



SAPIENZA
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Vibration Suppression Design for Virtual Compliance Control in Bilateral Teleoperation

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Locomotion and haptic interfaces for VR exploration

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Introduction

Problem statement

Goal

Development of a controller able to suppress the vibration and unwanted inputs in a bilateral control system.

Idea

Application of one degree of freedom *virtual* spring-damper system with an additional inertia.

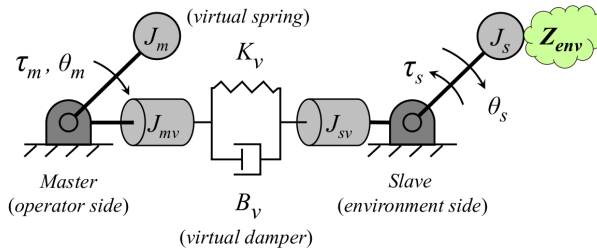
The disturbance suppression performances depend on the value of these virtual parameters, determined from the desired cut-off frequencies.

Goal of the teleoperation

1. **Stability** of the closed loop system irrespective to the behaviour of the human and the environment
2. **Transparency** of the teleoperation task: same forces and displacements on the two sides of the system

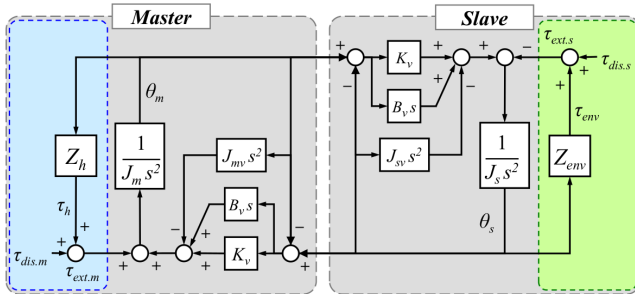
System Modeling

Inertia-Spring-Damper System



- Real inertiae: J_m, J_s
- Virtual inertiae: J_{mv}, J_{sv}
- Virtual damper and spring: B_v, K_v

Bilateral Control Scheme



Dynamics equations of the system:

$$\begin{aligned} (J_m + J_{mv})\ddot{\theta}_m + B_v(\dot{\theta}_m - \dot{\theta}_s) + K_v(\theta_m - \theta_s) &= \tau_m \\ (J_s + J_{sv})\ddot{\theta}_s + B_v(\dot{\theta}_s - \dot{\theta}_m) + K_v(\theta_s - \theta_m) &= -\tau_s \end{aligned}$$

Bilateral Controller

The virtual parameters are considered elements of the controller.
Equations are rearranged:

$$\begin{aligned}J_m s^2 \theta_m &= \tau_m - (B_v s + K_v)(\theta_m - \theta_s) - J_{mv} s^2 \theta_m \\J_s s^2 \theta_s &= -\tau_s - (B_v s + K_v)(\theta_s - \theta_m) - J_{sv} s^2 \theta_s\end{aligned}$$

where the external torques are action and reaction forces of the human and the environment.

System represented by:

$$\begin{bmatrix} \tau_m \\ \theta_s \end{bmatrix} = \underbrace{\begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix}}_{\text{Hybrid matrix}} \begin{bmatrix} \theta_m \\ -\tau_s \end{bmatrix}$$

Hybrid parameters H_{ij} :

$$H_{11} = \frac{1}{Z_s} [Z_m Z_s - (B_v s + K_v)^2]$$

$$H_{12} = -\frac{1}{Z_s} [B_v s + K_v]$$

$$H_{21} = \frac{1}{Z_s} [B_v s + K_v]$$

$$H_{22} = \frac{1}{Z_s}$$

where:

$$Z_m = (J_m + J_{mv})s^2 + B_v s + K_v$$

$$Z_s = (J_s + J_{sv})s^2 + B_v s + K_v$$

Condition of transparency: trasmitted impedance Z_t transferred to the operator should be equal to environment impedance Z_{env} :

$$\frac{\tau_m}{\theta_m} = Z_t = Z_{env} = \frac{\tau_s}{\theta_s}$$

Relationship between Z_t and Z_{env} :

$$Z_t = \left(\frac{-H_{12}H_{21}}{1 + H_{22}Z_{env}} \right) Z_{env} + H_{11}$$

Vibration Suppression Design

Parameters Selection and Design

Assume system disturbed by environment vibration noise.

Inspect H_{22} for the relationship of position response θ_s and external torque input τ_{ext} :

$$\frac{\theta_s}{\tau_{ext}} = \frac{1}{(J_s + J_{sv})s^2 + B_v s + K_v}$$

Virtual parameters J_v , B_v and K_v determined from the characteristic equation:

$$s^2 + (g_1 + g_2)s + (g_1 \cdot g_2) = 0$$

where the poles g_1 and g_2 represent the desired cut-off frequencies.

Parameters Selection and Design

Spring stiffness K_v influences the transparency.

It regulates the *compliance* of the system, achieving **rigid coupling** (high spring stiffness) or **spring coupling** (low spring stiffness).

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Virtual parameters B_v and J_v computed according to K_v :

$$B_v = \left(\frac{g_1 + g_2}{g_1 \cdot g_2} \right) K_v$$

and

$$J_{sv} = \left(\frac{1}{g_1 \cdot g_2} \right) K_v - J_s$$

Simulations

Simulation conditions

Simulation are done in ideal condition:

- the communication between master and slave have no critical aspects;
- instantaneous and loss-less signal transfer.

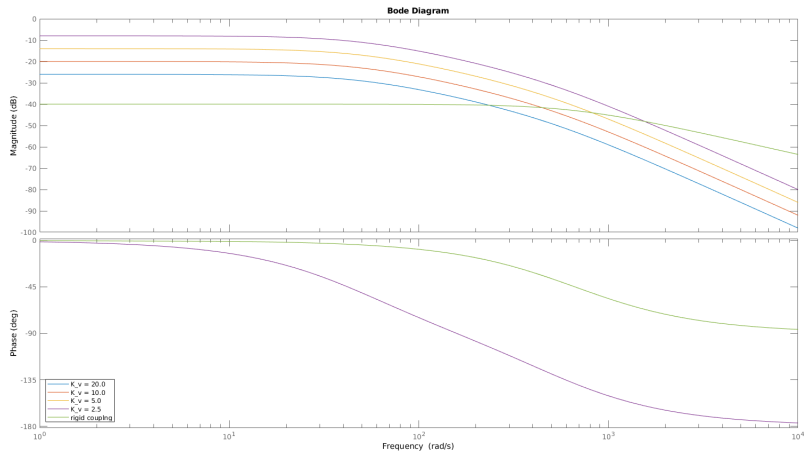
We will compare two different settings:

Behaviour	K_v	B_v	J_v
Virtual compliance	$20.0 \frac{\text{N m}}{\text{rad}}$	$4.4 \cdot 10^{-1} \frac{\text{N m}}{\text{rad/s}}$	$3 \cdot 10^{-4} \text{ kg m}^2$
Rigid coupling	$10^2 \frac{\text{N m}}{\text{rad}}$	$1.5 \cdot 10^{-1} \frac{\text{N m}}{\text{rad/s}}$	0 kg m^2

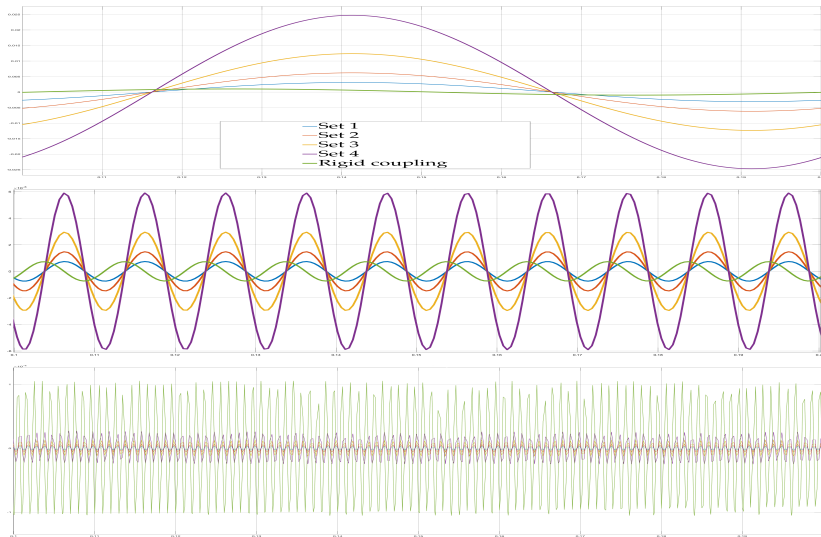
Table 1: Sets of chosen virtual parameters.

Disturbance response

$$\frac{\theta_s}{\tau_{ext}} = \frac{1}{(J_s + J_{sv})s^2 + B_v s + K_v}$$



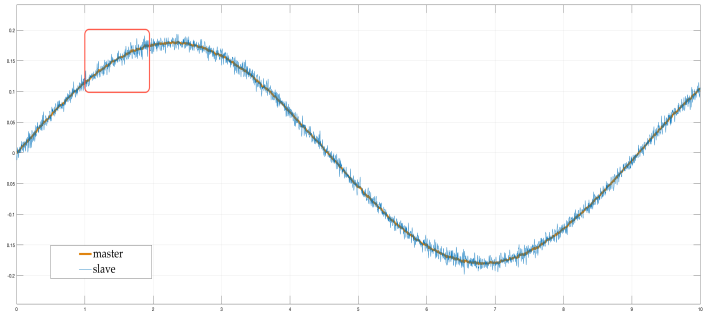
Vibration suppression



Noise frequency : 10 Hz | 10^2 Hz | 10^3 Hz

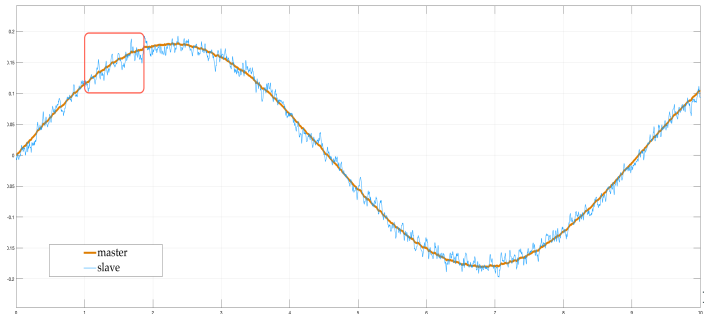
Free Motion: High frequency noise

**Rigid
coupling:**



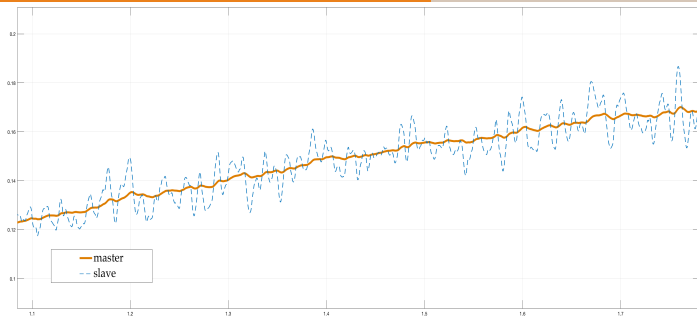
**High
frequency: 50
Hz**

**Virtual
compliance:**



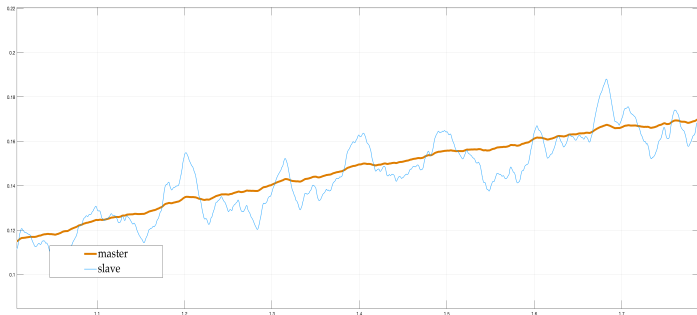
Free Motion: High frequency noise

**Rigid
coupling:**



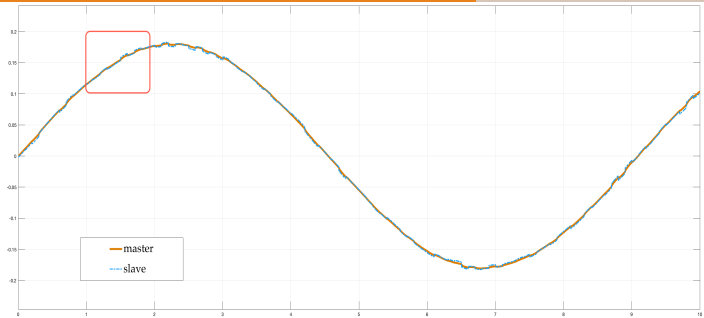
**High
frequency: 50
Hz**

**Virtual
compliance:**



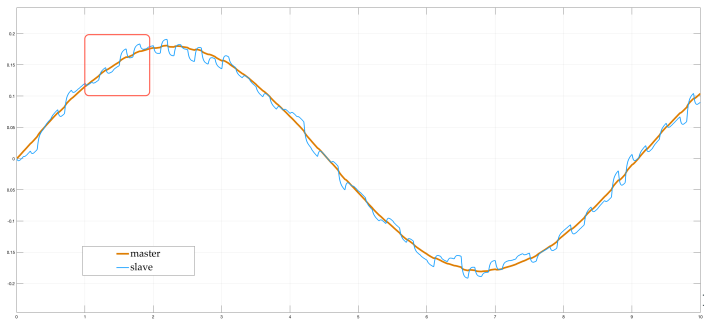
Free Motion: Low frequency noise 20 Hz

**Rigid
coupling:**



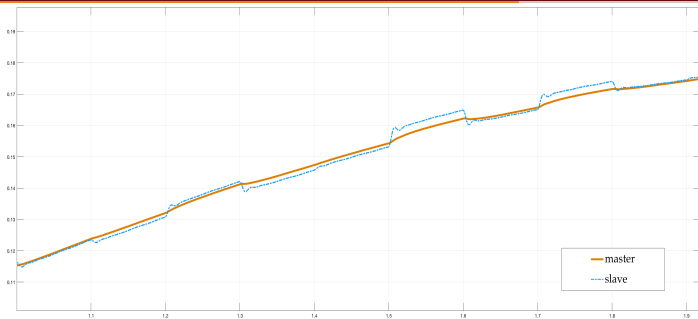
**Low
frequency: 20
Hz**

**Virtual
compliance:**

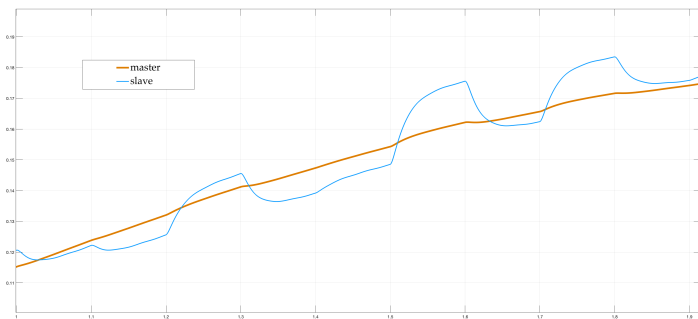


Free Motion: Low frequency noise 20 Hz

**Rigid
coupling:**



**Low
frequency: 20
Hz**



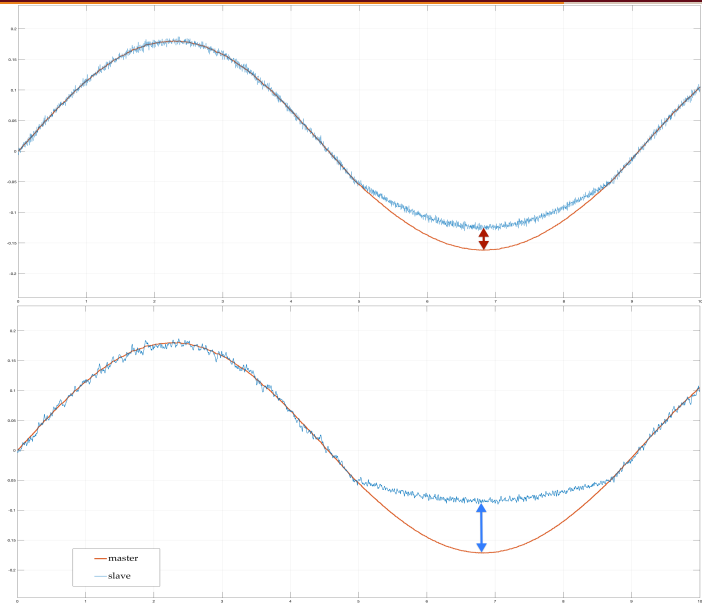
**Virtual
compliance:**

Task execution: In contact

**Rigid
coupling:**
lower error

**Position
tracking
error**

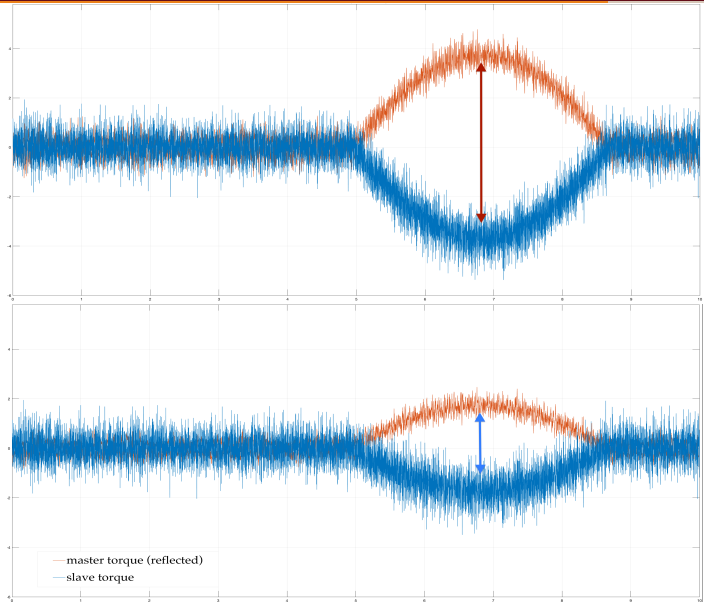
**Virtual
compliance:**
higher error



Task execution: In contact

Rigid coupling:
higher torque
noise

Virtual compliance:
lower torque
noise



Task execution: In contact

Virtual compliance :

Task execution: In contact

Rigid coupling:

Conclusions

Proposed solution:

- based on **virtual** spring-dumper system with additional inertia;
- desired **cut-off** frequencies obtained regulating the **virtual** spring stiffness.

Consequences:

- the proposed controller preserves the useful (*low*) frequency inputs and reject the noisy ones (*high*).
- In contact motion it leads to a position gap between master and slave. It can be used only in **soft material handling** tasks.

Thank you for your attention!