2.25 Fluid Mechanics

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Stream Functions for planar flow (satisfy $\nabla \cdot \vec{v} = 0$)								
Planar flow: Cartesian (x, y, \cancel{z})	$v_x = \frac{\partial \psi}{\partial y}$	$v_x = \frac{\partial \psi}{\partial y}$		$v_y = -\frac{\partial \psi}{\partial x}$				
Planar flow: Cylindrical $(r, \theta \not \Rightarrow)$	$v_r = \frac{1}{r} \frac{\partial \psi}{\partial \theta}$			$v_{\theta} = -\frac{\partial \psi}{\partial r}$				
Axisymmetric flow: Cylindrical	$v_r = -\frac{1}{r} \frac{\partial \psi}{\partial z}$		$v_{\theta} = 0$		$v_z = \frac{1}{r} \frac{\partial \psi}{\partial r}$			
Axisymmetric flow: Spherical ($v_r = \frac{1}{r^2 \sin \theta}$	$v_r = \frac{1}{r^2 \sin \theta} \frac{\partial \psi}{\partial \theta}$		$\frac{\partial \psi}{\partial r}$	$v_{\varphi} = 0$			
Potential Functions $(\vec{v} = \nabla \phi, \text{ requires } \nabla \times \vec{v} = 0, \nabla^2 \phi = 0)$								
Cartesian coordinates (x, y, z)	v_x	$= \frac{\partial \phi}{\partial x}$	$v_y =$	$\frac{\partial \phi}{\partial y}$	$v_z =$	$\frac{\partial \phi}{\partial z}$		
Cylindrical coordinates (r, θ, z)		$= \frac{\partial \phi}{\partial r}$		$\frac{1}{r}\frac{\partial \phi}{\partial \theta}$	$v_z =$	$\frac{\partial \phi}{\partial z}$		
Spherical coordinates (r, θ, φ)	v_r	$=\frac{\partial\phi}{\partial r}$	$v_{\theta} =$	$\frac{1}{r}\frac{\partial \phi}{\partial \theta}$	$v_{\varphi} =$	$\frac{1}{r\sin\theta}\frac{\partial\phi}{\partial\varphi}$		
uniform stream $ \begin{array}{ccccccccccccccccccccccccccccccccccc$		$\begin{aligned} z &= (U - iV)z \\ \phi &= Ux + Vy \\ \psi &= -Vx + Uz \\ 0 &= \frac{Q}{2\pi} \ln(z - z) \end{aligned}$	y		$egin{aligned} v_x &= 0 \ v_y &= 0 \end{aligned}$			
$\frac{r}{\theta} \qquad \text{shown for } Q > 0$		$b = \frac{Q}{2\pi} \ln r'$ $b = \frac{Q}{2\pi} \theta'$			$v_r = \frac{Q}{2\pi}$ $v_\theta = 0$	$\frac{1}{r'}$		
free vortex $z_0 \qquad \qquad r' \\ \theta' \qquad \qquad \text{shown for} \Gamma > 0$	ϕ	$= \frac{-i\Gamma}{2\pi} \ln(z - z)$ $= \frac{\Gamma}{2\pi} \theta'$ $= -\frac{\Gamma}{2\pi} \ln r'$	(z_0)		$v_r = 0$ $v_\theta = \frac{\Gamma}{2\pi}$	$\frac{1}{r'}$		
forced vortex $z_0 \qquad \qquad x' \qquad x' \qquad x' \qquad x' \qquad \qquad x' \qquad \qquad x' \qquad $		$V(\mathbf{z}) = \sharp$ $\phi = \sharp$ $\psi = -\frac{Kr'^2}{2}$			$v_r = 0$ $v_\theta = K$			

doublet (x-orientation) $z_0 = \int_{\theta'}^{r'} r'$	$W(z) = \frac{c}{z - z_0}$			
r	$\phi = \frac{c \cos \theta'}{r'}$	$v_r = -\frac{c \cos \theta'}{r'^2}$		
shown for $c > 0$	$\psi = -\frac{c \sin \theta'}{r'}$	$v_{\theta} = -\frac{c \sin \theta'}{r'^2}$		
doublet (y-orientation)	W(a) = ic			
	$W(z) = \frac{ic}{z - z_0}$	$c \sin \theta'$		
r shown for $c > 0$	$\phi = \frac{c \sin \theta'}{r'}$	$v_r = -\frac{c \sin \theta'}{r'^2}$		
	$\psi = \frac{c \cos \theta'}{r'}$	$v_{\theta} = \frac{c \cos \theta'}{r'^2}$		
sphere (axisymmetric flow) $z_0 \qquad \qquad r'$	$W(\mathbf{z}) = \phi + i\psi$	$v_r = U\cos\theta' \left(1 - \frac{R^3}{r'^3}\right)$		
$U \longrightarrow \theta'$	$\phi = U \cos \theta' \left(r' + \frac{R^3}{2r'^2} \right)$	$v_{\theta} = -U\sin\theta' \left(1 + \frac{R^3}{2r'^3}\right)$		
$ \frac{r}{\theta} \varphi \qquad \text{shown for } U > 0 $	$\psi = \frac{1}{2}U\sin^2\theta'\left(r'^2 - \frac{R^3}{r'}\right)$	$v_{arphi}=0$ sint $\left(1+rac{2r'^3}{2r'^3} ight)$		
shear flow	$\varphi = 20 \text{ sm} \cdot 0 \left(r - \frac{r'}{r'}\right)$	$v_{oldsymbol{arphi}} = v$		
, A	W(z) = #	$v_x = 2Ay'$		
y ²⁰	$\phi = \sharp$	$v_y = 0$		
x shown for $A > 0$	$\psi = Ay'^2$	$v_z = 0$		
stagnation point flow	$W(z) = \frac{1}{4} A(z - z)^2$	4/		
z_0	$W(z) = \frac{1}{2}A(z - z_0)^2$	$v_x = Ax'$		
$\begin{array}{c} y \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ $	$\phi = \frac{1}{2}A(x'^2 - y'^2)$	$v_y = -Ay'$		
	$\psi = Ax'y'$	$v_z = 0$		
Notes:				
z = x + iy		$W(\mathbf{z}) = \phi + i\psi$		
$z_0 = x_0 + iy_0$	$r' = [(x - x_0)^2 + (y - y_0)^2]^{\frac{1}{2}}$	$\frac{dW}{dz} = v_x - iv_y$		
$0 \le \theta < 2\pi =$	$\theta' = \tan^{-1} \left(\frac{y - y_0}{x - x_0} \right)$	$\frac{dW}{dz} = (v_r - iv_\theta)e^{-i\theta}$		
$v_x = v_r \cos \theta - v_\theta \sin \theta$	$v_r = v_x \cos \theta + v_y \sin \theta$			
$v_y = v_r \sin \theta + v_\theta \cos \theta$	$v_{\theta} = -v_x \sin \theta + v_y \cos \theta$			

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