WLOG, assume that $a \geq b > n$. Then, define positive integers $\alpha = n - a$ and $\beta = n - b$. We have

$$\begin{aligned}
\frac{1}{n} &= \frac{1}{a} + \frac{1}{b} \\
\frac{1}{n} &= \frac{1}{n-\alpha} + \frac{1}{n-\beta} \\
(n-\alpha)(n-\beta) &= n(n-\beta) + n(n-\alpha) \\
n^2 - (\alpha+\beta)n + \alpha\beta &= n^2 - \beta n + n^2 - \alpha n \\
\alpha\beta &= n^2
\end{aligned}$$

Thus, the number of solutions for $n$ is the number of divisors of $n^2$.