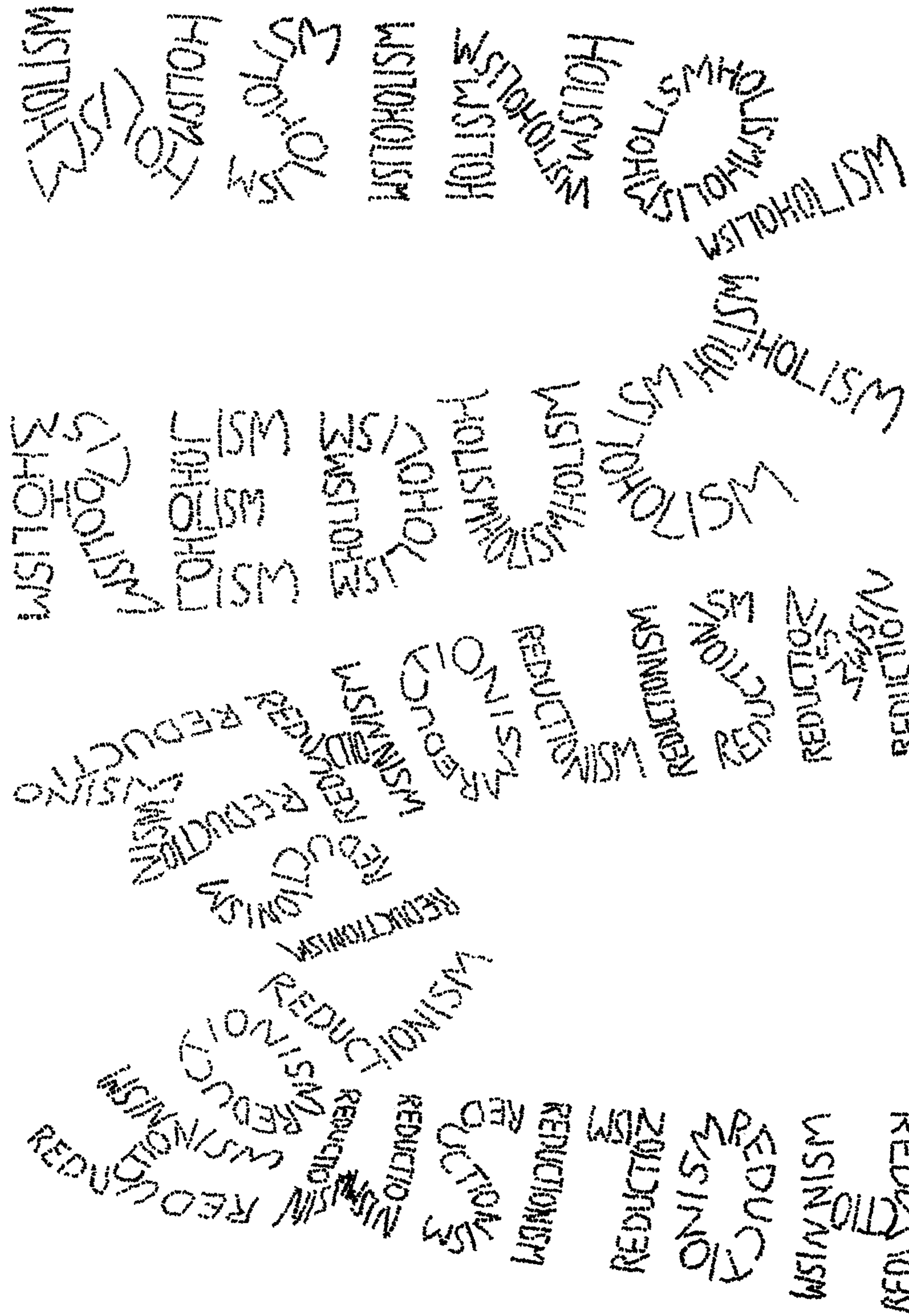

HOLISM
REDUCTIONISM

. . . *Ant Fugue*

. . . then, one by one, the four voices of the fugue chime in.)

Achilles: I know the rest of you won't believe this, but the answer to the question is staring us all in the face, hidden in the picture. It is simply one word—but what an important one: "MU"!

CCrab: I know the rest of you won't believe this, but the answer to the question is staring us all in the face, hidden in the picture. It is simply one word—but what an important one: "HOLISM"!

Achilles: Now hold on a minute. You must be seeing things. It's plain as day that the message of this picture is "MU", not "HOLISM"!

Crab: I beg your pardon, but my eyesight is extremely good. Please look again, and then tell me if the the picture doesn't say what I said it says!

Anteater: I know the rest of you won't believe this, but the answer to the question is staring us all in the face, hidden in the picture. It is simply one word—but what an important one: "REDUCTIONISM"!

Crab: Now hold on a minute. You must be seeing things. It's plain as day that the message of this picture is "HOLISM", not "REDUCTIONISM"!

Achilles: Another deluded one! Not "HOLISM", not "REDUCTIONISM", but "MU" is the message of this picture, and that much is certain.

Anteater: I beg your pardon, but my eyesight is extremely clear. Please look again, and then see if the picture doesn't say what I said it says.

Achilles: Don't you see that the picture is composed of two pieces, and that each of them is a single letter?

Crab: You are right about the two pieces, but you are wrong in your identification of what they are. The piece on the left is entirely composed of three copies of one word: "HOLISM"; and the piece on the right is composed of many copies, in smaller letters, of the same word. Why the letters are of different sizes in the two parts, I don't know, but I know what I see, and what I see is "HOLISM", plain as day. How you see anything else is beyond me.

Anteater: You are right about the two pieces, but you are wrong in your identification of what they are. The piece on the left is entirely composed of many copies of one word: "REDUCTIONISM"; and the piece on the right is composed of one single copy, in larger letters, of the same word. Why the letters are of different sizes in the two parts, I don't know, but I know what I see, and what I see is "REDUCTIONISM", plain as day. How you see anything else is beyond me.

Achilles: I know what is going on here. Each of you has seen letters which compose, or are composed of, other letters. In the left-hand piece,

FIGURE 60. [*Drawing by the author.*]

there are indeed three "HOLISM"'s, but each one of them is composed out of smaller copies of the word "REDUCTIONISM". And in complementary fashion, in the right-hand piece, there is indeed one "REDUCTIONISM", but it is composed out of smaller copies of the word "HOLISM". Now this is all fine and good, but in your silly squabble, the two of you have actually missed the forest for the trees. You see, what good is it to argue about whether "HOLISM" or "REDUCTIONISM" is right, when the proper way to understand the matter is to transcend the question, by answering "MU"?

Crab: I now see the picture as you have described it, Achilles, but I have no idea of what you mean by the strange expression "transcending the question".

Anteater: I now see the picture as you have described it, Achilles, but I have no idea of what you mean by the strange expression "MU".

Achilles: I will be glad to indulge both of you, if you will first oblige me, by telling me the meaning of these strange expressions, "HOLISM" and "REDUCTIONISM".

Crab: HOLISM is the most natural thing in the world to grasp. It's simply the belief that "the whole is greater than the sum of its parts". No one in his right mind could reject holism.

Anteater: REDUCTIONISM is the most natural thing in the world to grasp. It's simply the belief that "a whole can be understood completely if you understand its parts, and the nature of their 'sum'". No one in her left brain could reject reductionism.

Crab: I reject reductionism. I challenge you to tell me, for instance, how to understand a brain reductionistically. Any reductionistic explanation of a brain will inevitably fall far short of explaining where the consciousness experienced by a brain arises from.

Anteater: I reject holism. I challenge you to tell me, for instance, how a holistic description of an ant colony sheds any more light on it than is shed by a description of the ants inside it, and their roles, and their interrelationships. Any holistic explanation of an ant colony will inevitably fall far short of explaining where the consciousness experienced by an ant colony arises from.

Achilles: Oh, no! The last thing which I wanted to do was to provoke another argument. Anyway, now that I understand the controversy, I believe that my explanation of "MU" will help greatly. You see, "MU" is an ancient Zen answer which, when given to a question, UNASKS the question. Here, the question seems to be, "Should the world be understood via holism, or via reductionism?" And the answer of "MU" here rejects the premises of the question, which are that one or the other must be chosen. By unasking the question, it reveals a wider truth: that there is a larger context into which both holistic and reductionistic explanations fit.

Anteater: Absurd! Your "MU" is as silly as a cow's moo. I'll have none of this Zen wishy-washiness.

Crab: Ridiculous! Your "MU" is as silly as a kitten's mew. I'll have none of this Zen washy-wishiness.

Achilles: Oh, dear! We're getting nowhere fast. Why have you stayed so strangely silent, Mr. Tortoise? It makes me very uneasy. Surely you must somehow be capable of helping straighten out this mess?

Tortoise: I know the rest of you won't believe this, but the answer to the question is staring us all in the face, hidden in the picture. It is simply one word—but what an important one: "MU"!

(Just as he says this, the fourth voice in the fugue being played enters, exactly one octave below the first entry.)

Achilles: Oh, Mr. T, for once you have let me down. I was sure that you, who always see the most deeply into things, would be able to resolve this dilemma—but apparently, you have seen no further than I myself saw. Oh, well, I guess I should feel pleased to have seen as far as Mr. Tortoise, for once.

Tortoise: I beg your pardon, but my eyesight is extremely fine. Please look again, and then tell me if the picture doesn't say what I said it says.

Achilles: But of course it does! You have merely repeated my own original observation.

Tortoise: Perhaps "MU" exists in this picture on a deeper level than you imagine, Achilles—an octave lower (figuratively speaking). But for now I doubt that we can settle the dispute in the abstract. I would like to see both the holistic and reductionistic points of view laid out more explicitly; then there may be more of a basis for a decision. I would very much like to hear a reductionistic description of an ant colony, for instance.

Crab: Perhaps Dr. Anteater will tell you something of his experiences in that regard. After all, he is by profession something of an expert on that subject.

Tortoise: I am sure that we have much to learn from you, Dr. Anteater. Could you tell us more about ant colonies, from a reductionistic point of view?

Anteater: Gladly. As Mr. Crab mentioned to you, my profession has led me quite a long way into the understanding of ant colonies.

Achilles: I can imagine! The profession of anteater would seem to be synonymous with being an expert on ant colonies!

Anteater: I beg your pardon. "Anteater" is not my profession; it is my species. By profession, I am a colony surgeon. I specialize in correcting nervous disorders of the colony by the technique of surgical removal.

Achilles: Oh, I see. But what do you mean by "nervous disorders" of an ant colony?

Anteater: Most of my clients suffer from some sort of speech impairment. You know, colonies which have to grope for words in everyday situations. It can be quite tragic. I attempt to remedy the situation by, uhh—removing—the defective part of the colony. These operations

are sometimes quite involved, and of course years of study are required before one can perform them.

Achilles: But—isn't it true that, before one can suffer from speech impairment, one must have the faculty of speech?

Anteater: Right.

Achilles: Since ant colonies don't have that faculty, I am a little confused.

Crab: It's too bad, Achilles, that you weren't here last week, when Dr. Anteater and Aunt Hillary were my house guests. I should have thought of having you over then.

Achilles: Is Aunt Hillary your aunt, Mr. Crab?

Crab: Oh, no, she's not really anybody's aunt.

Anteater: But the poor dear insists that everybody should call her that, even strangers. It's just one of her many endearing quirks.

Crab: Yes, Aunt Hillary is quite eccentric, but such a merry old soul. It's a shame I didn't have you over to meet her last week.

Anteater: She's certainly one of the best-educated ant colonies I have ever had the good fortune to know. The two of us have spent many a long evening in conversation on the widest range of topics.

Achilles: I thought anteaters were devourers of ants, not patrons of ant-'intellectualism!

Anteater: Well, of course the two are not mutually inconsistent. I am on the best of terms with ant colonies. It's just ANTS that I eat, not colonies—and that is good for both parties: me, and the colony.

Achilles: How is it possible that—

Tortoise: How is it possible that—

Achilles: —having its ants eaten can do an ant colony any good?

Crab: How is it possible that—

Tortoise: —having a forest fire can do a forest any good?

Anteater: How is it possible that—

Crab: —having its branches pruned can do a tree any good?

Anteater: —having a haircut can do Achilles any good?

Tortoise: Probably the rest of you were too engrossed in the discussion to notice the lovely stretto which just occurred in this Bach fugue.

Achilles: What is a stretto?

Tortoise: Oh, I'm sorry; I thought you knew the term. It is where one theme repeatedly enters in one voice after another, with very little delay between entries.

Achilles: If I listen to enough fugues, soon I'll know all of these things and will be able to pick them out myself, without their having to be pointed out.

Tortoise: Pardon me, my friends. I am sorry to have interrupted. Dr. Anteater was trying to explain how eating ants is perfectly consistent with being a friend of an ant colony.

Achilles: Well, I can vaguely see how it might be possible for a limited and regulated amount of ant consumption to improve the overall health of

a colony—but what is far more perplexing is all this talk about having conversations with ant colonies. That's impossible. An ant colony is simply a bunch of individual ants running around at random looking for food and making a nest.

Anteater: You could put it that way if you want to insist on seeing the trees but missing the forest, Achilles. In fact, ant colonies, seen as wholes, are quite well-defined units, with their own qualities, at times including the mastery of language.

Achilles: I find it hard to imagine myself shouting something out loud in the middle of the forest, and hearing an ant colony answer back.

Anteater: Silly fellow! That's not the way it happens. Ant colonies don't converse out loud, but in writing. You know how ants form trails leading them hither and thither?

Achilles: Oh, yes—usually straight through the kitchen sink and into my peach jam.

Anteater: Actually, some trails contain information in coded form. If you know the system, you can read what they're saying just like a book.

Achilles: Remarkable. And can you communicate back to them?

Anteater: Without any trouble at all. That's how Aunt Hillary and I have conversations for hours. I take a stick and draw trails in the moist ground, and watch the ants follow my trails. Presently, a new trail starts getting formed somewhere. I greatly enjoy watching trails develop. As they are forming, I anticipate how they will continue (and more often I am wrong than right). When the trail is completed, I know what Aunt Hillary is thinking, and I in turn make my reply.

Achilles: There must be some amazingly smart ants in that colony, I'll say that.

Anteater: I think you are still having some difficulty realizing the difference in levels here. Just as you would never confuse an individual tree with a forest, so here you must not take an ant for the colony. You see, all the ants in Aunt Hillary are as dumb as can be. They couldn't converse to save their little thoraxes!

Achilles: Well then, where does the ability to converse come from? It must reside somewhere inside the colony! I don't understand how the ants can all be unintelligent, if Aunt Hillary can entertain you for hours with witty banter.

Tortoise: It seems to me that the situation is not unlike the composition of a human brain out of neurons. Certainly no one would insist that individual brain cells have to be intelligent beings on their own, in order to explain the fact that a person can have an intelligent conversation.

Achilles: Oh, no, clearly not. With brain cells, I see your point completely. Only . . . ants are a horse of another color. I mean, ants just roam about at will, completely randomly, chancing now and then upon a morsel of food . . . They are free to do what they want to do, and with that freedom, I don't see at all how their behavior, seen as a whole, can

amount to anything coherent—especially something so coherent as the brain behavior necessary for conversing.

Crab: It seems to me that the ants are free only within certain constraints. For example, they are free to wander, to brush against each other, to pick up small items, to work on trails, and so on. But they never step out of that small world, that ant-system, which they are in. It would never occur to them, for they don't have the mentality to imagine anything of the kind. Thus the ants are very reliable components, in the sense that you can depend on them to perform certain kinds of tasks in certain ways.

Achilles: But even so, within those limits they are still free, and they just act at random, running about incoherently without any regard for the thought mechanisms of a higher-level being which Dr. Anteater asserts they are merely components of.

Anteater: Ah, but you fail to recognize one thing, Achilles—the regularity of statistics.

Achilles: How is that?

Anteater: For example, even though ants as individuals wander about in what seems a random way, there are nevertheless overall trends, involving large numbers of ants, which can emerge from that chaos.

Achilles: Oh, I know what you mean. In fact, ant trails are a perfect example of such a phenomenon. There, you have really quite unpredictable motion on the part of any single ant—and yet, the trail itself seems to remain well-defined and stable. Certainly that must mean that the individual ants are not just running about totally at random.

Anteater: Exactly, Achilles. There is some degree of communication among the ants, just enough to keep them from wandering off completely at random. By this minimal communication they can remind each other that they are not alone but are cooperating with teammates. It takes a large number of ants, all reinforcing each other this way, to sustain any activity—such as trail-building—for any length of time. Now my very hazy understanding of the operation of brains leads me to believe that something similar pertains to the firing of neurons. Isn't it true, Mr. Crab, that it takes a group of neurons firing in order to make another neuron fire?

Crab: Definitely. Take the neurons in Achilles' brain, for example. Each neuron receives signals from neurons attached to its input lines, and if the sum total of inputs at any moment exceeds a critical threshold, then that neuron will fire and send its own output pulse rushing off to other neurons, which may in turn fire—and on down the line it goes. The neural flash swoops relentlessly in its Achillean path, in shapes stranger then the dash of a gnat-hungry swallow; every twist, every turn foreordained by the neural structure in Achilles' brain, until sensory input messages interfere.

Achilles: Normally, I think that I'M in control of what I think—but the way you put it turns it all inside out, so that it sounds as though "I" am just

what comes out of all this neural structure, and natural law. It makes what I consider my SELF sound at best like a by-product of an organism governed by natural law, and at worst, an artificial notion produced by my distorted perspective. In other words, you make me feel like I don't know who—or what—I am, if anything.

Tortoise: You'll come to understand much better as we go along. But Dr. Anteater—what do you make of this similarity?

Anteater: I knew there was something parallel going on in the two very different systems. Now I understand it much better. It seems that group phenomena which have coherence—trail-building, for example—will take place only when a certain threshold number of ants get involved. If an effort is initiated, perhaps at random, by a few ants in some locale, one of two things can happen: either it will fizzle out after a brief sputtering start—

Achilles: When there aren't enough ants to keep the thing rolling?

Anteater: Exactly. The other thing that can happen is that a critical mass of ants is present, and the thing will snowball, bringing more and more ants into the picture. In the latter case, a whole "team" is brought into being which works on a single project. That project might be trail-making, or food-gathering, or it might involve nest-keeping. Despite the extreme simplicity of this scheme on a small scale, it can give rise to very complex consequences on a larger scale.

Achilles: I can grasp the general idea of order emerging from chaos, as you sketch it, but that still is a long way from the ability to converse. After all, order also emerges from chaos when molecules of a gas bounce against each other randomly—yet all that results there is an amorphous mass with but three parameters to characterize it: volume, pressure, and temperature. Now that's a far cry from the ability to understand the world, or to talk about it!

Anteater: That highlights a very interesting difference between the explanation of the behavior of an ant colony and the explanation of the behavior of gas inside a container. One can explain the behavior of the gas simply by calculating the statistical properties of the motions of its molecules. There is no need to discuss any higher elements of structure than molecules, except the full gas itself. On the other hand, in an ant colony, you can't even begin to understand the activities of the colony unless you go through several layers of structure.

Achilles: I see what you mean. In a gas, one jump takes you from the lowest level—molecules—to the highest level—the full gas. There are no intermediate levels of organization. Now how do intermediate levels of organized activity arise in an ant colony?

Anteater: It has to do with the existence of several different varieties of ants inside any colony.

Achilles: Oh, yes. I think I have heard about that. They are called "castes", aren't they?

Anteater: That's correct. Aside from the queen, there are males, who do

practically nothing towards the upkeep of the nest, and then—

Achilles: And of course there are soldiers—Glorious Fighters Against Communism!

Crab: Hmm . . . I hardly think that could be right, Achilles. An ant colony is quite communistic internally, so why would its soldiers fight against communism? Or am I right, Dr. Anteater?

Anteater: Yes, about colonies you are right, Mr. Crab; they are indeed based on somewhat communistic principles. But about soldiers Achilles is somewhat naïve. In fact, the so-called "soldiers" are hardly adept at fighting at all. They are slow, ungainly ants with giant heads, who can snap with their strong jaws, but are hardly to be glorified. As in a true communistic state, it is rather the workers who are to be glorified. It is they who do most of the chores, such as food-gathering, hunting, and nursing of the young. It is even they who do most of the fighting.

Achilles: Bah. That is an absurd state of affairs. Soldiers who won't fight!

Anteater: Well, as I just said, they really aren't soldiers at all. It's the workers who are soldiers; the soldiers are just lazy fatheads.

Achilles: Oh, how disgraceful! Why, if I were an ant, I'd put some discipline in their ranks! I'd knock some sense into those fatheads!

Tortoise: If you were an ant? How could you be an ant? There is no way to map your brain onto an ant brain, so it seems to me to be a pretty fruitless question to worry over. More reasonable would be the proposition of mapping your brain onto an ant colony . . . But let us not get sidetracked. Let Dr. Anteater continue with his most illuminating description of castes and their role in the higher levels of organization.

Anteater: Very well. There are all sorts of tasks which must be accomplished in a colony, and individual ants develop specializations. Usually an ant's specialization changes as the ant ages. And of course it is also dependent on the ant's caste. At any one moment, in any small area of a colony, there are ants of all types present. Of course, one caste may be be very sparse in some places and very dense in others.

Crab: Is the density of a given caste, or specialization, just a random thing? Or is there a reason why ants of one type might be more heavily concentrated in certain areas, and less heavily in others?

Anteater: I'm glad you brought that up, since it is of crucial importance in understanding how a colony thinks. In fact, there evolves, over a long period of time, a very delicate distribution of castes inside a colony. And it is this distribution which allows the colony to have the complexity which underlies the ability to converse with me.

Achilles: It would seem to me that the constant motion of ants to and fro would completely prevent the possibility of a very delicate distribution. Any delicate distribution would be quickly destroyed by all the random motions of ants, just as any delicate pattern among molecules in a gas would not survive for an instant, due to the random bombardment from all sides.

Anteater: In an ant colony, the situation is quite the contrary. In fact, it is just exactly the constant to-ing and fro-ing of ants inside the colony

which adapts the caste distribution to varying situations, and thereby preserves the delicate caste distribution. You see, the caste distribution cannot remain as one single rigid pattern; rather, it must constantly be changing so as to reflect, in some manner, the real-world situation with which the colony is dealing, and it is precisely the motion inside the colony which updates the caste distribution, so as to keep it in line with the present circumstances facing the colony.

Tortoise: Could you give an example?

Anteater: Gladly. When I, an anteater, arrive to pay a visit to Aunt Hillary, all the foolish ants, upon sniffing my odor, go into a panic—which means, of course, that they begin running around completely differently from the way they were before I arrived.

Achilles: But that's understandable, since you're a dreaded enemy of the colony.

Anteater: Oh, no. I must reiterate that, far from being an enemy of the colony, I am Aunt Hillary's favorite companion. And Aunt Hillary is my favorite aunt. I grant you, I'm quite feared by all the individual ants in the colony—but that's another matter entirely. In any case, you see that the ants' action in response to my arrival completely changes the internal distribution of ants.

Achilles: That's clear.

Anteater: And that sort of thing is the updating which I spoke of. The new distribution reflects my presence. One can describe the change from old state to new as having added a "piece of knowledge" to the colony.

Achilles: How can you refer to the distribution of different types of ants inside a colony as a "piece of knowledge"?

Anteater: Now there's a vital point. It requires some elaboration. You see, what it comes down to is how you choose to describe the caste distribution. If you continue to think in terms of the lower levels—individual ants—then you miss the forest for the trees. That's just too microscopic a level, and when you think microscopically, you're bound to miss some large-scale features. You've got to find the proper high-level framework in which to describe the caste distribution—only then will it make sense how the caste distribution can encode many pieces of knowledge.

Achilles: Well, how DO you find the proper-sized units in which to describe the present state of the colony, then?

Anteater: All right. Let's begin at the bottom. When ants need to get something done, they form little "teams", which stick together to perform a chore. As I mentioned earlier, small groups of ants are constantly forming and unforming. Those which actually exist for a while are the teams, and the reason they don't fall apart is that there really is something for them to do.

Achilles: Earlier you said that a group will stick together if its size exceeds a certain threshold. Now you're saying that a group will stick together if there is something for it to do.

Anteater: They are equivalent statements. For instance, in food-gathering,

if there is an inconsequential amount of food somewhere which gets discovered by some wandering ant who then attempts to communicate its enthusiasm to other ants, the number of ants who respond will be proportional to the size of the food sample—and an inconsequential amount will not attract enough ants to surpass the threshold. Which is exactly what I meant by saying there is nothing to do—too little food ought to be ignored.

Achilles: I see. I assume that these "teams" are one of the levels of structure falling somewhere in between the single-ant level and the colony level.

Anteater: Precisely. There exists a special kind of team, which I call a "signal"—and all the higher levels of structure are based on signals. In fact, all the higher entities are collections of signals acting in concert. There are teams on higher levels whose members are not ants, but teams on lower levels. Eventually you reach the lowest-level teams—which is to say, signals—and below them, ants.

Achilles: Why do signals deserve their suggestive name?

Anteater: It comes from their function. The effect of signals is to transport ants of various specializations to appropriate parts of the colony. So the typical story of a signal is thus: it comes into existence by exceeding the threshold needed for survival, then it migrates for some distance through the colony, and at some point it more or less disintegrates into its individual members, leaving them on their own.

Achilles: It sounds like a wave, carrying sand dollars and seaweed from afar, and leaving them strewn, high and dry, on the shore.

Anteater: In a way that's analogous, since the team does indeed deposit something which it has carried from a distance, but whereas the water in the wave rolls back to the sea, there is no analogous carrier substance in the case of a signal, since the ants themselves compose it.

Tortoise: And I suppose that a signal loses its coherency just at some spot in the colony where ants of that type were needed in the first place.

Anteater: Naturally.

Achilles: Naturally? It's not so obvious to ME that a signal should always go just where it is needed. And even if it goes in the right direction, how does it figure out where to decompose? How does it know it has arrived?

Anteater: Those are extremely important matters, since they involve explaining the existence of purposeful behavior—or what seems to be purposeful behavior—on the part of signals. From the description, one would be inclined to characterize the signals' behavior as being oriented towards filling a need, and to call it "purposeful". But you can look at it otherwise.

Achilles: Oh, wait. Either the behavior IS purposeful, or it is NOT. I don't see how you can have it both ways.

Anteater: Let me explain my way of seeing things, and then see if you agree. Once a signal is formed, there is no awareness on its part that it

should head off in any particular direction. But here, the delicate caste distribution plays a crucial role. It is what determines the motion of signals through the colony, and also how long a signal will remain stable, and where it will "dissolve".

Achilles: So everything depends on the caste distribution, eh?

Anteater: Right. Let's say a signal is moving along. As it goes, the ants which compose it interact, either by direct contact or by exchange of scents, with ants of the local neighborhoods which it passes through. The contacts and scents provide information about local matters of urgency, such as nest-building, or nursing, or whatever. The signal will remain glued together as long as the local needs are different from what it can supply; but if it CAN contribute, it disintegrates, spilling a fresh team of usable ants onto the scene. Do you see now how the caste distribution acts as an overall guide of the teams inside the colony?

Achilles: I do see that.

Anteater: And do you see how this way of looking at things requires attributing no sense of purpose to the signal?

Achilles: I think so. Actually, I'm beginning to see things from two different vantage points. From an ant's-eye point of view, a signal has NO purpose. The typical ant in a signal is just meandering around the colony, in search of nothing in particular, until it finds that it feels like stopping. Its teammates usually agree, and at that moment the team unloads itself by crumbling apart, leaving just its members but none of its coherency. No planning is required, no looking ahead; nor is any search required, to determine the proper direction. But from the COLONY'S point of view, the team has just responded to a message which was written in the language of the caste distribution. Now from this perspective, it looks very much like purposeful activity.

Crab: What would happen if the caste distribution were entirely random? Would signals still band and disband?

Anteater: Certainly. But the colony would not last long, due to the meaninglessness of the caste distribution.

Crab: Precisely the point I wanted to make. Colonies survive because their caste distribution has meaning, and that meaning is a holistic aspect, invisible on lower levels. You lose explanatory power unless you take that higher level into account.

Anteater: I see your side; but I believe you see things too narrowly.

Crab: How so?

Anteater: Ant colonies have been subjected to the rigors of evolution for billions of years. A few mechanisms were selected for, and most were selected against. The end result was a set of mechanisms which make ant colonies work as we have been describing. If you could watch the whole process in a movie—running a billion or so times faster than life, of course—the emergence of various mechanisms would be seen as natural responses to external pressures, just as bubbles in boiling water are natural responses to an external heat source. I don't suppose you

see "meaning" and "purpose" in the bubbles in boiling water—or do you?

Crab: No, but—

Anteater: Now that's MY point. No matter how big a bubble is, it owes its existence to processes on the molecular level, and you can forget about any "higher-level laws". The same goes for ant colonies and their teams. By looking at things from the vast perspective of evolution, you can drain the whole colony of meaning and purpose. They become superfluous notions.

Achilles: Why, then, Dr. Anteater, did you tell me that you talked with Aunt Hillary? It now seems that you would deny that she can talk or think at all.

Anteater: I am not being inconsistent, Achilles. You see, I have as much difficulty as anyone else in seeing things on such a grandiose time scale, so I find it much easier to change points of view. When I do so, forgetting about evolution and seeing things in the here and now, the vocabulary of teleology comes back: the MEANING of the caste distribution and the PURPOSEFULNESS of signals. This not only happens when I think of ant colonies, but also when I think about my own brain and other brains. However, with some effort I can always remember the other point of view if necessary, and drain all these systems of meaning, too.

Crab: Evolution certainly works some miracles. You never know the next trick it will pull out of its sleeve. For instance, it wouldn't surprise me one bit if it were theoretically possible for two or more "signals" to pass through each other, each one unaware that the other one is also a signal; each one treating the other as if it were just part of the background population.

Anteater: It is better than theoretically possible; in fact it happens routinely!

Achilles: Hmm . . . What a strange image that conjures up in my mind. I can just imagine ants moving in four different directions, some black, some white, criss-crossing, together forming an orderly pattern, almost like—like—

Tortoise: A fugue, perhaps?

Achilles: Yes—that's it! An ant fugue!

Crab: An interesting image, Achilles. By the way, all that talk of boiling water made me think of tea. Who would like some more?

Achilles: I could do with another cup, Mr. C.

Crab: Very good.

Achilles: Do you suppose one could separate out the different visual "voices" of such an "ant fugue"? I know how hard it is for me—

Tortoise: Not for me, thank you.

Achilles: —to track a single voice—

Anteater: I'd like some, too, Mr. Crab—

Achilles: —in a musical fugue—

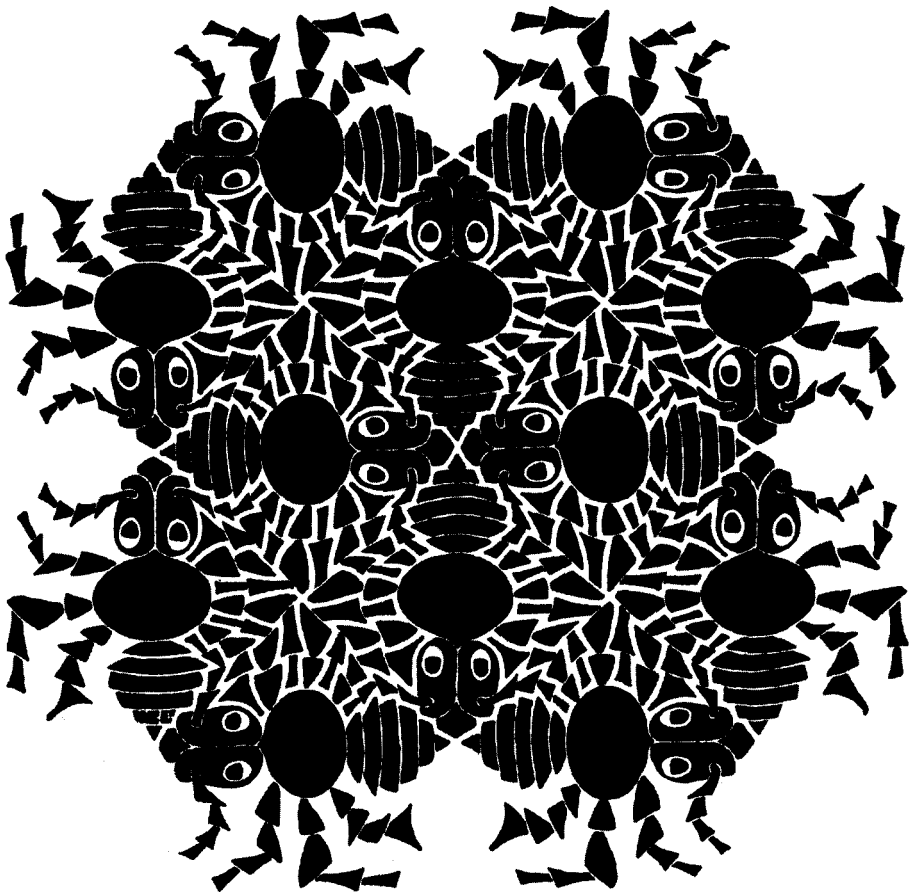


FIGURE 61. "Ant Fugue", *by M. C. Escher (woodcut, 1953).*

Anteater: —if it isn't too much trouble.

Achilles: —when all of them—

Crab: Not at all. Four cups of tea—

Tortoise: Three!

Achilles: —are going at once.

Crab: —coming right up!

Anteater: That's an interesting thought, Achilles. But it's unlikely that anyone could draw such a picture in a convincing way.

Achilles: That's too bad.

Tortoise: Perhaps you could answer this, Dr. Anteater. Does a signal, from its creation until its dissolution, always consist of the same set of ants?

Anteater: As a matter of fact, the individuals in a signal sometimes break off and get replaced by others of the same caste, if there are a few in the area. Most often, signals arrive at their disintegration points with nary an ant in common with their starting lineup.

Crab: I can see that the signals are constantly affecting the caste distribution throughout the colony, and are doing so in response to the internal needs of the colony—which in turn reflect the external situation which the colony is faced with. Therefore the caste distribution, as you said, Dr. Anteater, gets continually updated in a way which ultimately reflects the outer world.

Achilles: But what about those intermediate levels of structure? You were saying that the caste distribution should best be pictured not in terms of ants or signals, but in terms of teams whose members were other teams, whose members were other teams, and so on until you come down to the ant level. And you said that that was the key to understanding how it was possible to describe the caste distribution as encoding pieces of information about the world.

Anteater: Yes, we are coming to all that. I prefer to give teams of a sufficiently high level the name of "symbols". Mind you, this sense of the word has some significant differences from the usual sense. My "symbols" are ACTIVE SUBSYSTEMS of a complex system, and they are composed of lower-level active subsystems . . . They are therefore quite different from PASSIVE symbols, external to the system, such as letters of the alphabet or musical notes, which sit there immobile, waiting for an active system to process them.

Achilles: Oh, this is rather complicated, isn't it? I just had no idea that ant colonies had such an abstract structure.

Anteater: Yes, it's quite remarkable. But all these layers of structure are necessary for the storage of the kinds of knowledge which enable an organism to be "intelligent" in any reasonable sense of the word. Any system which has a mastery of language has essentially the same underlying sets of levels.

Achilles: Now just a cotton-picking minute. Are you insinuating that my brain consists of, at bottom, just a bunch of ants running around?

Anteater: Oh, hardly. You took me a little too literally. The lowest level may be utterly different. Indeed, the brains of anteaters, for instance, are not composed of ants. But when you go up a level or two in a brain, you reach a level whose elements have exact counterparts in other systems of equal intellectual strength—such as ant colonies.

Tortoise: That is why it would be reasonable to think of mapping your brain, Achilles, onto an ant colony, but not onto the brain of a mere ant.

Achilles: I appreciate the compliment. But how would such a mapping be carried out? For instance, what in my brain corresponds to the low-level teams which you call signals?

Anteater: Oh, I but dabble in brains, and therefore couldn't set up the map in its glorious detail. But—and correct me if I'm wrong, Mr. Crab—I would surmise that the brain counterpart to an ant colony's signal is the firing of a neuron; or perhaps it is a larger-scale event, such as a pattern of neural firings.

Crab: I would tend to agree. But don't you think that, for the purposes of our discussion, delineating the exact counterpart is not in itself crucial, desirable though it might be? It seems to me that the main idea is that such a correspondence does exist, even if we don't know exactly how to define it right now. I would only question one point, Dr. Anteater, which you raised, and that concerns the level at which one can have faith that the correspondence begins. You seemed to think that a SIGNAL might have a direct counterpart in a brain; whereas I feel that it is only at the level of your ACTIVE SYMBOLS and above that it is likely that a correspondence must exist.

Anteater: Your interpretation may very well be more accurate than mine, Mr. Crab. Thank you for bringing out that subtle point.

Achilles: What does a symbol do that a signal couldn't do?

Anteater: It is something like the difference between words and letters. Words, which are meaning-carrying entities, are composed of letters, which in themselves carry no meaning. This gives a good idea of the difference between symbols and signals. In fact it is a useful analogy, as long as you keep in mind the fact that words and letters are PASSIVE, symbols and signals are ACTIVE.

Achilles: I'll do so, but I'm not sure I understand why it is so vital to stress the difference between active and passive entities.

Anteater: The reason is that the meaning which you attribute to any passive symbol, such as a word on a page, actually derives from the meaning which is carried by corresponding active symbols in your brain. So that the meaning of passive symbols can only be properly understood when it is related to the meaning of active symbols.

Achilles: All right. But what is it that endows a SYMBOL—an active one, to be sure—with meaning, when you say that a SIGNAL, which is a perfectly good entity in its own right, has none?

Anteater: It all has to do with the way that symbols can cause other symbols to be triggered. When one symbol becomes active, it does not do so in isolation. It is floating about, indeed, in a medium, which is characterized by its caste distribution.

Crab: Of course, in a brain there is no such thing as a caste distribution, but the counterpart is the "brain state". There, you describe the states of all the neurons, and all the interconnections, and the threshold for firing of each neuron.

Anteater: Very well; let's lump "caste distribution" and "brain state" under a common heading, and call them just the "state". Now the state can be described on a low level or on a high level. A low-level description of the state of an ant colony would involve painfully specifying the location of each ant, its age and caste, and other similar items. A very detailed description, yielding practically no global insight as to WHY it is in that state. On the other hand, a description on a high level would involve specifying which symbols could be triggered by which combinations of other symbols, under what conditions, and so forth.

Achilles: What about a description on the level of signals, or teams?

Anteater: A description on that level would fall somewhere in between the low-level and symbol-level descriptions. It would contain a great deal of information about what is actually going on in specific locations throughout the colony, although certainly less than an ant-by-ant description, since teams consist of clumps of ants. A team-by-team description is like a summary of an ant-by-ant description. However, you have to add extra things which were not present in the ant-by-ant description—such as the relationships between teams, and the supply of various castes here and there. This extra complication is the price you pay for the right to summarize.

Achilles: It is interesting to me to compare the merits of the descriptions at various levels. The highest-level description seems to carry the most explanatory power, in that it gives you the most intuitive picture of the ant colony, although strangely enough, it leaves out seemingly the most important feature—the ants.

Anteater: But you see, despite appearances, the ants are not the most important feature. Admittedly, were it not for them, the colony wouldn't exist; but something equivalent—a brain—can exist, ant-free. So, at least from a high-level point of view, the ants are dispensable.

Achilles: I'm sure no ant would embrace your theory with eagerness.

Anteater: Well, I never met an ant with a high-level point of view.

Crab: What a counterintuitive picture you paint, Dr. Anteater. It seems that, if what you say is true, in order to grasp the whole structure, you have to describe it omitting any mention of its fundamental building blocks.

Anteater: Perhaps I can make it a little clearer by an analogy. Imagine you have before you a Charles Dickens novel.

Achilles: *The Pickwick Papers*—will that do?

Anteater: Excellently! And now imagine trying the following game: you must find a way of mapping letters onto ideas, so that the entire *Pickwick Papers* makes sense when you read it letter by letter.

Achilles: Hmm . . . You mean that every time I hit a word such as "the", I have to think of three definite concepts, one after another, with no room for variation?

Anteater: Exactly. They are the 't'-concept, the 'h'-concept, and the 'e'-concept—and every time, those concepts are as they were the preceding time.

Achilles: Well, it sounds like that would turn the experience of "reading" *The Pickwick Papers* into an indescribably boring nightmare. It would be an exercise in meaninglessness, no matter what concept I associated with each letter.

Anteater: Exactly. There is no natural mapping from the individual letters into the real world. The natural mapping occurs on a higher level—between words, and parts of the real world. If you wanted to describe the book, therefore, you would make no mention of the letter level.

Achilles: Of course not! I'd describe the plot and the characters, and so forth.

Anteater: So there you are. You would omit all mention of the building blocks, even though the book exists thanks to them. They are the medium, but not the message.

Achilles: All right—but what about ant colonies?

Anteater: Here, there are active signals instead of passive letters, and active symbols instead of passive words—but the idea carries over.

Achilles: Do you mean I couldn't establish a mapping between signals and things in the real world?

Anteater: You would find that you could not do it in such a way that the triggering of new signals would make any sense. Nor could you succeed on any lower level—for example the ant level. Only on the symbol level do the triggering patterns make sense. Imagine, for instance, that one day you were watching Aunt Hillary when I arrived to pay a call. You could watch as carefully as you wanted, and yet you would probably perceive nothing more than a rearrangement of ants.

Achilles: I'm sure that's accurate.

Anteater: And yet, as I watched, reading the higher level instead of the lower level, I would see several dormant symbols being awakened, those which translate into the thought, "Oh, here's that charming Dr. Anteater again—how pleasant!"—or words to that effect.

Achilles: That sounds like what happened when the four of us all found different levels to read in the MU-picture—or at least THREE of us did . . .

Tortoise: What an astonishing coincidence that there should be such a resemblance between that strange picture which I chanced upon in the *Well-Tempered Clavier*, and the trend of our conversation.

Achilles: Do you think it's just coincidence?

Tortoise: Of course.

Anteater: Well, I hope you can grasp now how the thoughts in Aunt Hillary emerge from the manipulation of symbols composed of signals composed of teams composed of lower-level teams, all the way down to ants.

Achilles: Why do you call it "symbol manipulation"? Who does the manipulating, if the symbols are themselves active? Who is the agent?

Anteater: This gets back to the question which you earlier raised about purpose. You're right that symbols themselves are active, but the activities which they follow are nevertheless not absolutely free. The activities of all symbols are strictly determined by the state of the full system in which they reside. Therefore, the full system is responsible for how its symbols trigger each other, and so it is quite reasonable to speak of the full system as the "agent". As the symbols operate, the state of the system gets slowly transformed, or updated. But there are many features which remain over time. It is this partially constant, partially varying system which is the agent. One can give a name to the

full system. For example, Aunt Hillary is the "who" who can be said to manipulate her symbols; and you are similar, Achilles.

Achilles: That's quite a strange characterization of the notion of who I am. I'm not sure I can fully understand it, but I will give it some thought.

Tortoise: It would be quite interesting to follow the symbols in your brain as you do that thinking about the symbols in your brain.

Achilles: That's too complicated for me. I have trouble enough just trying to picture how it is possible to look at an ant colony and read it on the symbol level. I can certainly imagine perceiving it at the ant level; and with a little trouble, I can imagine what it must be like to perceive it at the signal level; but what in the world can it be like to perceive an ant colony at the symbol level?

Anteater: One only learns through long practice. But when one is at my stage, one reads the top level of an ant colony as easily as you yourself read the "MU" in the MU-picture.

Achilles: Really? That must be an amazing experience.

Anteater: In a way—but it is also one which is quite familiar to you, Achilles.

Achilles: Familiar to me? What do you mean? I have never looked at an ant colony on anything but the ant level.

Anteater: Maybe not; but ant colonies are no different from brains in many respects.

Achilles: I have never seen nor read any brain either, however.

Anteater: What about your OWN brain? Aren't you aware of your own thoughts? Isn't that the essence of consciousness? What else are you doing but reading your own brain directly at the symbol level?

Achilles: I never thought of it that way. You mean that I bypass all the lower levels, and only see the topmost level?

Anteater: That's the way it is, with conscious systems. They perceive themselves on the symbol level only, and have no awareness of the lower levels, such as the signal levels.

Achilles: Does it follow that in a brain, there are active symbols which are constantly updating themselves so that they reflect the overall state of the brain itself, always on the symbol level?

Anteater: Certainly. In any conscious system there are symbols which represent the brain state, and they are themselves part of the very brain state which they symbolize. For consciousness requires a large degree of self-consciousness.

Achilles: That is a weird notion. It means that although there is frantic activity occurring in my brain at all times, I am only capable of registering that activity in one way—on the symbol level; and I am completely insensitive to the lower levels. It is like being able to read a Dickens novel by direct visual perception, without ever having learned the letters of the alphabet. I can't imagine anything as weird as that really happening.

Crab: But precisely that sort of thing DID happen when you read "MU",

without perceiving the lower levels "HOLISM" and "REDUCTIONISM".

Achilles: You're right—I bypassed the lower levels, and saw only the top. I wonder if I'm missing all sorts of meaning on lower levels of my brain as well, by reading only the symbol level. It's too bad that the top level doesn't contain all the information about the bottom level, so that by reading the top, one also learns what the bottom level says. But I guess it would be naïve to hope that the top level encodes anything from the bottom level—it probably doesn't percolate up. The MU-picture is the most striking possible example of that: there, the topmost level says only "MU", which bears no relation whatever to the lower levels!

Crab: That's absolutely true. *(Picks up the MU-picture, to inspect it more closely.)* Hmm . . . There's something strange about the smallest letters in this picture; they're very wiggly . . .

Anteater: Let me take a look. *(Peers closely at the MU-picture.)* I think there's yet another level, which all of us missed!

Tortoise: Speak for yourself, Dr. Anteater.

Achilles: Oh, no—that can't be! Let me see. *(Looks very carefully.)* I know the rest of you won't believe this, but the message of this picture is staring us all in the face, hidden in its depths. It is simply one word, repeated over and over again, like a mantra—but what an important one: "MU"! What do you know! It is the same as the top level! And none of us suspected it in the least.

Crab: We would never have noticed it if it hadn't been for you, Achilles.

Anteater: I wonder if the coincidence of the highest and lowest levels happened by chance? Or was it a purposeful act carried out by some creator?

Crab: How could one ever decide that?

Tortoise: I don't see any way to do so, since we have no idea why that particular picture is in the Crab's edition of the *Well-Tempered Clavier.*

Anteater: Although we have been having a lively discussion, I have still managed to listen with a good fraction of an ear to this very long and complex four-voice fugue. It is extraordinarily beautiful.

Tortoise: It certainly is. And now, in just a moment, comes an organ point.

Achilles: Isn't an organ point what happens when a piece of music slows down slightly, settles for a moment or two on a single note or chord, and then resumes at normal speed after a short silence?

Tortoise: No, you're thinking of a "fermata"—a sort of musical semicolon. Did you notice there was one of those in the prelude?

Achilles: I guess I must have missed it.

Tortoise: Well, you have another chance coming up to hear a fermata—in fact, there are a couple of them coming up, towards the end of this fugue.

Achilles: Oh, good. You'll point them out in advance, won't you?

Tortoise: If you like.

Achilles: But do tell me, what is an organ point?

Tortoise: An organ point is the sustaining of a single note by one of the

voices in a polyphonic piece (often the lowest voice), while the other voices continue their own independent lines. This organ point is on the note of G. Listen carefully, and you'll hear it.

Anteater: There occurred an incident one day when I visited with Aunt Hillary which reminds me of your suggestion of observing the symbols in Achilles' brain as they create thoughts which are about themselves.

Crab: Do tell us about it.

Anteater: Aunt Hillary had been feeling very lonely, and was very happy to have someone to talk to that day. So she gratefully told me to help myself to the juiciest ants I could find. (She's always been most generous with her ants.)

Achilles: Gee!

Anteater: It just happened that I had been watching the symbols which were carrying out her thoughts, because in them were some particularly juicy-looking ants.

Achilles: Gee!

Anteater: So I helped myself to a few of the fattest ants which had been parts of the higher-level symbols which I had been reading. Specifically, the symbols which they were part of were the ones which had expressed the thought, "Help yourself to any of the ants which look appetizing."

Achilles: Gee!

Anteater: Unfortunately for them, but fortunately for me, the little bugs didn't have the slightest inkling of what they were collectively telling me, on the symbol level.

Achilles: Gee! That is an amazing wraparound. They were completely unconscious of what they were participating in. Their acts could be seen as part of a pattern on a higher level, but of course they were completely unaware of that. Ah, what a pity—a supreme irony, in fact—that they missed it.

Crab: You are right, Mr. T—that was a lovely organ point.

Anteater: I had never heard one before, but that one was so conspicuous that no one could miss it. Very effective.

Achilles: What? Has the organ point already occurred? How can I not have noticed it, if it was so blatant?

Tortoise: Perhaps you were so wrapped up in what you were saying that you were completely unaware of it. Ah, what a pity—a supreme irony, in fact—that you missed it.

Crab: Tell me, does Aunt Hillary live in an anthill?

Anteater: Well, she owns a rather large piece of property. It used to belong to someone else, but that is rather a sad story. In any case, her estate is quite expansive. She lives rather sumptuously, compared to many other colonies.

Achilles: How does that jibe with the communistic nature of ant colonies which you earlier described to us? It sounds quite inconsistent, to me, to preach communism and to live in a fancy estate!

Anteater: The communism is on the ant level. In an ant colony all ants work for the common good, even to their own individual detriment at times. Now this is simply a built-in aspect of Aunt Hillary's structure, but for all I know, she may not even be aware of this internal communism. Most human beings are not aware of anything about their neurons; in fact they probably are quite content not to know anything about their brains, being somewhat squeamish creatures. Aunt Hillary is also somewhat squeamish; she gets rather antsy whenever she starts to think about ants at all. So she avoids thinking about them whenever possible. I truly doubt that she knows anything about the communistic society which is built into her very structure. She herself is a staunch believer in libertarianism—you know, laissez-faire and all that. So it makes perfect sense, to me at least, that she should live in a rather sumptuous manor.

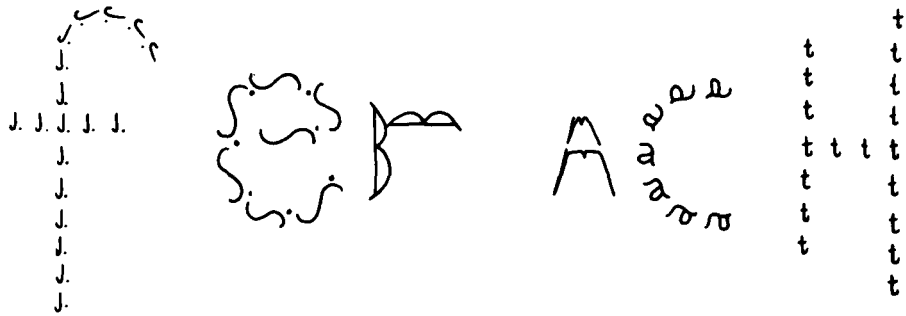


FIGURE 62. [*Drawing by the author.*]

Tortoise: As I turned the page just now, while following along in this lovely edition of the *Well-Tempered Clavier*, I noticed that the first of the two fermatas is coming up soon—so you might listen for it, Achilles.

Achilles: I will, I will.

Tortoise: Also, there's a most curious picture facing this page.

Crab: Another one? What next?

Tortoise: See for yourself. *(Passes the score over to the Crab.)*

Crab: Aha! It's just a few bunches of letters. Let's see—there are various numbers of the letters 'J', 'S', 'B', 'm', 'a', and 't'. It's strange, how the first three letters grow, and then the last three letters shrink again.

Anteater: May I see it?

Crab: Why, certainly.

Anteater: Oh, by concentrating on details, you have utterly missed the big picture. In reality, this group of letters is 'f', 'e', 'r', 'A', 'C', 'H', without any repetitions. First they get smaller, then they get bigger. Here, Achilles—what do you make of it?

Achilles: Let me see. Hmm. Well, I see it as a set of upper-case letters which grow as you move to the right.

Tortoise: Do they spell anything?

Achilles: Ah . . . "J. S. BACH". Oh! I understand now. It's Bach's name!

Tortoise: Strange that you should see it that way. I see it as a set of lower-case letters, shrinking as they move to the right, and . . . spelling out . . . the name of . . . *(Slows down slightly, especially drawing out the last few words. Then there is a brief silence. Suddenly he resumes as if nothing unusual had happened.)* —"fermat".

Achilles: Oh, you've got Fermat on the brain, I do believe. You see Fermat's Last Theorem everywhere.

Anteater: You were right, Mr. Tortoise—I just heard a charming little fermata in the fugue.

Crab: So did I.

Achilles: Do you mean everybody heard it but me? I'm beginning to feel stupid.

Tortoise: There, there, Achilles—don't feel bad. I'm sure you won't miss Fugue's Last Fermata (which is coming up quite soon). But, to return to our previous topic, Dr. Anteater, what is the very sad story which you alluded to, concerning the former owner of Aunt Hillary's property?

Anteater: The former owner was an extraordinary individual, one of the most creative ant colonies who ever lived. His name was Johant Sebastiant Fermant, and he was a mathematiciant by vocation, but a musiciant by avocation.

Achilles: How very versantile of him!

Anteater: At the height of his creative powers, he met with a most untimely demise. One day, a very hot summer day, he was out soaking up the warmth, when a freak thundershower—the kind that hits only once every hundred years or so—appeared from out of the blue, and thoroughly drenched J. S .F. Since the storm came utterly without warning, the ants got completely disoriented and confused. The intricate organization which had been so finely built up over decades, all went down the drain in a matter of minutes. It was tragic.

Achilles: Do you mean that all the ants drowned, which obviously would spell the end of poor J. S. F.?

Anteater: Actually, no. The ants managed to survive, every last one of them, by crawling onto various sticks and logs which floated above the raging torrents. But when the waters receded and left the ants back on their home grounds, there was no organization left. The caste distribution was utterly destroyed, and the ants themselves had no ability to reconstruct what had once before been such a finely tuned organization. They were as helpless as the pieces of Humpty Dumpty in putting themselves back together again. I myself tried, like all the king's horses and all the king's men, to put poor Fermant together again. I faithfully put out sugar and cheese, hoping against hope that somehow Fermant would reappear . . . *(Pulls out a handkerchief and wipes his eyes.)*

Achilles: How valiant of you! I never knew Anteaters had such big hearts.

Anteater: But it was all to no avail. He was gone, beyond reconstitution.

However, something very strange then began to take place: over the next few months, the ants which had been components of J. S. F. slowly regrouped, and built up a new organization. And thus was Aunt Hillary born.

Crab: Remarkable! Aunt Hillary is composed of the very same ants as Fermant was?

Anteater: Well, originally she was, yes. By now, some of the older ants have died, and been replaced. But there are still many holdovers from the J. S. F.-days.

Crab: And can't you recognize some of J. S. F.'s old traits coming to the fore, from time to time, in Aunt Hillary?

Anteater: Not a one. They have nothing in common. And there is no reason they should, as I see it. There are, after all, often several distinct ways to rearrange a group of parts to form a "sum". And Aunt Hillary was just a new "sum" of the old parts. Not MORE than the sum, mind you—just that particular KIND of sum.

Tortoise: Speaking of sums, I am reminded of number theory, where occasionally one will be able to take apart a theorem into its component symbols, rearrange them in a new order, and come up with a new theorem.

Anteater: I've never heard of such a phenomenon, although I confess to being a total ignoramus in the field.

Achilles: Nor have I heard of it—and I am rather well versed in the field, if I don't say so myself. I suspect Mr. T is just setting up one of his elaborate spoofs. I know him pretty well by now.

Anteater: Speaking of number theory, I am reminded of J. S. F. again, for number theory is one of the domains in which he excelled. In fact, he made some rather remarkable contributions to number theory. Aunt Hillary, on the other hand, is remarkably dull-witted in anything that has even the remotest connection with mathematics. Also, she has only a rather banal taste in music, whereas Sebastiant was extremely gifted in music.

Achilles: I am very fond of number theory. Could you possibly relate to us something of the nature of Sebastiant's contributions?

Anteater: Very well, then. *(Pauses for a moment to sip his tea, then resumes.)* Have you heard of Fourmi's infamous "Well-Tested Conjecture"?

Achilles: I'm not sure . . . It sounds strangely familiar, and yet I can't quite place it.

Anteater: It's a very simple idea. Lierre de Fourmi, a mathematiciant by vocation but lawyer by avocation, had been reading in his copy of the classic text *Arithmetica* by Di of Antus, and came across a page containing the equation

$$2^a + 2^b = 2^c$$

He immediately realized that this equation has infinitely many solutions a, b, c, and then wrote in the margin the following notorious comment:

FIGURE 63. During emigrations army ants sometimes create living bridges of their own bodies. In this photograph of such a bridge (de Fourmi Lierre), the workers of an Eciton burchelli *colony can be seen linking their legs and, along the top of the bridge, hooking their tarsal claws together to form irregular systems of chains. A symbiotic silverfish,* Trichatelura manni, *is seen crossing the bridge in the center.* [*From E. O. Wilson,* The Insect Societies *(Cambridge, Mass.: Harvard University Press, 1971), p. 62.*]

> The equation
>
> $$n^a + n^b = n^c$$
>
> has solutions in positive integers a, b, c, and n only when n = 2 (and then there are infinitely many triplets a, b, c which satisfy the equation); but there are no solutions for n > 2. I have discovered a truly marvelous proof of this statement, which, unfortunately, is so small that it would be well-nigh invisible if written in the margin.

Ever since that year, some three hundred days ago, mathematicians have been vainly trying to do one of two things: either to prove Fourmi's claim, and thereby vindicate Fourmi's reputation, which, although very high, has been somewhat tarnished by skeptics who think he never really found the proof he claimed to have found—or else to refute the claim, by finding a counterexample: a set of four integers a, b, c, and n, with n > 2, which satisfy the equation. Until very recently, every attempt in either direction had met with failure. To be sure, the Conjecture has been verified for many specific values of n—in particular, all n up to 125,000. But no one had succeeded in proving it for ALL n—no one, that is, until Johant Sebastiant Fermant came upon the scene. It was he who found the proof that cleared Fourmi's name.

It now goes under the name "Johant Sebastiant's Well-Tested Conjecture".

Achilles: Shouldn't it be called a "Theorem" rather than a "Conjecture", if it's finally been given a proper proof?

Anteater: Strictly speaking, you're right, but tradition has kept it this way.

Tortoise: What sort of music did Sebastiant do?

Anteater: He had great gifts for composition. Unfortunately, his greatest work is shrouded in mystery, for he never reached the point of publishing it. Some believe that he had it all in his mind; others are more unkind, saying that he probably never worked it out at all, but merely blustered about it.

Achilles: What was the nature of this magnum opus?

Anteater: It was to be a giant prelude and fugue; the fugue was to have twenty-four voices, and to involve twenty-four distinct subjects, one in each of the major and minor keys.

Achilles: It would certainly be hard to listen to a twenty-four-voice fugue as a whole!

Crab: Not to mention composing one!

Anteater: But all that we know of it is Sebastiant's description of it, which he wrote in the margin of his copy of Buxtehude's Preludes and Fugues for Organ. The last words which he wrote before his tragic demise were:

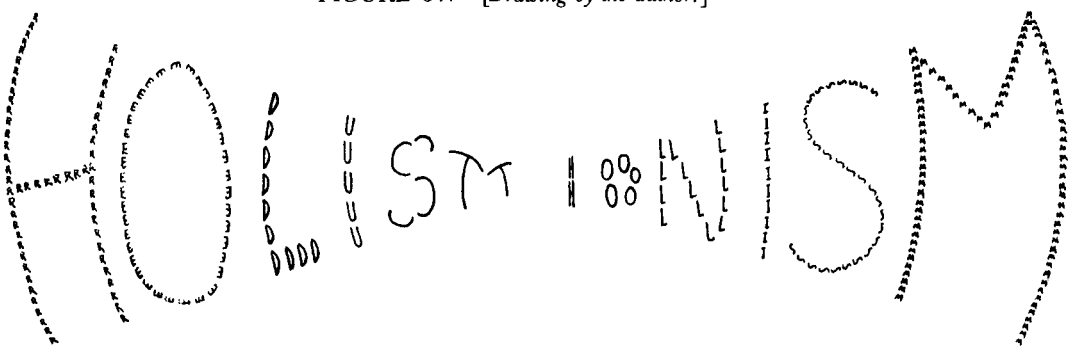
> I have composed a truly marvelous fugue. In it, I have added together the power of 24 keys, and the power of 24 themes; I came up with a fugue with the power of 24 voices. Unfortunately, this margin is too narrow to contain it.

And the unrealized masterpiece simply goes by the name, "Fermant's Last Fugue".

Achilles: Oh, that is unbearably tragic.

Tortoise: Speaking of fugues, this fugue which we have been listening to is nearly over. Towards the end, there occurs a strange new twist on its theme. *(Flips the page in the* Well-Tempered Clavier.*)* Well, what have we here? A new illustration—how appealing! *(Shows it to the Crab.)*

FIGURE 64. [*Drawing by the author.*]



Crab: Well, what have we here? Oh, I see: it's "HOLISMIONISM", written in large letters that first shrink and then grow back to their original size. But that doesn't make any sense, because it's not a word. Oh me, oh my! *(Passes it to the Anteater.)*

Anteater: Well, what have we here? Oh, I see: it's "REDUCTHOLISM", written in small letters that first grow and then shrink back to their original size. But that doesn't make any sense, because it's not a word. Oh my, oh me! *(Passes it to Achilles.)*

Achilles: I know the rest of you won't believe this, but in fact this picture consists of the word "HOLISM" written twice, with the letters continually shrinking as they proceed from left to right. *(Returns it to the Tortoise.)*

Tortoise: I know the rest of you won't believe this, but in fact this picture consists of the word "REDUCTIONISM" written once, with the letters continually growing as they proceed from left to right.

Achilles: At last—I heard the new twist on the theme this time! I am so glad that you pointed it out to me, Mr. Tortoise. Finally, I think I am beginning to grasp the art of listening to fugues.