a collection of input/output ports.
- A **channel** is a message queue between two tasks used for communication

Ian Foster's Design Methodology

1. **Partitioning** - the process of dividing the computations and data into pieces.
2. **Communication** - channels between tasks allow communication between them
 - Local - a task's computation requires values from a small number of other tasks
 - Global - many tasks must contribute values to perform a computation
3. **Agglomeration** - grouping tasks in order to improve performance and reduce overhead.
4. **Mapping** - assigning processes or tasks to specific processors or computers

Message Passing Interface (MPI)

- The most popular message-passing library standard
- There are many free implementations of MPI libraries, including OpenMPI and MPICH
- Integrates sequential language with functions that allow processes to communicate with each other

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The Sequential Algorithm

The Sieve of Eratosthenes

Summary: Finds all primes between 2 and n , inclusive

1. Create a list of natural numbers 2, 3, ... , n , none of which are marked

2. Set k equal to the first prime number, 2

while $k^2 \leq n$ **do**

- 3a. Mark all multiples of k between k^2 and n

- 3b. Set k to the smallest unmarked number greater than the current k

end while

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The Parallel Algorithm: Block Decomposition

- n is the size of array and p is the number of processors
- Divide the array into p contiguous blocks of roughly equal size
- If n is divisible by p , then there will be p blocks of size n/p . Otherwise, there will be p blocks of with sizes of either $\lfloor n/p \rfloor$ or $\lceil n/p \rceil$.
- Common data block computations include finding the first/last element controlled by a given process

The Parallel Algorithm: Block Decomposition, cont.

- Suppose n is the size of array and p is the number of processors
- First element controlled by process i :

$$\lfloor in/p \rfloor$$

- Last element controlled by process i :

$$\lfloor (i+1)n/p \rfloor - 1$$

Block Decomposition Example, $n = 121$, $p = 3$

Processor 0:

- First index: $\lfloor (0)(121)/3 \rfloor = 0$
- Last index: $\lfloor (0 + 1)121/3 \rfloor - 1 = \lfloor 40\frac{1}{3} \rfloor - 1 = 39$
- Size: last - first + 1 = $\lfloor n/p \rfloor = 40$.

Processor 1:

- First index: $\lfloor (1)(121)/3 \rfloor = \lfloor 40\frac{1}{3} \rfloor = 40$
- Last index: $\lfloor (1 + 1)121/3 \rfloor - 1 = \lfloor 80\frac{2}{3} \rfloor - 1 = 79$
- Size: last - first + 1 = 40 = $\lfloor n/p \rfloor$.

Processor 2:

- First index: $\lfloor (2)(121)/3 \rfloor = \lfloor 80\frac{2}{3} \rfloor = 80$
- Last index: $\lfloor (2 + 1)121/3 \rfloor - 1 = \lfloor 121 \rfloor - 1 = 120$
- Size: last - first + 1 = 41 = $\lceil n/p \rceil$.

Developing the Algorithm, Step 1

1. Create a list of natural numbers 2, 3, ... , n , none of which are marked

- Each task handles a specific block of the entire array
- To minimize communication between tasks, make task 0 responsible for finding the next value of k
- This can be done by ensuring $n/p > \sqrt{n}$
- Keep in mind the difference between the local index for the block and the global index for the entire array

2. Set k equal to the first prime number, 2

- k is set to the first prime number for all tasks
- After each iteration, each task must be told the next value of k

Developing the Algorithm, Step 3

While $k^2 \leq n$

3a. Mark all multiples of k between k^2 and n

- Must determine the first multiple of k in the given block (if it's greater than k^2)
- From the first multiple of k , mark every k^{th} element

3b. Set k to the smallest unmarked number greater than the current k

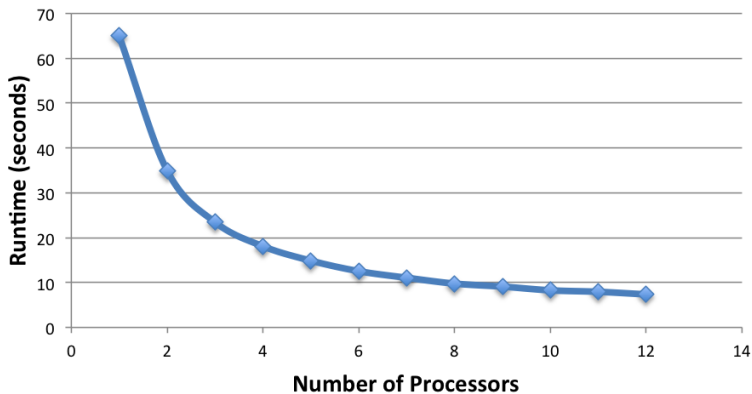
- The smallest unmarked number greater than the current k is always part of the block belonging to task 0
- Task 0 finds the next value of k , and broadcasts it to the rest of the tasks

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Sequential vs. Parallel

**Parallel Algorithm Runtimes for Finding Primes
between 2 and 1,000,000,000**



References

- ▶ MICHAEL J. QUINN.
Parallel Programming in C with MPI and OpenMP.
McGraw-Hill Science/Engineering/Math, 2003.