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## 1 Basic

### 1.1 .vimrc

```

sy on
se sw=4 ts=4 sts=4 et nu sc hls rnu
filet plugin indent on
no <F5> :!./a.out<CR>
no <F6> :!./a.out < input.txt<CR>
no <F9> :!g++ -O2 -std=gnu++14 -lm % -g -fsanitize=
    undefined -Wall -Wextra -Wshadow -Wno-unused-result
    <CR>

```

### 1.2 Misc

```

#include <random>
mt19937 rng(0x5EED);
int randint(int lb, int ub)
{ return uniform_int_distribution<int>(lb, ub)(rng); }

#define SECS (clock() / CLOCKS_PER_SEC)

struct KeyHasher {
    size_t operator()(const Key& k) const {
        return k.first + k.second * 100000;
    }
};
typedef unordered_map<Key,int,KeyHasher> map_t;

```

### 1.3 python-related

```

from fractions import Fraction
from decimal import Decimal, getcontext
getcontext().prec = 250 # set precision

itwo = Decimal(0.5)
two = Decimal(2)

N = 200
def angle(cosT):
    """given cos(theta) in decimal return theta"""
    for i in range(N):
        cosT = ((cosT + 1) / two) ** itwo
        sinT = (1 - cosT * cosT) ** itwo
        return sinT * (2 ** N)
pi = angle(Decimal(-1))

```

## 2 flow

### 2.1 ISAP

```

#define SZ(c) ((int)(c).size())
struct Maxflow {
    static const int MAXV = 20010;
    static const int INF = 1000000;
    struct Edge {
        int v, c, r;
        Edge(int _v, int _c, int _r):
            v(_v), c(_c), r(_r) {}
    };
    int s, t;
    vector<Edge> G[MAXV*2];
    int iter[MAXV*2], d[MAXV*2], gap[MAXV*2], tot;
    void init(int x) {
        tot = x+2;
        s = x+1, t = x+2;
        for(int i = 0; i <= tot; i++) {
            G[i].clear();
            iter[i] = d[i] = gap[i] = 0;
        }
    }
    void addEdge(int u, int v, int c) {
        G[u].push_back(Edge(v, c, SZ(G[v])));
    }

```

```

    G[v].push_back(Edge(u, 0, SZ(G[u]) - 1));
}
int dfs(int p, int flow) {
    if(p == t) return flow;
    for(int &i = iter[p]; i < SZ(G[p]); i++) {
        Edge &e = G[p][i];
        if(e.c > 0 && d[p] == d[e.v]+1) {
            int f = dfs(e.v, min(flow, e.c));
            if(f) {
                e.c -= f;
                G[e.v][e.r].c += f;
                return f;
            }
        }
    }
    if( (--gap[d[p]]) == 0) d[s] = tot;
    else {
        d[p]++;
        iter[p] = 0;
        ++gap[d[p]];
    }
    return 0;
}
int solve() {
    int res = 0;
    gap[0] = tot;
    for(res = 0; d[s] < tot; res += dfs(s, INF));
    return res;
}
} flow;

```

## 2.2 MinCostFlow

```

struct MinCostMaxFlow{
    typedef int Tcost;
    static const int MAXV = 20010;
    static const int INFf = 1000000;
    static const Tcost INFc = 1e9;
    struct Edge{
        int v, cap;
        Tcost w;
        int rev;
        Edge(){}
        Edge(int t2, int t3, Tcost t4, int t5)
            : v(t2), cap(t3), w(t4), rev(t5) {}
    };
    int V, s, t;
    vector<Edge> g[MAXV];
    void init(int n){
        V = n+2;
        s = n+1, t = n+2;
        for(int i = 0; i <= V; i++) g[i].clear();
    }
    void addEdge(int a, int b, int cap, Tcost w){
        g[a].push_back(Edge(b, cap, w, (int)g[b].size()));
        g[b].push_back(Edge(a, 0, -w, (int)g[a].size()-1));
    }
    Tcost d[MAXV];
    int id[MAXV], mom[MAXV];
    bool inqu[MAXV];
    queue<int> q;
    Tcost solve(){
        int mx = 0; Tcost mnc = 0;
        while(1){
            fill(d, d+1+V, INFc);
            fill(inqu, inqu+1+V, 0);
            fill(mom, mom+1+V, -1);
            mom[s] = s;
            d[s] = 0;
            q.push(s); inqu[s] = 1;
            while(q.size()){
                int u = q.front(); q.pop();
                inqu[u] = 0;
                for(int i = 0; i < (int) g[u].size(); i++){
                    Edge &e = g[u][i];
                    int v = e.v;
                    if(e.cap > 0 && d[v] > d[u]+e.w){
                        d[v] = d[u]+e.w;
                        mom[v] = u;
                        id[v] = i;

```

```

                        if(!inqu[v]) q.push(v), inqu[v] = 1;
                    }
                }
            }
            if(mom[t] == -1) break;
            int df = INFf;
            for(int u = t; u != s; u = mom[u])
                df = min(df, g[mom[u]][id[u]].cap);
            for(int u = t; u != s; u = mom[u]){
                Edge &e = g[mom[u]][id[u]];
                e.cap -= df;
                g[e.v][e.rev].cap += df;
            }
            mx += df;
            mnc += df*d[t];
        }
        return mnc;
    }
} flow;

```

## 2.3 Dinic

```

struct Dinic{
    static const int MXN = 10000;
    struct Edge{ int v, f, re; };
    int n, s, t, level[MXN];
    vector<Edge> E[MXN];
    void init(int _n, int _s, int _t){
        n = _n; s = _s; t = _t;
        for (int i=0; i<n; i++) E[i].clear();
    }
    void add_edge(int u, int v, int f){
        E[u].PB({v, f, SZ(E[v])});
        E[v].PB({u, 0, SZ(E[u])-1});
    }
    bool BFS(){
        for (int i=0; i<n; i++) level[i] = -1;
        queue<int> que;
        que.push(s);
        level[s] = 0;
        while (!que.empty()){
            int u = que.front(); que.pop();
            for (auto it : E[u]){
                if (it.f > 0 && level[it.v] == -1){
                    level[it.v] = level[u]+1;
                    que.push(it.v);
                }
            }
        }
        return level[t] != -1;
    }
    int DFS(int u, int nf){
        if (u == t) return nf;
        int res = 0;
        for (auto &it : E[u]){
            if (it.f > 0 && level[it.v] == level[u]+1){
                int tf = DFS(it.v, min(nf, it.f));
                res += tf; nf -= tf; it.f -= tf;
                E[it.v][it.re].f += tf;
                if (nf == 0) return res;
            }
        }
        if (!res) level[u] = -1;
        return res;
    }
    int flow(int res=0){
        while ( BFS() )
            res += DFS(s, 2147483647);
        return res;
    }
} flow;

```

## 2.4 Kuhn Munkres

```

int n1, nr;
int mat_x[MAX_N], mat_y[MAX_N];
bool vis_x[MAX_N], vis_y[MAX_N];

```

```

ll slack_y[MAX_N], W[MAX_N][MAX_N], l_x[MAX_N], l_y[
MAX_N];
bool dfs(int x) {
    vis_x[x] = 1;
    REP1(y, 1, nr) {
        if(vis_y[y]) continue;
        ll t = l_x[x] + l_y[y] - W[x][y];
        if(!t) {
            vis_y[y] = 1;
            if(!mat_y[y] || dfs(mat_y[y])) {
                mat_x[x] = y;
                mat_y[y] = x;
                return 1;
            }
        }
        else slack_y[y] = min(slack_y[y], t);
    }
    return 0;
}
inline void modify() {
    ll t = INF64;
    REP1(y, 1, nr) if(!vis_y[y]) t = min(t, slack_y[
y]);
    REP1(x, 1, nl) if(vis_x[x]) l_x[x] -= t;
    REP1(y, 1, nr) if(vis_y[y]) l_y[y] += t;
}
int main() {
    int m;
    RI(nl, nr, m);
    nr = max(nl, nr);
    while(m--) {
        int x, y;
        ll w;
        RI(x, y, w);
        W[x][y] = w;
        l_x[x] = max(l_x[x], w);
    }
    REP1(i, 1, nl) {
        while(1) {
            REP1(x, 1, nl) vis_x[x] = 0;
            REP1(y, 1, nr) vis_y[y] = 0, slack_y[y]
            = INF64;
            if(dfs(i)) break;
            else modify();
        }
    }
    ll ans = 0LL;
    REP1(x, 1, nl) ans += W[x][mat_x[x]];
    PL(ans);
    REP1(x, 1, nl) printf("%d%c", W[x][mat_x[x]] ?
    mat_x[x] : 0, "\n"[x == nl]);
    return 0;
}

```

## 2.5 DMST

```

/*
 * Edmond's algoirthm for Directed MST
 * runs in O(VE)
 */
const int MAXV = 10010;
const int MAXE = 10010;
const int INF = 2147483647;
struct Edge{
    int u, v, c;
    Edge(int x=0, int y=0, int z=0) : u(x), v(y), c(z){}
};
int V, E, root;
Edge edges[MAXE];
inline int newV(){ return ++ V; }
inline void addEdge(int u, int v, int c)
{ edges[++E] = Edge(u, v, c); }
bool con[MAXV];
int mnInW[MAXV], prv[MAXV], cyc[MAXV], vis[MAXV];
inline int DMST(){
    fill(con, con+V+1, 0);
    int r1 = 0, r2 = 0;
    while(1){
        fill(mnInW, mnInW+V+1, INF);
        fill(prv, prv+V+1, -1);
    }
}

```

```

REP(i, 1, E){
    int u=edges[i].u, v=edges[i].v, c=edges[i].c;
    if(u != v && v != root && c < mnInW[v])
        mnInW[v] = c, prv[v] = u;
}
fill(vis, vis+V+1, -1);
fill(cyc, cyc+V+1, -1);
r1 = 0;
bool jf = 0;
REP(i, 1, V){
    if(con[i]) continue;
    if(prv[i] == -1 && i != root) return -1;
    if(prv[i] > 0) r1 += mnInW[i];
    int s;
    for(s = i; s != -1 && vis[s] == -1; s = prv[s])
        vis[s] = i;
    if(s > 0 && vis[s] == i){
        // get a cycle
        jf = 1; int v = s;
        do{
            cyc[v] = s, con[v] = 1;
            r2 += mnInW[v]; v = prv[v];
        }while(v != s);
        con[s] = 0;
    }
}
if(!jf) break;
REP(i, 1, E){
    int &u = edges[i].u;
    int &v = edges[i].v;
    if(cyc[v] > 0) edges[i].c -= mnInW[edges[i].v];
    if(cyc[u] > 0) edges[i].u = cyc[edges[i].u];
    if(cyc[v] > 0) edges[i].v = cyc[edges[i].v];
    if(u == v) edges[i--] = edges[E--];
}
}
return r1+r2;
}

```

## 2.6 SW min-cut

```

// global min cut
struct SW{ // O(V^3)
    static const int MXN = 514;
    int n, vst[MXN], del[MXN];
    int edge[MXN][MXN], wei[MXN];
    void init(int _n){
        n = _n; FZ(edge); FZ(del);
    }
    void addEdge(int u, int v, int w){
        edge[u][v] += w; edge[v][u] += w;
    }
    void search(int &s, int &t){
        FZ(vst); FZ(wei);
        s = t = -1;
        while (true){
            int mx=-1, cur=0;
            for (int i=0; i<n; i++){
                if (!del[i] && !vst[i] && mx<wei[i])
                    cur = i, mx = wei[i];
            }
            if (mx == -1) break;
            vst[cur] = 1;
            s = t; t = cur;
            for (int i=0; i<n; i++){
                if (!vst[i] && !del[i]) wei[i] += edge[cur][i];
            }
        }
    }
    int solve(){
        int res = 2147483647;
        for (int i=0, x, y; i<n-1; i++){
            search(x, y);
            res = min(res, wei[y]);
            del[y] = 1;
            for (int j=0; j<n; j++){
                edge[x][j] = (edge[j][x] += edge[y][j]);
            }
        }
        return res;
    }
}graph;

```

## 2.7 Max Cost Circulation

```

struct MaxCostCirc {
    static const int MAXN = 33;
    int n, m;
    struct Edge { int v, w, c, r; };
    vector<Edge> g[ MAXN ];
    int dis[ MAXN ], prv[ MAXN ], prve[ MAXN ];
    bool vis[ MAXN ];
    int ans;
    void init( int _n, int _m ) : n(_n), m(_m) {}
    void adde( int u, int v, int w, int c ) {
        g[ u ].push_back( { v, w, c, SZ( g[ v ] ) } );
        g[ v ].push_back( { u, -w, 0, SZ( g[ u ] )-1 } );
    }
    bool poscyc() {
        fill( dis, dis+n+1, 0 );
        fill( prv, prv+n+1, 0 );
        fill( vis, vis+n+1, 0 );
        int tmp = -1;
        FOR( t, n+1 ) {
            REP( i, 1, n ) {
                FOR( j, SZ( g[ i ] ) ) {
                    Edge& e = g[ i ][ j ];
                    if( e.c && dis[ e.v ] < dis[ i ]+e.w ) {
                        dis[ e.v ] = dis[ i ]+e.w;
                        prv[ e.v ] = i;
                        prve[ e.v ] = j;
                        if( t == n ) {
                            tmp = i;
                            break;
                        }
                    }
                }
            }
        }
        if( tmp == -1 ) return 0;
        int cur = tmp;
        while( !vis[ cur ] ) {
            vis[ cur ] = 1;
            cur = prv[ cur ];
        }
        int now = cur, cost = 0, df = 100000;
        do{
            Edge &e = g[ prv[ now ] ][ prve[ now ] ];
            df = min( df, e.c );
            cost += e.w;
            now = prv[ now ];
        }while( now != cur );
        ans += df*cost; now = cur;
        do{
            Edge &e = g[ prv[ now ] ][ prve[ now ] ];
            Edge &re = g[ now ][ e.r ];
            e.c -= df;
            re.c += df;
            now = prv[ now ];
        }while( now != cur );
        return 1;
    }
} circ;

```

## 2.8 Max flow with lower/upper bound

```

// Max flow with lower/upper bound on edges
// source = 1, sink = n
int in[ N ], out[ N ];
int l[ M ], r[ M ], a[ M ], b[ M ];
int solve() {
    flow.init( n );
    for( int i = 0; i < m; i ++ ) {
        in[ r[ i ] ] += a[ i ];
        out[ l[ i ] ] += a[ i ];
        flow.addEdge( l[ i ], r[ i ], b[ i ] - a[ i ] );
        // flow from l[i] to r[i] must in [a[i], b[i]]
    }
    int nd = 0;
    for( int i = 1; i <= n; i ++ ) {
        if( in[ i ] < out[ i ] ) {
            flow.addEdge( i, flow.t, out[ i ] - in[ i ] );
            nd += out[ i ] - in[ i ];
        }
        if( out[ i ] < in[ i ] )

```

```

        flow.addEdge( flow.s, i, in[ i ] - out[ i ] );
    }
    // original sink to source
    flow.addEdge( n, 1, INF );
    if( flow.maxflow() != nd )
        // no solution
        return -1;
    int ans = flow.G[ 1 ].back().c; // source to sink
    flow.G[ 1 ].back().c = flow.G[ n ].back().c = 0;
    // take out super source and super sink
    for( size_t i = 0; i < flow.G[ flow.s ].size(); i ++ ) {
        flow.G[ flow.s ][ i ].c = 0;
        Edge &e = flow.G[ flow.s ][ i ];
        flow.G[ e.v ][ e.r ].c = 0;
    }
    for( size_t i = 0; i < flow.G[ flow.t ].size(); i ++ ) {
        flow.G[ flow.t ][ i ].c = 0;
        Edge &e = flow.G[ flow.t ][ i ];
        flow.G[ e.v ][ e.r ].c = 0;
    }
    flow.addEdge( flow.s, 1, INF );
    flow.addEdge( n, flow.t, INF );
    flow.reset();
    return ans + flow.maxflow();
}

```

## 2.9 Relabel to Front

```

// O(N^3), 0-base
struct Edge {
    int from, to, cap, flow;
    Edge( int _from, int _to, int _cap, int _flow = 0 ) :
        from(_from), to(_to), cap(_cap), flow(_flow) {}
};
struct PushRelabel {
    int n;
    vector<Edge> edges;
    vector<int> count, h, inQ, excess;
    vector<vector<int>> > G;
    queue<int> Q;
    PushRelabel( int _n ) :
        n(_n), count(_n<<1), G(_n), h(_n), inQ(_n), excess(_n) {}
    void addEdge( int from, int to, int cap ) {
        G[from].push_back( edges.size() );
        edges.push_back( Edge( from, to, cap ) );
        G[to].push_back( edges.size() );
        edges.push_back( Edge( to, from, 0 ) );
    }
    void enqueue( int u ) {
        if( !inQ[u] && excess[u] > 0 ) Q.push( u ), inQ[u] = true;
    }
    void Push( int EdgeIdx ) {
        Edge &e = edges[ EdgeIdx ];
        int toPush = min<int>( e.cap - e.flow, excess[e.from] );
        if( toPush > 0 && h[e.from] > h[e.to] ) {
            e.flow += toPush;
            excess[e.to] += toPush;
            excess[e.from] -= toPush;
            edges[ EdgeIdx^1 ].flow -= toPush;
            enqueue( e.to );
        }
    }
    void Relabel( int u ) {
        count[h[u]] --; h[u] = 2*n-2;
        for( size_t i = 0; i < G[u].size(); ++i ) {
            Edge &e = edges[ G[u][i] ];
            if( e.cap > e.flow ) h[u] = min( h[u], h[e.to] );
        }
        count[++h[u]] += 1;
    }
    void gapRelabel( int height ) {
        for( int u = 0; u < n; ++u ) if( h[u] >= height && h[u] < n ) {
            count[h[u]] --;
            count[h[u] = n] += 1;
        }
    }
};

```

```

    enqueue(u);
}
}
void Discharge(int u) {
    for (size_t i = 0; excess[u] > 0 && i < G[u].size()
        ; ++i)
        Push(G[u][i]);
    if (excess[u] > 0) {
        if (h[u] < n && count[h[u]] < 2) gapRelabel(h[u]);
        else Relabel(u);
    }
    else if (!Q.empty()) { // dequeue
        Q.pop();
        inQ[u] = false;
    }
}
int solve(int src, int snk) {
    h[src] = n; inQ[src] = inQ[snk] = true;
    count[0] = n - (count[n] = 1);
    for (size_t i = 0; i < G[src].size(); ++i) {
        excess[src] += edges[G[src][i]].cap;
        Push(G[src][i]);
    }
    while (!Q.empty())
        Discharge(Q.front());
    return excess[snk];
}
};

```

## 2.10 Flow Method

Maximize  $c^T x$  subject to  $Ax \leq b$ ,  $x \geq 0$ ;  
 with the corresponding symmetric dual problem,  
 Minimize  $b^T y$  subject to  $A^T y \geq c$ ,  $y \geq 0$ .

Maximize  $c^T x$  subject to  $Ax \leq b$ ;  
 with the corresponding asymmetric dual problem,  
 Minimize  $b^T y$  subject to  $A^T y = c$ ,  $y \geq 0$ .

Minimum vertex cover on bipartite graph =  
 Maximum matching on bipartite graph =  
 Max flow with source to one side, other side to sink

To reconstruct the minimum vertex cover, dfs from each  
 unmatched vertex on the left side **and** with unused edges  
 only. Equivalently, dfs from source with unused edges  
 only **and** without visiting sink. Then, a vertex is  
 chosen  
 iff. it is on the left side **and** without visited **or** on  
 the right side **and** visited through dfs.

Maximum density subgraph  $(\sum W_e + \sum W_v) / |V|$

Binary search on answer:  
 For a fixed  $D$ , construct a Max flow model as follow:  
 Let  $S$  be Sum of all weight( **or** inf)  
 1. from source to each node with cap =  $S$   
 2. For each  $(u,v,w)$  in  $E$ ,  $(u \rightarrow v, \text{cap}=w)$ ,  $(v \rightarrow u, \text{cap}=w)$   
 3. For each node  $v$ , from  $v$  to sink with cap =  $S + 2 * D$   
 -  $\text{deg}[v] - 2 * (W \text{ of } v)$   
 where  $\text{deg}[v] = \sum \text{weight of edge associated with } v$   
 If  $\text{maxflow} < S * |V|$ ,  $D$  is an answer.

Requiring subgraph: all vertex can be reached from  
 source with  
 edge whose cap  $> 0$ .

## 3 Math

### 3.1 FFT

```

// const int MAXN = 262144;
// (must be 2^k)
// before any usage, run pre_fft() first

```

```

typedef long double ld;
typedef complex<ld> cplx;
const ld PI = acos(-1);
const cplx I(0, 1);
cplx omega[MAXN+1];
void pre_fft(){
    for(int i=0; i<=MAXN; i++)
        omega[i] = exp(i * 2 * PI / MAXN * I);
}
// n must be 2^k
void fft(int n, cplx a[], bool inv=false){
    int basic = MAXN / n;
    int theta = basic;
    for (int m = n; m >= 2; m >= 1) {
        int mh = m >> 1;
        for (int i = 0; i < mh; i++) {
            cplx w = omega[inv ? MAXN-(i*theta%MAXN)
                : i*theta%MAXN];
            for (int j = i; j < n; j += m) {
                int k = j + mh;
                cplx x = a[j] - a[k];
                a[j] += a[k];
                a[k] = w * x;
            }
        }
        theta = (theta * 2) % MAXN;
    }
    int i = 0;
    for (int j = 1; j < n - 1; j++) {
        for (int k = n >> 1; k > (i ^= k); k >= 1);
        if (j < i) swap(a[i], a[j]);
    }
    if(inv) for (i = 0; i < n; i++) a[i] /= n;
}

```

### 3.2 NTT

```

typedef long long LL;
// Remember coefficient are mod P
/* p=a*2^n+1
    n    2^n          p          a    root
    16   65536       65537       1    3
    20   1048576    7340033     7    3 */
// (must be 2^k)
template<LL P, LL root, int MAXN>
struct NTT{
    static LL bigmod(LL a, LL b) {
        LL res = 1;
        for (LL bs = a; b; b >= 1, bs = (bs * bs) % P)
            if (b&1) res=(res*bs)%P;
        return res;
    }
    static LL inv(LL a, LL b) {
        if(a==1)return 1;
        return (((LL)(a-inv(b%a,a))*b+1)/a)%b;
    }
    LL omega[MAXN+1];
    NTT() {
        omega[0] = 1;
        LL r = bigmod(root, (P-1)/MAXN);
        for (int i=1; i<=MAXN; i++)
            omega[i] = (omega[i-1]*r)%P;
    }
    // n must be 2^k
    void tran(int n, LL a[], bool inv_ntt=false){
        int basic = MAXN / n, theta = basic;
        for (int m = n; m >= 2; m >= 1) {
            int mh = m >> 1;
            for (int i = 0; i < mh; i++) {
                LL w = omega[i*theta%MAXN];
                for (int j = i; j < n; j += m) {
                    int k = j + mh;
                    LL x = a[j] - a[k];
                    if (x < 0) x += P;
                    a[j] += a[k];
                    if (a[j] > P) a[j] -= P;
                    a[k] = (w * x) % P;
                }
            }
        }
        theta = (theta * 2) % MAXN;
    }
}

```

```

    }
    int i = 0;
    for (int j = 1; j < n - 1; j++) {
        for (int k = n >> 1; k > (i ^ k); k >>= 1);
        if (j < i) swap(a[i], a[j]);
    }
    if (inv_ntt) {
        LL ni = inv(n, P);
        reverse(a + 1, a + n);
        for (i = 0; i < n; i++)
            a[i] = (a[i] * ni) % P;
    }
};
const LL P=2013265921, root=31;
const int MAXN=4194304;
NTT<P, root, MAXN> ntt;

```

### 3.3 Fast Walsh Transform

```

/* xor convolution:
 * x = (x0,x1) , y = (y0,y1)
 * z = ( x0y0 + x1y1 , x0y1 + x1y0 )
 * =>
 * x' = ( x0+x1 , x0-x1 ) , y' = ( y0+y1 , y0-y1 )
 * z' = ( ( x0+x1 )( y0+y1 ) , ( x0-x1 )( y0-y1 ) )
 * z = (1/2) * z'
 * or convolution:
 * x = (x0, x0+x1), inv = (x0, x1-x0) w/o final div
 * and convolution:
 * x = (x0+x1, x1), inv = (x0-x1, x1) w/o final div */
typedef long long LL;
const int MAXN = (1<<20)+10;
const LL MOD = 1e9+7;
inline LL pw( LL x , LL k ) {
    LL res = 1;
    for( LL bs = x ; k ; k >>= 1, bs = (bs * bs)%MOD )
        if( k&1 ) res = ( res * bs ) % MOD;
    return res;
}
inline LL inv( LL x ) {
    return pw( x , MOD-2 );
}
inline void fwt( LL x[ MAXN ] , int N , bool inv=0 ) {
    for( int d = 1 ; d < N ; d <= 1 ) {
        int d2 = d<<1;
        for( int s = 0 ; s < N ; s += d2 )
            for( int i = s , j = s+d ; i < s+d ; i++, j++ ){
                LL ta = x[ i ] , tb = x[ j ];
                x[ i ] = ta+tb;
                x[ j ] = ta-tb;
                if( x[ i ] >= MOD ) x[ i ] -= MOD;
                if( x[ j ] < 0 ) x[ j ] += MOD;
            }
    }
    if( inv )
        for( int i = 0 ; i < N ; i++ ) {
            x[ i ] *= inv( N );
            x[ i ] %= MOD;
        }
}

```

### 3.4 Poly operator

```

struct PolyOp {
#define FOR(i, c) for (int i = 0; i < (c); ++i)
    NTT<P, root, MAXN> ntt;
    static int nxt2k(int x) {
        int i = 1; for (; i < x; i <= 1); return i;
    }
    void Mul(int n, LL a[], int m, LL b[], LL c[]) {
        static LL aa[MAXN], bb[MAXN];
        int N = nxt2k(n+m);
        copy(a, a+n, aa); fill(aa+n, aa+N, 0);
        copy(b, b+m, bb); fill(bb+m, bb+N, 0);
        ntt(N, aa); ntt(N, bb);
        FOR(i, N) c[i] = aa[i] * bb[i] % P;
        ntt(N, c, 1);
    }
};

```

```

}
void Inv(int n, LL a[], LL b[]) {
    // ab = aa^-1 = 1 mod x^(n/2)
    // (b - a^-1)^2 = 0 mod x^n
    // bb - a^-2 + 2 ba^-1 = 0
    // bba - a^-1 + 2b = 0
    // bba + 2b = a^-1
    static LL tmp[MAXN];
    if (n == 1) {b[0] = ntt.inv(a[0], P); return;}
    Inv((n+1)/2, a, b);
    int N = nxt2k(n*2);
    copy(a, a+n, tmp);
    fill(tmp+n, tmp+N, 0);
    fill(b+n, b+N, 0);
    ntt(N, tmp); ntt(N, b);
    FOR(i, N) {
        LL t1 = (2 - b[i] * tmp[i]) % P;
        if (t1 < 0) t1 += P;
        b[i] = b[i] * t1 % P;
    }
    ntt(N, b, 1);
    fill(b+n, b+N, 0);
}
void Div(int n, LL a[], int m, LL b[], LL d[], LL r
    []) {
    // Ra = Rb * Rd mod x^(n-m+1)
    // Rd = Ra * Rb^-1 mod
    static LL aa[MAXN], bb[MAXN], ta[MAXN], tb[MAXN];
    if (n < m) {copy(a, a+n, r); fill(r+n, r+m, 0);
        return;}
    // d: n-1 - (m-1) = n-m (n-m+1 terms)
    copy(a, a+n, aa); copy(b, b+m, bb);
    reverse(aa, aa+n); reverse(bb, bb+m);
    Inv(n-m+1, bb, tb);
    Mul(n-m+1, ta, n-m+1, tb, d);
    fill(d+n-m+1, d+n, 0); reverse(d, d+n-m+1);
    // r: m-1 - 1 = m-2 (m-1 terms)
    Mul(m, b, n-m+1, d, ta);
    FOR(i, n) { r[i] = a[i] - ta[i]; if (r[i] < 0) r[i]
        += P; }
}
void dx(int n, LL a[], LL b[]) { REP(i, 1, n-1) b[i]
    -1] = i * a[i] % P; }
void Sx(int n, LL a[], LL b[]) {
    b[0] = 0;
    FOR(i, n) b[i+1] = a[i] * ntt.iv[i+1] % P;
}
void Ln(int n, LL a[], LL b[]) {
    // Integral a' a^-1 dx
    static LL a1[MAXN], a2[MAXN], b1[MAXN];
    int N = nxt2k(n*2);
    dx(n, a, a1); Inv(n, a, a2);
    Mul(n-1, a1, n, a2, b1);
    Sx(n+n-1-1, b1, b);
    fill(b+n, b+N, 0);
}
void Exp(int n, LL a[], LL b[]) {
    // Newton method to solve g(a(x)) = ln b(x) - a(x)
    // = 0
    // b' = b - g(b(x)) / g'(b(x))
    // b' = b (1 - lnb + a)
    static LL lnb[MAXN], c[MAXN], tmp[MAXN];
    assert(a[0] == 0); // dont know exp(a[0]) mod P
    if (n == 1) {b[0] = 1; return;}
    Exp((n+1)/2, a, b);
    fill(b+(n+1)/2, b+n, 0);
    Ln(n, b, lnb);
    fill(c, c+n, 0); c[0] = 1;
    FOR(i, n) {
        c[i] += a[i] - lnb[i];
        if (c[i] < 0) c[i] += P;
        if (c[i] >= P) c[i] -= P;
    }
    Mul(n, b, n, c, tmp);
    copy(tmp, tmp+n, b);
}
} polyop;

```

### 3.5 Linear Recurrence



```

LL n, m, dp[ N + N ];
// dp[ n ] = \sum_{i=0}^{m-1} A_i * dp[ n - i - 1 ]
void pre_dp( const vector<LL>& A ){
    dp[ 0 ] = 1;
    LL bdr = min( m + m , n );
    for( LL i = 1 ; i <= bdr ; i ++ ){
        dp[ i ] = 0;
        for( LL j = i - 1 ; j >= max( 0LL , i - m ) ; j -- )
            dp[ i ] = add( dp[ i ] , mul( dp[ j ] , A[ i - j - 1 ] ) );
    }
}
vector<LL> Mul( vector<LL>& v1, vector<LL>& v2 ){
    int _sz1 = (int)v1.size(), _sz2 = (int)v2.size();
    assert( _sz1 == m ); assert( _sz2 == m );
    vector<LL> _v( m + m );
    for( int i = 0 ; i < m + m ; i ++ ) _v[ i ] = 0;
    // expand
    for( int i = 0 ; i < _sz1 ; i ++ )
        for( int j = 0 ; j < _sz2 ; j ++ )
            _v[ i + j + 1 ] = add( _v[ i + j + 1 ] , mul( v1[ i ] , v2[ j ] ) );
    // shrink
    for( int i = 0 ; i < m ; i ++ )
        for( int j = 1 ; j <= m ; j ++ )
            _v[ i + j ] = add( _v[ i + j ] , _v[ i ] );
    for( int i = 0 ; i < m ; i ++ ) _v[ i ] = _v[ i + m ];
    _v.resize( m );
    return _v;
}
vector<LL> I, A;
LL solve(){ // A should be filled
    pre_dp( A );
    if( n <= m + m ) return dp[ n ];
    I.resize( m );
    for( int i = 0 ; i < m ; i ++ ) I[ i ] = 1;
    LL dlt = ( n - m ) / m, rdlt = dlt * m;
    while( dlt ){
        if( dlt & 1LL ) I = Mul( I , A );
        A = Mul( A , A ); dlt >>= 1;
    }
    LL ans = 0;
    for( int i = 0 ; i < m ; i ++ )
        ans = add( ans, mul( I[ i ], dp[ n - i - 1 - rdlt ] ) );
    return ans;
}

```

### 3.6 Miller Rabin

```

// n < 4,759,123,141      3 : 2, 7, 61
// n < 1,122,004,669,633  4 : 2, 13, 23, 1662803
// n < 3,474,749,660,383  6 : pimes <= 13
// n < 2^64               7 :
// 2, 325, 9375, 28178, 450775, 9780504, 1795265022
// Make sure testing integer is in range [2, n-2] if
// you want to use magic.
bool witness(LL a, LL n, LL u, int t){
    LL x = mypow(a, u, n);
    for( int i = 0; i < t; i ++ ) {
        LL nx = mul(x, x, n);
        if( nx == 1 && x != 1 && x != n-1 ) return 1;
        x = nx;
    }
    return x != 1;
}
bool miller_rabin(LL n, int s=100) {
    // iterate s times of witness on n
    // return 1 if prime, 0 otherwise
    if( n < 2 ) return 0;
    if( !(n & 1) ) return n == 2;
    LL u = n-1; int t = 0;
    // n-1 = u*2^t
    while( !(u & 1) ) u >>= 1, t++;
    while( s-- ){
        LL a = randll() % (n-1) + 1;
        if( witness(a, n, u, t) ) return 0;
    }
    return 1;
}

```

### 3.7 Simplex

```

const int MAXN = 111;
const int MAXM = 111;
const double eps = 1E-10;
double a[MAXN][MAXM], b[MAXN], c[MAXN], d[MAXN][MAXM];
double x[MAXN];
int ix[MAXN + MAXM]; // !!! array all indexed from 0
// max{cx} subject to {Ax<=b, x>=0}
// n: constraints, m: vars !!!
// x[] is the optimal solution vector
// usage :
// value = simplex(a, b, c, N, M);
double simplex(double a[MAXN][MAXM], double b[MAXN],
               double c[MAXN], int n, int m){
    ++m;
    int r = n, s = m - 1;
    memset(d, 0, sizeof(d));
    for( int i = 0; i < n + m; ++i ) ix[i] = i;
    for( int i = 0; i < n; ++i ) {
        for( int j = 0; j < m - 1; ++j ) d[i][j] = -a[i][j];
        d[i][m - 1] = 1;
        d[i][m] = b[i];
        if( d[r][m] > d[i][m] ) r = i;
    }
    for( int j = 0; j < m - 1; ++j ) d[n][j] = c[j];
    d[n + 1][m - 1] = -1;
    for( double dd;; ) {
        if( r < n ) {
            int t = ix[s]; ix[s] = ix[r + m]; ix[r + m] = t;
            d[r][s] = 1.0 / d[r][s];
            for( int j = 0; j <= m; ++j )
                if( j != s ) d[r][j] *= -d[r][s];
            for( int i = 0; i <= n + 1; ++i ) if( i != r ) {
                for( int j = 0; j <= m; ++j ) if( j != s )
                    d[i][j] += d[r][j] * d[i][s];
                d[i][s] *= d[r][s];
            }
        }
        r = -1; s = -1;
        for( int j = 0; j < m; ++j )
            if( s < 0 || ix[s] > ix[j] ) {
                if( d[n + 1][j] > eps ||
                    (d[n + 1][j] > -eps && d[n][j] > eps) )
                    s = j;
            }
        if( s < 0 ) break;
        for( int i = 0; i < n; ++i ) if( d[i][s] < -eps ) {
            if( r < 0 ||
                (dd = d[r][m] / d[r][s] - d[i][m] / d[i][s]) < -eps ||
                (dd < eps && ix[r + m] > ix[i + m]) )
                r = i;
        }
        if( r < 0 ) return -1; // not bounded
    }
    if( d[n + 1][m] < -eps ) return -1; // not executable
    double ans = 0;
    for( int i = 0; i < m; i ++ ) x[i] = 0;
    for( int i = m; i < n + m; ++i ) { // the missing
        enumerated x[i] = 0
        if( ix[i] < m - 1 ) {
            ans += d[i - m][m] * c[ix[i]];
            x[ix[i]] = d[i - m][m];
        }
    }
    return ans;
}

```

### 3.8 Faulhaber

```

/* faulhaber 's formula -
 * cal power sum formula of all p=1~k in O(k^2) */
#define MAXK 2500
const int mod = 1000000007;
int b[MAXK]; // bernoulli number
int inv[MAXK+1]; // inverse
int cm[MAXK+1][MAXK+1]; // combinatorics
int co[MAXK][MAXK+2]; // coeeficient of x^j when p=i

```

```

inline int getinv(int x) {
    int a=x,b=mod,a0=1,a1=0,b0=0,b1=1;
    while(b) {
        int q,t;
        q=a/b; t=b; b=a-b*q; a=t;
        t=b0; b0=a0-b0*q; a0=t;
        t=b1; b1=a1-b1*q; a1=t;
    }
    return a0<0?a0+mod:a0;
}
inline void pre() {
    /* combinational */
    for(int i=0;i<=MAXK;i++) {
        cm[i][0]=cm[i][i]=1;
        for(int j=1;j<i;j++)
            cm[i][j]=add(cm[i-1][j-1],cm[i-1][j]);
    }
    /* inverse */
    for(int i=1;i<=MAXK;i++) inv[i]=getinv(i);
    /* bernoulli */
    b[0]=1; b[1]=getinv(2); // with b[1] = 1/2
    for(int i=2;i<MAXK;i++) {
        if(i&1) { b[i]=0; continue; }
        b[i]=1;
        for(int j=0;j<i;j++)
            b[i]=sub(b[i],
                mul(cm[i][j],mul(b[j], inv[i-j+1])));
    }
    /* faulhaber */
    // sigma_x=1~n {x^p} =
    // 1/(p+1) * sigma_j=0~p {C(p+1,j)*Bj*n^(p-j+1)}
    for(int i=1;i<MAXK;i++) {
        co[i][0]=0;
        for(int j=0;j<=i;j++)
            co[i][i-j+1]=mul(inv[i+1], mul(cm[i+1][j], b[j]));
    }
}
/* sample usage: return f(n,p) = sigma_x=1~n (x^p) */
inline int solve(int n,int p) {
    int sol=0,m=n;
    for(int i=1;i<=p+1;i++) {
        sol=add(sol,mul(co[p][i],m));
        m = mul(m, n);
    }
    return sol;
}

```

### 3.9 Chinese Remainder

```

LL solve(LL x1, LL m1, LL x2, LL m2) {
    LL g = __gcd(m1, m2);
    if((x2 - x1) % g) return -1; // no sol
    m1 /= g; m2 /= g;
    pair<LL,LL> p = gcd(m1, m2);
    LL lcm = m1 * m2 * g;
    LL res = p.first * (x2 - x1) * m1 + x1;
    return (res % lcm + lcm) % lcm;
}

```

### 3.10 Pollard Rho

```

// does not work when n is prime
LL f(LL x, LL mod){ return add(mul(x,x,mod),1,mod); }
LL pollard_rho(LL n) {
    if(!(n&1)) return 2;
    while(true){
        LL y=2, x=rand()%(n-1)+1, res=1;
        for(int sz=2; res==1; sz*=2) {
            for(int i=0; i<sz && res<=1; i++) {
                x = f(x, n);
                res = __gcd(abs(x-y), n);
            }
            y = x;
        }
        if (res!=0 && res!=n) return res;
    }
}

```

### 3.11 ax+by=gcd

```

PII gcd(int a, int b){
    if(b == 0) return {1, 0};
    PII q = gcd(b, a % b);
    return {q.second, q.first - q.second * (a / b)};
}

```

### 3.12 Discrete sqrt

```

void calcH(int &t, int &h, const int p) {
    int tmp=p-1; for(t=0;(tmp&1)==0;tmp/=2) t++; h=tmp;
}
// solve equation x^2 mod p = a
bool solve(int a, int p, int &x, int &y) {
    if(p == 2) { x = y = 1; return true; }
    int p2 = p / 2, tmp = mypow(a, p2, p);
    if (tmp == p - 1) return false;
    if ((p + 1) % 4 == 0) {
        x=mypow(a,(p+1)/4,p); y=p-x; return true;
    } else {
        int t, h, b, pb; calcH(t, h, p);
        if (t >= 2) {
            do {b = rand() % (p - 2) + 2;
                while (mypow(b, p / 2, p) != p - 1);
                pb = mypow(b, h, p);
            } int s = mypow(a, h / 2, p);
            for (int step = 2; step <= t; step++) {
                int ss = (((LL)(s * s) % p) * a) % p;
                for(int i=0;i<t-step;i++) ss=mul(ss,ss,p);
                if (ss + 1 == p) s = (s * pb) % p;
                pb = ((LL)pb * pb) % p;
            } x = ((LL)s * a) % p; y = p - x;
        } return true;
    }
}

```

### 3.13 SchreierSims

```

// time: O(n^2 lg^3 |G| + t n lg |G|)
// mem : O(n^2 lg |G| + tn)
// t : number of generator
namespace SchreierSimsAlgorithm{
    typedef vector<int> Permu;
    Permu inv( const Permu& p ){
        Permu ret( p.size() );
        for( int i = 0; i < int(p.size()); i ++ )
            ret[ p[ i ] ] = i;
        return ret;
    }
    Permu operator*( const Permu& a, const Permu& b ){
        Permu ret( a.size() );
        for( int i = 0; i < (int)a.size(); i ++ )
            ret[ i ] = b[ a[ i ] ];
        return ret;
    }
    typedef vector<Permu> Bucket;
    typedef vector<int> Table;
    typedef pair<int,int> pii;
    int n, m;
    vector<Bucket> bkts, bktsInv;
    vector<Table> lookup;
    int fastFilter( const Permu &g, bool addToG = 1 ){
        n = bkts.size();
        Permu p;
        for( int i = 0; i < n; i ++ ){
            int res = lookup[ i ][ p[ i ] ];
            if( res == -1 ){
                if( addToG ){
                    bkts[ i ].push_back( p );
                    bktsInv[ i ].push_back( inv( p ) );
                    lookup[ i ][ p[ i ] ] = (int)bkts[i].size()-1;
                }
                return i;
            }
            p = p * bktsInv[i][res];
        }
        return -1;
    }
}

```



```

}
long long calcTotalSize(){
    long long ret = 1;
    for( int i = 0 ; i < n ; i ++ )
        ret *= bkts[i].size();
    return ret;
}
bool inGroup( const Permu &g ){
    return fastFilter( g, false ) == -1;
}
void solve( const Bucket &gen, int _n ){
    n = _n, m = gen.size(); // m perm[0..n-1]s
    //clear all
    bkts.clear();
    bktsInv.clear();
    lookup.clear();
}
for(int i = 0 ; i < n ; i ++ ){
    lookup[i].resize(n);
    fill(lookup[i].begin(), lookup[i].end(), -1);
}
Permu id( n );
for(int i = 0 ; i < n ; i ++ ) id[i] = i;
for(int i = 0 ; i < n ; i ++ ){
    bkts[i].push_back(id);
    bktsInv[i].push_back(id);
    lookup[i][i] = 0;
}
for(int i = 0 ; i < m ; i ++ )
    fastFilter( gen[i] );
queue< pair<pii,pii> > toUpd;
for(int i = 0 ; i < n ; i ++ )
    for(int j = i ; j < n ; j ++ )
        for(int k = 0 ; k < (int)bkts[i].size(); k ++ )
            for(int l = 0 ; l < (int)bkts[j].size(); l ++ )
                toUpd.push( {pii(i,k), pii(j,l)} );
while( !toUpd.empty() ){
    pii a = toUpd.front().first;
    pii b = toUpd.front().second;
    toUpd.pop();
    int res = fastFilter(bkts[a.first][a.second] *
                        bkts[b.first][b.second]);
    if(res == -1) continue;
    pii newPair(res, (int)bkts[res].size() - 1);
    for(int i = 0 ; i < n ; i ++ )
        for(int j = 0 ; j < (int)bkts[i].size(); ++j){
            if(i <= res)
                toUpd.push(make_pair(pii(i, j), newPair));
            if(res <= i)
                toUpd.push(make_pair(newPair, pii(i, j)));
        }
}
}
}
}

```

### 3.14 Romberg

```

// Estimates the definite integral of
// \int_a^b f(x) dx
template<class T>
double romberg( T& f, double a, double b, double eps=1e-8 ){
    vector<double> t; double h=b-a, last, curr; int k=1, i=1;
    t.push_back(h*(f(a)+f(b))/2);
    do{ last=t.back(); curr=0; double x=a+h/2;
        for(int j=0; j<k; j++) curr+=f(x), x+=h;
        curr=(t[0] + h*curr)/2; double k1=4.0/3.0, k2=1.0/3.0;
        for(int j=0; j<i; j++){ double temp=k1*curr-k2*t[j];
            t[j]=curr; curr=temp; k2/=4*k1-k2; k1=k2+1;
        } t.push_back(curr); k*=2; h/=2; i++;
    }while( fabs(last-curr) > eps);
    return t.back();
}

```

### 3.15 Prefix Inverse

```

void solve( int m ){
    inv[ 1 ] = 1;
    for( int i = 2 ; i < m ; i ++ )
        inv[ i ] = ((LL)(m - m / i) * inv[m % i]) % m;
}

```

### 3.16 Roots of Polynomial

```

const double eps = 1e-12;
const double inf = 1e+12;
double a[ 10 ], x[ 10 ];
int n;
int sign( double x ){return (x < -eps)?(-1):(x>eps);}
double f(double a[], int n, double x){
    double tmp=1, sum=0;
    for(int i=0; i<n; i++){
        sum=sum+a[i]*tmp; tmp=tmp*x;
    }
    return sum;
}
double binary(double l, double r, double a[], int n){
    int sl=sign(f(a,n,l)), sr=sign(f(a,n,r));
    if(sl==0) return l; if(sr==0) return r;
    if(sl*sr>0) return inf;
    while(r-l>eps){
        double mid=(l+r)/2;
        int ss=sign(f(a,n,mid));
        if(ss==0) return mid;
        if(ss*sl>0) l=mid; else r=mid;
    }
    return l;
}
void solve(int n, double a[], double x[], int &nx){
    if(n==1){ x[1]=-a[0]/a[1]; nx=1; return; }
    double da[10], dx[10]; int ndx;
    for(int i=n; i>=1; i--) da[i-1]=a[i]*i;
    solve(n-1, da, dx, ndx);
    nx=0;
    if(ndx==0){
        double tmp=binary(-inf, inf, a, n);
        if (tmp<inf) x[++nx]=tmp;
        return;
    }
    double tmp;
    tmp=binary(-inf, dx[1], a, n);
    if(tmp<inf) x[++nx]=tmp;
    for(int i=1; i<=ndx-1; i++){
        tmp=binary(dx[i], dx[i+1], a, n);
        if(tmp<inf) x[++nx]=tmp;
    }
    tmp=binary(dx[ndx], inf, a, n);
    if(tmp<inf) x[++nx]=tmp;
}
int main() {
    scanf("%d", &n);
    for(int i=n; i>=0; i--) scanf("%lf", &a[i]);
    int nx;
    solve(n, a, x, nx);
    for(int i=1; i<=nx; i++) printf("%.6f\n", x[i]);
}

```

### 3.17 Result

- Lucas' Theorem :  
For  $n, m \in \mathbb{Z}^+$  and prime  $P$ ,  $C(m, n) \bmod P = \prod C(m_i, n_i)$  where  $m_i$  is the  $i$ -th digit of  $m$  in base  $P$ .
- Stirling Numbers(permutation  $|P| = n$  with  $k$  cycles):  
 $S(n, k) = \text{coefficient of } x^k \text{ in } \Pi_{i=0}^{n-1} (x+i)$
- Stirling Numbers(Partition  $n$  elements into  $k$  non-empty set):  
$$S(n, k) = \frac{1}{k!} \sum_{j=0}^k (-1)^{k-j} \binom{k}{j} j^n$$
- Pick's Theorem :  $A = i + b/2 - 1$
- Kirchhoff's theorem :  
 $A_{ii} = \deg(i), A_{ij} = (i, j) \in E ? - 1 : 0$ , Deleting any one row, one column, and cal the  $\det(A)$

## 4 Geometry

### 4.1 halfPlaneIntersection

### 4.2 Intersection of 2 lines

```
Pt interPnt( Line l1, Line l2, bool &res ){
    Pt p1, p2, q1, q2;
    tie(p1, p2) = l1; tie(q1, q2) = l2;
    double f1 = (p2 - p1) ^ (q1 - p1);
    double f2 = (p2 - p1) ^ (p1 - q2);
    double f = (f1 + f2);
    if( fabs(f) < eps){ res=0; return {0, 0}; }
    res = true;
    return q1 * (f2 / f) + q2 * (f1 / f);
}

bool isin( Line l0, Line l1, Line l2 ){
    // Check inter(l1, l2) in l0
    bool res; Pt p = interPnt(l1, l2, res);
    return ( (l0.SE - l0.FI) ^ (p - l0.FI) ) > eps;
}

/* If no solution, check: 1. ret.size() < 3
 * Or more precisely, 2. interPnt(ret[0], ret[1])
 * in all the lines. (use (l.S - l.F) ^ (p - l.F) > 0
 */
/* --- Line.FI --- Line.SE --- */
vector<Line> halfPlaneInter( vector<Line> lines ){
    int sz = lines.size();
    vector<double> ata(sz), ord(sz);
    for( int i=0; i<sz; i++) {
        ord[i] = i;
        Pt d = lines[i].SE - lines[i].FI;
        ata[i] = atan2(d.Y, d.X);
    }
    sort( ord.begin(), ord.end(), [&](int i, int j) {
        if( fabs(ata[i] - ata[j]) < eps )
            return ( (lines[i].SE - lines[i].FI) ^
                    (lines[j].SE - lines[i].FI) ) < 0;
        return ata[i] < ata[j];
    });
    vector<Line> fin;
    for( int i=0; i<sz; i++)
        if (!i or fabs(ata[ord[i]] - ata[ord[i-1]]) > eps)
            fin.pb(lines[ord[i]]);
    deque<Line> dq;
    for( int i=0; i<(int)(fin.size()); i++) {
        while((int)(dq.size()) >= 2 and
            not isin(fin[i], dq[(int)(dq.size())-2],
                    dq[(int)(dq.size())-1]))
            dq.pop_back();
        while((int)(dq.size()) >= 2 and
            not isin(fin[i], dq[0], dq[1]))
            dq.pop_front();
        dq.push_back(fin[i]);
    }
    while( (int)(dq.size()) >= 3 and
        not isin(dq[0], dq[(int)(dq.size())-2],
                dq[(int)(dq.size())-1]))
        dq.pop_back();
    while( (int)(dq.size()) >= 3 and
        not isin(dq[(int)(dq.size())-1], dq[0], dq[1]))
        dq.pop_front();
    vector<Line> res(dq.begin(), dq.end());
    return res;
}
```

### 4.3 Intersection of 2 segments

```
int ori( const Pt& o , const Pt& a , const Pt& b ){
    LL ret = ( a - o ) ^ ( b - o );
    return (ret > 0) - (ret < 0);
}

// p1 == p2 || q1 == q2 need to be handled
bool banana( const Pt& p1 , const Pt& p2 ,
              const Pt& q1 , const Pt& q2 ){
    if( ( ( p2 - p1 ) ^ ( q2 - q1 ) ) == 0 ){ // parallel
        if( ori( p1 , p2 , q1 ) ) return false;
        return ( ( p1 - q1 ) * ( p2 - q1 ) ) <= 0 ||
```

```
( ( p1 - q2 ) * ( p2 - q2 ) ) <= 0 ||
( ( q1 - p1 ) * ( q2 - p1 ) ) <= 0 ||
( ( q1 - p2 ) * ( q2 - p2 ) ) <= 0;
}
return (ori( p1, p2, q1 ) * ori( p1, p2, q2 ) <= 0) &&
(ori( q1, q2, p1 ) * ori( q1, q2, p2 ) <= 0);
}
```

### 4.4 Intersection of circle and segment

```
bool Inter( const Pt& p1 , const Pt& p2 , Circle& cc ){
    Pt dp = p2 - p1;
    double a = dp * dp;
    double b = 2 * ( dp * ( p1 - cc.O ) );
    double c = cc.O * cc.O + p1 * p1 - 2 * ( cc.O * p1 )
        - cc.R * cc.R;
    double bb4ac = b * b - 4 * a * c;
    return !( fabs( a ) < eps or bb4ac < 0 );
}
```

### 4.5 Intersection of 2 circles

### 4.6 Circle cover

```
#define N 1021
struct CircleCover{
    int C; Circ c[ N ];
    bool g[ N ][ N ], overlap[ N ][ N ];
    // Area[i] : area covered by at least i circles
    D Area[ N ];
    void init( int _C ){ C = _C; }
    bool Cinter( Circ& a , Circ& b , Pt& p1 , Pt& p2 ){
        Pt o1 = a.O , o2 = b.O;
        D r1 = a.R , r2 = b.R;
        if( norm( o1 - o2 ) > r1 + r2 ) return false;
        if( norm( o1 - o2 ) < max(r1, r2) - min(r1, r2) )
            return true;
        D d2 = ( o1 - o2 ) * ( o1 - o2 );
        D d = sqrt(d2);
        if( d > r1 + r2 ) return false;
        Pt u = (o1+o2)*0.5 + (o1-o2)*((r2*r2-r1*r1)/(2*d2));
        D A = sqrt((r1+r2+d)*(r1-r2+d)*(r1+r2-d)*(-r1+r2+d));
        Pt v = Pt( o1.Y-o2.Y , -o1.X + o2.X ) * A / (2*d2);
        p1 = u + v; p2 = u - v;
        return true;
    }
    struct Teve {
        Pt p; D ang; int add;
        Teve() {}
        Teve(Pt _a, D _b, int _c):p(_a), ang(_b), add(_c){}
        bool operator<(const Teve &a)const {
            return ang < a.ang;
        }
    } eve[ N * 2 ];
    // strict: x = 0, otherwise x = -1
    bool disjunct( Circ& a, Circ &b, int x )
    {return sign( norm( a.O - b.O ) - a.R - b.R ) > x;}
    bool contain( Circ& a, Circ &b, int x )
    {return sign( a.R - b.R - norm( a.O - b.O ) ) > x;}
    bool contain(int i, int j){
        /* c[j] is non-strictly in c[i]. */
        return (sign(c[i].R - c[j].R) > 0 ||
            (sign(c[i].R - c[j].R) == 0 && i < j) ) &&
            contain(c[i], c[j], -1);
    }
    void solve(){
        for( int i = 0 ; i <= C + 1 ; i ++ )
            Area[ i ] = 0;
        for( int i = 0 ; i < C ; i ++ )
            for( int j = 0 ; j < C ; j ++ )
                overlap[i][j] = contain(i, j);
        for( int i = 0 ; i < C ; i ++ )
            for( int j = 0 ; j < C ; j ++ )
                g[i][j] = !(overlap[i][j] || overlap[j][i] ||
                    disjunct(c[i], c[j], -1));
        for( int i = 0 ; i < C ; i ++ ){
            int E = 0, cnt = 1;
            for( int j = 0 ; j < C ; j ++ )
```

```

    if( j != i && overlap[j][i] )
        cnt ++;
    for( int j = 0 ; j < C ; j ++ )
        if( i != j && g[i][j] ){
            Pt aa, bb;
            CCinter(c[i], c[j], aa, bb);
            D A=atan2(aa.Y - c[i].O.Y, aa.X - c[i].O.X);
            D B=atan2(bb.Y - c[i].O.Y, bb.X - c[i].O.X);
            eve[E ++] = Teve(bb, B, 1);
            eve[E ++] = Teve(aa, A, -1);
            if(B > A) cnt ++;
        }
    if( E == 0 ) Area[ cnt ] += pi * c[i].R * c[i].R;
    else{
        sort( eve , eve + E );
        eve[E] = eve[0];
        for( int j = 0 ; j < E ; j ++ ){
            cnt += eve[j].add;
            Area[cnt] += (eve[j].p ^ eve[j + 1].p) * .5;
            D theta = eve[j + 1].ang - eve[j].ang;
            if (theta < 0) theta += 2. * pi;
            Area[cnt] +=
                (theta - sin(theta)) * c[i].R*c[i].R * .5;
        }
    }
}
};

```

#### 4.7 Convex Hull trick

```

/* Given a convexhull, answer queries in O(\lg N)
CH should not contain identical points, the area should
be > 0, min pair(x, y) should be listed first */
double det( const Pt& p1 , const Pt& p2 )
{ return p1.X * p2.Y - p1.Y * p2.X; }
struct Conv{
    int n;
    vector<Pt> a;
    vector<Pt> upper, lower;
    Conv(vector<Pt> _a) : a(_a){
        n = a.size();
        int ptr = 0;
        for(int i=1; i<n; ++i) if (a[ptr] < a[i]) ptr = i;
        for(int i=0; i<=ptr; ++i) lower.push_back(a[i]);
        for(int i=ptr; i<n; ++i) upper.push_back(a[i]);
        upper.push_back(a[0]);
    }
    int sign( LL x ){ // fixed when changed to double
        return x < 0 ? -1 : x > 0; }
    pair<LL,int> get_tang(vector<Pt> &conv, Pt vec){
        int l = 0, r = (int)conv.size() - 2;
        for( ; l + 1 < r; ){
            int mid = (l + r) / 2;
            if(sign(det(conv[mid+1]-conv[mid],vec))>0)r=mid;
            else l = mid;
        }
        return max(make_pair(det(vec, conv[r]), r),
            make_pair(det(vec, conv[0]), 0));
    }
    void upd_tang(const Pt &p, int id, int &i0, int &i1){
        if(det(a[i0] - p, a[id] - p) > 0) i0 = id;
        if(det(a[i1] - p, a[id] - p) < 0) i1 = id;
    }
    void bi_search(int l, int r, Pt p, int &i0, int &i1){
        if(l == r) return;
        upd_tang(p, l % n, i0, i1);
        int sl=sign(det(a[l % n] - p, a[(l + 1) % n] - p));
        for( ; l + 1 < r; ){
            int mid = (l + r) / 2;
            int smid=sign(det(a[mid%n]-p, a[(mid+1)%n]-p));
            if (smid == sl) l = mid;
            else r = mid;
        }
        upd_tang(p, r % n, i0, i1);
    }
    int bi_search(Pt u, Pt v, int l, int r) {
        int sl = sign(det(v - u, a[l % n] - u));
        for( ; l + 1 < r; ){
            int mid = (l + r) / 2;

```

```

            int smid = sign(det(v - u, a[mid % n] - u));
            if (smid == sl) l = mid;
            else r = mid;
        }
        return l % n;
    }
    // 1. whether a given point is inside the CH
    bool contain(Pt p) {
        if (p.X < lower[0].X || p.X > lower.back().X)
            return 0;
        int id = lower_bound(lower.begin(), lower.end(), Pt
            (p.X, -INF)) - lower.begin();
        if (lower[id].X == p.X) {
            if (lower[id].Y > p.Y) return 0;
        }else if(det(lower[id-1]-p,lower[id]-p)<0)return 0;
        id = lower_bound(upper.begin(), upper.end(), Pt(p.X
            , INF), greater<Pt>()) - upper.begin();
        if (upper[id].X == p.X) {
            if (upper[id].Y < p.Y) return 0;
        }else if(det(upper[id-1]-p,upper[id]-p)<0)return 0;
        return 1;
    }
    // 2. Find 2 tang pts on CH of a given outside point
    // return true with i0, i1 as index of tangent points
    // return false if inside CH
    bool get_tang(Pt p, int &i0, int &i1) {
        if (contain(p)) return false;
        i0 = i1 = 0;
        int id = lower_bound(lower.begin(), lower.end(), p)
            - lower.begin();
        bi_search(0, id, p, i0, i1);
        bi_search(id, (int)lower.size(), p, i0, i1);
        id = lower_bound(upper.begin(), upper.end(), p,
            greater<Pt>()) - upper.begin();
        bi_search((int)lower.size() - 1, (int)lower.size()
            - 1 + id, p, i0, i1);
        bi_search((int)lower.size() - 1 + id, (int)lower.
            size() - 1 + (int)upper.size(), p, i0, i1);
        return true;
    }
    // 3. Find tangent points of a given vector
    // ret the idx of vertex has max cross value with vec
    int get_tang(Pt vec){
        pair<LL, int> ret = get_tang(upper, vec);
        ret.second = (ret.second+(int)lower.size()-1)%n;
        ret = max(ret, get_tang(lower, vec));
        return ret.second;
    }
    // 4. Find intersection point of a given line
    // return 1 and intersection is on edge (i, next(i))
    // return 0 if no strictly intersection
    bool get_intersection(Pt u, Pt v, int &i0, int &i1){
        int p0 = get_tang(u - v), p1 = get_tang(v - u);
        if(sign(det(v-u,a[p0]-u))*sign(det(v-u,a[p1]-u))<0){
            if (p0 > p1) swap(p0, p1);
            i0 = bi_search(u, v, p0, p1);
            i1 = bi_search(u, v, p1, p0 + n);
            return 1;
        }
        return 0;
    }
};

```

#### 4.8 Tangent line of two circles

```

vector<Line> go( const Cir& c1 , const Cir& c2 , int
    sign1 ){
    // sign1 = 1 for outer tang, -1 for inter tang
    vector<Line> ret;
    double d_sq = norm2( c1.O - c2.O );
    if( d_sq < eps ) return ret;
    double d = sqrt( d_sq );
    Pt v = ( c2.O - c1.O ) / d;
    double c = ( c1.R - sign1 * c2.R ) / d;
    if( c * c > 1 ) return ret;
    double h = sqrt( max( 0.0 , 1.0 - c * c ) );
    for( int sign2 = 1 ; sign2 >= -1 ; sign2 -= 2 ){
        Pt n = { v.X * c - sign2 * h * v.Y ,
            v.Y * c + sign2 * h * v.X };
        Pt p1 = c1.O + n * c1.R;

```

```

    Pt p2 = c2.0 + n * ( c2.R * sign1 );
    if( fabs( p1.X - p2.X ) < eps and
        fabs( p1.Y - p2.Y ) < eps )
        p2 = p1 + perp( c2.0 - c1.0 );
    ret.push_back( { p1 , p2 } );
}
return ret;
}

```

## 4.9 KD Tree

```

const int MXN = 100005;
struct KDTree {
    struct Nd {
        int x,y,x1,y1,x2,y2;
        int id,f;
        Nd *L, *R;
    } tree[MXN];
    int n;
    Nd *root;
    LL dis2(int x1, int y1, int x2, int y2) {
        LL dx = x1-x2; LL dy = y1-y2;
        return dx*dx+dy*dy;
    }
    static bool cmpx(Nd& a, Nd& b){ return a.x<b.x; }
    static bool cmpy(Nd& a, Nd& b){ return a.y<b.y; }
    void init(vector<pair<int,int>> ip) {
        n = ip.size();
        for (int i=0; i<n; i++) {
            tree[i].id = i;
            tree[i].x = ip[i].first;
            tree[i].y = ip[i].second;
        }
        root = build_tree(0, n-1, 0);
    }
    Nd* build_tree(int L, int R, int dep) {
        if (L>R) return nullptr;
        int M = (L+R)/2;
        tree[M].f = dep%2;
        nth_element(tree+L, tree+M, tree+R+1,
            tree[M].f ? cmpy : cmpx);
        tree[M].x1 = tree[M].x2 = tree[M].x;
        tree[M].y1 = tree[M].y2 = tree[M].y;

        tree[M].L = build_tree(L, M-1, dep+1);
        if (tree[M].L) {
            tree[M].x1 = min(tree[M].x1, tree[M].L->x1);
            tree[M].x2 = max(tree[M].x2, tree[M].L->x2);
            tree[M].y1 = min(tree[M].y1, tree[M].L->y1);
            tree[M].y2 = max(tree[M].y2, tree[M].L->y2);
        }

        tree[M].R = build_tree(M+1, R, dep+1);
        if (tree[M].R) {
            tree[M].x1 = min(tree[M].x1, tree[M].R->x1);
            tree[M].x2 = max(tree[M].x2, tree[M].R->x2);
            tree[M].y1 = min(tree[M].y1, tree[M].R->y1);
            tree[M].y2 = max(tree[M].y2, tree[M].R->y2);
        }
        return tree+M;
    }
    int touch(Nd* r, int x, int y, LL d2){
        LL dis = sqrt(d2)+1;
        if (x<r->x1-dis || x>r->x2+dis ||
            y<r->y1-dis || y>r->y2+dis)
            return 0;
        return 1;
    }
    void nearest(Nd* r, int x, int y, int &mID, LL &md2){
        if (!r || !touch(r, x, y, md2)) return;
        LL d2 = dis2(r->x, r->y, x, y);
        if (d2 < md2 || (d2 == md2 && mID < r->id)) {
            mID = r->id; md2 = d2;
        }
        // search order depends on split dim
        if ((r->f == 0 && x < r->x) ||
            (r->f == 1 && y < r->y)) {
            nearest(r->L, x, y, mID, md2);
            nearest(r->R, x, y, mID, md2);
        } else {

```

```

            nearest(r->R, x, y, mID, md2);
            nearest(r->L, x, y, mID, md2);
        }
    }
    int query(int x, int y) {
        int id = 1029384756;
        LL d2 = 102938475612345678LL;
        nearest(root, x, y, id, d2);
        return id;
    }
}tree;

```

## 4.10 Lower Concave Hull

```

/****
    maintain a "concave hull" that support the following
    1. insertion of a line
    2. query of height(y) on specific x on the hull
****/
/* set as needed */
typedef long double LD;
const LD eps=1e-9;
const LD inf=1e19;
class Seg {
public:
    LD m,c,x1,x2; // y=mx+c
    bool flag;
    Seg(
        LD _m,LD _c,LD _x1=-inf,LD _x2=inf,bool _flag=0)
        :m(_m),c(_c),x1(_x1),x2(_x2),flag(_flag) {}
    LD evaly(LD x) const { return m*x+c; }
    const bool operator<(LD x) const{return x2-eps<x;}
    const bool operator<(const Seg &b) const {
        if(flag||b.flag) return *this<b.x1;
        return m+eps<b.m;
    }
};
class LowerConcaveHull { // maintain a hull like: \_/_/
public:
    set<Seg> hull;
    /* functions */
    LD xintersection(Seg a,Seg b)
    { return (a.c-b.c)/(b.m-a.m); }
    inline set<Seg>::iterator replace(set<Seg> &
        hull,set<Seg>::iterator it,Seg s) {
        hull.erase(it);
        return hull.insert(s).first;
    }
    void insert(Seg s) {
        // insert a line and update hull
        set<Seg>::iterator it=hull.find(s);
        // check for same slope
        if(it!=hull.end()) {
            if(it->c+eps>=s.c) return;
            hull.erase(it);
        }
        // check if below whole hull
        it=hull.lower_bound(s);
        if(it!=hull.end()&&
            s.evaly(it->x1)<=it->evaly(it->x1)+eps) return;
        // update right hull
        while(it!=hull.end()) {
            LD x=xintersection(s,*it);
            if(x>=it->x2-eps) hull.erase(it++);
            else {
                s.x2=x;
                it=replace(hull,it,Seg(it->m,it->c,x,it->x2));
                break;
            }
        }
        // update left hull
        while(it!=hull.begin()) {
            LD x=xintersection(s,*(--it));
            if(x<=it->x1+eps) hull.erase(it++);
            else {
                s.x1=x;
                it=replace(hull,it,Seg(it->m,it->c,it->x1,x));
                break;
            }
        }
    }
}

```

```

    // insert s
    hull.insert(s);
}
void insert(LD m, LD c) { insert(Seg(m, c)); }
LD query(LD x) { // return y @ given x
    set<Seg>::iterator it =
        hull.lower_bound(Seg(0.0, 0.0, x, x, 1));
    return it->evaly(x);
}
};

```

#### 4.11 Delaunay Triangulation

/\* Delaunay Triangulation:  
Given a sets of points on 2D plane, find a triangulation such that no points will strictly inside circumcircle of any triangle.

find : return a triangle contain given point  
add\_point : add a point into triangulation

A Triangle is in triangulation iff. its has\_chd is 0.  
Region of triangle u: iterate each u.edge[i].tri,  
each points are u.p[(i+1)%3], u.p[(i+2)%3]

```

calculation involves  $O(|V|^6)$  */
const int N = 100000 + 5;
const type inf = 2e3;
type eps = 1e-6; // 0 when integer
type sqr(type x) { return x*x; }
// return p4 is in circumcircle of tri(p1,p2,p3)
bool in_cc(const Pt& p1, const Pt& p2, const Pt& p3,
    const Pt& p4){
    type u11 = p1.X - p4.X; type u12 = p1.Y - p4.Y;
    type u21 = p2.X - p4.X; type u22 = p2.Y - p4.Y;
    type u31 = p3.X - p4.X; type u32 = p3.Y - p4.Y;
    type u13 = sqr(p1.X)-sqr(p4.X)+sqr(p1.Y)-sqr(p4.Y);
    type u23 = sqr(p2.X)-sqr(p4.X)+sqr(p2.Y)-sqr(p4.Y);
    type u33 = sqr(p3.X)-sqr(p4.X)+sqr(p3.Y)-sqr(p4.Y);
    type det = -u13*u22*u31 + u12*u23*u31 + u13*u21*u32
        -u11*u23*u32 - u12*u21*u33 + u11*u22*u33;
    return det > eps;
}
type side(const Pt& a, const Pt& b, const Pt& p)
{ return (b - a) ^ (p - a); }
typedef int SdRef;
struct Tri;
typedef Tri* TriRef;
struct Edge {
    TriRef tri; SdRef side;
    Edge():tri(0), side(0){}
    Edge(TriRef _tri, SdRef _side):tri(_tri), side(_side)
    {}
};
struct Tri {
    Pt p[3];
    Edge edge[3];
    TriRef chd[3];
    Tri() {}
    Tri(const Pt& p0, const Pt& p1, const Pt& p2) {
        p[0] = p0; p[1] = p1; p[2] = p2;
        chd[0] = chd[1] = chd[2] = 0;
    }
    bool has_chd() const { return chd[0] != 0; }
    int num_chd() const {
        return chd[0] == 0 ? 0
            : chd[1] == 0 ? 1
            : chd[2] == 0 ? 2 : 3;
    }
    bool contains(Pt const& q) const {
        for( int i = 0 ; i < 3 ; i ++ )
            if( side(p[i], p[(i + 1) % 3] , q) < -eps )
                return false;
        return true;
    }
} pool[ N * 10 ], *tris;
void edge( Edge a, Edge b ){
    if(a.tri) a.tri->edge[a.side] = b;
    if(b.tri) b.tri->edge[b.side] = a;
}

```

```

struct Trig { // Triangulation
    Trig(){
        the_root = // Tri should at least contain all
            points
            new(tris++)Tri(Pt(-inf,-inf),Pt(+inf+inf,-inf),Pt
                (-inf,+inf+inf));
    }
    TriRef find(Pt p) const { return find(the_root, p); }
    void add_point(const Pt& p) { add_point(find(the_root,
        p), p); }
    TriRef the_root;
    static TriRef find(TriRef root, const Pt& p) {
        while( true ){
            if( !root->has_chd() )
                return root;
            for( int i = 0 ; i < 3 && root->chd[i] ; ++i )
                if (root->chd[i]->contains(p)) {
                    root = root->chd[i];
                    break;
                }
        }
        assert( false ); // "point not found"
    }
    void add_point(TriRef root, Pt const& p) {
        TriRef tab, tbc, tca;
        /* split it into three triangles */
        tab=new(tris++) Tri(root->p[0], root->p[1], p);
        tbc=new(tris++) Tri(root->p[1], root->p[2], p);
        tca=new(tris++) Tri(root->p[2], root->p[0], p);
        edge(Edge(tab,0), Edge(tbc,1));
        edge(Edge(tbc,0), Edge(tca,1));
        edge(Edge(tca,0), Edge(tab,1));
        edge(Edge(tab,2), root->edge[2]);
        edge(Edge(tbc,2), root->edge[0]);
        edge(Edge(tca,2), root->edge[1]);
        root->chd[0] = tab;
        root->chd[1] = tbc;
        root->chd[2] = tca;
        flip(tab,2);
        flip(tbc,2);
        flip(tca,2);
    }
    void flip(TriRef tri, SdRef pi) {
        TriRef trj = tri->edge[pi].tri;
        int pj = tri->edge[pi].side;
        if (!trj) return;
        if (!in_cc(tri->p[0], tri->p[1], tri->p[2], trj->p[pj]
            )) return;
        /* flip edge between tri, trj */
        TriRef trk = new(tris++) Tri(tri->p[(pi+1)%3], trj
            ->p[pj], tri->p[pi]);
        TriRef trl = new(tris++) Tri(trj->p[(pj+1)%3], tri
            ->p[pi], trj->p[pj]);
        edge(Edge(trk,0), Edge(trl,0));
        edge(Edge(trk,1), tri->edge[(pi+2)%3]);
        edge(Edge(trk,2), trj->edge[(pj+1)%3]);
        edge(Edge(trl,1), trj->edge[(pj+2)%3]);
        edge(Edge(trl,2), tri->edge[(pi+1)%3]);
        tri->chd[0]=trk; tri->chd[1]=trl; tri->chd[2]=0;
        trj->chd[0]=trk; trj->chd[1]=trl; trj->chd[2]=0;
        flip(trk,1); flip(trk,2);
        flip(trl,1); flip(trl,2);
    }
};
vector<TriRef> triang;
set<TriRef> vst;
void go( TriRef now ){
    if( vst.find( now ) != vst.end() )
        return;
    vst.insert( now );
    if( !now->has_chd() ){
        triang.push_back( now );
        return;
    }
    for( int i = 0 ; i < now->num_chd() ; i ++ )
        go( now->chd[ i ] );
}
void build( int n , Pt* ps ){
    tris = pool;
    random_shuffle(ps, ps + n);
    Trig tri;
    for(int i = 0; i < n; ++ i)

```



```

    tri.add_point(ps[i]);
    go( tri.the_root );
}

```

## 4.12 Min Enclosing Circle

```

struct Mec{
    // return pair of center and r
    static const int N = 101010;
    int n;
    Pt p[ N ], cen;
    double r2;
    void init( int _n , Pt _p[] ){
        n = _n;
        memcpy( p , _p , sizeof(Pt) * n );
    }
    double sqr(double a){ return a*a; }
    Pt center(Pt p0, Pt p1, Pt p2) {
        Pt a = p1-p0;
        Pt b = p2-p0;
        double c1=norm2( a ) * 0.5;
        double c2=norm2( b ) * 0.5;
        double d = a ^ b;
        double x = p0.X + (c1 * b.Y - c2 * a.Y) / d;
        double y = p0.Y + (a.X * c2 - b.X * c1) / d;
        return Pt(x,y);
    }
    pair<Pt,double> solve(){
        random_shuffle(p,p+n);
        r2=0;
        for (int i=0; i<n; i++){
            if (norm2(cen-p[i]) <= r2) continue;
            cen = p[i];
            r2 = 0;
            for (int j=0; j<i; j++){
                if (norm2(cen-p[j]) <= r2) continue;
                cen=Pt((p[i].X+p[j].X)/2,(p[i].Y+p[j].Y)/2);
                r2 = norm2(cen-p[j]);
                for (int k=0; k<j; k++){
                    if (norm2(cen-p[k]) <= r2) continue;
                    cen = center(p[i],p[j],p[k]);
                    r2 = norm2(cen-p[k]);
                }
            }
        }
        return {cen,sqrt(r2)};
    }
} mec;

```

## 4.13 Minkowski sum

```

vector<Pt> minkowski(vector<Pt> p, vector<Pt> q){
    int n = p.size() , m = q.size();
    Pt c = Pt(0, 0);
    for( int i = 0; i < m; i ++ ) c = c + q[i];
    c = c / m;
    for( int i = 0; i < m; i ++ ) q[i] = q[i] - c;
    int cur = -1;
    for( int i = 0; i < m; i ++ )
        if( (q[i] ^ (p[0] - p[n-1])) > -eps)
            if( cur == -1 || (q[i] ^ (p[0] - p[n-1])) >
                (q[cur] ^ (p[0] - p[n-1])) )
                cur = i;
    vector<Pt> h;
    p.push_back(p[0]);
    for( int i = 0; i < n; i ++ )
        while( true ){
            h.push_back(p[i] + q[cur]);
            int nxt = (cur + 1 == m ? 0 : cur + 1);
            if((q[cur] ^ (p[i+1] - p[i])) < -eps) cur = nxt;
            else if( (q[nxt] ^ (p[i+1] - p[i])) >
                (q[cur] ^ (p[i+1] - p[i])) ) cur = nxt;
            else break;
        }
    for(auto &&i : h) i = i + c;
    return convex_hull(h);
}

```

## 4.14 Min dist on Cuboid

```

typedef LL T;
T r;
void turn(T i, T j, T x, T y, T z,
          T x0, T y0, T L, T W, T H) {
    if (z==0) { T R = x*x+y*y; if (R<r) r=R; return; }
    if(i>=0 && i< 2) turn(i+1, j, x0+L+z, y, x0+L-x,
                          x0+L, y0, H, W, L);
    if(j>=0 && j< 2) turn(i, j+1, x, y0+W+z, y0+W-y,
                          x0, y0+W, L, H, W);
    if(i<=0 && i>-2) turn(i-1, j, x0-z, y, x-x0,
                          x0-H, y0, H, W, L);
    if(j<=0 && j>-2) turn(i, j-1, x, y0-z, y-y0,
                          x0, y0-H, L, H, W);
}
T solve(T L, T W, T H,
        T x1, T y1, T z1, T x2, T y2, T z2){
    if( z1!=0 && z1!=H ){
        if( y1==0 || y1==W )
            swap(y1,z1), swap(y2,z2), swap(W,H);
    }else swap(x1,z1), swap(x2,z2), swap(L,H);
    if (z1==H) z1=0, z2=H-z2;
    r=INF; turn(0,0,x2-x1,y2-y1,z2,-x1,-y1,L,W,H);
    return r;
}

```

## 4.15 Heart of Triangle

```

Pt inCenter( Pt &A, Pt &B, Pt &C) { // 內心
    double a = norm(B-C), b = norm(C-A), c = norm(A-B);
    return (A * a + B * b + C * c) / (a + b + c);
}
Pt circumCenter( Pt &a, Pt &b, Pt &c) { // 外心
    Pt bb = b - a, cc = c - a;
    double db=norm2(bb), dc=norm2(cc), d=2*(bb ^ cc);
    return a-Pt(bb.Y*dc-cc.Y*db, cc.X*db-bb.X*dc) / d;
}
Pt othroCenter( Pt &a, Pt &b, Pt &c) { // 垂心
    Pt ba = b - a, ca = c - a, bc = b - c;
    double Y = ba.Y * ca.Y * bc.Y,
           A = ca.X * ba.Y - ba.X * ca.Y,
           x0= (Y+ca.X*ba.Y*b.X-ba.X*ca.Y*c.X) / A,
           y0= -ba.X * (x0 - c.X) / ba.Y + ca.Y;
    return Pt(x0, y0);
}

```

# 5 Graph

## 5.1 HeavyLightDecomp

```

#define REP(i, s, e) for(int i = (s); i <= (e); i++)
#define REPD(i, s, e) for(int i = (s); i >= (e); i--)
const int MAXN = 100010;
const int LOG = 19;
struct HLD{
    int n;
    vector<int> g[MAXN];
    int sz[MAXN], dep[MAXN];
    int ts, tid[MAXN], tdi[MAXN], tl[MAXN], tr[MAXN];
    // ts : timestamp , useless after yutruLi
    // tid[ u ] : pos. of node u in the seq.
    // tdi[ i ] : node at pos i of the seq.
    // tl , tr[ u ] : subtree interval in the seq. of
    // node u
    int prt[MAXN][LOG], head[MAXN];
    // head[ u ] : head of the chain contains u
    void dfsz(int u, int p){
        dep[u] = dep[p] + 1;
        prt[u][0] = p; sz[u] = 1; head[u] = u;
        for(int& v:g[u]) if(v != p){
            dep[v] = dep[u] + 1;
            dfsz(v, u);
            sz[u] += sz[v];
        }
    }
}

```



```

void dfs1(int u){
    ts++;
    tid[u] = tl[u] = tr[u] = ts;
    tdi[tid[u]] = u;
    sort(ALL(g[u]),
        [&](int a, int b){return sz[a] > sz[b];});
    bool flag = 1;
    for(int& v:g[u]) if(v != prt[u][0]){
        if(flag) head[v] = head[u], flag = 0;
        dfs1(v);
        tr[u] = tr[v];
    }
}
inline int lca(int a, int b){
    if(dep[a] > dep[b]) swap(a, b);
    int diff = dep[b] - dep[a];
    REPD(k, LOG-1, 0) if(diff & (1<<k)){
        b = prt[b][k];
    }
    if(a == b) return a;
    REPD(k, LOG-1, 0) if(prt[a][k] != prt[b][k]){
        a = prt[a][k]; b = prt[b][k];
    }
    return prt[a][0];
}
void init(int _n){
    n = _n; REP(i, 1, n) g[i].clear();
}
void addEdge(int u, int v){
    g[u].push_back(v);
    g[v].push_back(u);
}
void yutruli(){
    dfssz(1, 0);
    ts = 0;
    dfs1(1);
    REP(k, 1, LOG-1) REP(i, 1, n)
        prt[i][k] = prt[prt[i][k-1]][k-1];
}
vector< PII > getPath(int u, int v){
    vector< PII > res;
    while(tid[u] < tid[head[v]]){
        res.push_back(PII(tid[head[v]], tid[v]));
        v = prt[head[v]][0];
    }
    res.push_back(PII(tid[u], tid[v]));
    reverse(ALL(res));
    return res;
}
/* res : list of intervals from u to v
 * u must be ancestor of v
 * usage :
 * vector< PII > path = tree.getPath(u, v)
 * for( PII tp : path ){
 *     int l, r; tie(l, r) = tp;
 *     upd(l, r);
 *     uu = tree.tdi[l], vv = tree.tdi[r];
 *     uu ~> vv is a heavy path on tree
 * }
 */
}
} tree;

```

## 5.2 DominatorTree

```

const int MAXN = 100010;
struct DominatorTree{
#define REP(i,s,e) for(int i=(s);i<=(e);i++)
#define REPD(i,s,e) for(int i=(s);i>=(e);i--)
    int n, m, s;
    vector< int > g[ MAXN ], pred[ MAXN ];
    vector< int > cov[ MAXN ];
    int dfn[ MAXN ], nfd[ MAXN ], ts;
    int par[ MAXN ];
    int sdom[ MAXN ], idom[ MAXN ];
    int mom[ MAXN ], mn[ MAXN ];
    inline bool cmp(int u, int v){
        return dfn[u] < dfn[v];
    }
    int eval(int u){
        if(mom[u] == u) return u;

```

```

        int res = eval(mom[u]);
        if(cmp(sdom[mn[mom[u]]], sdom[mn[u]]))
            mn[u] = mn[mom[u]];
        return mom[u] = res;
    }
    void init(int _n, int _m, int _s){
        ts = 0; n = _n; m = _m; s = _s;
        REP(i, 1, n) g[i].clear(), pred[i].clear();
    }
    void addEdge(int u, int v){
        g[u].push_back(v);
        pred[v].push_back(u);
    }
    void dfs(int u){
        ts++;
        dfn[u] = ts;
        nfd[ts] = u;
        for(int v : g[u]) if(dfn[v] == 0){
            par[v] = u;
            dfs(v);
        }
    }
    void build(){
        REP(i, 1, n){
            dfn[i] = nfd[i] = 0;
            cov[i].clear();
            mom[i] = mn[i] = sdom[i] = i;
        }
        dfs(s);
        REPD(i, n, 2){
            int u = nfd[i];
            if(u == 0) continue;
            for(int v : pred[u]) if(dfn[v]){
                eval(v);
                if(cmp(sdom[mn[v]], sdom[u]))
                    sdom[u] = sdom[mn[v]];
            }
            cov[sdom[u]].push_back(u);
            mom[u] = par[u];
            for(int w : cov[par[u]]){
                eval(w);
                if(cmp(sdom[mn[w]], par[u]))
                    idom[w] = mn[w];
                else idom[w] = par[u];
            }
            cov[par[u]].clear();
        }
        REP(i, 2, n){
            int u = nfd[i];
            if(u == 0) continue;
            if(idom[u] != sdom[u])
                idom[u] = idom[idom[u]];
        }
    }
} domT;

```

## 5.3 MaxClique

```

#define N 111
struct MaxClique{ // 0-base
    typedef bitset< N > Int;
    Int linkto[ N ], v[ N ];
    int n;
    void init(int _n){
        n = _n;
        for(int i = 0; i < n; i++){
            linkto[i].reset();
            v[i].reset();
        }
    }
    void addEdge(int a, int b){
        v[a][b] = v[b][a] = 1;
    }
    int popcount(const Int& val){
        return val.count();
    }
    int lowbit(const Int& val){
        return val._Find_first();
    }
    int ans, stk[ N ];
    int id[ N ], di[ N ], deg[ N ];
    Int cans;

```

```

void maxclique(int elem_num, Int candi){
    if(elem_num > ans){
        ans = elem_num;
        cans.reset();
        for( int i = 0 ; i < elem_num ; i ++ )
            cans[ id[ stk[ i ] ] ] = 1;
    }
    int potential = elem_num + popcount(candi);
    if(potential <= ans) return;
    int pivot = lowbit(candi);
    Int smaller_candi = candi & (~linkto[pivot]);
    while(smaller_candi.count() && potential > ans){
        int next = lowbit(smaller_candi);
        candi[next] = !candi[next];
        smaller_candi[ next ] = !smaller_candi[ next ];
        potential --;
        if(next == pivot || (smaller_candi & linkto[next]
            ).count() ){
            stk[elem_num] = next;
            maxclique(elem_num + 1, candi & linkto[next]);
        }
    }
}
int solve(){
    for( int i = 0 ; i < n ; i ++ ){
        id[ i ] = i;
        deg[ i ] = v[ i ].count();
    }
    sort( id , id + n , [&](int id1, int id2){
        return deg[id1] > deg[id2]; } );
    for( int i = 0 ; i < n ; i ++ )
        di[ id[ i ] ] = i;
    for( int i = 0 ; i < n ; i ++ )
        for( int j = 0 ; j < n ; j ++ )
            if( v[ i ][ j ] )
                linkto[ di[ i ] ][ di[ j ] ] = 1;
    Int cand; cand.reset();
    for( int i = 0 ; i < n ; i ++ )
        cand[ i ] = 1;
    ans = 1;
    cans.reset(); cans[ 0 ] = 1;
    maxclique(0, cand);
    return ans;
}
} solver;

```

## 5.4 Strongly Connected Component

```

struct Scc{
    int n, nScc, vst[MXN], bln[MXN];
    vector<int> E[MXN], rE[MXN], vec;
    void init(int _n){
        n = _n;
        for (int i=0; i<MXN; i++)
            E[i].clear(), rE[i].clear();
    }
    void addEdge(int u, int v){
        E[u].PB(v); rE[v].PB(u);
    }
    void DFS(int u){
        vst[u]=1;
        for (auto v : E[u]) if (!vst[v]) DFS(v);
        vec.PB(u);
    }
    void rDFS(int u){
        vst[u] = 1; bln[u] = nScc;
        for (auto v : rE[u]) if (!vst[v]) rDFS(v);
    }
    void solve(){
        nScc = 0;
        vec.clear();
        FZ(vst);
        for (int i=0; i<n; i++)
            if (!vst[i]) DFS(i);
        reverse(vec.begin(),vec.end());
        FZ(vst);
        for (auto v : vec)
            if (!vst[v]){
                rDFS(v); nScc++;
            }
    }
}

```

```

}
};

```

## 5.5 Dynamic MST

```

/* Dynamic MST  $O(Q \lg^2 Q)$ 
(qx[i], qy[i]) -> chg weight of edge No.qx[i] to qy[i]
delete an edge: (i, \infty)
add an edge: change from \infty to specific value
*/
const int SZ=M+3*MXQ;
int a[N],*tz;
int find(int xx){
    int root=xx; while(a[root]) root=a[root];
    int next; while((next=a[xx])){a[xx]=root; xx=next;}
    return root;
}
bool cmp(int aa,int bb){ return tz[aa]<tz[bb]; }
int kx[N],ky[N],kt, vd[N],id[M], app[M];
bool extra[M];
void solve(int *qx,int *qy,int Q,int n,int *x,int *y,
    int *z,int m1,long long ans){
    if(Q==1){
        for(int i=1;i<=n;i++) a[i]=0;
        z[ qx[0] ]=qy[0]; tz = z;
        for(int i=0;i<m1;i++) id[i]=i;
        sort(id,id+m1,cmp); int ri,rj;
        for(int i=0;i<m1;i++){
            ri=find(x[id[i]]); rj=find(y[id[i]]);
            if(ri!=rj){ ans+=z[id[i]]; a[ri]=rj; }
        }
        printf("%lld\n",ans);
        return;
    }
    int ri,rj;
    //contract
    kt=0;
    for(int i=1;i<=n;i++) a[i]=0;
    for(int i=0;i<Q;i++){
        ri=find(x[qx[i]]); rj=find(y[qx[i]]); if(ri!=rj) a[ri]=rj;
    }
    int tm=0;
    for(int i=0;i<m1;i++) extra[i]=true;
    for(int i=0;i<Q;i++) extra[ qx[i] ]=false;
    for(int i=0;i<m1;i++) if(extra[i]) id[tm++]=i;
    tz=z; sort(id,id+tm,cmp);
    for(int i=0;i<tm;i++){
        ri=find(x[id[i]]); rj=find(y[id[i]]);
        if(ri!=rj){
            a[ri]=rj; ans += z[id[i]];
            kx[kt]=x[id[i]]; ky[kt]=y[id[i]]; kt++;
        }
    }
    for(int i=1;i<=n;i++) a[i]=0;
    for(int i=0;i<kt;i++) a[ find(kx[i]) ]=find(ky[i]);
    int n2=0;
    for(int i=1;i<=n;i++) if(a[i]==0)
        vd[i]=++n2;
    for(int i=1;i<=n;i++) if(a[i])
        vd[i]=vd[find(i)];
    int m2=0, *Nx=x+m1, *Ny=y+m1, *Nz=z+m1;
    for(int i=0;i<m1;i++) app[i]=-1;
    for(int i=0;i<Q;i++) if(app[qx[i]]==-1){
        Nx[m2]=vd[ x[ qx[i] ] ]; Ny[m2]=vd[ y[ qx[i] ] ];
        Nz[m2]=z[ qx[i] ];
        app[qx[i]]=m2; m2++;
    }
    for(int i=0;i<Q;i++){ z[ qx[i] ]=qy[i]; qx[i]=app[qx[i]]; }
    for(int i=1;i<=n2;i++) a[i]=0;
    for(int i=0;i<tm;i++){
        ri=find(vd[ x[id[i]] ]); rj=find(vd[ y[id[i]] ]);
        if(ri!=rj){
            a[ri]=rj; Nx[m2]=vd[ x[id[i]] ];
            Ny[m2]=vd[ y[id[i]] ]; Nz[m2]=z[id[i]]; m2++;
        }
    }
    int mid=Q/2;
    solve(qx,qy,mid,n2,Nx,Ny,Nz,m2,ans);
}

```

```

    solve(qx+mid,qy+mid,Q-mid,n2,Nx,Ny,Nz,m2,ans);
}
int x[SZ],y[SZ],z[SZ],qx[MXQ],qy[MXQ],n,m,Q;
void init(){
    scanf("%d%d",&n,&m);
    for(int i=0;i<m;i++) scanf("%d%d%d",x+i,y+i,z+i);
    scanf("%d",&Q);
    for(int i=0;i<Q;i++){ scanf("%d%d",qx+i,qy+i); qx[i]
        ]--; }
}
void work(){ if(Q) solve(qx,qy,Q,n,x,y,z,m,0); }
int main(){init(); work(); }

```

## 5.6 Maximum General graph Matching

```

const int N = 514, E = (2e5) * 2;
struct Graph{
    int to[E],bro[E],head[N],e;
    int lnk[N],vis[N],stp,n;
    void init( int _n ){
        stp = 0; e = 1; n = _n;
        for( int i = 1 ; i <= n ; i ++ )
            lnk[i] = vis[i] = 0;
    }
    void add_edge(int u,int v){
        to[e]=v,bro[e]=head[u],head[u]=e++;
        to[e]=u,bro[e]=head[v],head[v]=e++;
    }
    bool dfs(int x){
        vis[x]=stp;
        for(int i=head[x];i;i=bro[i]){
            int v=to[i];
            if(!lnk[v]){
                lnk[x]=v,lnk[v]=x;
                return true;
            }else if(vis[lnk[v]]<stp){
                int w=lnk[v];
                lnk[x]=v,lnk[v]=x,lnk[w]=0;
                if(dfs(w)){
                    return true;
                }
                lnk[w]=v,lnk[v]=w,lnk[x]=0;
            }
        }
        return false;
    }
    int solve(){
        int ans = 0;
        for(int i=1;i<=n;i++){
            if(!lnk[i]){
                stp++; ans += dfs(i);
            }
        }
        return ans;
    }
} graph;

```

## 5.7 Minimum General Weighted Matching

```

struct Graph {
    // Minimum General Weighted Matching (Perfect Match)
    static const int MXN = 105;
    int n, edge[MXN][MXN];
    int match[MXN],dis[MXN],onstk[MXN];
    vector<int> stk;
    void init(int _n) {
        n = _n;
        for( int i = 0 ; i < n ; i ++ )
            for( int j = 0 ; j < n ; j ++ )
                edge[ i ][ j ] = 0;
    }
    void add_edge(int u, int v, int w)
    { edge[u][v] = edge[v][u] = w; }
    bool SPFA(int u){
        if (onstk[u]) return true;
        stk.PB(u);
        onstk[u] = 1;
        for (int v=0; v<n; v++){
            if (u != v && match[u] != v && !onstk[v]){

```

```

                int m = match[v];
                if (dis[m] > dis[u] - edge[v][m] + edge[u][v]){
                    dis[m] = dis[u] - edge[v][m] + edge[u][v];
                    onstk[v] = 1;
                    stk.PB(v);
                    if (SPFA(m)) return true;
                    stk.pop_back();
                    onstk[v] = 0;
                }
            }
        }
        onstk[u] = 0;
        stk.pop_back();
        return false;
    }
    int solve() {
        // find a match
        for (int i=0; i<n; i+=2){
            match[i] = i+1;
            match[i+1] = i;
        }
        while (true){
            int found = 0;
            for( int i = 0 ; i < n ; i ++ )
                onstk[ i ] = dis[ i ] = 0;
            for (int i=0; i<n; i++){
                stk.clear();
                if (!onstk[i] && SPFA(i)){
                    found = 1;
                    while (SZ(stk)>=2){
                        int u = stk.back(); stk.pop_back();
                        int v = stk.back(); stk.pop_back();
                        match[u] = v;
                        match[v] = u;
                    }
                }
            }
            if (!found) break;
        }
        int ret = 0;
        for (int i=0; i<n; i++)
            ret += edge[i][match[i]];
        ret /= 2;
        return ret;
    }
}graph;

```

## 5.8 Maximum General Weighted Matching

```

struct WeightGraph {
    static const int INF = INT_MAX;
    static const int N = 514;
    struct edge{
        int u,v,w; edge(){
            edge(int ui,int vi,int wi)
                :u(ui),v(vi),w(wi){}
        };
    };
    int n,n_x;
    edge g[N*2][N*2];
    int lab[N*2];
    int match[N*2],slack[N*2],st[N*2],pa[N*2];
    int flo_from[N*2][N+1],S[N*2],vis[N*2];
    vector<int> flo[N*2];
    queue<int> q;
    int e_delta(const edge &e){
        return lab[e.u]+lab[e.v]-g[e.u][e.v].w*2;
    }
    void update_slack(int u,int x){
        if(!slack[x]||e_delta(g[u][x])<e_delta(g[slack[x]]
            [x]))slack[x]=u;
    }
    void set_slack(int x){
        slack[x]=0;
        for(int u=1;u<=n;++u)
            if(g[u][x].w>0&&st[u]!=x&&S[st[u]]==0)
                update_slack(u,x);
    }
    void q_push(int x){
        if(x<=n)q.push(x);
        else for(size_t i=0;i<flo[x].size();i++)

```

```

    q_push(flo[x][i]);
}
void set_st(int x, int b){
    st[x]=b;
    if(x>n)for(size_t i=0;i<flo[x].size();++i)
        set_st(flo[x][i],b);
}
int get_pr(int b, int xr){
    int pr=find(flo[b].begin(), flo[b].end(), xr)-flo[b].begin();
    if(pr%2==1){
        reverse(flo[b].begin()+1, flo[b].end());
        return (int)flo[b].size()-pr;
    }else return pr;
}
void set_match(int u, int v){
    match[u]=g[u][v].v;
    if(u<n) return;
    edge e=g[u][v];
    int xr=flo_from[u][e.u], pr=get_pr(u, xr);
    for(int i=0; i<pr; ++i) set_match(flo[u][i], flo[u][i+1]);
    set_match(xr, v);
    rotate(flo[u].begin(), flo[u].begin()+pr, flo[u].end());
}
void augment(int u, int v){
    for(;;){
        int xnv=st[match[u]];
        set_match(u, v);
        if(!xnv) return;
        set_match(xnv, st[pa[xnv]]);
        u=st[pa[xnv]], v=xnv;
    }
}
int get_lca(int u, int v){
    static int t=0;
    for(++t; u!=v; swap(u, v)){
        if(u==0) continue;
        if(vis[u]==t) return u;
        vis[u]=t;
        u=st[match[u]];
        if(u==st[pa[u]]);
    }
    return 0;
}
void add_blossom(int u, int lca, int v){
    int b=n+1;
    while(b<=n_x&&st[b])++b;
    if(b>n_x)++n_x;
    lab[b]=0, S[b]=0;
    match[b]=match[lca];
    flo[b].clear();
    flo[b].push_back(lca);
    for(int x=u, y; x!=lca; x=st[pa[y]])
        flo[b].push_back(x), flo[b].push_back(y=st[match[x]]), q_push(y);
    reverse(flo[b].begin()+1, flo[b].end());
    for(int x=v, y; x!=lca; x=st[pa[y]])
        flo[b].push_back(x), flo[b].push_back(y=st[match[x]]), q_push(y);
    set_st(b, b);
    for(int x=1; x<=n_x; ++x) g[b][x].w=g[x][b].w=0;
    for(int x=1; x<=n; ++x) flo_from[b][x]=0;
    for(size_t i=0; i<flo[b].size();++i){
        int xs=flo[b][i];
        for(int x=1; x<=n_x; ++x)
            if(g[b][x].w==0 || e_delta(g[xs][x])<e_delta(g[b][x]))
                g[b][x]=g[xs][x], g[x][b]=g[x][xs];
        for(int x=1; x<=n; ++x)
            if(flo_from[xs][x]) flo_from[b][x]=xs;
    }
    set_slack(b);
}
void expand_blossom(int b){
    for(size_t i=0; i<flo[b].size();++i)
        set_st(flo[b][i], flo[b][i]);
    int xr=flo_from[b][g[b][pa[b]].u], pr=get_pr(b, xr);
    for(int i=0; i<pr; i+=2){
        int xs=flo[b][i], xns=flo[b][i+1];
        pa[xs]=g[xns][xs].u;

```

```

        S[xs]=1, S[xns]=0;
        slack[xs]=0, set_slack(xns);
        q_push(xns);
    }
    S[xr]=1, pa[xr]=pa[b];
    for(size_t i=pr+1; i<flo[b].size();++i){
        int xs=flo[b][i];
        S[xs]=-1, set_slack(xs);
    }
    st[b]=0;
}
bool on_found_edge(const edge &e){
    int u=st[e.u], v=st[e.v];
    if(S[v]==-1){
        pa[v]=e.u, S[v]=1;
        int nu=st[match[v]];
        slack[v]=slack[nu]=0;
        S[nu]=0, q_push(nu);
    }else if(S[v]==0){
        int lca=get_lca(u, v);
        if(!lca) return augment(u, v), augment(v, u), true;
        else add_blossom(u, lca, v);
    }
    return false;
}
bool matching(){
    memset(S+1, -1, sizeof(int)*n_x);
    memset(slack+1, 0, sizeof(int)*n_x);
    q=queue<int>();
    for(int x=1; x<=n_x; ++x)
        if(st[x]==x&&!match[x]) pa[x]=0, S[x]=0, q_push(x);
    if(q.empty()) return false;
    for(;;){
        while(q.size()){
            int u=q.front(); q.pop();
            if(S[st[u]]==1) continue;
            for(int v=1; v<=n; ++v)
                if(g[u][v].w>0&&st[u]!=st[v]){
                    if(e_delta(g[u][v])==0){
                        if(on_found_edge(g[u][v])) return true;
                    }else update_slack(u, st[v]);
                }
        }
        int d=INF;
        for(int b=n+1; b<=n_x; ++b)
            if(st[b]==b&&S[b]==1) d=min(d, lab[b]/2);
        for(int x=1; x<=n_x; ++x)
            if(st[x]==x&&slack[x]){
                if(S[x]==-1) d=min(d, e_delta(g[slack[x]][x]));
                else if(S[x]==0) d=min(d, e_delta(g[slack[x]][x])/2);
            }
        for(int u=1; u<=n; ++u){
            if(S[st[u]]==0){
                if(lab[u]<=d) return 0;
                lab[u]-=d;
            }else if(S[st[u]]==1) lab[u]+=d;
        }
        for(int b=n+1; b<=n_x; ++b)
            if(st[b]==b){
                if(S[st[b]]==0) lab[b]+=d*2;
                else if(S[st[b]]==1) lab[b]-=d*2;
            }
        q=queue<int>();
        for(int x=1; x<=n_x; ++x)
            if(st[x]==x&&slack[x]&&st[slack[x]]!=x&&e_delta(g[slack[x]][x])==0)
                if(on_found_edge(g[slack[x]][x])) return true;
        for(int b=n+1; b<=n_x; ++b)
            if(st[b]==b&&S[b]==1&&lab[b]==0) expand_blossom(b);
    }
    return false;
}
pair<long long, int> solve(){
    memset(match+1, 0, sizeof(int)*n);
    n_x=n;
    int n_matches=0;
    long long tot_weight=0;
    for(int u=0; u<=n; ++u) st[u]=u, flo[u].clear();
    int w_max=0;
    for(int u=1; u<=n; ++u)

```

```

    for(int v=1;v<=n;v++){
        flo_from[u][v]=(u==v?0);
        w_max=max(w_max,g[u][v].w);
    }
    for(int u=1;u<=n;u++)lab[u]=w_max;
    while(matching())n_matches++;
    for(int u=1;u<=n;u++)
        if(match[u]&&match[u]<u)
            tot_weight+=g[u][match[u]].w;
    return make_pair(tot_weight,n_matches);
}
void add_edge( int ui , int vi , int wi ){
    g[ui][vi].w = g[vi][ui].w = wi;
}
void init( int _n ){
    n = _n;
    for(int u=1;u<=n;u++)
        for(int v=1;v<=n;v++)
            g[u][v]=edge(u,v,0);
}
} graph;

```

## 5.9 Minimum Steiner Tree

```

// Minimum Steiner Tree
// O(V^3AT + V^2 2^AT)
struct SteinerTree{
#define V 33
#define T 8
#define INF 1023456789
    int n , dst[V][V] , dp[1 << T][V] , tdst[V];
    void init( int _n ){
        n = _n;
        for( int i = 0 ; i < n ; i ++ ){
            for( int j = 0 ; j < n ; j ++ ){
                dst[ i ][ j ] = INF;
                dst[ i ][ i ] = 0;
            }
        }
        void add_edge( int ui , int vi , int wi ){
            dst[ ui ][ vi ] = min( dst[ ui ][ vi ] , wi );
            dst[ vi ][ ui ] = min( dst[ vi ][ ui ] , wi );
        }
        void shortest_path(){
            for( int k = 0 ; k < n ; k ++ )
                for( int i = 0 ; i < n ; i ++ )
                    for( int j = 0 ; j < n ; j ++ )
                        dst[ i ][ j ] = min( dst[ i ][ j ] ,
                            dst[ i ][ k ] + dst[ k ][ j ] );
        }
        int solve( const vector<int>& ter ){
            int t = (int)ter.size();
            for( int i = 0 ; i < ( 1 << t ) ; i ++ )
                for( int j = 0 ; j < n ; j ++ )
                    dp[ i ][ j ] = INF;
            for( int i = 0 ; i < n ; i ++ )
                dp[ 0 ][ i ] = 0;
            for( int msk = 1 ; msk < ( 1 << t ) ; msk ++ ){
                if( msk == ( msk & (-msk) ) ){
                    int who = __lg( msk );
                    for( int i = 0 ; i < n ; i ++ )
                        dp[ msk ][ i ] = dst[ ter[ who ] ][ i ];
                    continue;
                }
                for( int i = 0 ; i < n ; i ++ )
                    for( int submsk = ( msk - 1 ) & msk ; submsk ;
                        submsk = ( submsk - 1 ) & msk )
                        dp[ msk ][ i ] = min( dp[ msk ][ i ] ,
                            dp[ submsk ][ i ] +
                            dp[ msk ^ submsk ][ i ] );
                for( int i = 0 ; i < n ; i ++ ){
                    tdst[ i ] = INF;
                    for( int j = 0 ; j < n ; j ++ )
                        tdst[ i ] = min( tdst[ i ] ,
                            dp[ msk ][ j ] + dst[ j ][ i ] );
                }
                for( int i = 0 ; i < n ; i ++ )
                    dp[ msk ][ i ] = tdst[ i ];
            }
            int ans = INF;

```

```

        for( int i = 0 ; i < n ; i ++ )
            ans = min( ans , dp[ ( 1 << t ) - 1 ][ i ] );
        return ans;
    }
} solver;

```

## 5.10 BCC based on vertex

```

struct BccVertex {
    int n,nScc,step,dfn[MXN],low[MXN];
    vector<int> E[MXN],sccv[MXN];
    int top,stk[MXN];
    void init(int _n) {
        n = _n; nScc = step = 0;
        for (int i=0; i<n; i++) E[i].clear();
    }
    void addEdge(int u, int v)
    { E[u].PB(v); E[v].PB(u); }
    void DFS(int u, int f) {
        dfn[u] = low[u] = step++;
        stk[top++] = u;
        for (auto v:E[u]) {
            if (v == f) continue;
            if (dfn[v] == -1) {
                DFS(v,u);
                low[u] = min(low[u], low[v]);
                if (low[v] >= dfn[u]) {
                    int z;
                    sccv[nScc].clear();
                    do {
                        z = stk[--top];
                        sccv[nScc].PB(z);
                    } while (z != v);
                    sccv[nScc++].PB(u);
                }
            } else
                low[u] = min(low[u],dfn[v]);
        }
    }
    vector<vector<int>> solve() {
        vector<vector<int>> res;
        for (int i=0; i<n; i++)
            dfn[i] = low[i] = -1;
        for (int i=0; i<n; i++)
            if (dfn[i] == -1) {
                top = 0;
                DFS(i,i);
            }
        REP(i,nScc) res.PB(sccv[i]);
        return res;
    }
}graph;

```

## 5.11 Min Mean Cycle

```

/* minimum mean cycle O(VE) */
struct MMC{
#define E 101010
#define V 1021
#define inf 1e9
#define eps 1e-6
    struct Edge { int v,u; double c; };
    int n, m, prv[V][V], prve[V][V], vst[V];
    Edge e[E];
    vector<int> edgeID, cycle, rho;
    double d[V][V];
    void init( int _n )
    { n = _n; m = 0; }
    // WARNING: TYPE matters
    void addEdge( int vi , int ui , double ci )
    { e[ m ++ ] = { vi , ui , ci }; }
    void bellman_ford() {
        for(int i=0; i<n; i++) d[0][i]=0;
        for(int i=0; i<n; i++) {
            fill(d[i+1], d[i+1]+n, inf);
            for(int j=0; j<m; j++) {
                int v = e[j].v, u = e[j].u;
                if(d[i][v]<inf && d[i+1][u]>d[i][v]+e[j].c) {

```

```

        d[i+1][u] = d[i][v]+e[j].c;
        prv[i+1][u] = v;
        prve[i+1][u] = j;
    }
}
}
double solve(){
    // returns inf if no cycle, mmc otherwise
    double mmc=inf;
    int st = -1;
    bellman_ford();
    for(int i=0; i<n; i++) {
        double avg=-inf;
        for(int k=0; k<n; k++) {
            if(d[n][i]<inf-eps) avg=max(avg,(d[n][i]-d[k][i])/(n-k));
            else avg=max(avg,inf);
        }
        if (avg < mmc) tie(mmc, st) = tie(avg, i);
    }
    FZ(vst); edgeID.clear(); cycle.clear(); rho.clear();
    for (int i=n; !vst[st]; st=prv[i--][st]) {
        vst[st]++;
        edgeID.PB(prve[i][st]);
        rho.PB(st);
    }
    while (vst[st] != 2) {
        int v = rho.back(); rho.pop_back();
        cycle.PB(v);
        vst[v]++;
    }
    reverse(ALL(edgeID));
    edgeID.resize(SZ(cycle));
    return mmc;
}
} mmc;

```

## 5.12 Directed Graph Min Cost Cycle

```

// works in O(N M)
#define INF 1000000000000000LL
#define N 5010
#define M 200010
struct edge{
    int to; LL w;
    edge(int a=0, LL b=0): to(a), w(b){}
};
struct node{
    LL d; int u, next;
    node(LL a=0, int b=0, int c=0): d(a), u(b), next(c){}
}b[M];
struct DirectedGraphMinCycle{
    vector<edge> g[N], grev[N];
    LL dp[N][N], p[N], d[N], mu;
    bool inq[N];
    int n, bn, bsz, hd[N];
    void b_insert(LL d, int u){
        int i = d/mu;
        if(i >= bn) return;
        b[++bsz] = node(d, u, hd[i]);
        hd[i] = bsz;
    }
    void init( int _n ){
        n = _n;
        for( int i = 1 ; i <= n ; i ++ )
            g[ i ].clear();
    }
    void addEdge( int ai , int bi , LL ci )
    { g[ai].push_back(edge(bi,ci)); }
    LL solve(){
        fill(dp[0], dp[0]+n+1, 0);
        for(int i=1; i<=n; i++){
            fill(dp[i]+1, dp[i]+n+1, INF);
            for(int j=1; j<=n; j++) if(dp[i-1][j] < INF){
                for(int k=0; k<(int)g[j].size(); k++){
                    dp[i][g[j][k].to] = min(dp[i][g[j][k].to],
                                            dp[i-1][j]+g[j][k].w);
                }
            }
        }
    }
}

```

```

}
mu=INF; LL bunbo=1;
for(int i=1; i<=n; i++) if(dp[n][i] < INF){
    LL a=-INF, b=1;
    for(int j=0; j<=n-1; j++) if(dp[j][i] < INF){
        if(a*(n-j) < b*(dp[n][i]-dp[j][i])){
            a = dp[n][i]-dp[j][i];
            b = n-j;
        }
    }
    if(mu*b > bunbo*a)
        mu = a, bunbo = b;
}
if(mu < 0) return -1; // negative cycle
if(mu == INF) return INF; // no cycle
if(mu == 0) return 0;
for(int i=1; i<=n; i++){
    for(int j=0; j<(int)g[i].size(); j++){
        g[i][j].w *= bunbo;
    }
    memset(p, 0, sizeof(p));
    queue<int> q;
    for(int i=1; i<=n; i++){
        q.push(i);
        inq[i] = true;
    }
    while(!q.empty()){
        int i=q.front(); q.pop(); inq[i]=false;
        for(int j=0; j<(int)g[i].size(); j++){
            if(p[g[i][j].to] > p[i]+g[i][j].w-mu){
                p[g[i][j].to] = p[i]+g[i][j].w-mu;
                if(!inq[g[i][j].to]){
                    q.push(g[i][j].to);
                    inq[g[i][j].to] = true;
                }
            }
        }
    }
}
for(int i=1; i<=n; i++) grev[i].clear();
for(int i=1; i<=n; i++){
    for(int j=0; j<(int)g[i].size(); j++){
        g[i][j].w += p[i]-p[g[i][j].to];
        grev[g[i][j].to].push_back(edge(i, g[i][j].w));
    }
}
LL mldc = n*mu;
for(int i=1; i<=n; i++){
    bn=mldc/mu, bsz=0;
    memset(hd, 0, sizeof(hd));
    fill(d+i+1, d+n+1, INF);
    b_insert(d[i]=0, i);
    for(int j=0; j<=bn-1; j++) for(int k=hd[j]; k; k=
        b[k].next){
        int u = b[k].u;
        LL du = b[k].d;
        if(du > d[u]) continue;
        for(int l=0; l<(int)g[u].size(); l++) if(g[u][l]
            .to > i){
            if(d[g[u][l].to] > du + g[u][l].w){
                d[g[u][l].to] = du + g[u][l].w;
                b_insert(d[g[u][l].to], g[u][l].to);
            }
        }
    }
}
for(int j=0; j<(int)grev[i].size(); j++) if(grev[
    i][j].to > i)
    mldc=min(mldc,d[grev[i][j].to] + grev[i][j].w);
}
return mldc / bunbo;
}
} graph;

```

## 5.13 K-th Shortest Path

```

// time: O(|E| \lg |E| + |V| \lg |V| + K)
// memory: O(|E| \lg |E| + |V|)
struct KSP{ // 1-base
    struct nd{
        int u, v, d;
        nd(int ui = 0, int vi = 0, int di = INF)
        { u = ui; v = vi; d = di; }
    };
};

```



```

struct heap{
    nd* edge; int dep; heap* chd[4];
};
static int cmp(heap* a, heap* b)
{ return a->edge->d > b->edge->d; }
struct node{
    int v; LL d; heap* H; nd* E;
    node(){
        node(LL _d, int _v, nd* _E)
        { d = _d; v = _v; E = _E; }
        node(heap* _H, LL _d)
        { H = _H; d = _d; }
        friend bool operator<(node a, node b)
        { return a.d > b.d; }
    };
};
int n, k, s, t, dst[ N ];
nd *nxt[ N ];
vector<nd*> g[ N ]; rg[ N ];
heap *nullNd, *head[ N ];
void init( int _n , int _k , int _s , int _t ){
    n = _n; k = _k; s = _s; t = _t;
    for( int i = 1 ; i <= n ; i ++ ){
        g[ i ].clear(); rg[ i ].clear();
        nxt[ i ] = head[ i ] = NULL;
        dst[ i ] = -1;
    }
}
void addEdge( int ui , int vi , int di ){
    nd* e = new nd(ui, vi, di);
    g[ ui ].push_back( e );
    rg[ vi ].push_back( e );
}
queue<int> dfsQ;
void dijkstra(){
    while(dfsQ.size()) dfsQ.pop();
    priority_queue<node> Q;
    Q.push(node(0, t, NULL));
    while (!Q.empty()){
        node p = Q.top(); Q.pop();
        if(dst[p.v] != -1) continue;
        dst[ p.v ] = p.d;
        nxt[ p.v ] = p.E;
        dfsQ.push( p.v );
        for(auto e: rg[ p.v ])
            Q.push(node(p.d + e->d, e->u, e));
    }
}
heap* merge(heap* curNd, heap* newNd){
    if(curNd == nullNd) return newNd;
    heap* root = new heap;
    memcpy(root, curNd, sizeof(heap));
    if(newNd->edge->d < curNd->edge->d){
        root->edge = newNd->edge;
        root->chd[2] = newNd->chd[2];
        root->chd[3] = newNd->chd[3];
        newNd->edge = curNd->edge;
        newNd->chd[2] = curNd->chd[2];
        newNd->chd[3] = curNd->chd[3];
    }
    if(root->chd[0]->dep < root->chd[1]->dep)
        root->chd[0] = merge(root->chd[0], newNd);
    else
        root->chd[1] = merge(root->chd[1], newNd);
    root->dep = max(root->chd[0]->dep, root->chd[1]->
        dep) + 1;
    return root;
}
vector<heap*> V;
void build(){
    nullNd = new heap;
    nullNd->dep = 0;
    nullNd->edge = new nd;
    fill(nullNd->chd, nullNd->chd+4, nullNd);
    while(not dfsQ.empty()){
        int u = dfsQ.front(); dfsQ.pop();
        if(!nxt[ u ]) head[ u ] = nullNd;
        else head[ u ] = head[nxt[ u ]->v];
        V.clear();
        for( auto& e : g[ u ] ){
            int v = e->v;
            if( dst[ v ] == -1 ) continue;
            e->d += dst[ v ] - dst[ u ];
        }
    }
}

```

```

        if( nxt[ u ] != e ){
            heap* p = new heap;
            fill(p->chd, p->chd+4, nullNd);
            p->dep = 1;
            p->edge = e;
            V.push_back(p);
        }
    }
    if(V.empty()) continue;
    make_heap(V.begin(), V.end(), cmp);
#define L(X) ((X<<1)+1)
#define R(X) ((X<<1)+2)
    for( size_t i = 0 ; i < V.size() ; i ++ ){
        if(L(i) < V.size()) V[i]->chd[2] = V[L(i)];
        else V[i]->chd[2]=nullNd;
        if(R(i) < V.size()) V[i]->chd[3] = V[R(i)];
        else V[i]->chd[3]=nullNd;
    }
    head[u] = merge(head[u], V.front());
}
vector<LL> ans;
void first_KC(){
    ans.clear();
    priority_queue<node> Q;
    if( dst[ s ] == -1 ) return;
    ans.push_back( dst[ s ] );
    if( head[s] != nullNd )
        Q.push(node(head[s], dst[s]+head[s]->edge->d));
    for( int _ = 1 ; _ < k and not Q.empty() ; _ ++ ){
        node p = Q.top(); q: Q.pop();
        ans.push_back( p.d );
        if(head[ p.H->edge->v ] != nullNd){
            q.H = head[ p.H->edge->v ];
            q.d = p.d + q.H->edge->d;
            Q.push(q);
        }
        for( int i = 0 ; i < 4 ; i ++ )
            if( p.H->chd[ i ] != nullNd ){
                q.H = p.H->chd[ i ];
                q.d = p.d - p.H->edge->d + p.H->chd[ i ]->
                    edge->d;
                Q.push( q );
            }
    }
}
void solve(){
    dijkstra();
    build();
    first_KC();
}
} solver;

```

## 6 String

### 6.1 PalTree

```

/*
 * sfail: compressed fail links with same diff
 * 0(lgn): length of sfail link path
 */
const int MAXN = 1e6+10;
struct PalT{
    int tot, lst;
    int nxt[MAXN][26], len[MAXN];
    int fail[MAXN], diff[MAXN], sfail[MAXN];
    char* s;
    int newNode(int l, int _fail) {
        int res = ++tot;
        fill(nxt[res], nxt[res]+26, 0);
        len[res] = l, fail[res] = _fail;
        diff[res] = l - len[_fail];
        if (diff[res] == diff[_fail])
            sfail[res] = sfail[_fail];
        else
            sfail[res] = _fail;
        return res;
    }
}

```

```

void push(int p) {
    int np = lst;
    int c = s[p] - 'a';
    while (p - len[np] - 1 < 0 || s[p] != s[p - len[np] - 1])
        np = fail[np];
    if ((lst = nxt[np][c])) return;
    int nq_f = 0;
    if (len[np] + 2 == 1) nq_f = 2;
    else {
        int tf = fail[np];
        while (p - len[tf] - 1 < 0 || s[p] != s[p - len[tf] - 1])
            tf = fail[tf];
        nq_f = nxt[tf][c];
    }
    int nq = newNode(len[np] + 2, nq_f);
    nxt[np][c] = nq;
    lst = nq;
}

void init(char* _s) {
    s = _s;
    tot = 0;
    newNode(-1, 1);
    newNode(0, 1);
    diff[2] = 0;
    lst = 2;
}
} palt;

```

## 6.2 SAIS

```

const int N = 300010;
struct SA {
#define REP(i, n) for (int i = 0; i < int(n); i++)
#define REP1(i, a, b) for (int i = (a); i <= int(b); i++)
    bool _t[N * 2];
    int _s[N * 2], _sa[N * 2], _c[N * 2], x[N], _p[N], _q[N * 2],
        hei[N], r[N];
    int operator [] (int i) { return _sa[i]; }
    void build(int* s, int n, int m) {
        memcpy(_s, s, sizeof(int) * n);
        sais(_s, _sa, _p, _q, _t, _c, n, m);
        mkhei(n);
    }
    void mkhei(int n) {
        REP(i, n) r[_sa[i]] = i;
        hei[0] = 0;
        REP(i, n) if (r[i]) {
            int ans = i > 0 ? max(hei[r[i] - 1] - 1, 0) : 0;
            while (_s[i + ans] == _s[_sa[r[i] - 1] + ans]) ans++;
            hei[r[i]] = ans;
        }
    }
    void sais(int* s, int* sa, int* p, int* q, bool* t,
        int* c, int n, int z) {
        bool uniq = t[n - 1] = true, neq;
        int nn = 0, nmzx = -1, *nsa = sa + n, *ns = s + n,
            lst = -1;
#define MS0(x, n) memset((x), 0, n * sizeof(*(x)))
#define MAGIC(XD) MS0(sa, n); \
        memcpy(x, c, sizeof(int) * z); \
        XD; \
        memcpy(x + 1, c, sizeof(int) * (z - 1)); \
        REP(i, n) if (sa[i] && !t[sa[i] - 1]) sa[x[sa[i] - 1] - 1]++ = sa[i] - 1; \
        memcpy(x, c, sizeof(int) * z); \
        for (int i = n - 1; i >= 0; i--) if (sa[i] && t[sa[i] - 1]) sa[--x[sa[i] - 1]] = sa[i] - 1;
        MS0(c, z);
        REP(i, n) uniq &= ++c[s[i]] < 2;
        REP(i, z - 1) c[i + 1] += c[i];
        if (uniq) { REP(i, n) sa[--c[s[i]]] = i; return; }
        for (int i = n - 2; i >= 0; i--) t[i] = (s[i] == s[i + 1] ? t[i + 1] : s[i] < s[i + 1]);
        MAGIC(CREP1(i, 1, n - 1) if (t[i] && !t[i - 1]) sa[--x[s[i]]] = p[q[i] - nn++] = i);
        REP(i, n) if (sa[i] && t[sa[i]] && !t[sa[i] - 1]) {
            neq = lst < 0 || memcmp(s + sa[i], s + lst, (p[q[sa[i]] + 1] - sa[i]) * sizeof(int));
            ns[q[lst = sa[i]]] = nmzx + neq;
        }
    }
}

```

```

    sais(ns, nsa, p + nn, q + n, t + n, c + z, nn, nmzx + 1);
    MAGIC(for (int i = nn - 1; i >= 0; i--) sa[--x[s[p[nsa[i]]]]] = p[nsa[i]]);
}
} sa;
int H[N], SA[N];
void suffix_array(int* ip, int len) {
    // should padding a zero in the back
    // ip is int array, len is array length
    // ip[0..n-1] != 0, and ip[len] = 0
    ip[len++] = 0;
    sa.build(ip, len, 128);
    for (int i = 0; i < len; i++) {
        H[i] = sa.hei[i + 1];
        SA[i] = sa._sa[i + 1];
    }
    // resulting height, sa array \in [0, len)
}

```

## 6.3 SuffixAutomata

```

const int MAXM = 1000010;
struct SAM {
    int tot, root, lst, mom[MAXM], mx[MAXM];
    int acc[MAXM], nxt[MAXM][33];
    int newNode() {
        int res = ++tot;
        fill(nxt[res], nxt[res] + 33, 0);
        mom[res] = mx[res] = acc[res] = 0;
        return res;
    }
    void init() {
        tot = 0;
        root = newNode();
        mom[root] = 0, mx[root] = 0;
        lst = root;
    }
    void push(int c) {
        int p = lst;
        int np = newNode();
        mx[np] = mx[p] + 1;
        for (; p && nxt[p][c] == 0; p = mom[p])
            nxt[p][c] = np;
        if (p == 0) mom[np] = root;
        else {
            int q = nxt[p][c];
            if (mx[p] + 1 == mx[q]) mom[np] = q;
            else {
                int nq = newNode();
                mx[nq] = mx[p] + 1;
                for (int i = 0; i < 33; i++)
                    nxt[nq][i] = nxt[q][i];
                mom[nq] = mom[q];
                mom[q] = nq;
                mom[np] = nq;
                for (; p && nxt[p][c] == q; p = mom[p])
                    nxt[p][c] = nq;
            }
        }
        lst = np;
    }
    void push(char* str) {
        for (int i = 0; str[i]; i++)
            push(str[i] - 'a' + 1);
    }
} sam;

```

## 6.4 Aho-Corasick

```

struct ACautomata {
    struct Node {
        int cnt, dp;
        Node* go[26], *fail;
        Node() {
            cnt = 0; dp = -1; fail = 0;
            memset(go, 0, sizeof(go));
        }
    }
}

```

```

};
Node *root, pool[1048576];
int nMem;
Node* new_Node(){
    pool[nMem] = Node();
    return &pool[nMem++];
}
void init()
{ nMem = 0; root = new_Node(); }
void add(const string &str)
{ insert(root, str, 0); }
void insert(Node *cur, const string &str, int pos){
    if (pos >= (int)str.size())
        { cur->cnt++; return; }
    int c = str[pos] - 'a';
    if (cur->go[c] == 0)
        cur->go[c] = new_Node();
    insert(cur->go[c], str, pos+1);
}
void make_fail(){
    queue<Node*> que;
    que.push(root);
    while (!que.empty()){
        Node* fr=que.front();
        que.pop();
        for (int i=0; i<26; i++){
            if (fr->go[i]){
                Node *ptr = fr->fail;
                while (ptr && !ptr->go[i]) ptr = ptr->fail;
                if (!ptr) fr->go[i]->fail = root;
                else fr->go[i]->fail = ptr->go[i];
                que.push(fr->go[i]);
            }
        }
    }
}
};

```

## 6.5 Z Value

```

char s[MAXN];
int len, z[MAXN];
void Z_value() {
    int i, j, left, right;
    left=right=0; z[0]=len;
    for(i=1; i<len; i++) {
        j=max(min(z[i-left], right-i), 0);
        for(; i+j<len && s[i+j]==s[j]; j++);
        z[i]=j;
        if(i+z[i]>right) {
            right=i+z[i];
            left=i;
        }
    }
}

```

## 6.6 BWT

```

struct BurrowsWheeler{
#define SIGMA 26
#define BASE 'a'
    vector<int> v[ SIGMA ];
    void BWT(char* ori, char* res){
        // make ori -> ori + ori
        // then build suffix array
    }
    void iBWT(char* ori, char* res){
        for( int i = 0 ; i < SIGMA ; i ++ )
            v[ i ].clear();
        int len = strlen( ori );
        for( int i = 0 ; i < len ; i ++ )
            v[ ori[i] - BASE ].push_back( i );
        vector<int> a;
        for( int i = 0 , ptr = 0 ; i < SIGMA ; i ++ )
            for( auto j : v[ i ] ){
                a.push_back( j );
                ori[ ptr ++ ] = BASE + i;
            }
    }
}

```

```

for( int i = 0 , ptr = 0 ; i < len ; i ++ ){
    res[ i ] = ori[ a[ ptr ] ];
    ptr = a[ ptr ];
}
res[ len ] = 0;
}
} bwt;

```

## 6.7 ZValue Palindrome

```

int len, zv[MAX*2];
char ip[MAX], op[MAX*2];
int main(){
    cin >> ip; len = strlen(ip);
    int l2 = len*2 - 1;
    for(int i=0; i<l2; i++){
        if(i&1) op[i] = '@';
        else op[i] = ip[i/2];
    }
    int l=0, r=0; zv[0] = 1;
    for(int i=1; i<l2; i++){
        if( i > r ){
            l = r = i;
            while( l>0 && r<l2-1 && op[l-1] == op[r+1] )
                l--, r++;
            zv[i] = (r-l+1);
        }else{
            int md = (l+r)/2, j = md + md - i;
            zv[i] = zv[j];
            int q = zv[i] / 2, nr = i + q;
            if( nr == r ){
                l = i + i - r;
                while( l>0 && r<l2-1 && op[l-1] == op[r+1] )
                    l--, r++;
                zv[i] = r - l + 1;
            }else if( nr > r )
                zv[i] = (r - i) * 2 + 1;
        }
    }
}

```

## 6.8 Smallest Rotation

```

string mcp(string s){
    int n = s.length();
    s += s;
    int i=0, j=1;
    while (i<n && j<n){
        int k = 0;
        while (k < n && s[i+k] == s[j+k]) k++;
        if (s[i+k] <= s[j+k]) j += k+1;
        else i += k+1;
        if (i == j) j++;
    }
    int ans = i < n ? i : j;
    return s.substr(ans, n);
}

```

## 6.9 Cyclic LCS

```

#define L 0
#define LU 1
#define U 2
const int mov[3][2]={0,-1, -1,-1, -1,0};
int al,bl;
char a[MAXL*2], b[MAXL*2]; // 0-indexed
int dp[MAXL*2][MAXL];
char pred[MAXL*2][MAXL];
inline int lcs_length(int r) {
    int i=r+al, j=bl, l=0;
    while(i>r) {
        char dir=pred[i][j];
        if(dir==LU) l++;
        i+=mov[dir][0];
        j+=mov[dir][1];
    }
    return l;
}

```

```

}
inline void reroot(int r) { // r = new base row
    int i=r,j=1;
    while(j<=bl&&pred[i][j]!=LU) j++;
    if(j>bl) return;
    pred[i][j]=L;
    while(i<2*al&&j<=bl) {
        if(pred[i+1][j]==U) {
            i++;
            pred[i][j]=L;
        } else if(j<bl&&pred[i+1][j+1]==LU) {
            i++;
            j++;
            pred[i][j]=L;
        } else {
            j++;
        }
    }
}
int cyclic_lcs() {
    // a, b, al, bl should be properly filled
    // note: a WILL be altered in process
    // -- concatenated after itself
    char tmp[MAXL];
    if(al>bl) {
        swap(al,bl);
        strcpy(tmp,a);
        strcpy(a,b);
        strcpy(b,tmp);
    }
    strcpy(tmp,a);
    strcat(a,tmp);
    // basic lcs
    for(int i=0;i<=2*al;i++) {
        dp[i][0]=0;
        pred[i][0]=U;
    }
    for(int j=0;j<=bl;j++) {
        dp[0][j]=0;
        pred[0][j]=L;
    }
    for(int i=1;i<=2*al;i++) {
        for(int j=1;j<=bl;j++) {
            if(a[i-1]==b[j-1]) dp[i][j]=dp[i-1][j-1]+1;
            else dp[i][j]=max(dp[i-1][j],dp[i][j-1]);
            if(dp[i][j-1]==dp[i][j]) pred[i][j]=L;
            else if(a[i-1]==b[j-1]) pred[i][j]=LU;
            else pred[i][j]=U;
        }
    }
    // do cyclic lcs
    int clcs=0;
    for(int i=0;i<al;i++) {
        clcs=max(clcs,lcs_length(i));
        reroot(i+1);
    }
    // recover a
    a[al]='\0';
    return clcs;
}

```

## 7 Data Structure

### 7.1 Treap

```

struct Treap{
    int sz , val , pri , tag;
    Treap *l , *r;
    Treap( int _val ){
        val = _val; sz = 1;
        pri = rand(); l = r = NULL; tag = 0;
    }
};
void push( Treap * a ){
    if( a->tag ){
        Treap *swp = a->l; a->l = a->r; a->r = swp;
        int swp2;
        if( a->l ) a->l->tag ^= 1;

```

```

        if( a->r ) a->r->tag ^= 1;
        a->tag = 0;
    }
}
int Size( Treap * a ){ return a ? a->sz : 0; }
void pull( Treap * a ){
    a->sz = Size( a->l ) + Size( a->r ) + 1;
}
Treap* merge( Treap *a , Treap *b ){
    if( !a || !b ) return a ? a : b;
    if( a->pri > b->pri ){
        push( a );
        a->r = merge( a->r , b );
        pull( a );
        return a;
    }else{
        push( b );
        b->l = merge( a , b->l );
        pull( b );
        return b;
    }
}
void split( Treap *t , int k , Treap*&a , Treap*&b ){
    if( !t ){ a = b = NULL; return; }
    push( t );
    if( Size( t->l ) + 1 <= k ){
        a = t;
        split( t->r , k - Size( t->l ) - 1 , a->r , b );
        pull( a );
    }else{
        b = t;
        split( t->l , k , a , b->l );
        pull( b );
    }
}

```

### 7.2 Link-Cut Tree

```

const int MXN = 100005;
const int MEM = 100005;
struct Splay {
    static Splay nil, mem[MEM], *pmem;
    Splay *ch[2], *f;
    int val, rev, size;
    Splay( int _val=-1 ) : val(_val), rev(0), size(1) {
        f = ch[0] = ch[1] = &nil; }
    bool isr() {
        return f->ch[0] != this && f->ch[1] != this; }
    int dir() {
        return f->ch[0] == this ? 0 : 1; }
    void setCh(Splay *c, int d){
        ch[d] = c;
        if (c != &nil) c->f = this;
        pull();
    }
    void push(){
        if( !rev ) return;
        swap(ch[0], ch[1]);
        if (ch[0] != &nil) ch[0]->rev ^= 1;
        if (ch[1] != &nil) ch[1]->rev ^= 1;
        rev=0;
    }
    void pull(){
        size = ch[0]->size + ch[1]->size + 1;
        if (ch[0] != &nil) ch[0]->f = this;
        if (ch[1] != &nil) ch[1]->f = this;
    }
} Splay::nil, Splay::mem[MEM], *Splay::pmem = Splay::mem;
Splay *nil = &Splay::nil;
void rotate(Splay *x){
    Splay *p = x->f;
    int d = x->dir();
    if (!p->isr()) p->f->setCh(x, p->dir());
    else x->f = p->f;
    p->setCh(x->ch[!d], d);
    x->setCh(p, !d);
    p->pull(); x->pull();
}
vector<Splay*> splayVec;

```

```

void splay(Splay *x){
    splayVec.clear();
    for (Splay *q=x;; q=q->f){
        splayVec.push_back(q);
        if (q->isr()) break;
    }
    reverse(begin(splayVec), end(splayVec));
    for (auto it : splayVec) it->push();
    while (!x->isr()) {
        if (x->f->isr()) rotate(x);
        else if (x->dir()==x->f->dir())
            rotate(x->f), rotate(x);
        else rotate(x), rotate(x);
    }
}
int id(Splay *x) { return x - Splay::mem + 1; }
Splay* access(Splay *x){
    Splay *q = nil;
    for (;x!=nil;x=x->f){
        splay(x);
        x->setCh(q, 1);
        q = x;
    }
    return q;
}
void chroot(Splay *x){
    access(x);
    splay(x);
    x->rev ^= 1;
    x->push(); x->pull();
}
void link(Splay *x, Splay *y){
    access(x);
    splay(x);
    chroot(y);
    x->setCh(y, 1);
}
void cut_p(Splay *y) {
    access(y);
    splay(y);
    y->push();
    y->ch[0] = y->ch[0]->f = nil;
}
void cut(Splay *x, Splay *y){
    chroot(x);
    cut_p(y);
}
Splay* get_root(Splay *x) {
    access(x);
    splay(x);
    for(; x->ch[0] != nil; x = x->ch[0])
        x->push();
    splay(x);
    return x;
}
bool conn(Splay *x, Splay *y) {
    x = get_root(x);
    y = get_root(y);
    return x == y;
}
Splay* lca(Splay *x, Splay *y) {
    access(x);
    access(y);
    splay(x);
    if (x->f == nil) return x;
    else return x->f;
}

```

### 7.3 Black Magic

```

#include <bits/extc++.h>
using namespace __gnu_pbds;
typedef tree<int, null_type, less<int>, rb_tree_tag,
    tree_order_statistics_node_update> set_t;
#include <ext/pb_ds/assoc_container.hpp>
typedef cc_hash_table<int, int> umap_t;
typedef priority_queue<int> heap;
#include <ext/rope>
using namespace __gnu_cxx;
int main(){

```

```

// Insert some entries into s.
set_t s; s.insert(12); s.insert(505);
// The order of the keys should be: 12, 505.
assert(*s.find_by_order(0) == 12);
assert(*s.find_by_order(3) == 505);
// The order of the keys should be: 12, 505.
assert(s.order_of_key(12) == 0);
assert(s.order_of_key(505) == 1);
// Erase an entry.
s.erase(12);
// The order of the keys should be: 505.
assert(*s.find_by_order(0) == 505);
// The order of the keys should be: 505.
assert(s.order_of_key(505) == 0);

heap h1, h2; h1.join(h2);

rope<char> r[2];
r[1] = r[0]; // persistenet
string t = "abc";
r[1].insert(0, t.c_str());
r[1].erase(1, 1);
cout << r[1].substr(0, 2);
}

```

## 8 Others

### 8.1 Find max tangent(x,y is increasing)

```

typedef long long LL;
const int MAXN = 100010;
struct Coord{
    LL x, y;
    Coord operator - (Coord ag) const{
        Coord res;
        res.x = x - ag.x;
        res.y = y - ag.y;
        return res;
    }
}sum[MAXN], pnt[MAXN], ans, calc;

inline bool cross(Coord a, Coord b, Coord c){
    return (c.y-a.y)*(c.x-b.x) > (c.x-a.x)*(c.y-b.y);
}

int main(){
    int n, l, np, st, ed, now;
    scanf("%d %d\n", &n, &l);
    sum[0].x = sum[0].y = np = st = ed = 0;
    for (int i = 1, v; i <= n; i++){
        scanf("%d", &v);
        sum[i].y = sum[i-1].y + v;
        sum[i].x = i;
    }
    ans.x = now = 1;
    ans.y = -1;
    for (int i = 0; i <= n-1; i++){
        while (np > 1 &&
            cross(pnt[np-2], pnt[np-1], sum[i]))
            np--;
        if (np < now && np != 0) now = np;
        pnt[np++] = sum[i];
        while (now < np &&
            !cross(pnt[now-1], pnt[now], sum[i+1]))
            now++;
        calc = sum[i+1] - pnt[now-1];
        if (ans.y * calc.x < ans.x * calc.y){
            ans = calc;
            st = pnt[now-1].x;
            ed = i+1;
        }
    }
    double res = (sum[ed].y - sum[st].y) /
        (sum[ed].x - sum[st].x);
    printf("%f\n", res);
    return 0;
}

```