

# **Vibration Suppression Design for Virtual Compliance Control in Bilateral Teleoperation**

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# Introduction

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# Problem statement

Application of one degree of freedom inertia-spring-damper system for **vibration suppression** in a bilateral control system.

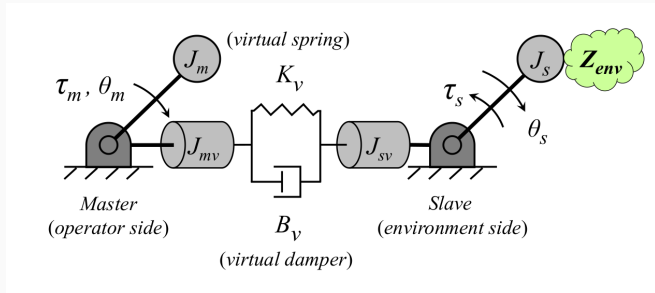
System compliance for different input frequency controlled by the value of virtual elements (based on the cut-off frequencies and stiffness of the virtual spring).

1. **Stability** of the closed loop system irrespective to the behaviour of the human and the environment
2. **Transparency** of the teleoperation task: same forces and displacements on the two sides of the system

# System Modeling

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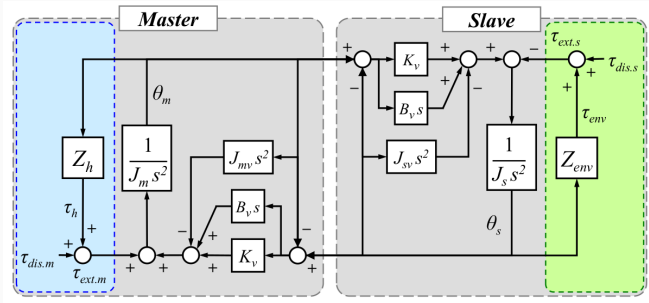
# Inertia-Spring-Damper System



Position error based torque reflection to the operator at the master side.

- Real inertias:  $J_m, J_s$
- Virtual inertias:  $J_{mv}, J_{sv}$
- Virtual damper and spring:  $B_v, K_v$

# Bilateral Control Scheme



Dynamics equations in frequency domain:

$$\begin{aligned} (J_m + J_{mv})s^2\theta_m + (B_v s + K_v)(\theta_m - \theta_s) &= \tau_m \\ (J_s + J_{sv})s^2\theta_s + (B_v s + K_v)(\theta_s - \theta_m) &= -\tau_s \end{aligned}$$



# System Analysis

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# Bilateral Controller

External torques as action and reaction torque of human and environment impedance,  $Z_h$  and  $Z_{env}$ :

$$\begin{aligned}J_ms^2\theta_m &= \tau_m - (B_vs + K_v)(\theta_m - \theta_s) - J_{mv}s^2\theta_m \\J_ss^2\theta_s &= -\tau_s - (B_vs + K_v)(\theta_s - \theta_m) - J_{sv}s^2\theta_s\end{aligned}$$

System represented by  $2 \times 2$  hybrid matrix as:

$$\begin{bmatrix} \tau_m \\ \theta_s \end{bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{bmatrix} \theta_m \\ -\tau_s \end{bmatrix}$$

Hybrid parameters  $H_{ij}$ :

$$H_{11} = \frac{1}{Z_s} [Z_m Z_s - (B_v s + K_v)^2]$$

$$H_{12} = -\frac{1}{Z_s} [B_v s + K_v]$$

$$H_{21} = \frac{1}{Z_s} [B_v s + K_v]$$

$$H_{22} = \frac{1}{Z_s}$$

where:

$$Z_m = (J_m + J_{mv})s^2 + B_v s + K_v$$

$$Z_s = (J_s + J_{sv})s^2 + B_v s + K_v$$

*Condition of transparency:* Transmitted impedance  $Z_t$  transferred to the operator should be equal to environment impedance  $Z_{env}$ :

$$\frac{\tau_m}{\theta_m} = Z_t = Z_{env} = \frac{\tau_s}{\theta_s}$$

Relationship between  $Z_t$  and  $Z_{env}$ :

$$Z_t = \left( \frac{-H_{12}H_{21}}{1 + H_{22}Z_{env}} \right) Z_{env} + H_{11}$$

Hybrid parameters for the perfect transparency condition, derived as:

$$\begin{bmatrix} \tau_m \\ \theta_s \end{bmatrix} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} \theta_m \\ -\tau_s \end{bmatrix}$$

# **Vibration Suppression Design**

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# Parameters Selection and Design

Assume system disturbed by environment vibration noise.

Inspect  $H_{22}$  for the relationship of position response  $\theta^{res}$  and external torque input  $\tau_{ext}$ :

$$\frac{\theta_s}{\tau_{ext}} = \frac{1}{(J_s + J_{sv})s^2 + B_v s + K_v}$$

Virtual parameters  $J_v$ ,  $B_v$  and  $K_v$  determined from the second-order characteristic equation:

$$s^2 + (g_1 + g_2)s + (g_1 * g_2) = 0$$

established from two poles  $g_1$  and  $g_2$ , representing the desired cut-off frequencies of the system for disturbance suppression.

# Parameters Selection and Design

Spring stiffness  $K_v$  influences the behaviour of the system.

It regulates the *compliance* of this, achieving **rigid coupling** (high spring stiffness) or **spring coupling** (low spring stiffness).

Virtual parameters  $B_v$  and  $J_v$  computed according to  $K_v$ :

$$B_v = \left( \frac{g_1 + g_2}{g_1 * g_2} \right) K_v$$

and

$$J_{sv} = \left( \frac{1}{g_1 * g_2} \right) K_v - J_s$$

# Vibration Suppression Performance Analysis



# Results

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VIDEO

# Conclusion

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# Summary

**Thank you for your attention!**