

# MPC Controller for Trajectory Tracking Control of Quadcopter

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**Abstract**—This paper focuses on trajectory tracking of quadcopter using a Model Predictive control (MPC). The main motive in using MPC is the ability to consider control and state constraints that occur in practical problems. In addition, MPC techniques consider a clear-cut performance criterion to be lessened during the control law computation. The trajectory tracking problem is solved using two approaches: PID controller and Linear MPC. The modeling of Quadcopter is developed using kinematic and dynamic equations. By making suitable assumptions, a simplified model is obtained, which is taken as the reference model for MPC. Simulation results are provided in order to show the effectiveness of both schemes. For comparison purpose only one degree of freedom, i.e., altitude is considered. The same methodology can be extended to other degrees of freedom (DOF). The results show that LMPC was able to achieve good tracking than the PID controller

**Index Terms**—Quadrotor, MPC, Trajectory Tracking, Model, PID

## I. INTRODUCTION

Nowadays, Quadcopter are widely used mainly because of its wide range of applications. Some of the applications include wild fire surveillance[1], inspection of nuclear power plant[2], marine operations[3] etc. All these applications are carried out with ease and it requires less human exertion. The previously mentioned augmented arrangement of conceivable applications forces new requirements with in the control and navigation for designing unmanned frameworks equipped for working in challenging situations and adapting to complex missions.

Among the manned and unmanned aerial vehicles, rotorcrafts and helicopters are considered as the best solution due to its vertical takeoff and landing and furthermore because of its forceful maneuverability. Due to its high maneuverability it can be controlled effectively utilizing remote with in the field of vision. In any case, rotorcraft UAVs have scientific issues that must be tended to all together to have the capacity to fly self-sufficiently and proficiently. UAVs are characterized by aggressive dynamics due to its low moment of inertia. This puts strict requirement in state estimation and controller implementation.

Quadcopter is a device with intense mixture of electronics, mechanical and mainly principles of aviation. Due to this electro-mechanical design, automatic control becomes

challenging. This becomes more challenging if the disturbance or perturbation due to wind is considered. Therefore while designing the control law, it has to ensure that constraints are considered and also it provides efficient control action.

The problem of control design in unmanned quadcopters are mainly focused in following areas: (a) PID controllers with linear quadratic (LQ)-regulators [4-6], (b) sliding mode controller which is a nonlinear control method [7], backstepping control method [8-10], and integral predictive-nonlinear  $H_\infty$  control approach[11], (c) dynamic inversion based techniques[12], (d) constrained finite time optimal control schemes [13]. Also, in most papers of quadcopters, main focus is on effect of disturbance such as in [14,15], that deals with simulation results and not on experimental studies[16, 17].

In this paper trajectory tracking of Quadcopter using a MPC is considered. The Quadcopter is defined by a set of nonlinear equations. By making suitable assumptions we can simplify it into linear equations and the MPC controller is designed for linear approximate model. The results are compared with a PID controller which is very simple and it works well. It is implemented using nested control loops. The inner most loop controls the angular velocities of each axis of the quadrotor. The outer loop controls the position of the Quadcopter.

In the following, the motion equation of Quadcopter is given in section 2. Then, the MPC controller for desired trajectory is shown in section 3. In section 4, PID controller for desired altitude is shown and also the comparison result.

## II. QUADCOPTER DYNAMICS

### A. Quadcopter Configuration

Quadcopter will work on the basis of changing torque and thrust. Each rotor consists of a brushless dc motor with fixed pitch. The arrangement of motor is in such a way that the forward pair will rotate in clock wise and horizontal pair will rotate in anticlockwise. This will result in reaction torque which is exactly opposed by each other if all four are rotating in same speed. The elimination of rotating moment will lead the vehicle to have a constant heading while hovering. To create a non-zero torque, yaw is controlled by varying speeds of pairs of motor. Altitude is controlled by varying the thrust from each motor by equal amount. For movement in lateral

directions, the relative speeds of each motor in the lateral pair are varied to create a desired lateral thrust offset. The quadrotors have six degree of freedom, which includes three translational and three rotational movements. But it has only four control inputs, i.e., speed of four rotors. Therefore it can be called as an under actuated system.

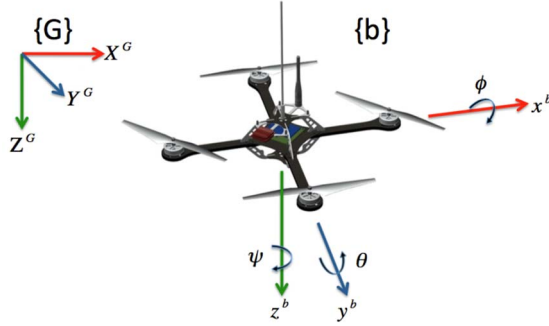


Fig.1. Basic quadcopter system

Fig 1 shows the Quadcopter structure for developing the model. The position of quadrotor is described in global coordinate system where as the velocity and angular velocity are defined in the quadrotor body coordinate system. The body co-ordinate system is defined on the body of Quadcopter whereas global co-ordinate system rotates with earth around its spin axis.

### B. Kinematic Equations

The state variables for velocity are in the body frame but the state variables for position are in the global frame. In order to transform the variables between coordinate system, i.e., from global coordinate system into body coordinate system, a rotation matrix,  $R_G^b$  is defined.

$$(1) \quad R_G^b = R(\Phi)R(\theta)R(\psi)$$

$$R(\Psi) = \begin{bmatrix} c(\Psi) & s(\Psi) & 0 \\ -s(\Psi) & c(\Psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \quad (2)$$

$$R(\theta) = \begin{bmatrix} c(\theta) & 0 & -s(\theta) \\ 0 & 1 & 0 \\ s(\theta) & 0 & c(\theta) \end{bmatrix} \quad (3)$$

$$(4) \quad R(\phi) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & c(\phi) & s(\phi) \\ 0 & -s(\phi) & c(\phi) \end{bmatrix}$$

$$R_G^b = \begin{bmatrix} c(\psi)c(\theta) & s(\psi)c(\theta) & -s(\theta) \\ c(\psi)s(\phi)s(\theta) - c(\phi)s(\psi) & s(\phi)s(\psi)s(\theta) + c(\phi)c(\psi) & c(\theta)s(\phi) \\ c(\phi)c(\psi)s(\theta) + s(\phi)s(\psi) & c(\phi)s(\psi)s(\theta) - c(\psi)s(\phi) & c(\phi)c(\theta) \end{bmatrix} \quad (5)$$

Here c and s represents cos and sin respectively.

Similarly, to transform the variables from body co-ordinate system to global coordinate system, a transformation matrix  $R_b^G$  is defined.

$$R_b^G = R(\phi)^T R(\theta)^T R(\psi)^T \quad (6)$$

$$R_b^G = \begin{bmatrix} c(\psi)c(\theta) & c(\psi)s(\phi)s(\theta) - c(\phi)s(\psi) & c(\phi)c(\psi)s(\theta) + s(\phi)s(\psi) \\ s(\psi)c(\theta) & s(\phi)s(\psi)s(\theta) + c(\phi)c(\psi) & c(\phi)s(\psi)s(\theta) - c(\psi)s(\phi) \\ -s(\theta) & c(\theta)s(\phi) & c(\phi)c(\theta) \end{bmatrix} \quad (7)$$

The angular rates are defined in body co-ordinate system and Euler angles are defined in intermediate co-ordinate system. To obtain the relationship between angular rates and time derivative of Euler angles, a rotation matrix is derived. The angular velocities are the vectors that are pointing towards axis of rotation. It is not equal to time derivative of Euler angles. The derivation will assume that time derivative of each Euler angle is small.

$$(8) \quad \omega = \begin{bmatrix} p \\ q \\ r \end{bmatrix} = R(\phi)R(\theta) \begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix} + R(\phi) \begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix}$$

$$(9) \quad \omega = \begin{bmatrix} p \\ q \\ r \end{bmatrix} = \begin{bmatrix} 1 & 0 & -s(\theta) \\ 0 & c(\phi) & s(\phi)c(\theta) \\ 0 & -s(\phi) & c(\phi)c(\theta) \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = S \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}$$

$$(10) \quad \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & s(\phi)\tan\theta & c(\phi)\tan\theta \\ 0 & c(\phi) & -s(\phi) \\ 0 & \frac{s(\phi)}{c(\theta)} & \frac{c(\phi)}{c(\theta)} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

If we consider the Euler angles to be small, then the matrix S becomes an identity matrix. This implies that angular rate will be equal to time derivative of Euler angles.

### C. Equations of Motion

The equations that define the Quadcopter structure are described here. Global frame is used for translational position because it is the same coordinate frame as of GPS sensor. Similarly, the body frame is chosen for attitude since the IMU (accelerometer, magnetometer, and gyroscope) make measurements in the body frame.

$$\begin{bmatrix} \dot{X}^G \\ \dot{Y}^G \\ \dot{Z}^G \end{bmatrix} = \begin{bmatrix} c(\psi)c(\theta) & c(\psi)s(\phi)s(\theta) - c(\phi)s(\psi) & c(\phi)c(\psi)s(\theta) + s(\phi)s(\psi) \\ s(\psi)c(\theta) & s(\phi)s(\psi)s(\theta) + c(\phi)c(\psi) & c(\phi)s(\psi)s(\theta) - c(\psi)s(\phi) \\ -s(\theta) & c(\theta)s(\phi) & c(\phi)c(\theta) \end{bmatrix} \begin{bmatrix} \dot{x}^b \\ \dot{y}^b \\ \dot{z}^b \end{bmatrix}$$

Equation (11) represents translational velocity.

$$(12) \begin{bmatrix} \dot{X}^G \\ \dot{Y}^G \\ \dot{Z}^G \end{bmatrix} = \begin{bmatrix} \frac{1}{m}(-[c(\phi)c(\psi)s(\theta) + s(\phi)s(\psi)]F_T^b - K_{dx}\dot{X}^G) \\ \frac{1}{m}(-[c(\phi)s(\psi)s(\theta) - c(\psi)s(\phi)]F_T^b - K_{dy}\dot{Y}^G) \\ \frac{1}{m}(-[c(\phi)c(\theta)]F_T^b - K_{dz}\dot{Z}^G) + g \end{bmatrix}$$

Equation (12) represents translational acceleration.  $K_{dx}, K_{dy}, K_{dz}$  represents the drag coefficient.  $F_T^b$  represents the total thrust force in body co-ordinate system.  $m$  is the mass of Quadcopter.

$$(13) \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \begin{bmatrix} 1 & s(\phi)\tan\theta & c(\phi)\tan\theta \\ 0 & c(\phi) & -s(\phi) \\ 0 & \frac{s(\phi)}{c(\phi)} & \frac{c(\phi)}{c(\theta)} \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

Equation (13) represents angular velocity.

$$(14) \begin{bmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} \frac{1}{J_x}[(J_y - J_z)qr - J_r q(\omega_1 - \omega_2 + \omega_3 - \omega_4) + lK_T(\omega_1^2 - \omega_2^2)] \\ \frac{1}{J_y}[(J_z - J_x)pr + J_r p(\omega_1 - \omega_2 + \omega_3 - \omega_4) + lK_T(\omega_1^2 - \omega_3^2)] \\ \frac{1}{J_z}[(J_z - J_y)pq + K_d(\omega_1^2 - \omega_2^2 + \omega_3^2 - \omega_4^2)] \end{bmatrix}$$

Equation (14) represents the angular acceleration.  $\omega$  is the motor angular velocity.  $J$  represents the moment of inertia.

### III. MODEL PREDICTIVE CONTROLLER

MPC is an optimal control strategy, i.e., based on numerical optimization. Depending on current measurements and prediction of future values of the output MPC calculations are done. The aim of the MPC control calculations is to find a successive control moves, which is the input i.e., called as manipulated input, so that the predicted response moves to the bench mark or set point in an optimal manner. The reference model required for trajectory tracking is obtained by making suitable assumptions.

#### A. Reference Model

The motion equation that describes the Quadcopter are complex, nonlinear and highly coupled and therefore it will be difficult to find the accurate behavior and also to perform the control in simulation. So the primary elements that is required while hovering, desired operational state is only considered. The elements that become significant during high speeds are neglected. The model equations described in equations (12) and (14) are rewritten along with the control inputs.

$$(15) \begin{bmatrix} \ddot{X}^G \\ \ddot{Y}^G \\ \ddot{Z}^G \end{bmatrix} = \begin{bmatrix} \frac{1}{m}(-[c(\phi)c(\psi)s(\theta) + s(\phi)s(\psi)]\mu_1 - K_{dx}\dot{X}^G) \\ \frac{1}{m}(-[c(\phi)s(\psi)s(\theta) - c(\psi)s(\phi)]\mu_1 - K_{dy}\dot{Y}^G) \\ \frac{1}{m}(-[c(\phi)c(\theta)]\mu_1 - K_{dz}\dot{Z}^G) + g \end{bmatrix}$$

$$(16) \begin{bmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} \frac{1}{J_x}[(J_y - J_z)qr - J_r q(\omega_1 - \omega_2 + \omega_3 - \omega_4) + u_2] \\ \frac{1}{J_y}[(J_z - J_x)pr + J_r p(\omega_1 - \omega_2 + \omega_3 - \omega_4) + u_3] \\ \frac{1}{J_z}[(J_z - J_y)pq + u_4] \end{bmatrix}$$

The model is simplified using some basic assumptions so that quad rotor can operate within a stable hover with small attitude angles and minimum rotational and translational velocity and accelerations. The aerodynamic forces and moments are assumed to be negligible as there are no aerodynamic lifting surfaces. The neglected effects are considered as disturbance and it can be compensated while designing the control system. The assumptions are described mathematically and can be written as follows.

$$\dot{X}_G = \dot{Y}_G = \dot{Z}_G = 0$$

$$\dot{\psi} = \dot{\theta} = \dot{\phi} = 0$$

$$\phi = \theta = 0$$

$$\cos\phi \approx \cos\theta \approx 1$$

$$\sin\phi \approx \sin\theta \approx 0$$

Equation (9) can be simplified into

$$\begin{bmatrix} p \\ q \\ r \end{bmatrix} \approx \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \approx \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \quad (17)$$

Equation (12) can be simplified as given below.

$$\begin{bmatrix} \ddot{X}^G \\ \ddot{Y}^G \\ \ddot{Z}^G \end{bmatrix} = \begin{bmatrix} \frac{1}{m}(-[c(\phi)c(\psi)s(\theta) + s(\phi)s(\psi)]\mu_1) \\ \frac{1}{m}(-[c(\phi)s(\psi)s(\theta) - c(\psi)s(\phi)]\mu_1) \\ \frac{1}{m}(-[c(\phi)c(\theta)]\mu_1) + g \end{bmatrix} \quad (18)$$

$$\psi = 0$$

$$\cos\psi = 1$$

$$\sin\psi = 0$$

Now the equations (14) and (18) can be simplified as follows.

$$\begin{bmatrix} \ddot{X}^G \\ \ddot{Y}^G \\ \ddot{Z}^G \end{bmatrix} = \begin{bmatrix} \frac{1}{m}(-[\theta c(\psi) + \phi s(\psi)]\mu_1) \\ \frac{1}{m}(-[\theta s(\psi) - \phi c(\psi)]\mu_1) \\ -\frac{1}{m}(u_1) + g \end{bmatrix} = \begin{bmatrix} -\frac{1}{m}\phi u_1 \\ \frac{1}{m}\theta u_1 \\ -\frac{1}{m}(u_1) + g \end{bmatrix} \quad (19)$$

$$\begin{bmatrix} \ddot{\phi} \\ \ddot{\theta} \\ \ddot{\psi} \end{bmatrix} = \begin{bmatrix} \frac{1}{J_x}(u_2) \\ \frac{1}{J_y}(u_3) \\ \frac{1}{J_z}(u_4) \end{bmatrix} \quad (20)$$

Even though the equations depicted above simplify the control design, the accurate, nonlinear equations can be used for assessing the robustness of control system. The translational movement has been decoupled from attitude by assuming that heading remains about zero degrees. By changing the roll and pitch values, movement along x and y axis can be controlled. Also, for having a stable altitude, the ratio of thrust vector to quad rotor's mass must be equal to the acceleration due to gravity.

#### IV. PID CONTROLLER

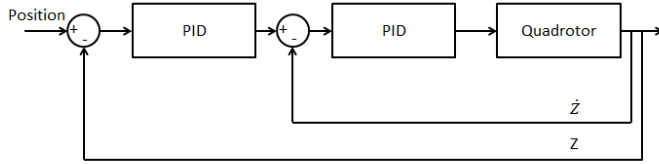


Fig.2 Trajectory tracking using PID

The above diagram shows tracking in z direction. Quadrotor control is implemented using nested control loops. The angular velocities of each of the axes of the quadrotor are controlled by the inner control loop. Since the quadrotor is having very fast dynamics, the inner loop has to be executed at a very high frequency. The control loop comes after the inner loop is responsible for the attitude and altitude control of the quad rotor. Light variations in the attitude are correlated to translational acceleration directly, i.e., large and undesired translational displacements will be generated by even small errors in the attitudes. Another advantage in executing the loop for rate control at high frequency is that, the sensors such as accelerometers and gyroscope also runs at high frequency. Thus an additional benefit of accurate and most recent measurements can be obtained while the control values are being calculated. The desired values of altitude and attitude can be communicated to the altitude/attitude controller from a far way controller or the outer loop controller

#### V. SIMULATION AND RESULTS

To demonstrate the feasibility of the developed controller, the simulation is done using MATLAB coding. The parameters used in LMPC for altitude tracking is given below.

Table1: Parameters used in LMPC

Input constraints	
Maximum U	[9.6,9.6,9.6,9.6]
Minimum U	[13,13,13,13]
Output Constraints	
Minimum Y	$\left[-\frac{\pi}{6}, -\frac{\pi}{6}, -\frac{\pi}{6}, -1, -1, -1\right]$
Maximum Y	$\left[\frac{\pi}{6}, \frac{\pi}{6}, \frac{\pi}{6}, 5, 5, 5\right]$
Reference Signal	[0,0,0,0,1]
Prediction Horizon	10
Simulation Steps	60
Control Horizon	2

Here the reference is given only for the altitude z. The simulation diagram obtained corresponding to the attitude control is given below.

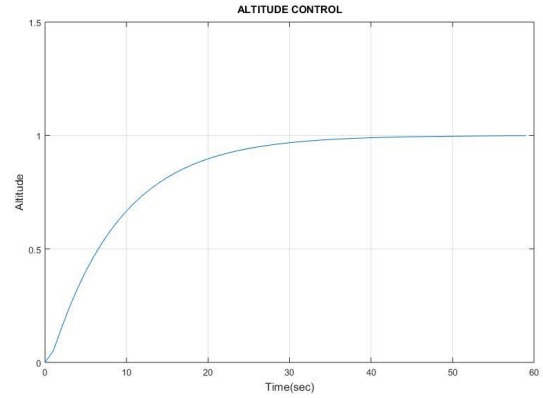
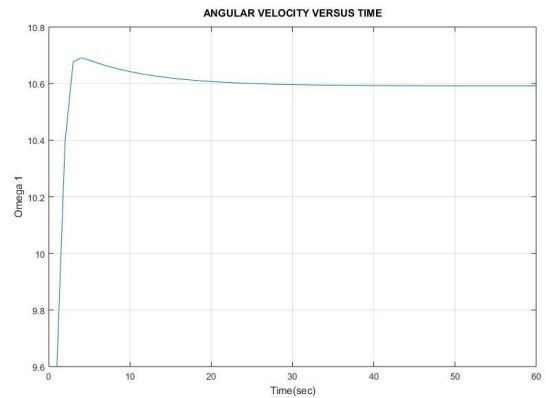
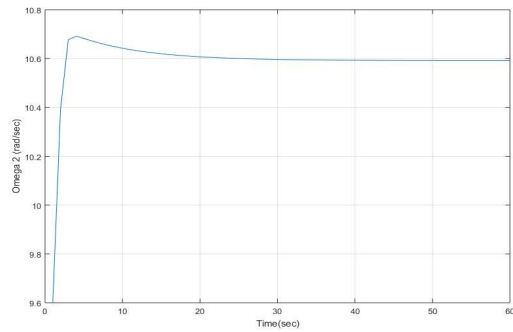


Fig.3 Altitude control using LMPC

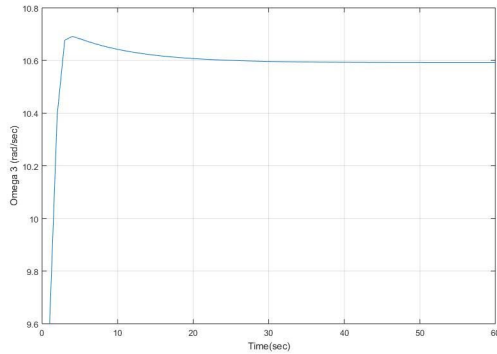
The corresponding motor angular velocities for achieving  $z=1$  is also plotted.



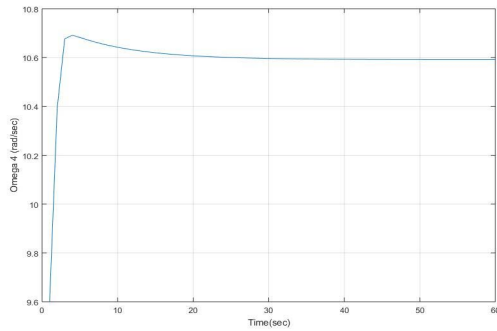
(a)



(b)



(c)



(d)

Fig.4 Angular velocities of motor(a) Motor 1 (b) Motor 2 (c)Motor 3 (d)Motor 4

From Fig.4, we can see that all the motor angular velocities are same for altitude control,  $z=1$ . The upward motion of Quadcopter is called thrust. For achieving thrust all the motors should increase its speed in same manner. We had given the input constraint from 9.6 to 13. From the graph, we can see that the input constraints are satisfied.

The same altitude control is performed using a PID control. Here we use two PIDs. One for finding velocity along  $z$  direction and other for positioning. The simulation is done by auto tuning of PIDs. The values of control gain obtained for inner and outer loop are given below.

Table.2 PID values for  $Z$ 

$K_P$	$K_I$	$K_D$
5.8824	1.515	2.30

Table.3 PID values for  $\dot{Z}$ 

$K_P$	$K_I$	$K_D$
3.27	4.1	5.01

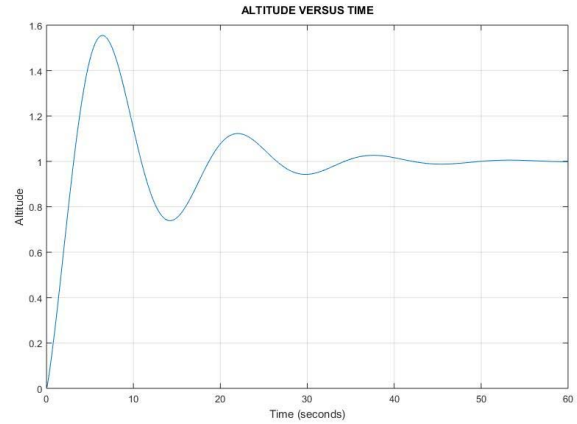


Fig.5Altitude control using PID

Now the performance of LMPC and PID is compared. By comparison we can see that LMPC has much better control characteristics when compared to PID. The time domain specifications of both controllers are tabulated and shown in Table 4.

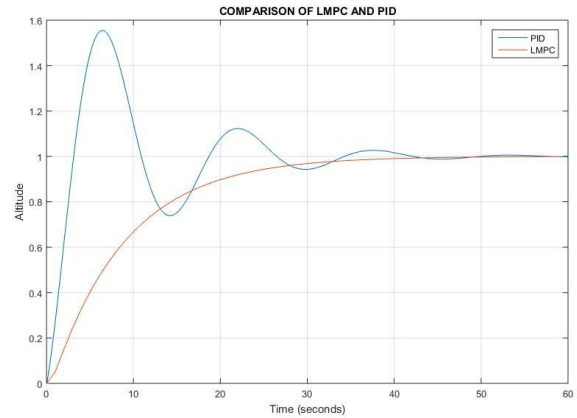


Fig.6Comparison of LMPC and PID

Table.4 Comparison between LMPC and PID

Controller	Rise time	Settling time	Peak Overshoot(%)
LMPC	4	50	55
PID	25	40	0

## VI. CONCLUSION AND FUTURE SCOPE

This paper proposes a model predictive control for altitude hold of quad copter. This system is then compared with PID controller. It is found that response of MPC has better settling time when compared to that with PID. MPC gives this response with no peak overshoot while the system with PID control has considerable peak overshoot. The future work may include using of Multi Parameter Model Predictive Control.

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