A number n is squarefree if and only if $|\mu(n)| = 1$, where μ is the Mobius function. Computing the Mobius function in linear time (using a linear sieve) would yield an $\mathcal{O}(n)$, which is too slow.

Instead, consider enumerating the square divisor of a number. Using principle of inclusion-exclusion, we can subtract squarefree numbers that have an odd number of factors, and add squarefree numbers that have an even number of factors. In other words, the answer is

$$\sum_{i=1}^{\sqrt{n}} \mu(i) \cdot \left\lfloor \frac{n}{i^2} \right\rfloor .$$