CSCI 499 Final, Spring 2019

Due Date: Thursday, May 2nd, 4pm

You may submit at instructor office hours, or in the homework dropbox (Box 16), lobby of PHE.

Late submissions will not be accepted.

Please staple this page to the front of your submission.

You are allowed the following:

- You may use the Sipser book, the Davis book, your notes, and any resources on Blackboard.
- You may ask the instructor or TAs for clarification on the problems (no hints will be provided).

You are **not** allowed any other resources, including:

- You may **not** discuss the exam with your classmates.
- You may **not** look on the internet for assistance.
- You may **not** talk to people about the exam other than the instructor or TA.
- You may **not** use textbooks or resources other than the official resources specified above.

I have read, understood, and agree to abide by the above policy. In addition, my submitted work subscribes to the highest standard of academic honesty: I did not knowingly divert from either the word or spirit of the above policy.

_____

Signature

_____

Name, Printed

_____

USC ID Number

Question	Points	Possible
1		10
2		10
3		10
4		10
5		10
6		10
Total		60

1. Suppose you start with any positive integer x. If x is even, divide x by 2. Otherwise, if x is odd, multiply x by 3 and add 1. Repeat this until the value of x is one. The **Collatz Conjecture** states that all possible inputs eventually reach 1.

   Suppose that A_{TM} were decidable by a TM H. Use H to describe a TM that will output 'yes' if the Collatz Conjecture is true, and 'no' if the Conjecture is false.

2. When answering this question, note that it is not known whether $EQ_{NFA} \in P$. The input to the following problems are DFA, so the size of the DFA is the size of the input.

   (a) (7 pts) Show that $EQ_{DFA} \in P$, and analyze the runtime of your algorithm.

   (b) (3 pts) A language A is **star-closed** if $A = A^*$. Show that the problem of testing whether a DFA recognizes a star-closed language is $\in P$, and analyze the runtime of your algorithm.

3. Let $U = \{\langle M, x, \#^t \rangle \mid M$ is an NTM that accepts input x within t steps on at least one branch$\}$. M may or may not be a decider. Show that U is NP-complete.

4. Let $UPATH = \{\langle G, s, t \rangle \mid G$ is an undirected graph with a path from s to $t\}$, and let $BIPARTITE = \{\langle G \rangle \mid G$ is an undirected graph with no odd-length cycles$\}$. Rather surprisingly, $UPATH \in L$. Use this fact to prove that $BIPARTITE \in L$.

5. Let $A_{LBA} = \{\langle M, w \rangle \mid M$ is an LBA that accepts $w\}$. Prove that A_{LBA} is PSPACE-complete.

6. A language is **prefix-closed** if all prefixes of every string in the language are also in the language. Prove that any infinite, prefix-closed, context-free language must have an infinite regular subset.