MATH 2418: Linear Algebra

Assignment# 7

Due: Tuesday, 10/17/2023, 11:59pm Term: Fall 2023

[Last Name] [First Name] [Net ID] [Lab Section]

Recommended Problems:(Do not turn in)
Sec 3.4: 1, 2, 3, 5, 9, 11, 12, 13, 16, 19, 20, 27, 22, 31, 35
Sec 3.5: 1, 2, 3, 4, 6, 11, 14, 17, 19, 23, 25.

1. Consider $\mathbf{v}_1 = \begin{bmatrix} 1 \\ 3 \\ 3 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 5 \\ 5 \\ 4 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 2 \\ -4 \\ -1 \end{bmatrix}$

   (a) Show that $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ are linearly independent.

   (b) Find the dimension of Span($\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$).

   (c) Is the vector $\mathbf{v} = (3, 5, 7)$ in the Span($\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$)? If so, express the vector $\mathbf{v}$ as a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$.

2. Consider the 3×5 matrix $A =$, with columns

$$u_1 = \begin{bmatrix} 1 \\ 4 \\ 4 \end{bmatrix},\ u_2 = \begin{bmatrix} 5 \\ 1 \\ 2 \end{bmatrix},\ u_3 = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix},\ u_4 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix},\ u_5 = \begin{bmatrix} 1 \\ 2 \\ 5 \end{bmatrix}$$

a) Find a set of vectors in $\{u_1, u_2, u_3, u_4, u_5\}$ which is a basis of the column space of A.

b) Find a basis of the row space of A.

c) Find the rank of A.

3. For each of the following matrices, find all real numbers c, d, and e such that matrices C and D have same rank and then find the rank.

   (a) $C = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & e & 0 & 0 \\ c & 0 & 0 & 0 \\ 0 & 0 & 0 & e \end{bmatrix}$, $D = \begin{bmatrix} d \\ d \\ d \\ d \end{bmatrix}$.

   (b) $C = \begin{bmatrix} 1 & 0 & 1 & 2 \\ 0 & c & 0 & 0 \end{bmatrix}$, $D = \begin{bmatrix} d & 0 \\ 0 & e \end{bmatrix}$.

   (c) $C = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & d & 0 & 0 \\ 0 & 0 & 0 & c \end{bmatrix}$, $D = \begin{bmatrix} d & e \\ c & d \end{bmatrix}$.

4. If $\mathbf{V}$ is the subspace spanned by $(1, 0, 1)$ and $(2, 1, 0)$, find a matrix A that has $\mathbf{V}$ as its row space. Find a matrix B that has $\mathbf{V}$ as its nullspace.

5. Without using elimination, find dimensions and bases for the four subspaces for the matrix

$$A = \begin{bmatrix} 0 & 3 & 2 & 2 \\ 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 1 \end{bmatrix}.$$

6. Find a basis for each of the four subspaces associated with the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 2 & 1 & 0 \\ 2 & 3 & 2 & 1 \end{bmatrix}$$

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