MATH 2418: Linear Algebra

Assignment# 5

Due : Tuesday 09/26; 11:59pm   Term : Fall 2023

[Last Name]	[First Name]	[Net ID]	[Lab Section]

Recommended Problems (do not turn in)

Sec 2.4: 1, 2, 3, 4, 5, 6, 7, 8, 9, 13, 14, 16, 18, 19, 20, 22. **Sec 2.5:** 1, 2, 3, 4, 5, 6, 7, 9, 10, 12, 13, 15, 16, 17, 18.

1. Find the LDL^T decomposition of $A = \begin{bmatrix} 1 & 3 & 2 \\ 3 & 7 & 5 \\ 2 & 5 & 8 \end{bmatrix}$.

2. Find a permutation matrix P so that $PA = LU$.

$$A = \begin{bmatrix} 1 & 2 & 4 \\ 3 & 6 & 8 \\ 0 & 9 & 0 \end{bmatrix}$$

3. (a) Let $A = \begin{bmatrix} 2 & 0 \\ 1 & -2 \end{bmatrix}$, find A^{-1}, $(A^{-1})^T$ and $(A^T)^{-1}$. Is $(A^{-1})^T = (A^T)^{-1}$?

(b) Let $A = \begin{bmatrix} 1 & 0 \\ 1 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 3 & 2 \\ 0 & 1 \end{bmatrix}$, find $A^T B^T$, $B^T A^T$ and $(AB)^T$. Is $(AB)^T = (A)^T (B)^T$?

(c) If A and B are invertible, express $[(AB)^{-1}]^T$ with A^{-1} and B^{-1}.

4. For $y(x) = x^3 - x^2$ and $h = 1/10$, compute these approximations to $\frac{dy}{dx}$ at $x = 0$:

   Centered $\frac{y(h)-y(-h)}{2h}$ **Forward** $\frac{y(h)-y(0)}{h}$ **Backward** $\frac{y(0)-y(-h)}{h}$

5. The derivative of $y = e^x + \sin x$ at $x = 0$ is $\frac{dy}{dx} = 2$. Find the centered-forward-backward approximation to $\frac{dy}{dx}(0)$ using $h = 1$. Which is closest to 2?

6. Solve $-u'' = \cos 4\pi x$ with fixed-fixed conditions $u(0) = u(1) = 0$. Use K_3 to compute $u_1, \ldots, u_n$:

$$u_{i+1} - 2u_i + u_{i-1} = h^2 \cos 4\pi i h \text{ with } u_0 = u_{n+1} = 0.$$