WaterRower Processing Algorithm

Explanatory Notes

1.0 General Remarks

The objective of the analysis that has been carried out was to establish the measurands necessary to properly establish the work rate of a rower on the WaterRower device and to devise a processing algorithm to allow the data to be presented to the rower in an appropriate and meaningful manner. The algorithm described herein has been created with the objective of enabling the useful work to be calculated on a stroke-by-stroke basis. With this correctly established it is a simple matter to calculate the average work rate based on any desired time interval or number of strokes. Rowers generally like to see feedback presented in terms of a boat speed or time to cover a specified distance. Hence a further algorithm has been devised that enables the work done to be converted to boat kinetic energy and hence to these desired parameters.

The basic assumption of the suggested data gathering procedure and of the subsequent data processing algorithm is that the speed of the paddle accurately represents the speed of the water. Based on this assumption the rotational energy stored in the WaterRower has been assumed to be the sum of that associated with the shaft, spokes, paddle blades (the paddle assembly) and the water. It is recognised that at certain points during the stroke, such as at the beginning of the drive and recovery phases, this assumption may not hold true as the speed of the water may differ from that of the paddle assembly. However, as will become apparent the algorithm relies on data taken at the end of the drive and recovery phases where it is expected that the speeds of the water and paddle assembly will be matched.

The required measurand is the paddle speed, from which, must also be deduced the stroke rate and the duration of the drive and recovery phases. The WaterRower is now provided with a toothed wheel and optical detector such that a pulse train containing 57 pulses per revolution can be recorded for the purposes of paddle speed measurement. Herein there are no instructions for deriving speed from the output pulse train. However, it should be noted that the processing algorithm relies on an accurate evaluation of the maximum and minimum speeds of the paddle during each stroke and of the duration of the drive and recovery phases. It is recommended therefore that, if the speed is to be deduced from the time taken for a number of teeth to pass the detector or as the average frequency based on the passage of a number of teeth, that the number be kept as small as possible to minimise errors due to smoothing effects.

2.0 Processing

2.1 Work Input

Knowledge of the rotational speed of the paddle at the beginning and end of the drive phase would enable the amount of rotational kinetic energy added to the system to be evaluated. However, this is not a good estimate of the work done by the rower since in addition to providing this additional energy, he or she must also overcome the resistance of the paddle due to friction and viscous effects. Hence a more sophisticated approach is required.

The work rate or power of the rower during the drive phase is given by the expression:

$$\dot{W} = Tw \qquad \text{.....1}$$

where $\dot{W}$ is the work rate or power (measured in Watts) T is the torque applied to the paddle shaft (measured in Nm) and w is its angular velocity (measured in rad/s). Hence if the torque and angular velocity were known at each point during the drive then the work done could be deduced from the following integral:

$$W = \int_0^{t_{drive}} Tw \, dt \qquad \text{.....2}$$

where W is the work done (measured in Joules) and t_{drive} is the duration of the drive phase of the stroke. The angular velocity, w, is the principal measurand and is therefore known. Hence, what remains to be established is the torque. Newton's second law expressed in terms of circular motion states that torque is equal to the rate of change of angular momentum, i.e.:

$$T = I\dot{w} \qquad \text{.....3}$$

where I is the moment of inertia (measured in kg.m^2) and $\dot{w}$ is the rate of change of angular velocity. The moment of inertia is a property of the paddle assembly and the water. It is simple to calculate (see full report) and remains unchanged. The angular velocity, w, is measured throughout the stroke and its rate of change may therefore relatively easily be derived. One can therefore expect that, by the above method, the work per stroke can reliably be calculated.

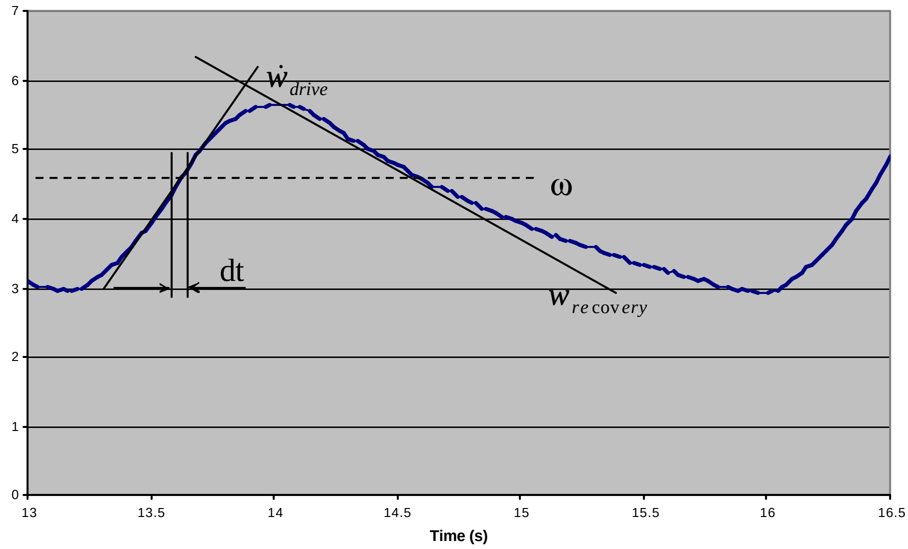

Figure 1 Typical paddle speed profile for a single stroke

Consider the paddle speed profile shown in figure 1. For the period, dt, we can deduce the rate of change of angular velocity and, assuming that we know the moment of inertia, calculate the torque required to accelerate the paddle at that rate using equation 3. However, as already discussed, in addition to providing the energy necessary to accelerate the paddle and water, the rower must also overcome the resistance. Conveniently, we may deduce the resistance torque by observing the rate of deceleration of the system at the average speed associated with the period dt, i.e. $w_{recovery}$. Hence, the work done during the period dt is given by the expression:

$$dW = I(\dot{w}_{drive} - \dot{w}_{recovery})w\ dt \qquad \text{…..4}$$

In order to calculate the work done during the drive period (and hence the stroke work done by the rower) equation 4 should be integrated over the drive period as in equation 2. The process may be simplified if it can be assumed that $\dot{w}_{drive}$ and $\dot{w}_{recovery}$ are constants. Such an assumption is analogous to treating the acceleration and deceleration portions of the stroke as linear, i.e. as shown in figure 2. Analysis has shown (see full report) that the error associated

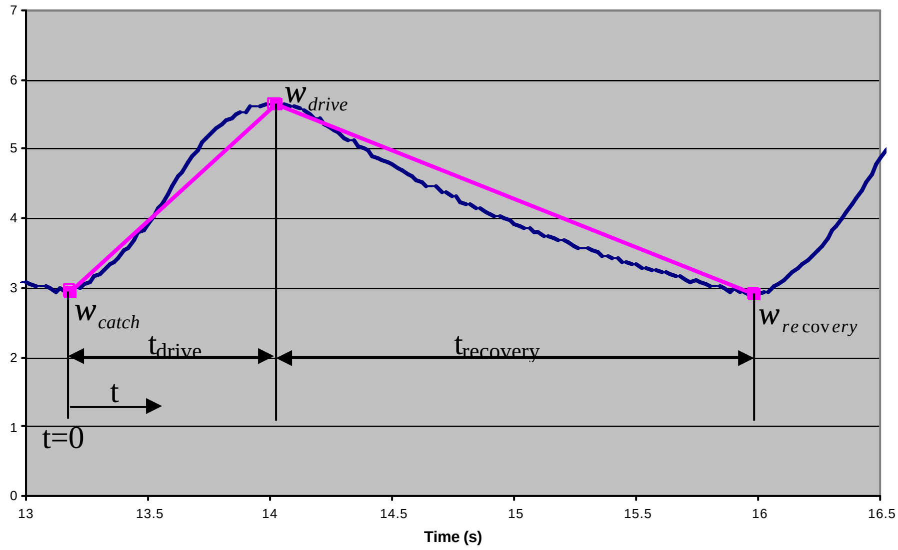

Figure 2 Linearised stroke profile

with such an assumption is small, of the order of 1% relative to the more complex approach associated with equation 4. Based on this simplified approach, the speed of the paddle during the drive phase can be described by the equation:

$$w = w_{catch} + t\frac{(w_{drive} - w_{catch})}{t_{drive}} \qquad \text{…..5}$$

The acceleration and deceleration rates of the system are then given by:

$$\dot{w}_{drive} = \frac{w_{drive} - w_{catch}}{t_{drive}} \qquad \text{…..6}$$

and

$$\dot{w}_{recovery} = \frac{w_{recovery} - w_{drive}}{t_{recovery}} \qquad \text{…..7}$$

From these the torque applied during the drive may be deduced from:

$$T = I\left(\dot{w}_{drive} - \dot{w}_{catch}\right) \qquad .....8$$

and the work done during the drive from:

$$W = \frac{t_{drive}}{2} T\left(w_{drive} + w_{catch}\right) \qquad .....9$$

2.2 Boat Calculations

Having established the work input to the device during the drive, what remains is to calculate how a rowing shell might respond to such a work rate so as to allow the data to be presented to the rower in the desired format. In a manner similar to that described above, the work done by a rower must accelerate the shell and at the same time overcome its resistance to motion. Resistance of a rowing shell is dependent on a number or phenomena including skin friction, wave drag and form drag. Of these, skin friction is generally regarded as having the largest contribution and hence it is the only form of drag to have been considered in this study. Analysis has suggested (see main report for details) that the drag of the virtual shell can be deduced from a fixed value of the drag coefficient defined as:

$$C_D = \frac{F_{drag}/A}{\frac{1}{2} r V^2} \qquad .....10$$

Where F_{drag} is the drag of the shell, A is the wetted surface area of the shell, r is the density of water and V is the velocity of the shell. The drag coefficient value used was $C_D = 0.0025$. However, if the results derived using this value are unsatisfactory, the value may be modified as part of a calibration exercise without changing the essential physics of the calculations.

With the WaterRower we have data concerning the speed of the paddle at the catch, the end of the drive and at the end of the recovery. With respect to shell calculations we have no such luxury. One approach might be to relate the speed of the paddle to an equivalent shell speed. However, in the current programme a different approach has been adopted. Careful analysis (see main report) of data derived from a mathematical model of the shell, in which the shell was cruising at constant average speed, has allowed an empirical relationship to be derived that relates the fraction of the work done that is converted to shell kinetic energy to the duration of the drive phase relative to the total stroke duration. The empirical relationship is as follows:

$$h = 1 - g \qquad .....11$$

where h is the fraction of work done converted to shell kinetic energy and g is the fraction of the stroke made up by the drive i.e. $t_{drive}/(t_{drive}+t_{recovery})$. Hence once the total work for a given stroke has been determined using equation 9, the change in kinetic energy of the shell can then be found by multiplying through by h. If the speed of the shell is known at the start of the stroke (0m/s prior to first stroke) then the speed of the shell at the end of the drive will be given by:

$$V_{drive} = \sqrt{V_{catch}^{\;2} + \frac{2hW}{m}} \qquad .....12$$

where V_{catch} is the shell speed at the beginning of the stroke and m is the mass of the shell and rower. The shell speed will then decrease during the recovery due to skin friction drag and at the end of the speed at the end of the recovery will be:

$$V_{recovery} = \left(\frac{1}{V_{drive}} + \frac{r\,AC_D}{2m} \right)^{-1} \qquad \text{.....13}$$

Once these speeds have been deduced, it is a simple matter to calculate the average speed for the duration of the stroke and hence the distance travelled.