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Notes on constructing linking algorithms for searches of interstellar objects

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1 INTRODUCTION

In these notes, we construct an algorithm consisting of a linking algorithm Holman et al. (2018) and orbit fitting routing, Orbfit, that can find interstellar, hyperbolic, orbits (IOs) from observation files like the Minor Planet Center's Isolated Tracklet File (ITF). We first validate our algorithm. We construct mock, plausible IO observations and integrating such orbits in time. We carefully study the errors of our reconstructed orbits from our algorithm.

2 LINKING AND ORBFIT VALIDATION ON HYPERBOLIC OBJECT MOCK DATA

We first create a set of mock observations of Interstellar Objects (IOs) on hyperbolic orbits that plausibly might pass through the Solar System and have a magnitude and distance form Earth such that they would be observable by one of the Earth-based observatories. Our goal is to have a simple test of our linking and orbit fitting algorithm's ability to reconstruct the mock orbits, and in this test, we are not concerned with constructing a realistic population of IOs. To obtain these observations, we use the format used by the Minor Planet Center, obs80[1]. The Minor Planet Center is the official organization to collect observations of minor planets under International Astronomical Union.

In order to generate the mock observations, we are guided by the observations of the first discovered IO, 1I/'Oumuamua. Its first observation is,

```
0001IK17U010 C2017 10 14.43936 04 49 12.95 -02 29
47.4 19.0 GU@6548703
```

We note a few important parts. The right ascension (RA) is $r_0 =$ 04 hrs, 49 min, 12.95 s in a J2000 equatorial frame with origin at the Sun. The declination (Dec), in the same frame is $-2°$, 29 min, and 47.4 s. The observation time, t_0, in Coordinate Universal Time (UTC) is day 14.43936 of the 10th month (October) of 2017. 703 indicates an observatory code, in this case for the Catalina Sky Survey (CSS). The CSS focuses on searches for near-Earth objects (NEOs). A library lists the geocentric positions of all observatories in an International Terrestrial Reference System. Given a time in UTC, we can obtain a rotation matrix that gives coordinates in the J2000 frame described above.

In our mock IOs, the first observation has time t_0. We set an initial velocity vector randomly. The eccentricity is chosen to be 500 linearly spaced values between 1.1 and 3.0. A velocity magnitude can the be derived, given the positions. We are constructing an IO orbit which we will integrate in time, and we set a guess for the observatory–IO distance at $s_0 = 0.2$ au, similar to the 'Oumuamua–observatory distance of 0.16 au at t_0. We set a guess for the RA and Dec of the IO at t_0 to be the same as 'Oumuamua's RA and dec at t_0. We denote them r_0 and d_0, respectively. However, we need the RA, Dec, and distance corrected for the light travel time, when the IO is at a time slightly earlier than t_0. We describe the light travel time correction in more detail below. The initial semi-major axes, a, of the 500 particles, range from -10.5 to 0.038 au, with mean -1.20 au. a tends to increase with eccentricity. The pericenter, $a(e-1)$, has no clear trend, though.

To produce mock observations, we use the Rebound framework (Rein & Liu 2012). We set a simple model with only Newtonian gravity and the eight planets. We discuss the validity of this model below. There will be up to 192 observations, with the same observation times and same observatories as 'Oumuamua. There are 192 obs80 ground-based observations of 'Oumuamua, and the observations span 38 days. The observations are comprised of 26 distinct ground-based observatories. There are also 29 two-line satellite observations for 'Oumuamua. The second line gives the satellite position in an equatorial geocentric vector in km. Because satellite observations are 13% of the 'Oumuamua observations, we do not incorporate them into our mock obs80 observations. The satellite observations span 42 days.

Rebound queries the positions of the planets and Sun in a J2000 ecliptic coordinate frame, with a time specified in Barycentric Dynamical Time (TDB). Rebound queries the coordinates to the nearest minute, so there is an error in their coordinates. We tried inputting directly the ephemeris of Sun and Planets at t_0, with a query from JPL Horizons at the exact time, but our errors were not significantly affected, meaning the error in our IO is dominated by other errors like model errors. We can then transform to the center of mass frame and compare to the Horizons Solar System barycenter at t_0. The position errors committed in the Sun–barycenter vector are $(7.89438731 \times 10^{-7}, 6.45942086 \times 10^{-8}, -9.74932653 \times 10^{-8})$, while the velocity errors committed are $(1.63523304 \times 10^{-11}, 3.82691243 \times 10^{-12}, 2.83652865 \times 10^{-12})$ in units of au and day. Time rounding errors and a lack of massive bodies in our model are two sources of error here. In order to use Rebound, various coordinate transformations must be used. First, t_0 must be converted to TDB. We use algorithms including those in the NOVAS python

[1] https://www.minorplanetcenter.net/iau/info/OpticalObs.html

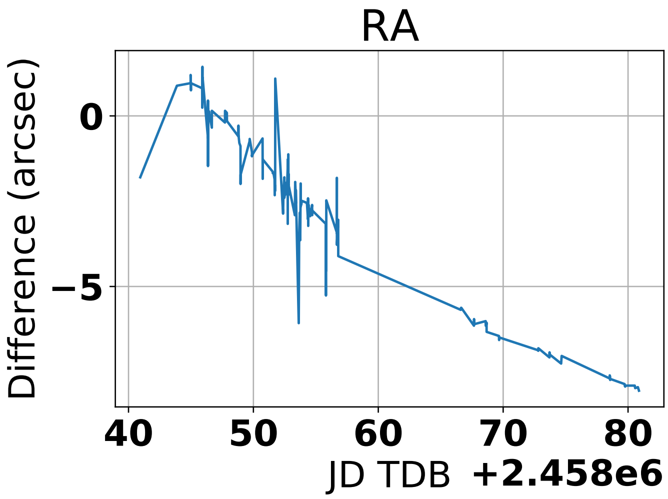

Figure 1. Test

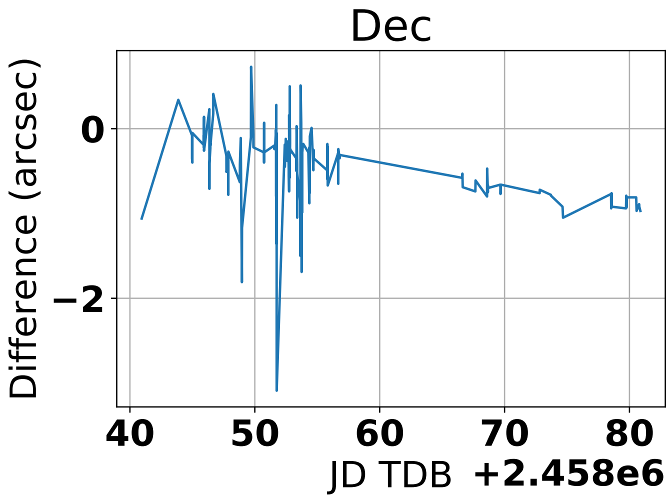

Figure 2. Test

package[2], and first compute the intermediate quantity of terrestrial time (TT). In TDB Julian days, $t_0 = 2458040.940160722$. Changing any of these significant figures affects the result of the Horizons query. r_0 and d_0 are converted to ecliptic J2000 Cartesian coordinates using the assumed observatory–IO separation, s_0. A Sun–IO vector is constructed, after obtaining observatory–Sun vector. Finally, a barycentric IO vector is constructed from the queried initial Sun coordinates.

We must choose the integrator to produce observations; we select `IAS15` from Rebound (Rein & Spiegel 2015), with adaptive stepsize, set by a tolerance. The initial stepsize guess has an effect on our results, and finally we set it to $h = 0.5$ days, which is accurate. Using this integrator, we integrate to 192 times to produce mock obs80 observations. The mass of the IO is set to 0. Once we integrate to the observation time t, we must correct the IO position, using the light travel time $dt = s/c$, where s is the observatory–IO vector, and c is the speed of light. We integrate backwards to $t - dt$ and a new light travel time is computed, dt_1. We then integrate to $t - dt_1$ to obtain new IO coordinates. This process is repeated until the light travel time fractional difference converges to a tolerance of 10^{-8}. This typically requires calculating 3 light time travel delays before convergence (in one test, the average was 2.995). To produce a mock observation, an RA and dec must then be computed using the IO–observatory vector, and coordinate transformations to equatorial coordinates.

We then check whether the observation is physically possible. If the dot product of the observatory–object vector and geocenter–observatory vector is negative, we discard it. We checked, as stated above, that inputting the exact Sun and planet ephemerides do not impact accuracy in this test. We compare our RA and dec as a function of time in Figs. 1 and 2, where differences from actual values are computed in arcsec.

The errors are on the order of arcsec. Already, the difference in the first obs80 line in RA and dec is −2.7" and −1.1, respectively; recall an integration must be performed to correct for the light travel time. We confirmed the model and forces we use cause these errors with the following tests. First, we tried simply integrating the model forward for 100 days and comparing the simulated IO's geocentric vector g_1 to the Horizons geocentric vector g_2. If $d = g_1 - g_2$, we computed $d_\perp = d - (d \cdot \hat{g}_2)\hat{g}_2$, from which we get an angle of 9". Including the most massive asteroids, Ceres, Vesta, and Pallas, did not significantly affect this error.

To further reduce this error, we repeated the 100 day test using an ephemeris-quality integrator instead (Holman, in prep) with initial conditions queried exactly from JPL, with no time rounding. Using the same model of planets plus Sun, the error was about 400". However, when we include Solar radiation pressure, the Yarkovsky effect, and Einstein–Infeld–Hoffman relativistic terms (Farnocchia et al. 2015; Marsden et al. 1973), fit with JPL non-gravitational parameters for 'Oumuamua, the error is reduced to mas. Asteroid masses were not needed. So we have determined model errors are a main cause of our errors. However, we do not need such high precision for testing our linking and orbit fitting algorithms on mock data.

An additional source of errors with Rebound is the time resolution of a minute when querying astronomical bodies. We tried perturbing the first output times by a minute, and the error in the RA and dec at t_0 jumped to 18.0" and −10.0", respectively. We can also directly input the initial conditions of 'Oumuamua, rather than querying through Rebound. Doing this improves the RA and dec errors to −1.8" and −1.1", respectively. Fig. 1 and 2 use this direct input of initial conditions.

Having produced our mock observations with 192 obs80 lines, we can now test our linking and orbit fitting routines, despite the small model errors described above. 16 clusters are fit by our linking algorithm (Holman et al. 2018); these clusters are candidate objects. We successfully fit an orbit to all clusters using Orbfit. All clusters are fit with weighted root mean squared (RMS) residuals less than 1". For 15 of the clusters, the residuals are < 0.004". The remaining cluster's residual is 0.826", and an eccentricity fit which is clearly wrong of 0.58. By contrast, 'Oumuamua's true eccentricity is $e_0 = 1.20113 \pm 0.00002$. The cluster is comprised of 10 observations spanning three days. For the remaining clusters, a magnitude of the percent difference with e_0 is calculated, p. The median is 0.00014, while the maximum and minimum p are 0.0071 and 9.8×10^{-6}, respectively, indicating our fits were accurate, and we

[2] `https://pypi.org/project/novas/`

were able to successfully determine 'Oumuamua's orbit from our mock observations.

2.1 Mock IO tests

Having established the validity of our linking and orbit fitting routines in recovering the orbit of our mock 'Oumuamua, we are prepared to analyze the 500 mock obs80 IO observation data sets, each spanning 38 days.

We run a wide parameter search of γ, $\dot{\gamma}$ (Holman et al. 2018): $\gamma \in (0.2, 0.65)$ au^{-1} and $\dot{\gamma} \in (-0.004, 0.004)$ day^{-1}. The range of γ, $\dot{\gamma}$ does not significantly affect our cluster links, even if they are fine-tuned with a-priori knowledge of the orbits. A typical γ for our mock orbits is $\gamma \in (1, 10)$ au^{-1} and a typical $\dot{\gamma}$ is 0.01 day^{-1}. A clustering radius (Holman et al. 2018) of 0.0006 is used.

In total, 96,834 clusters are found from linking. For 221 IOs, almost half of our set, no clusters were found. This is easily explained as IOs for which many observations had to be discarded. The mean number of discarded observations per IO in this set was 0.86, the minimum was 0.60, and the maximum was 0.95. This contrasts with the remaining IOs with clusters, in which the mean, minimum, and maximum discarded observations were, respectively, 0.20, 0, and 0.79. For the remaining IOs with clusters, at most, one of the IOs had 1470 clusters. The median number of clusters found was 7.5. Having obtained candidate clusters, we apply Orbfit. Orbfit produced orbits successfully for 45,840 clusters, less than half of the original amount. From these, we keep the clusters with RMS < 1"; the number is 45,075.

From these remaining clusters with small residuals, we can calculate how closely our linking and orbit fitting routine reproduces the original IO population. We again calculate p, the absolute value of the percent difference with the true eccentricity. The mean p is 0.014, while the median p is 0.033. Only keeping clusters for which the RMS is 0.005" or less, the mean and median decrease by a factor 10 to 0.0018 and 0.00032, respectively. For high eccentricity clusters, for which $e > 1.7$, the fits become poorer. The mean and median p become 0.022 and 0.0040, respectively. The original mean and median are 0.014 and 0.0040, respectively. The maximum p is 2.44. Fig. 3 shows the distribution of $|p| = |(e_t - e)/e_t|$. Here, e_t is the true eccentricity and e is the fitted eccentricity. While most fits are good, a small peak with large p is seen, indicating a subset of clusters were fit poorly despite small RMS.

3 CONCLUSION

In these notes, we have presented linking and orbit fitting algorithms that can find hyperbolic orbits in datasets in the obs80 format. We have verified our orbit fitting algorithm with a set of mock plausible interstellar object observations. One promising dataset in which to look for IOs it the MPC's Isolated Tracklet File (ITF)[3]. Applying the linking and orbit fitting to the ITF, we have found several hyperbolic arcs, but all are too short to conclude they are actual IOs.

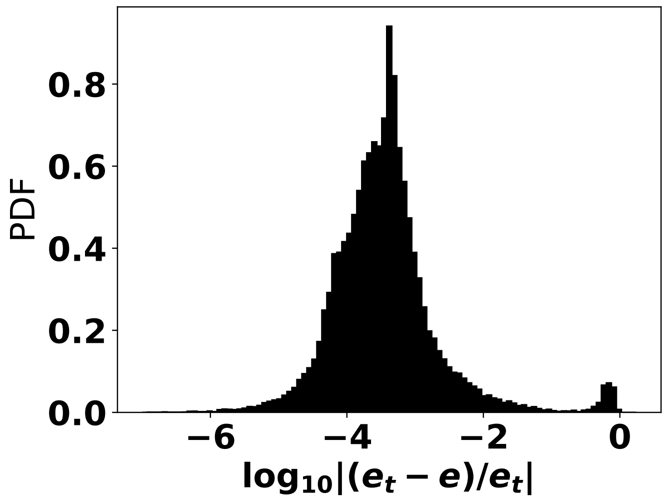

Figure 3. Test

REFERENCES

Farnocchia D., Chesley S. R., Milani A., Gronchi G. F., Chodas P. W., 2015, in , Asteroids IV. pp 815–834, doi:10.2458/azu_uapress_9780816532131-ch041
Holman M. J., Payne M. J., Blankley P., Janssen R., Kuindersma S., 2018, AJ, 156, 135
Marsden B. G., Sekanina Z., Yeomans D. K., 1973, AJ, 78, 211
Rein H., Liu S. F., 2012, A&A, 537, A128
Rein H., Spiegel D. S., 2015, MNRAS, 446, 1424

[3] http://www.minorplanetcenter.net/iau/ITF/itf.txt.gz