Radiating Shock experiment documentation

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1 Introduction

The radiating shock experiment generates equilibrium solutions of the one dimensional equations of radiating hydrodynamic or ideal magnetohydrodynamic flows and tests them for instability.

2 Derivations

To begin with we state the equations of Ideal MHD with a polytropic equation of state,

$$\partial_t \rho + \nabla(\rho v) = 0 \quad (1a)$$
$$\partial_t(\rho \mathbf{v}) + \nabla(\rho \mathbf{v}\mathbf{v} + \mathbf{I}P - BB) = 0 \quad (1b)$$
$$\partial_t B + \nabla(vB - Bv) = 0 \quad (1c)$$
$$\partial_t E + \nabla(\mathbf{v}(E+P) - (\mathbf{v}\cdot\mathbf{B})\mathbf{B}) = 0 \quad (1d)$$

Where we define ρ as the matter density, $\mathbf{v}$ its velocity, P the total (gas plus magnetic) pressure, $\mathbf{I}$ the identity tensor and $\mathbf{B}$ the magnetic field (with $\mu_0 = 1$).

Assuming we have structure only in the x direction (∂_y and $\partial_z == 0$), a series of conserved quantities associated with the above system (now a set of coupled ODEs) now clearly arises by going to the integral form: Whatever initial values we specified for the above variables define unchanging fluxes. Thus,

$$p_x = \rho_0 v x_0 \quad (2a)$$
$$f_x = \rho_0 v x_0^2 + P_0 - B x_0^2 \quad (2b)$$
$$f_y = \rho v x_0 v y_0 - b x_0 b y_0 \quad (2c)$$
$$0 = v x_0 b y_0 - b x_0 v y_0 \quad (2d)$$
$$bx = bx_0 \quad (2e)$$

Nominally one might argue the inclusion of f_z terms, but without loss of generality we may consider flows in the $x-y$ plane; Without transverse structure nothing will cause the flow to rotate itself out of the $x-y$ plane. So the first three invariants arise from conservation of mass and momentum, the fourth from the z part of the induction equation and the last from the divergence constraint.

If the flow were adiabatic, there would be a 6th invariant associated with the energy flux and we would have rederived the Rankine-Hugoniot conditions. However we let there be a radiation function $\Gamma = \beta \rho^{2-\theta} P^{\theta}$ which breaks this.

Choosing vx as the variable, we may use 2 a-d to solve for ρ, vy, by and P_{gas} in terms of vx and the invariants:

$$\rho = p_x / vx \quad (3a)$$
$$vy = f_y b x_0 / (p_x vx - b x_0^2) \quad (3b)$$
$$by = b x_0 f_y / (p_x vx - b x_0^2) \quad (3c)$$
$$P_{gas} = f_x - p_x vx + (b x_0^2 - by(vx)^2)/2 \quad (3d)$$

There is now just one, highly nonlinear, ODE to solve numerically. It is formulated by

$$\partial_{vx}(1.\mathrm{d})\partial_x vx = -\Gamma \quad (4)$$

By choosing vx to be some parameterized trial function of x, we may solve the system as accurately as may be desired. Imogen uses the Taylor series since we know this will work for a region around any ordinary point of the ODE; The RadiatingFlowSolver.m class contains expansions up to 4th order for the MHD case and up to 6th order for the hydrodynamic case. It (configurably) uses Pade approximants which may improve the error prefactor by up to 100x.

However, these unwieldy expressions (the 4th term in the MHD expression took Mathematica 4 hours to derive in that form) are only actually used by RadiatingFlowSolver to bootstrap a linear multistep method, the 5th order Adams-Bashforth integrator, which requires only evaluation of the 1st derivative. This is both faster and avoids expressions whose length and term count is rapidly approaching the point that they require rearrangement of terms and compensated summation just to evaluate.

2.1 Endpoint behavior

There are three possible outcomes in the general integration case. If it is hydrodynamic ($\mathbf{B} = 0$) with a reasonable value of θ, nothing interferes with the positive feedback loop of density and radiation rate and the solution reaches zero velocity (infinite density) in finite distance.

Normally a temperature cutoff is imposed on the radiation, such that $\Gamma(T < T_{crit}) = 0$ in which case the flow abruptly reverts to adiabatic (uniform flow) behavior thereafter. The critical vx is given by solving the quadratic (HD) or quartic (MHD) polynomial $P_{gas}/\rho = T_crit$ for vx.

If the flow is magnetized, the character of the ODE will abruptly change when it becomes a low beta plasma. Even with no thermal radiation limit, the flow conditions will abruptly (in a few percent of the flow's elapsed distance) rearrange themselves in a $\beta = 0$ flow supported by magnetic pressure. The weaker the initial magnetization, the harder the "knee" corner where the flow abruptly stops decelerating will be. The magnetic limit is a special case of the thermal cutoff, defined by solving $P_{gas} = 0$ for vx.

RadiatingFlowSolver allows the user to select no cutoff, magnetization cutoff times a value slightly greater than unity, or thermal cutoff with arbitrary temperature ratio (selecting a low thermal cutoff for a magnetized flow approaches the magnetization cutoff itself).

3 Files

Files involved with this experiment include:

- run/run_RadiatingHydroShock.m
- experiment/RadiatingShock/*
- experiment/experimentUtils/RadiatingFlowSolver.
- experiment/experimentUtils/pade.m
- experiment/experimentUtils/solveCubic.m
- experiment/experimentUtils/solveQuartic.m

4 Utilization