Binomial Heaps

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Comp 401: Senior Seminar

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Outline

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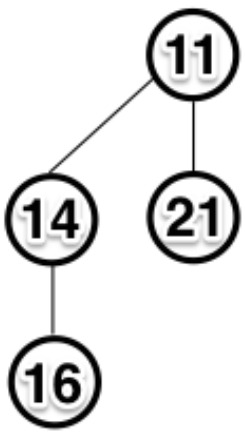
Binomial Trees

- A Binomial Tree is a specific type of tree that includes the following specifications:
  1. The order or rank of the binomial tree is the number of children of the root node.
  2. A Binomial Tree of order 0 is a single node.
  3. A Binomial Tree of order k has k child nodes, all of which are the roots of binomial trees of orders $k-1$, $k-2$, ..., 2, 1, 0 from left to right.

Binomial Trees: Examples

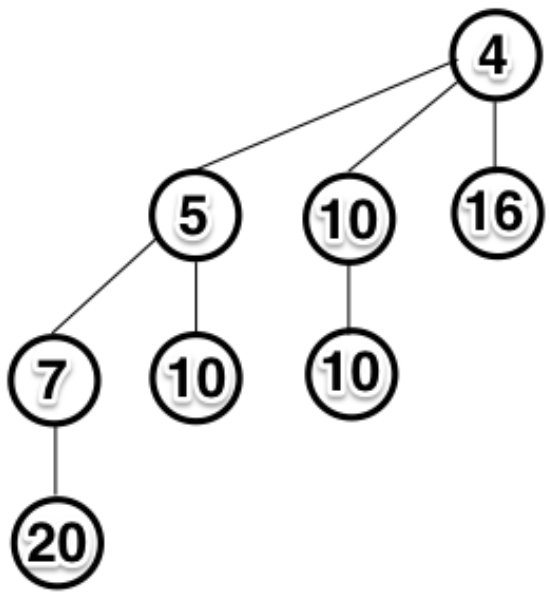
- If a binomial tree has order k, the orders of the k child nodes decrease from left to right from $k-1$ to 0.

$k = 2$



$k = 0$

$k = 3$

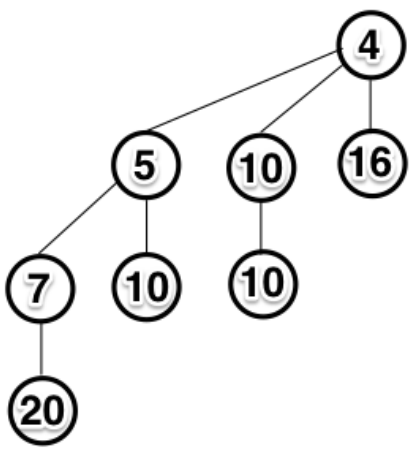


$k = 1$

Binomial Trees

- If a Binomial Tree has an order k:
  1. The tree has 2^k nodes.
  2. The height of the tree is k.
  3. There are $\binom{k}{d}$ nodes at depth d.
- $\binom{k}{d} = \frac{k!}{d!(k-d)!}$ is known as the Binomial Coefficient.

Example: $k = 3$, $d = 2$



$$\binom{k}{d} = \binom{3}{2} = \frac{3!}{2!(3-2)!} = \frac{6}{2*1} = \frac{6}{2} = 3$$

Outline

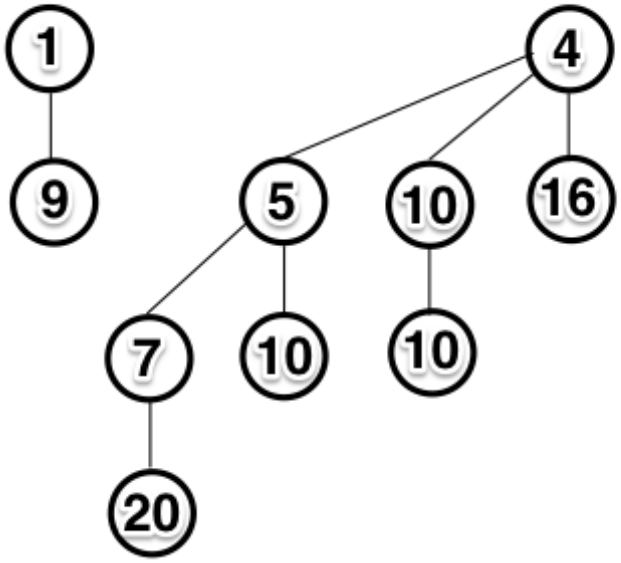
Binomial Heaps

- A Binomial Heap is a collection of binomial trees that satisfy the following two binomial heap properties:
  1. The key of any node is greater than or equal to the key of its parent (minimum-heap property).
  2. There cannot be two binomial trees of the same order.

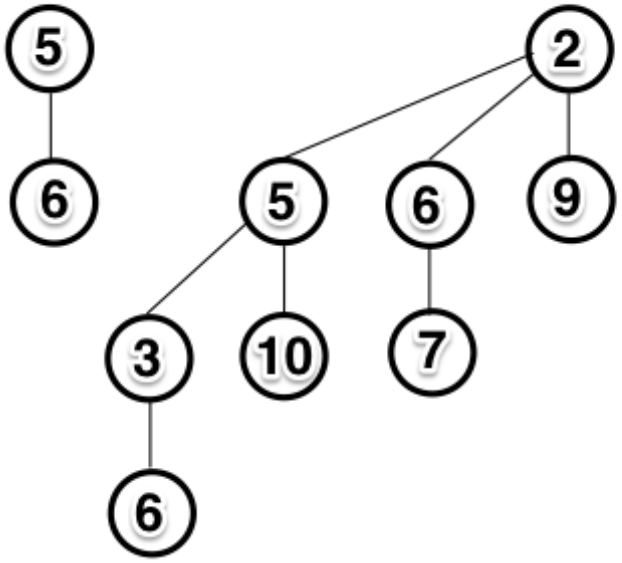
Binomial Heap: Property #1 (minimum-heap)

- The first property (minimum-heap) ensures that the root is the smallest key in each binomial tree.
- The smallest key of the entire heap is one of the roots.

Min-Heap Property ✓



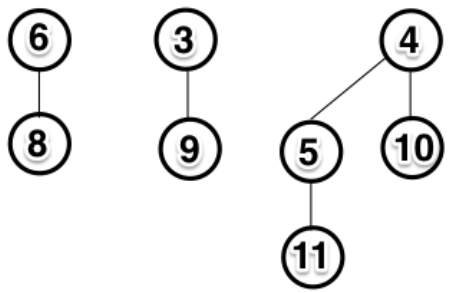
Min-Heap Property ×



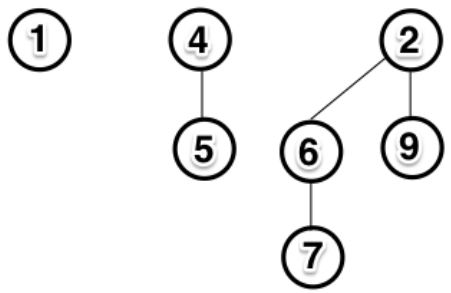
Binomial Heap: Property #2

- The order of each binomial tree must be unique.

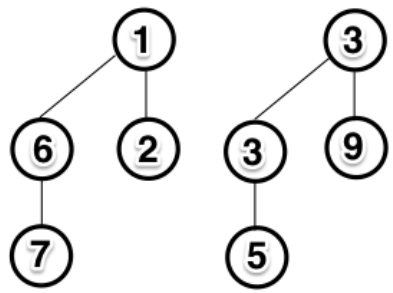
Property #2 ×



Property #2 ✓

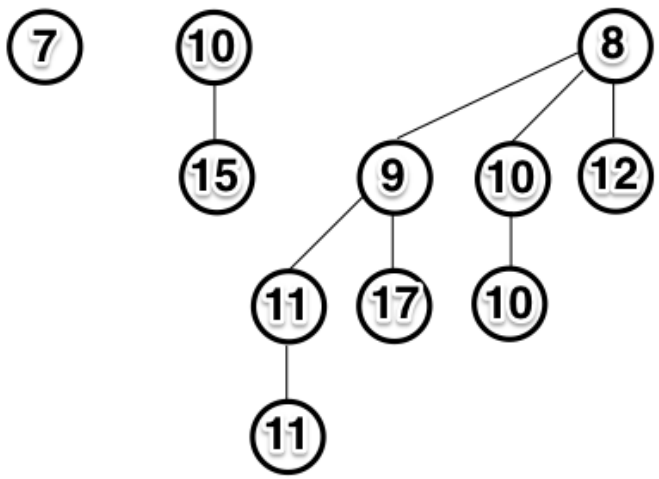


Property #2 ×



Binomial Heap: Property #2, Cont.

- The second property ensures that if a binomial heap has n nodes, then it will have at most $\lfloor log\ n \rfloor + 1$ binomial trees.
- The total number of nodes can also be thought of as a binary string, where each binomial tree represents a bit.



Outline

Standard Functions: Merge

Standard Functions: Join

Standard Functions: Insert

Standard Functions: DeleteMinimum

Standard Functions: DecreaseKey

Standard Functions: Delete

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Uses of Binomial Heaps

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Implementation

- A Binomial Heap is a collection of binomial trees that satisfy the following two binomial heap properties:
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