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UNIT - II

SET THEORY

Set:- Any collection of well defined objects is called a set. and it is denoted by the letters (A, B, C, ...) and represented the elements in { }.

Eg:- Collection of students in a class is a set.

Collection of all integers less than 10

{1, 2, 3, 4, 5, 6, 7, 8, 9, 10}.

There are two types of set notations.

1. Roster Form
2. Set builder form

Eg:- List the even numbers between 1 to 20 by using roster form and set builder form.

Roster form $E = \{2, 4, 6, 8, 10, 12, 14, 16, 18, 20\}$.

Set Builder form

$E = \{x / x \text{ is even number} \leq 20\}$

or

$= \{x / x = 2n, 1 \leq n \leq 20\}$

Empty Set or Null Set:

A ~~s~~ set consisting of no elements is called empty set or null set and it is denoted by the symbol ϕ or { }.

Eg:- $\{x, x \in N, 4 < x < 5\} = \phi$

$\{x, x \in R, x^2 + 1 = 0\} = \phi$.

Equal Sets: Two sets A and B are equal if and only if they have the same elements A and B are equal.

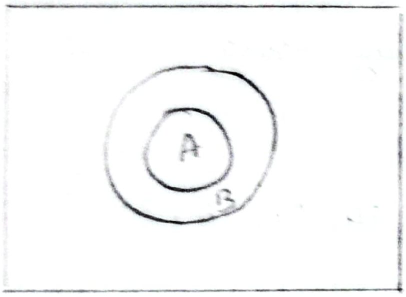
$$\forall x (x \in A \leftrightarrow x \in B)$$

A set A is a proper subset of set B if

i) $A \subseteq B$

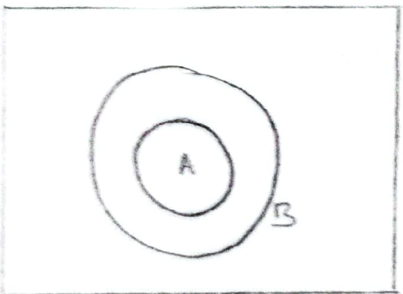
ii) B will have atleast one elements.

where $A \subset B$ the symbol '$\subset$' stands for a proper subset.



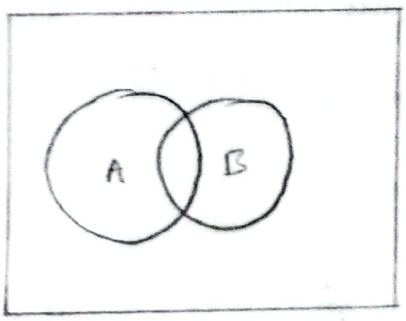
$A \subseteq B$

A is a subset of B.



$A \subset B$

A is proper subset of B



$A \not\subseteq B$

A is not a subset of B.

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Note: Every set is a subset of itself. Two sets A and B are equal if any only if $A \subseteq B$ and $B \subseteq A$.

- The null set ϕ is a subset of every set A.
- For any sets A, B, C if $A \subseteq B$ and $B \subseteq C$ then $A \subseteq C$.
- For any sets A, B, C if $A = B$ and $B = C$ then $A = C$.

Universal Set:

- All sets that we consider are ~~only~~ sub-sets of a certain set U.
- The set U is called the universal set.
- Universal set is not unique.

Power Set:

- The set of all subsets of a set is called a power set and is denoted by $P(A)$. where A is set. The power set of A has 2^n elements.

Example: i) Consider the set $A = \{a, b\}$. Find $P(A)$.

$\therefore P(A) = \{\{a\}, \{b\}, \{a\,b\}, \phi\}$.

- ii) $A = \{1, 2, 3\}$

$\therefore P(A) = \{\phi, \{1\}, \{2\}, \{3\}, \{1,2\}, \{2,3\}, \{1,3\}, \underset{A}{\{1,2,3\}}\}$

iii) Find powerset of empty set.

The empty set has exactly one subset namely itself.

$P(\phi) = \phi$.

The set ϕ has exactly two subsets namly ϕ and $\{\phi\}$.

$P(\{\phi\}) = \{\phi, \{\phi\}\}$.

Cardiniality of Set:

Let S be a set if there are exactly 'n' distinct elements in S where n is a non negitive integer, we say that S is a finite set. and that n is the cardinal of set S.

The cardinality of set S is dentoted by $|S|$.

Example: i) Let A be set of odd +ve integers lessthan 10, Then, find cardinality of A.

$A = \{1, 3, 5, 7, 9\}$. $|A| = 5$.

ii) Let S be the set of letters in english alphabet. Then cardinality of $S = |S| = 26$.

iii) ϕ is a null set (no elements). cardinality of ϕ is zero "0".

Catesion Product of two sets :

Let A and B are two non empty sets. Then the cartision product of A and B is defined as $A \times B = \{(a,b) / a \in A, b \in B\}$

Example: If $A = \{1,2,3\}$ $\therefore$ $A \times B = \{(1,a), (2,a), (3,a), (1,b), (2,b), (3,b)\}$.
$B = \{a, b\}$

ii) What is the cartesion product of $A = \{1,2\}$, $B = \{a,b,c\}$. also check if $A \times B = B \times A$.

$A \times B = \{(1,a), (1,b), (1,c), (2,a), (2,b), (2,c)\}$ $B \times A = \{(a,1), (a,2), (b,1), (b,2), (c,1), (c,2)\}$

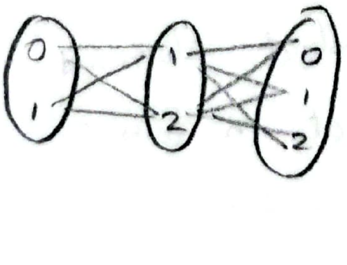
$A \times B \neq B \times A$.

iii) What is the cartesion product of $A = \{0,1\}$, $B = \{1,2\}$, $C = \{0,1,2\}$.
Find:

$A \times B \times C$

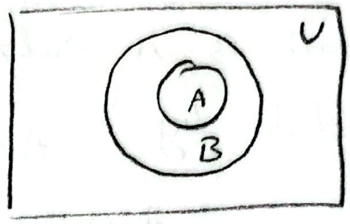
The cartesion product $A \times B \times C$ consiste of all ordered. where $a \in A$, $b \in B$, $c \in C$.

Hence, $A \times B \times C = \{(0,1,0), (0,2,0), (1,1,0), (1,2,0), (0,1,1), (0,2,1), (1,1,1), (1,2,1), (0,1,2), (0,2,2), (1,1,2), (1,2,2)\}$.



Sub Set : Set A is said to be subset of the set the set B if each element of Set A is also element of set B.

It is denoted by $A \subseteq B$.



$A \subseteq B$

Example: $A = \{1, 2, 3\}$, $B = \{1, 2, 3, 4, 5\}$

$\therefore A \subseteq B$.

$N = \{$ Set of all Natural Numbers $\}$

$I =$ set of all integers

$N \subseteq I$.

$W =$ Set of whole numbers.

$N \subseteq W$.

Cartesion Product "n" sets :

Let $A_1, A_2, A_3, \dots, A_n$ be the n sets. The cartesion product of $A_1, A_2, \dots, A_n$ is defined as. $A_1 \times A_2 \times \dots \times A_n = \{(a_1, a_2, a_3, \dots, a_n) \mid a_1 \in A_1, a_2 \in A_2, \dots, a_n \in A_n\}$

H.W

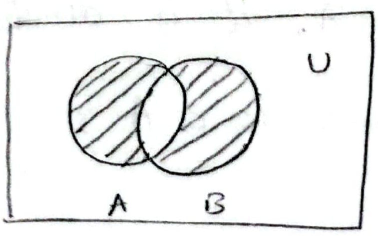
If $A = \{a, b, c\}$, $B = \{x, y\}$, $C = \{0, 1\}$. Find: i) $A \times B \times C$

iii. $C \times B \times A$

8/4/2024 Separation On Sets :

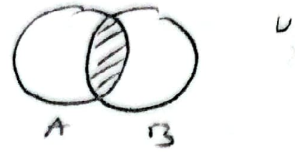
Union of sets: ~~Consists~~ Consider 2 sets A and B, then the set consisting of all the elements, that belongs to A or B is called the union of A and B. And denoted by "$A \cup B$".

$A \cup B = \{x / x \in A \text{ (or) } x \in B\}$



Intersection of Sets: Given two sets A and B, then the side consisting of an elements that belongs to both A and B is called the intersection of A and B. and denoted by "$A \cap B$".

$\therefore$ $A \cap B = \{x / x \in A \text{ and } B\}$.



$A \cap B \subset A$ and $A \cap B \subset B$.

Eg:- If $A = \{1, 2, 3, 4, 5, 6, 7\}$ and

$B = \{4, 5, 8, 9\}$

$A \cap B = \{4, 5\}$

(2) $A = \{1, 2, 3\}$ and $B = \{5, 7\}$.

$A \cap B = \{\emptyset\}$

$\therefore$ A and B are dis joint sets.

Note: 2 sets A and B are said to be disjoint when ever $A \cap B = \emptyset$.

Complement of a set: Given a universal set U and a set 'A' contained in U. The set elements that belongs to U but not to A is called complement of A, it is denoted by $\bar{A}$.

$\therefore$ $\bar{A} = \{x / x \in U \text{ and } x \in A\}$

Q If $A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$ and $A \cap B = \{7\}$. Find $A \Delta B$.

$A \Delta B = (A \cup B) - (A \cap B)$

$= \{1, 2, 3, 4, 5, 6, 7\} - \{7\}$

$A \Delta B = \{1, 2, 3, 4, 5, 6\}$

Q If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $A = \{1, 2, 4, 6, 8\}$ and $B = \{2, 4, 5, 9\}$. Find and compute i) $\bar{A}$ ii) $\bar{B}$ iii) $\bar{A} \cup \bar{B}$ iv) $\overline{A \cup B}$ v) $\bar{A} \cap \bar{B}$ vi) $\overline{A \cap B}$ vii) $B - A$ viii) $A - B$. ix) $A \Delta B$ x)

i) $\bar{A} = U - A$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{1, 2, 4, 6, 8\}$

$= \{3, 5, 7, 9\}$

ii) $\bar{B} = U - B$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{2, 4, 5, 9\}$

$= \{1, 3, 6, 7, 8\}$

iii) $\bar{A} \cup \bar{B} = \bar{A} \cup \bar{B}$

$= \{3, 5, 7, 9\} \cup \{1, 3, 6, 7, 8\}$

$= \{1, 3, 5, 6, 7, 8, 9\}$

iv) $\overline{A \cup B} = U - (A \cup B)$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - [\{1, 2, 4, 6, 8\} \cup \{2, 4, 5, 9\}]$

$= \{1, 2, 3, 4, 5, 6, 7, 8, 9\} - \{1, 2, 4, 5, 6, 8, 9\}$

$= \{3, 7\}$

v) $\bar{A} \cap \bar{B} = \bar{A} \cap \bar{B}$

$= \{3, 5, 7, 9\} \cap \{1, 3, 6, 7, 8\}$

$= \{3, 7\}$.

vi) $\overline{A \cap B} = U - [A \cap B\}$

$= U - [\{1,2,4,6,8\} \cap \{2,4,5,9\}]$

$= U - \{2,4\}$

$= \{1,2,3,4,5,6,7,8,9\} - \{2,4\}$

$= \{1,3,5,6,7,8,9\}$

vii) $B - A = \{2,4,5,9\} - \{1,2,4,6,8\}$

$= \{5,9\}$

viii) $A - B = \{1,2,4,6,8\} - \{2,4,5,9\}$

$= \cancel{5,9}\ \{1,6,8\}$

ix) $A \Delta B = (A \cup B) - (A \cap B)$

$= \{1,2,5,4,6,8,9\} - \{2,4\}$

$= \{1,5,6,8,9\}$

Determine sets A and B Given that $A - B = \{1,2,4\}$, $B - A = \{7,8\}$ and $A \cup B = \{1,2,3,4,7,8,9\}$. Find A and B

$A = (A \cup B) - (B - A)$

$B = (A \cup B) - (A - B)$

$A = \{1,2,4,5,7,8,9\} - \{7,8\}$

$= \{1,2,4,5,9\}$

$B = \{1,2,4,5,7,8,9\} - \{1,2,4\}$

$= \{5,7,8,9\}$.

Laws of set theory :-

1. commutative law:

$A \cup B = B \cup A$

$A \cap B = B \cap A$

2. Associative law:

$A \cup B \cup C = (A \cup B) \cup (A \cup C)$

$A \cap B \cap C = (A \cap B) \cap (A \cap C)$

3. Idempotent Law:

$A \cup A = A$ $\quad$ $A \cap A = A$

$B \cup B = B$ $\quad$ $B \cap B = B$

4. Identity law:

$A \cup \emptyset = A$

$A \cap \quad = A$

5. Double Complement:

$\overline{(\bar{A})} = A.$

6. Inverse Law:

$A \cup \bar{A} = U$

$A \cap \bar{B} = \emptyset$

7. Demorgons law:

$\overline{A \cup B} = \bar{A} \cap \bar{B}$

$\overline{A \cap B} \quad \bar{A} \cup \bar{B}$

8. Domination law:

$A \cup U = U$

$A \cup \emptyset = \emptyset$

9. Absorption Law:

$A \cup (A \cap B) = A$

$A \cap (A \cup B) = A$

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Properties of Relations:

Reflexive: A relation on a set 'A' is said to be reflexive if $a R a, \forall a \in A$, It means every element of A refers to a A (itself).

Eg:- $A = \{1,2,3\}$

$R = \{(1,1),(1,2),(1,3),(2,2),(2,3),(3,3)\}$

$2 \leq 2$

$a \leq a$

PO Set

Equivalance Relation

Hasse Diagram

Irreflexive: A relation on a set 'A' is irreflexive if for every $a \in R$, $(a,a) \notin R$ i.e, if there is no $a \in A$ such that $a R a$.

Defined $A = \{1,2,3\}$.

Note: Symmetric and anti-symmetric are not negative of each other.

$A = \{1,2,3\}$

$R = \{(1,3), (3,1) (2,3)\}$ is neither symmetric nor anti symmetric.

Transitive: A relation R on a set A is said to be transitive if when ~~error either~~ aRb and bRc then aRc.

i.e, if when (a,b) and $(b,c) \in R$ then $(a,c) \in R$.

Eg:- $a \leq b,\ b \leq c \Rightarrow a \leq c$.

Equivalance Relation:

A relation R on a set A is called an equivalance relation if R is Reflexive, Symmetric and transitive. i.e, R is an equivalence relation if it has the following these property

- $aRa, \ \forall a \in A$
- if aRb then bRa, $a, b \in A$
- if aRb and bRc then aRc.

Partial ordering relation:

A relation R on a set A is called partial ordering or partial order relation if R is reflexive, anti symmetric and transitive i.e R is a partial order relation on A if it has the following properties

- $aRa \quad \forall a \in A$
- aRb and $bRa \Rightarrow a = b$
- aRb and $bRc \Rightarrow aRc$.

Partial Ordered Set (poset): A set together with a partial ordered relation R is called a partially orders set or poset $\geq t$

Eg:- • $a \geq a$, $\forall a \in \mathbb{Z}$ Reflexive

• $a \geq b$ and $b \geq a \Rightarrow a = b$ anti symmetric

• $a \geq b$ and $b \geq c \Rightarrow a \geq c$ Transitive

$\therefore (\mathbb{Z}, \geq)$ is poset.

Equivalance class: If R is an equivalance ~~class~~ relation on a set A. The set of all elements of A that are related to an element a of 'A' is called the equivalance class of a and denoted by

$$[a]_R \text{ or } [a]$$

i.e, $[a] = \{x / (a,x) \in R\}$.

Quotient set: The collection of all equivalence classes of elements of A under an equivalence relation R is dented by A/R and is called the quotient set of A by R.

Eg:- Let $A = \{1,2,3\}$

$R = \{(1,1)(1,2), (2,1), (2,2), (3,3)\}$

Here, it is Reflexive, Symmetric, transitive.

Here, R is an equivalance relation.

$[1] = \{1,2\}$; $[2] = \{1,2\}$; $[3] = \{3\}$.

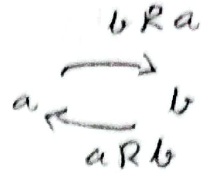
the equivalence classes $[1], [2], [3]$ of A under R.

Here from $A/R = \{[1], [2], [3]\}$.

Representation of relations by graphs:

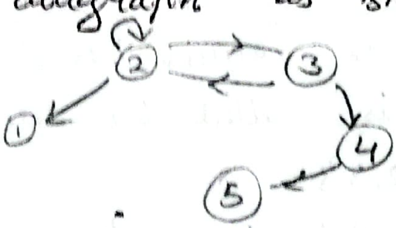
• Let R be a relation on a set A to represent R graphically each elements of A is represented by a point. These points are nodes or vertices.

- whenever the element a is related to the element 'b' an arc is drawn from point 'a' to point 'b'. These arcs called arcs edges.
- The arcs start from first element of the related of the related pair and go to the second element.



- The direction is indicated by arrow arc.
- The resulting diagram is called directed graph or diagraph of R.
- The edge of the form (a,a) represented by using an arc from the vertex a back to itself is called a loop.

Eg:- Write the relation as a set of ordered pairs from the diagraph as shown.



R = { (2,2), (2,3), (2,1)
(3,2), (3,4), (4,5) }.

Hasse Diagram for Partial Orderings:

- The simplified form of diagraph of a partial ordering on a finite set that contains efficient information about the partial ordering is called a Hasse diagram.
- In this diagram each element is represented by a small circle or a dot. we have the following
  - Since the partial ordering is reflexive relation its diagraph has loops at all vertices we need not show these loops since they must be present.
  - Since the partial ordering is transitive we need not show those edges that must present due to transitivity..

Pg:- 7, 8.

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verify the following equation $X = \{1,2,3,4\}$ is an equidance relation or not.

Given: $R\{(1,1), (1,4), (4,1), (2,2), (2,3)$
$(2,4), (3,3), (3,2), (4,3), (4,4)\}$.

Solution: A relation which is reflexive, symmetric and transitive is called and equivalance relation.

i) Reflexive: relation R is reflexive as
$(1,1), (2,2), (3,3), (4,4) \in R$
i.e, $(a,a) \in R$.

ii) Symmetric: Relation R is symmetric as
$(2,3) \in R \Rightarrow (3,2) \in R$
i.e, $(a,b) \in R \Rightarrow (b,a) \in R$.

iii) Transitive: Relation R is not Transitive as
$(2,3) \in R, (3,4) \in R \Rightarrow (2,4) \in R$.

$X = \{1,2,3,4,5,6,7\}$
$R = \{(x,y) / (x-y)$ is divisible by 3 in X $\}$.

Show That R is an equidance relation.

Solution:

We have $X = \{1,2,3,4,5,6,7\}$

$R = \{(1,1), (2,2), (3,3), (4,4), (5,5), (6,6), (7,7),$
$(1,4), (4,1), (1,7), (7,1), (2,5), (5,2), (3,6), (6,3),$
$(4,7), (7,4)\}$.

All the ordered pairs satisfies the relation $X-y$ is divisible by 3.

i) Reflexive: $(1,1), (2,2), (3,3) \ldots$
$(a,a) \in R$

ii) Symmetric: $(1,4) \in R \Rightarrow (4,1) \in R$.

iii) Transitive: $(1,4) \in R, (4,7) \in R \Rightarrow (1,7) \in R$.

$\therefore$ It is Equilance Relation.

(H.W)

Consider the following relation $\{1, 2, 3, 4, 5, 6\}$
$R = \{ i, j \,/\, i - j = 2 \}$ is R Transitive ? reflective ? Symmetric ?.

$R = \{ (6,4), (5,3), (4,2), (3,1) \}$.

Draw the ~~diagram~~ diagraph by the given relation
$R = \{ (2,2), (2,3), (2,1), (3,2), (3,4), (4,5) \}$

Show that the relation $R = \{ (1,1), (1,2), (1,3), (2,2), (3,2), (3,3)$
$(4,2)(4,3), (4,4) \}$ is a poset
and draw its hasse diagram. Here set is $\{ 1, 2, 3, 4 \}$.

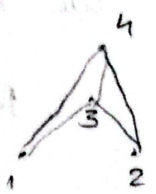
Solution:

$R = \{ (1,1), (2,2), (3,3), (4,4),$
$(1,2), (1,3), (3,2), (4,2), (4,3) \}$

Set $\{ 1, 2, 3, 4 \}$.

Reflective : $(1,1), (2,2), (3,3), (4,4) \in R$
i.e, $(a,a) \in R$

Hesse Diagram :
$(X, \leq)$



Anti-Symmetric :

$(1,2) \in R$, $(2,1) \notin R$.
i.e, $(a,b) \in R$; $(b,a) \notin R$.

Transitive : $(1,3) \in R$ $(3,2) \in R \Rightarrow (1,2) \in R$
$(a,b) \in R$, $(b,c) \in R \Rightarrow (a,c) \in R$.

$\therefore$ R is po set..