The initial constraints of 45 suggests a meet-in-the-middle or an enumerative approach. For the first 80 value, we can try to preprocess the list to reduce it to a manageable size.

Assume we have some fraction a/b with $0 \leq a < b$ and $0 < b$. Adding a term x^2 will result in $\frac{ax^2+b}{bx^2}$. If $\gcd(b, x) = 1$, then at least one more term is required to cancel the x^2 from the denominator.

This fact immediately eliminates any prime p above 40, as there is no other number with a factor of p in the set.

More generally, consider for some prime p the set $\{x^{-2} | x \in [2, 80] \wedge p \setminus x\}$. If for all subsets, the denominator of the simplified sum retains a factor of p^2, then all numbers in the set are impossible. Moreover, over all primes, if a number fails to show up in any valid set (a set with no factor of p^2), then that number is necessarily impossible. These sets can be checked via brute force in $\mathcal{O}(2^{\lfloor 80/p \rfloor})$.

The resulting set, having eliminated numbers described above, has around 40 elements. This is enough to process with a meet-in-the-middle algorithm: split the remaining numbers into two sets of equal size, process and store all possible sums of subsets, and count the pairs that add to $1/2$ using a two-pointer algorithm.