Voronoi-Based Coverage Control of Pan/Tilt/Zoom Camera Networks

Multirobot Systems - Students presentation

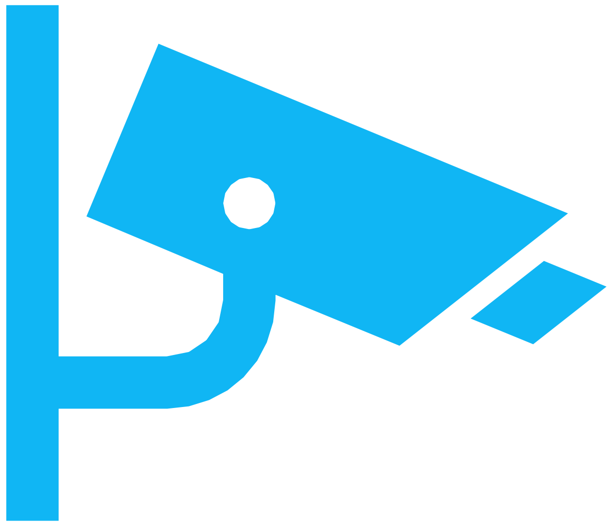
Cerilli, Ghini, Umili

Introduction

A reactive coverage control for PTZ camera networks is provided, given an event distribution over a convex environment.

Introduction

- *Conic Voronoi diagrams* introduced to solve camera network allocation
- Greedy gradient algorithm for continuous- and discrete-time first-order PTZ camera dynamics



Spatial Sensing Models for PTZ Cameras

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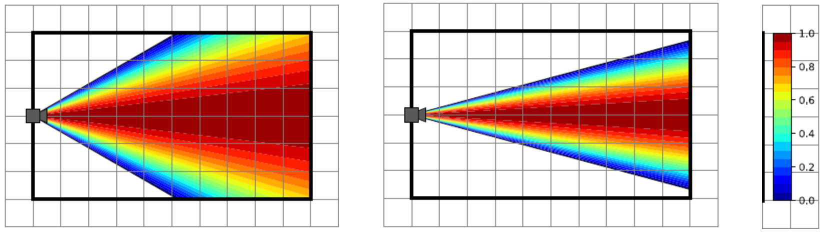
Perspective quality

- Conic field of view in $n \geq 2$
- Fixed position $p \in \mathbb{R}^n$
- Variable optical-axis direction $v \in \mathbb{S}^{n-1}$
- Adjustable angle of view $2\alpha \in (0, \pi)$
- Event at location $x \in \mathbb{R}^n$

$$q_{\text{pers}}(\mathrm{x}) := \frac{1}{1-\cos\alpha}\left(\frac{(\mathrm{x}-\mathrm{p})^T \mathrm{v}}{\|\mathrm{x}-\mathrm{p}\|} - \cos\alpha\right)$$



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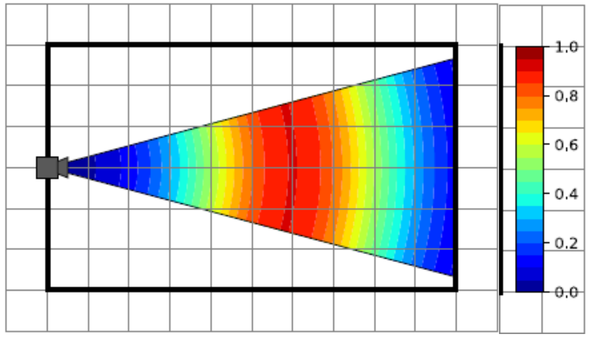
Resolution Quality

UNLIMITED RANGE

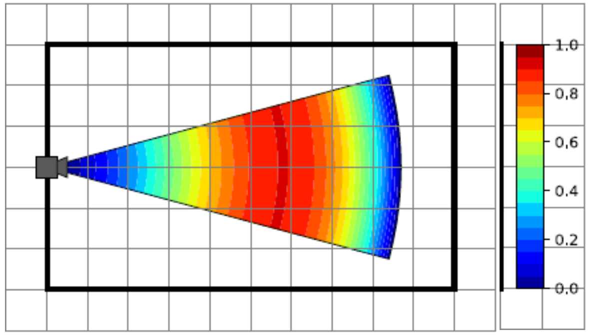
$$Q_{\text{res}}(\mathrm{x}) := \exp\left(-\frac{\left(\|\mathrm{x}-\mathrm{p}\| - \frac{N}{2r^*\alpha}\right)^2}{2\sigma^2}\right)$$



- Desired sensing depth:
  $$\frac{N}{2r^*\alpha}$$
- Spatial resolution variability:
  $\sigma > 0$, $\kappa > 0$, $\lambda \geq 0$

LIMITED RANGE

$$\widehat{Q}_{\text{res}}(\mathrm{x}) := \frac{\|\mathrm{x}-\mathrm{p}\|^\lambda}{\left(\frac{N}{2r^*\alpha}\right)^{\lambda+1}}\left(\frac{N}{2r^*\alpha} - \lambda\left(\|\mathrm{x}-\mathrm{p}\| - \frac{N}{2r^*\alpha}\right)\right)$$



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Simplified resolution quality

UNLIMITED RANGE

$$q_{\text{res}}(\mathrm{x}) := \cos^{\kappa}\alpha \, \exp\left(-\frac{(\|\mathrm{x}-\mathrm{p}\|-R)^2}{2\sigma^2}\right)$$

LIMITED RANGE

$$\widehat{q}_{\text{res}}(\mathrm{x}) := \frac{\|\mathrm{x}-\mathrm{p}\|^{\lambda}}{R^{\lambda+1}}\Big(R\cos\alpha - \lambda\big(\|\mathrm{x}-\mathrm{p}\| - R\cos\alpha\big)\Big)$$

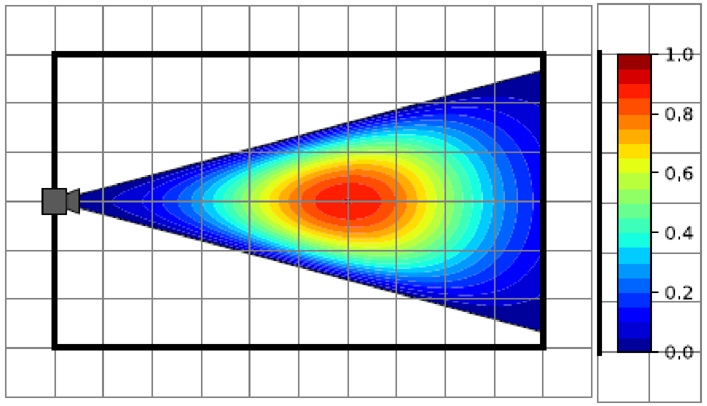
- Fixed desired sensing range: $R > 0$
- Depth at which $\hat{q}_{res}$ is miximized depends on camera's angle view

Spatial sensing quality

UNLIMITED RANGE

$$q(\mathrm{x}) := q_{\mathrm{pers}}(\mathrm{x})\, q_{\mathrm{res}}(\mathrm{x})$$

$$C := \left\{ \mathrm{x} \in \mathbb{R}^n \middle| q_{\mathrm{pers}}(\mathrm{x}) \geq 0 \right\} = \left\{ \mathrm{x} \in \mathbb{R}^n \middle| \frac{(\mathrm{x}-\mathrm{p})^T \mathrm{v}}{\|\mathrm{x}-\mathrm{p}\|} \geq \cos\alpha \right\}$$

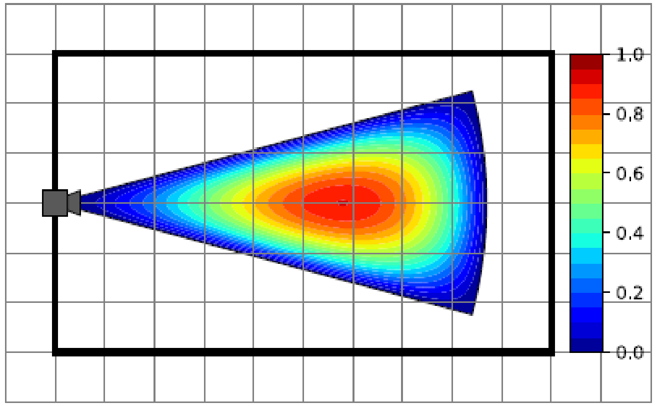


Spatial sensing quality

LIMITED RANGE

$$\widehat{q}\,(\mathrm{x})\ := q_{\mathrm{pers}}\ (\mathrm{x})\,\widehat{q}_{\mathrm{res}}\ (\mathrm{x})$$

$$\widehat{C} := \left\{ \mathrm{x} \in \mathbb{R}^n \,\middle|\, q_{\mathrm{pers}}(\mathrm{x}) \geq 0,\, q_{\mathrm{res}}(\mathrm{x}) \geq 0 \right\},$$
$$= \left\{ \mathrm{x} \in \mathbb{R}^n \,\middle|\, \frac{(\mathrm{x}-\mathrm{p})^{\mathrm{T}}\mathrm{v}}{\|\mathrm{x}-\mathrm{p}\|} \geq \cos\alpha,\, \|\mathrm{x}-\mathrm{p}\| \leq \frac{\lambda+1}{\lambda} R\cos\alpha \right\}$$



Optimal Sensor Allocation in PTZ Camera Networks

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Coverage objective

GIVEN:

- A convex bounded environment *W* in $\mathbb{R}^n$
- An event distribution function $\Phi : W \rightarrow \mathbb{R}^+$
- *m* identical PTZ cameras described by
  - fixed locations $p := (p_1, p_2, \dots, p_m) \in (\mathbb{R}^n)^m$
  - optical-axis directions $v := (v_1, v_2, \dots, v_m) \in (\mathrm{S}^{n-1})^m$
  - (halves the) angles of view $\alpha := (\alpha_1, \alpha_2, \dots, \alpha_m) \in (0, \frac{\pi}{2})^m$

FIND:

A partition $P = \{P_1, P_2, \dots, P_m\}$ of *W* so to maximize the total spatial sensing quality of cameras

$$H(\mathcal{P}) := \sum_{i=1}^{m} \int_{P_i \cap C_i} q_i(\mathrm{x}) \, \phi(\mathrm{x}) \, \mathrm{dx}$$

Conic Voronoi diagrams

OPTIMAL ALLOCATION STRATEGY

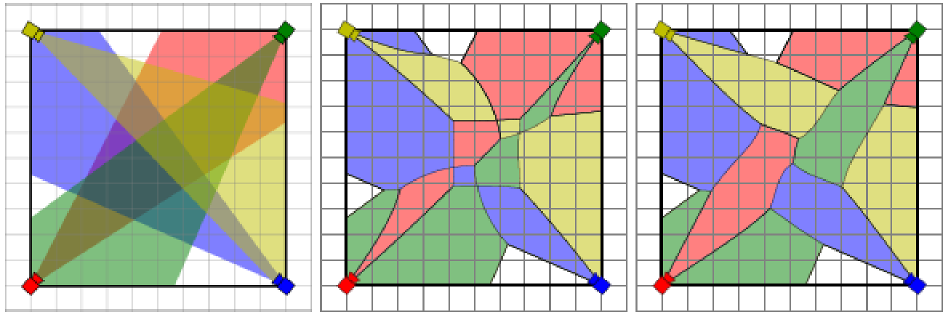
Assign each event location *x* to the i-th cameras such as

$$i = argmax_i \; q_i(x)$$

CONIC VORONOI DIAGRAM OF W

V = $\{V_1, V_2, \dots, V_m\}$ partition of the visible $W \cap \bigcup_{i=1}^{m} C_i$

$$V_i := \left\{ \mathrm{x} \in W \cap C_i \middle| q_i(\mathrm{x}) \geq q_j(\mathrm{x}) \quad \forall j \neq i \right\}$$



UTILITY FUNCTION IN NEW FORM

$$H_{\mathcal{V}} = \sum_{i=1}^{m} \int_{V_i} q_i(\mathrm{x}) \, \phi(\mathrm{x}) \, \mathrm{dx}$$

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Locally Optimal Coverage Configurations

UNLIMITED-RANGE VISUAL SENSING

- Mass
- Centroidal perspective
- Centroidal aperture
- Centroidal angle of view

$$\mu_{V_i} := \int_{V_i} e^{-\frac{(\|x-p_i\|-R)^2}{2\sigma^2}} \phi(x)\,dx$$

$$v_{V_i} := \frac{1}{\mu_{V_i}} \int_{V_i} \frac{x-p_i}{\|x-p_i\|} e^{-\frac{(\|x-p_i\|-R)^2}{2\sigma^2}} \phi(x)\,dx$$

$$\delta_{V_i} := 1 - {v_{V_i}}^T v_i$$

$$\alpha_{V_i} := \arccos\left(1 - \frac{(\kappa-1)\delta_{V_i} + \sqrt{(\kappa-1)^2 {\delta_{V_i}}^2 + 4\kappa\delta_{V_i}}}{2\kappa}\right)$$

THEOREM

For unlimited-range visual sensing, a PTZ camera network configuration is locally optimal in the sense of H_V if and only if all cameras look towards their centroidal perspectives with centroidal angles of view.

Locally Optimal Coverage Configurations

LIMITED-RANGE VISUAL SENSING

- Mass
- Centroidal perspective
- Centroidal aperture
- Centroidal angle of view

$$\widehat{\mu}_{V_i} := \int_{V_i} \frac{\|\mathrm{x} - \mathrm{p}_i\|^\lambda}{R^\lambda} \phi(\mathrm{x})\, \mathrm{dx}$$

$$\widehat{\mathrm{v}}_{V_i} := \frac{1}{\widehat{\mu}_{V_i}} \int_{V_i} \frac{\mathrm{x}-\mathrm{p}_i}{\|\mathrm{x}-\mathrm{p}_i\|} \left(\cos\alpha_i - \frac{\lambda\|\mathrm{x}-\mathrm{p}_i\|}{(\lambda+1)R}\right) \frac{\|\mathrm{x}-\mathrm{p}_i\|^\lambda}{R^\lambda} \phi(\mathrm{x})\, \mathrm{dx}$$

$$\widehat{\delta}_{V_i} := \frac{1}{\widehat{\mu}_{V_i}} \int_{V_i} \left(1 - \frac{(\mathrm{x}-\mathrm{p}_i)^{\mathrm{T}}\mathrm{v}_i}{\|\mathrm{x}-\mathrm{p}_i\|}\right)\left(1 - \frac{\lambda\|\mathrm{x}-\mathrm{p}_i\|}{(\lambda+1)R}\right) \frac{\|\mathrm{x}-\mathrm{p}_i\|^\lambda}{R^\lambda} \phi(\mathrm{x}) \mathrm{dx}.$$

$$\widehat{\alpha}_{V_i} := \arccos\left(1 - \sqrt{\widehat{\delta}_{V_i}}\right)$$

THEOREM

For limited-range visual sensing, at a locally optimal PTZ camera coverage configuration of H_V , all cameras are directed at the centroidal perspectives of their respective Voronoi cells with the associated centroidal angles-of-views.

Coverage control of PTZ cameras

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Continuous-Time Camera Dynamics

“MOVE-TO-CENTROIDAL-PERSPECTIVE” AND “MOVE-TO-CENTROIDAL-ANGLE-OF-VIEW” CONTROL LAWS

$$\dot{\mathrm{v}}_i = K_{\mathrm{v}} \left(\mathbf{I} - \mathrm{v}_i {\mathrm{v}_i}^{\mathrm{T}} \right) \mathrm{v}_{V_i}$$

$$\dot{\alpha}_i = -K_{\alpha} \left(\alpha_i - \alpha_{V_i} \right)$$

THEOREM

The continuously differentiable laws leave camera’s angles of view, $2\alpha i$, positively invariant in $(0, \pi)$, and asymptotically bring a PTZ camera network to a locally optimal coverage configuration of H_V while strictly increasing the total coverage quality H_V along the way.

Discrete-time Camera Dynamics

"MOVE-TO-CENTROIDAL-PERSPECTIVE" AND "MOVE-TO-CENTROIDAL-ANGLE-OF-VIEW" CONTROL LAWS

$$\mathrm{v}_i[k+1] = \frac{\mathrm{v}_{V_i(\mathbf{v}[k],\boldsymbol{\alpha}[k])}}{\left\|\mathrm{v}_{V_i(\mathbf{v}[k],\boldsymbol{\alpha}[k])}\right\|}$$

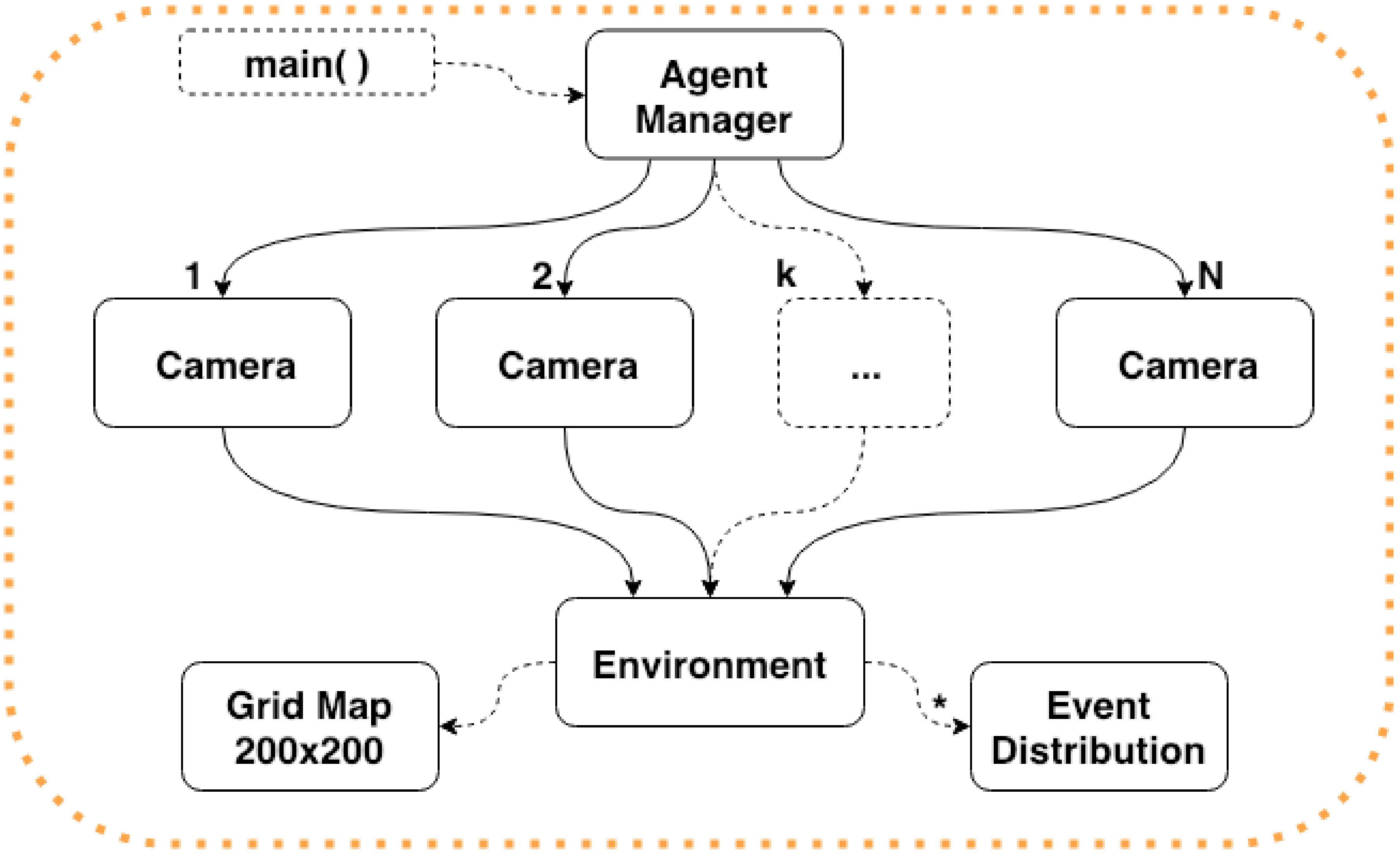
$$\alpha_i[k+1] = \alpha_{V_i(\mathbf{v}[k+1],\boldsymbol{\alpha}[k])}$$

THEOREM

The total coverage quality H_V of a PTZ camera network increases at each iteration of the laws until asymptotically reaching a locally optimal coverage configuration. Further, each iteration yields a valid camera angle of view, 2αi, in (0, π).

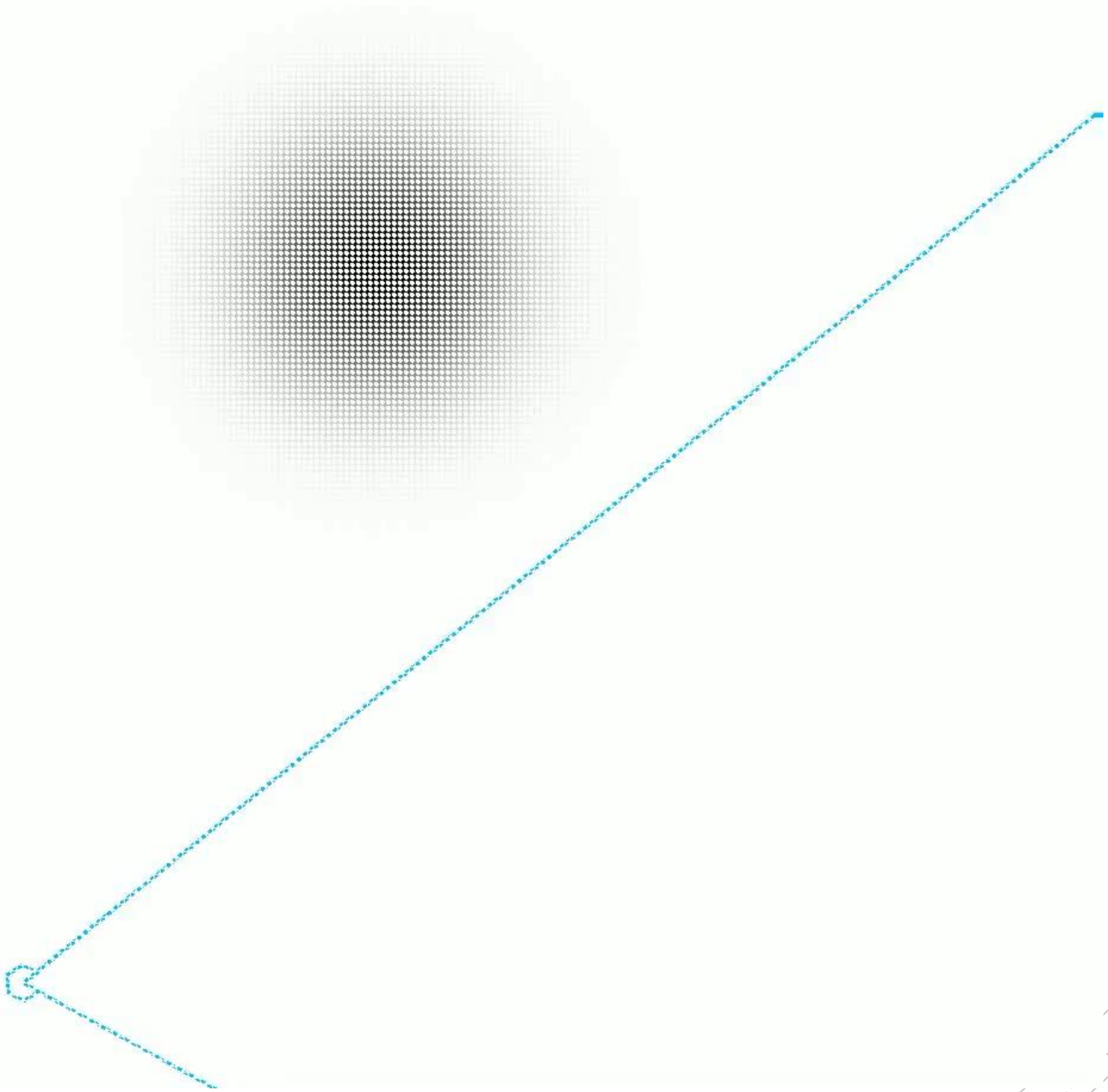
Numerical simulations

Implementation Logic



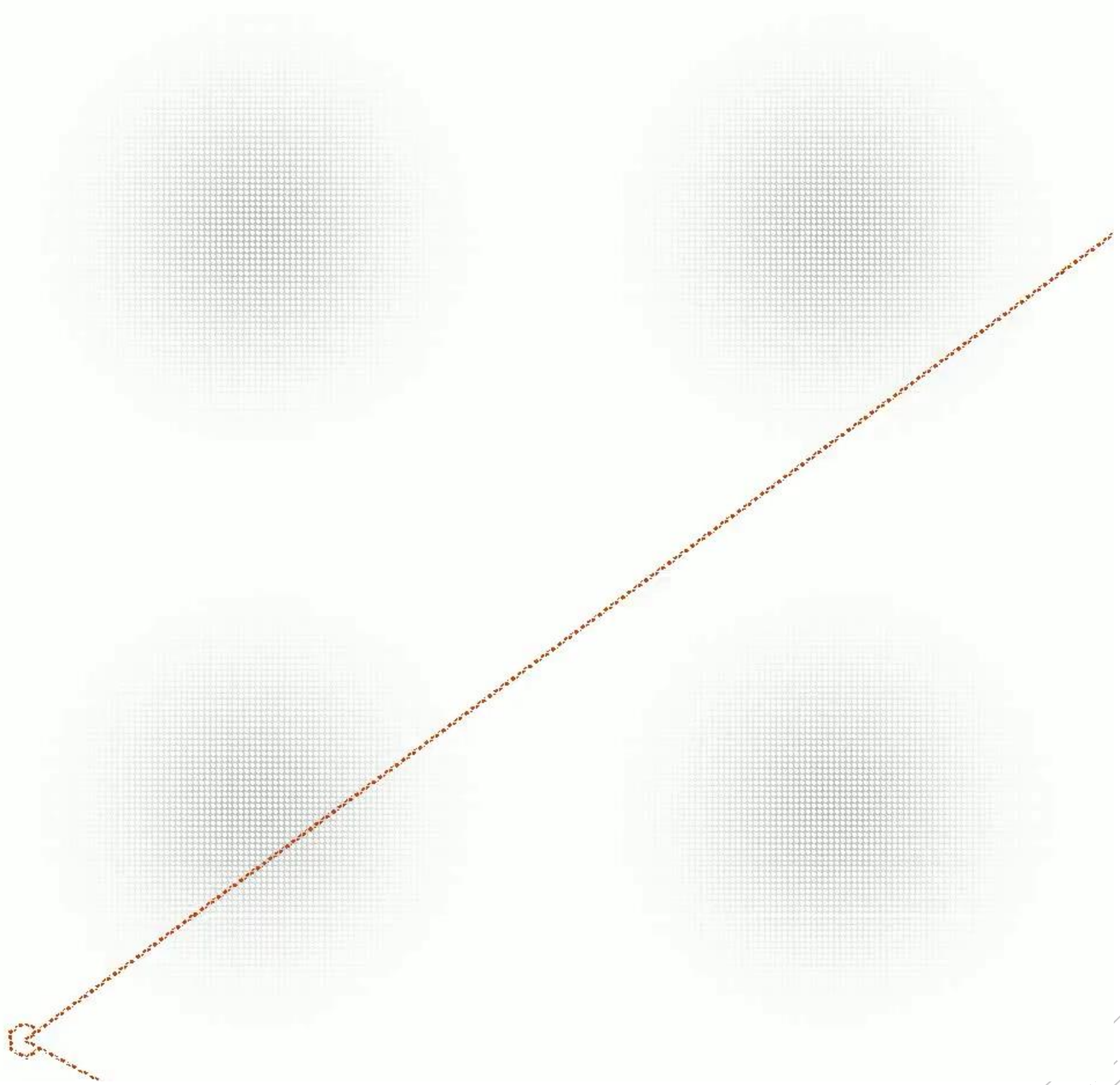
Single camera convergence

R = 7 $\kappa = 3$
$\sigma = 2$ $\lambda = 2$



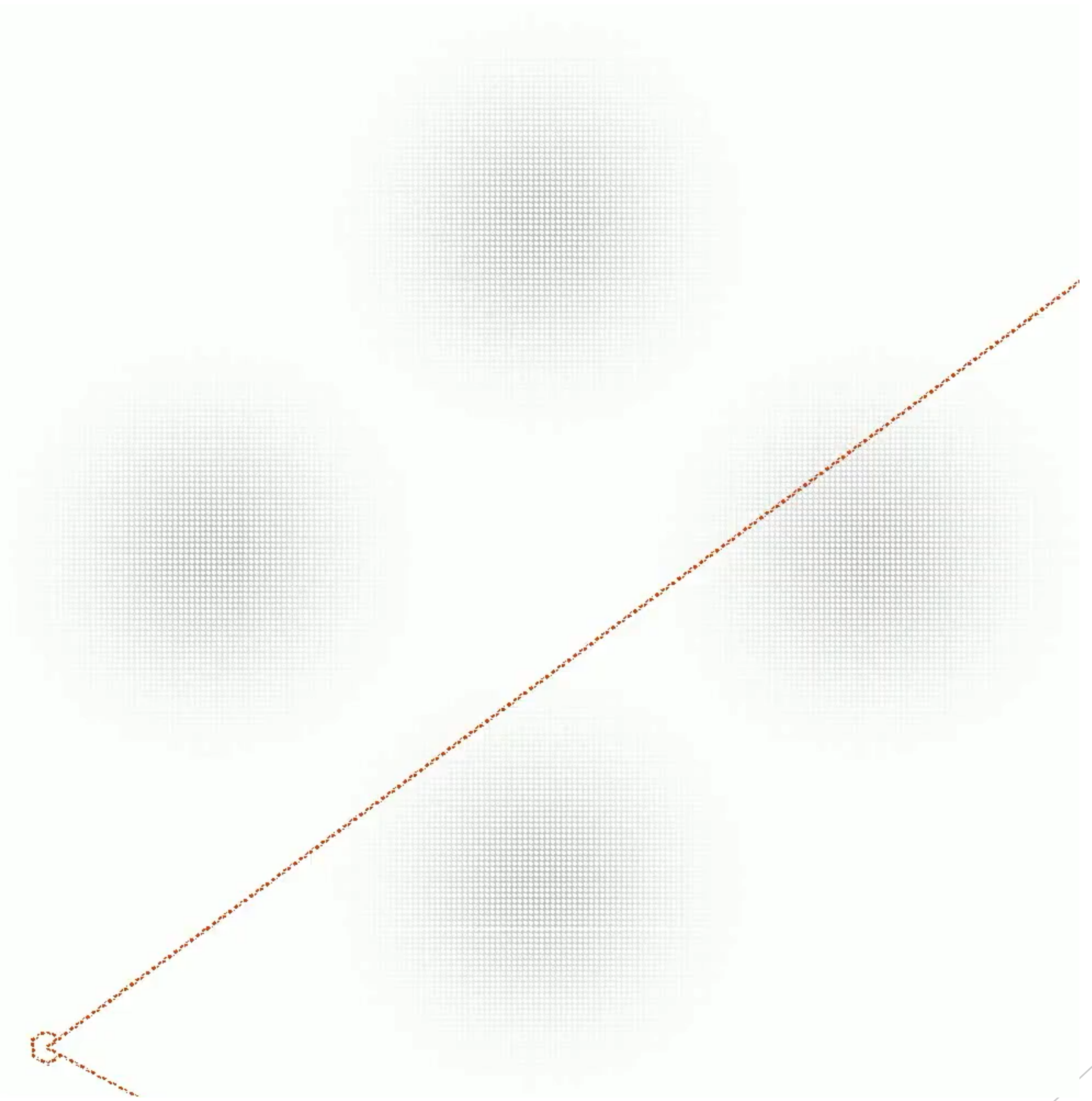
Each cam covers the farthest event

$R = 7$ $\kappa = 3$
$\sigma = 2$ $\lambda = 2$



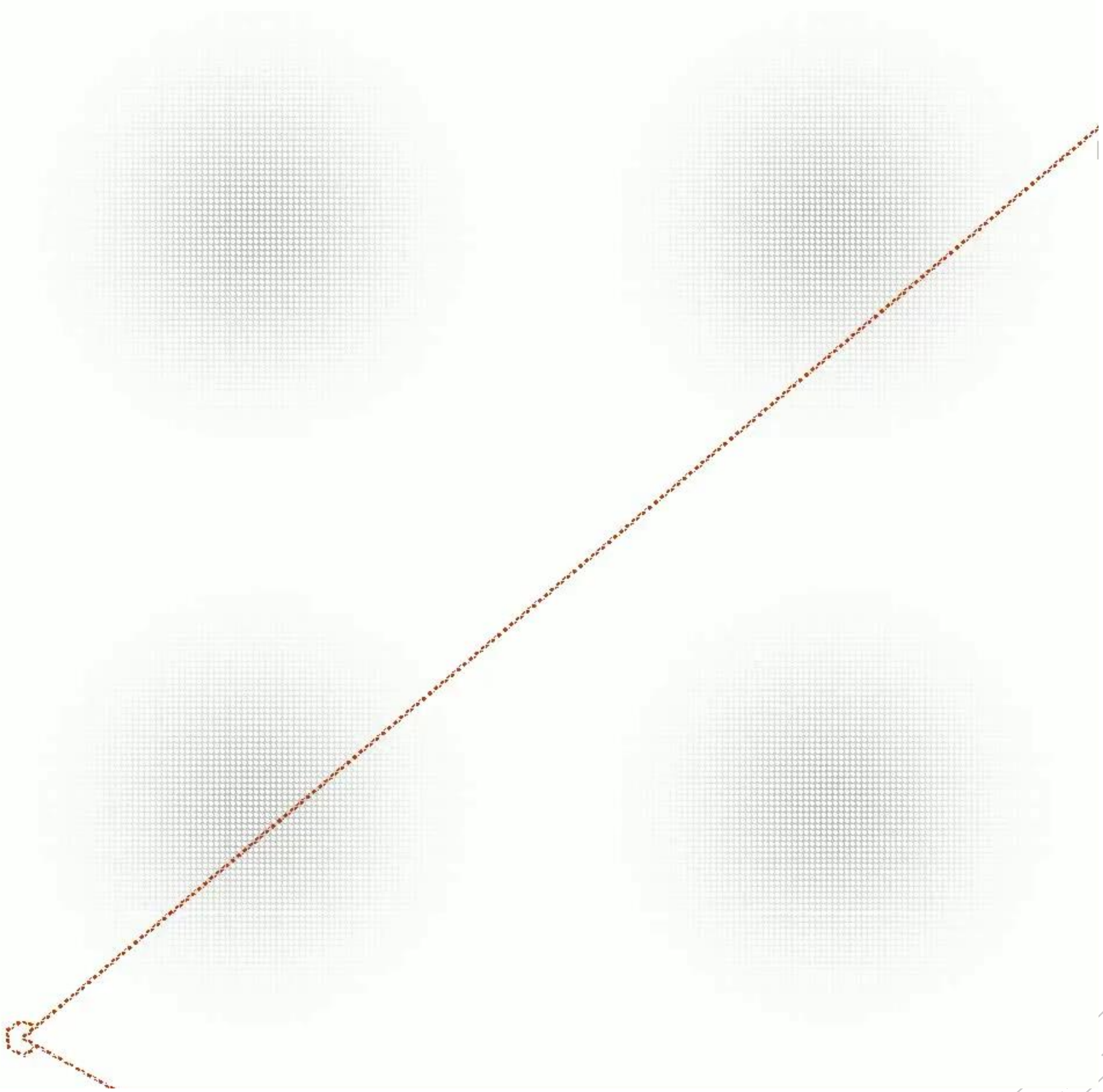
Each camera covers two events

R = 7 $\kappa = 3$
$\sigma = 2$ $\lambda = 2$



Cameras cover the event on their left

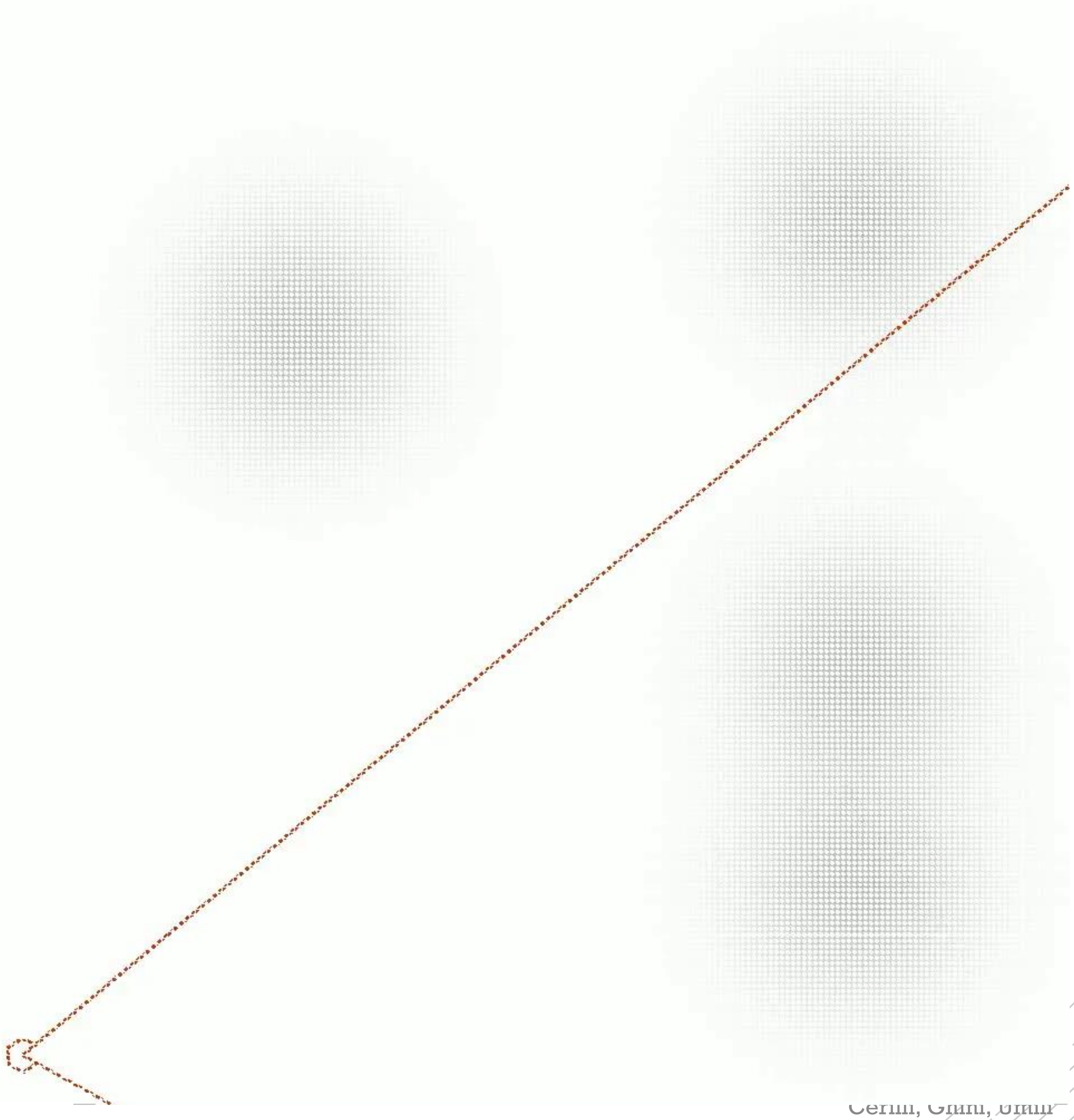
$R = 7$ $\kappa = 3$
$\sigma = 2$ $\lambda = 2$



Complex event distribution coverage

$R = 7$ $\kappa = 3$

$\sigma = 2$ $\lambda = 2$

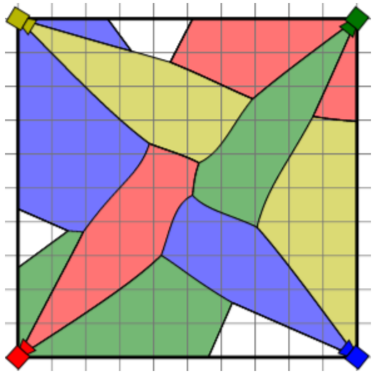


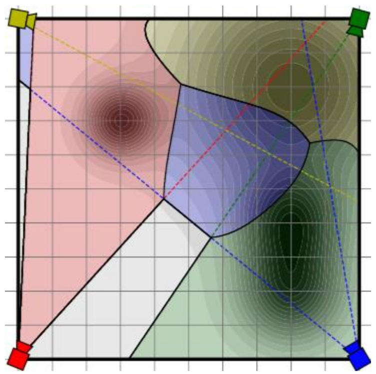
Conclusions

Contributions

- Defined two new quality measures:
  - Perspective quality
  - Resolution quality

- Introduced a greedy control law that, given an optimal *conic Voronoi* allocation, converges asymptotically to a locally optimal coverage configuration





Future improvements

- Online vision-based event distribution estimation
- Extension to non-convex environments and environments with visual occlusions
- Optimal coverage control in mobile PTZ camera networks for active visual monitoring