Radiating Shock experiment documentation

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1 Introduction

The radiating shock experiment generates equilibrium solutions of the one dimensional equations of radiating hydrodynamic or ideal magnetohydrodynamic flows and plugs them, with small perturbations in front of the shock, into Imogen.

2 Derivations

To begin with we state the equations of Ideal MHD with a polytropic equation of state,

$$\begin{aligned}
&\partial_t \rho + \nabla(\rho v) &= 0 \quad &(1a)\\
&\partial_t(\rho \mathbf{v}) + \nabla(\rho \mathbf{v}\mathbf{v} + \mathbf{I}P - BB) &= 0 \quad &(1b)\\
&\partial_t B + \nabla(vB - Bv) &= 0 \quad &(1c)\\
&\partial_t E + \nabla(\mathbf{v}(E+P) - (\mathbf{v}\cdot\mathbf{B})\mathbf{B}) &= 0 \quad &(1d)
\end{aligned}$$

Where we define ρ as the matter density, $\mathbf{v}$ its velocity, P the total (gas plus magnetic) pressure, $\mathbf{I}$ the identity tensor and $\mathbf{B}$ the magnetic field (with $\mu_0 = 1$).

Assuming an equilibrium solution ($\partial_t \rightarrow 0$) and that we have structure only in the x direction ($\partial_y \rightarrow 0$ and $\partial_z \rightarrow 0$), a series of conserved quantities associated with the above system (now a set of coupled ODEs) now clearly arises by going to the integral form: The derivatives of the fluxes are zero so the fluxes themselves must be constants.

$$\begin{aligned}
p_x &= \rho_0 v x_0 &(2a)\\
f_x &= \rho_0 v x_0^2 + P_0 - B x_0^2 &(2b)\\
f_y &= \rho v x_0 v y_0 - b x_0 b y_0 &(2c)\\
0 &= v x_0 b y_0 - b x_0 v y_0 &(2d)\\
bx &= b x_0 &(2e)
\end{aligned}$$

Thus the fluxes of mass and momentum are conserved, $\mathbf{v}$ and $\mathbf{B}$ are coupled by the induction equation, and bx is constant due to the divergence constraint.

There naturally exist z direction terms as well, but because the equilibrium flow lacks any structure that could cause it to twist as it goes, the angle the transverse terms form to the x-y plane is another conserved quantity*. In other words, we are free to rotate the system into the x-y plane without loss of generality.

If the flow were adiabatic, there would be a 6th invariant associated with the energy flux and we would have a restatement of the Rankine-Hugoniot conditions. However we let there exist a radiation function $\Gamma(\rho, P)$ which breaks the conservation of energy flux.

Choosing vx as the variable, we may use 2.a-2.d to solve for ρ, vy, by and P_{gas} in terms of the invariants and vx only:

$$\begin{aligned}
\rho &= p_x / vx &(3a)\\
vy &= f_y b x_0 / (p_x vx - b x_0^2) &(3b)\\
by &= b x_0 f_y / (p_x vx - b x_0^2) &(3c)\\
P_{gas} &= f_x - p_x vx + (b x_0^2 - by(vx)^2)/2 &(3d)
\end{aligned}$$

There is now just one nonlinear, ODE to solve numerically. It is formulated by

$$\partial_{vx}(1.\mathrm{d})\partial_x vx = -\Gamma(\rho, P) \tag{4}$$

By electing that vx be a parameterized trial function of x, the system may be numerically integrated as accurately as desired provided the trial basis is well chosen. The Taylor series basis always (mathematically) works at any ordinary point of an ODE. Imogen's RadiatingFlowSolver contains the first 4 terms for the MHD case and the first 6 terms for the hydrodynamic case. The use of Pade approximant transforms is found to considerably improve the convergence properties of the series.

However, the direct series expansions are unwieldy (the 4th MHD term took 4 hours for Mathematica to simplify and is still over 2000 characters long) and so the solver only uses them to provide the necessary number of past history points to bootstrap a 5th order Adams-Bashforth linear multistep method.

2.1 Endpoint behavior

There are three possible outcomes in the general integration case. If it is hydrodynamic ($\mathbf{B} = \mathbf{0}$) with $\theta > -3$, the solution reaches zero velocity / infinite density in a finite distance; the velocity curve approaches this point with infinite derivative.

Normally a temperature cutoff is imposed on the radiation, such that $\Gamma(T < T_{crit}) \to 0$ which makes the flow revert to adiabatic (uniform flow) behavior thereafter. The critical vx is given by solving the quadratic (HD) or quartic (MHD) polynomial $P_{gas}/\rho = T_{crit}$ for vx.

If the flow is magnetized, the character of the ODE will abruptly change when it becomes a low beta plasma. Even with no thermal radiation limit, the flow conditions will abruptly (in a few percent of the flow's elapsed distance) rearrange themselves in a $\beta = 0$ flow supported by magnetic pressure. The weaker the initial magnetization, the harder the "knee" corner where the flow abruptly stops decelerating will be. The magnetic limit is a special case of the thermal cutoff, defined by solving $P_{gas} = 0$, a cubic equation, for vx.

RadiatingFlowSolver allows the user to select no cutoff, magnetization cutoff times a value slightly greater than unity, or thermal cutoff with arbitrary temperature ratio (selecting a low thermal cutoff for a magnetized flow approaches the magnetization cutoff itself).

The integrator itself actually terminates on one of three conditions:

- Too many restarts - Every time a physically invalid vx occurs, the engine backs up, divides the space step by 16, and resumes. This is only allowed to happen 7 times. This is the condition that terminates a pure hydrodynamic flow, typically ending with $vx < 1e-5$.
- Too many steps - A hard cutoff of 20K iterations is imposed
- Reached cutoff condition

2.2 Assumptions

The RadiatingShock experiment's flow solver assumes that the radiation function is parameterized in the form of

$$\Lambda(\rho, P) = \beta\rho^{2-\theta}P^{\theta} \tag{5}$$

I.e. a standard power law radiation model. $\theta = 1/2$ corresponds to free-free scattering/Bremsstrahlung. However, the accompanying Mathematica notebooks themselves contain a line which replaces this with a stub for an arbitrary Λ (though the expressions for velocity will, of course, become even more complex).

The assumption of a polytropic equation of state is built into the energy flux function used to derive all the equations used by the equilibrim solver; Relaxing it in the derivation notebooks would require rewriting equation 4 in them.

3 Files

Files involved with this experiment include:

- run/run_RadiatingHydroShock.m

- experiment/RadiatingShock/*
- experiment/experimentUtils/RadiatingFlowSolver.m
- experiment/experimentUtils/pade.m
- experiment/experimentUtils/solveCubic.m
- experiment/experimentUtils/solveQuartic.m

4 Simulation Parameters

Run parameters relevant to this simulation class are:

- .gamma (5/3) - Sets the polytropic index γ.
- .fractionPreshock (1/4) - This fraction with the lowest X values starts in uniform adiabatic preshock flow
- .fractionCold (1/10) - This fraction at the highest X values starts in the condition the flow had when the integrator terminated.
- .Tcutoff (1) - HACK
- .theta (0) - Sets the inclination of the preshock flow to the shock normal
- .sonicMach (3) - Sets the ratio of X velocity to preshock adiabatic soundspeed
- .alfvenMach (-1) - Sets the ratio of X velocity to preshock Alfven speed. If less than 0, a hydrodynamic flow is set up.
- .radBeta (1) - Sets the radiation parameter β in $\Lambda = \beta \rho^{2-\theta} P^{\theta}$
- .radTheta (1/2) - Sets the θ parameter in the Λ equation above.

Numerical parameters of note include:

- .perturbationType - must be one of .RANDOM (white noise), .COSINE (y-plane monochromatic), .COSINE_2D (yz-plane monochromatic) to select the type of perturbation used to seed instability
- .seedAmplitude (5e-4) - Sets the magnitude of the sinusoid or of all Fourier coefficients used to generate white noise in the density right upstream of the shock
- .randomSeed_spectrumLimit (64) - Cuts off the white noise spectrum at this wavenumber.

Note that slow MHD shocks are almost unconditionally unstable to the corrugation instability; It will exhibit itself in around 20K timesteps at cfl=.35 for seed amplitudes of 1e-5 or so, independent of resolution.