

Consider taking the log of both sides of the inequality:

$$p^q q^p < a^b$$
$$q \log p + p \log q < b \log a$$

We can iterate over all p and binary search for the maximum allowed value of q .