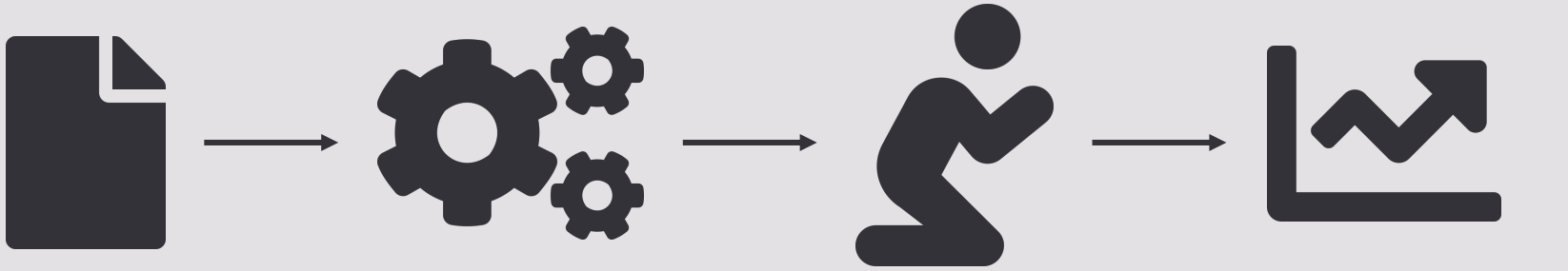
ELEVATE

A Language for Expressing Optimization Strategies

Bastian Hagedorn (University of Münster)
Martin Lücke (University of Münster)
Johannes Lenfers (University of Münster)
Michel Steuwer (University of Glasgow)

MOTIVATION

How do we optimize programs today?



MOTIVATION

How do we optimize programs today?

General Purpose Compilers

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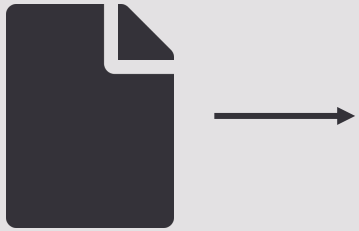
General Purpose Compilers



C++

Code Generation Options

-O0, **-O1**, **-O2**, **-O3**, **-Ofast**, **-Os**, **-Oz**, **-Og**, **-O**, **-O4**

Specify which optimization level to use:

-O0 Means “no optimization”: this level compiles the fastest and generates the most debuggable code.

-O1 Somewhere between **-O0** and **-O2**.

-O2 Moderate level of optimization which enables most optimizations.

-O3 Like **-O2**, except that it enables optimizations that take longer to perform or that may generate larger code (in an attempt to make the program run faster).

-Ofast Enables all the optimizations from **-O3** along with other aggressive optimizations that may violate strict compliance with language standards.

-Os Like **-O2** with extra optimizations to reduce code size.

-Oz Like **-Os** (and thus **-O2**), but reduces code size further.

-Og Like **-O1**. In future versions, this option might disable different optimizations in order to improve debuggability.

-O Equivalent to **-O2**.

-O4 and higher

Currently equivalent to **-O3**

Code Generation Options

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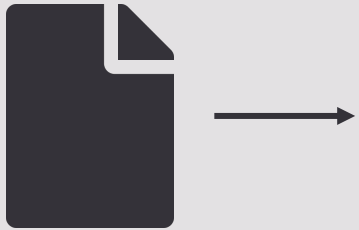

C++

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"... in an attempt to make the program run faster"

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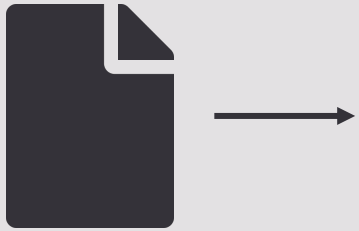

C++


```
-targetlibinfo -tti -tbaa -scoped-noalias -assumption-cache-tracker -profile-summary-info -forceattrs -inferattrs -callsite-splitting -i
psccp -called-value-propagation -globalopt -domtree -mem2reg -deadargelim -domtree -basicaa -aa -loops -lazy-branch-prob -lazy-block-fre
q -opt-remark-emitter -instcombine -simplifycfg -basiccg -globals-aa -prune-eh -inline -functionattrs -argpromotion -domtree -sroa -basi
caa -aa -memoryssa -early-cse-memssa -speculative-execution -basicaa -aa -lazy-value-info -jump-threading -correlated-propagation -simpl
ifycfg -domtree -aggressive-instcombine -basicaa -aa -loops -lazy-branch-prob -lazy-block-freq -opt-remark-emitter -instcombine -libcall
s-shrinkwrap -loops -branch-prob -block-freq -lazy-branch-prob -lazy-block-freq -opt-remark-emitter -pgo-memop-opt -basicaa -aa -loops -
lazy-branch-prob -lazy-block-freq -opt-remark-emitter -tailcallelim -simplifycfg -reassociate -domtree -loops -loop-simplify -lcssa-veri
fication -lcssa -basicaa -aa -scalar-evolution -loop-rotate -licm -loop-unswitch -simplifycfg -domtree -basicaa -aa -loops -lazy-branch-
prob -lazy-block-freq -opt-remark-emitter -instcombine -loop-simplify -lcssa-verification -lcssa -scalar-evolution -indvars -loop-idiom
-loop-deletion -loop-unroll -mldst-motion -phi-values -basicaa -aa -memdep -lazy-branch-prob -lazy-block-freq -opt-remark-emitter -gvn -
phi-values -basicaa -aa -memdep -memcpyopt -sccp -demanded-bits -bdce -basicaa -aa -loops -lazy-branch-prob -lazy-block-freq -opt-remark
-emitter -instcombine -lazy-value-info -jump-threading -correlated-propagation -basicaa -aa -phi-values -memdep -dse -loops -loop-simpli
fy -lcssa-verification -lcssa -basicaa -aa -scalar-evolution -licm -postdomtree -adce -simplifycfg -domtree -basicaa -aa -loops -lazy-br
anch-prob -lazy-block-freq -opt-remark-emitter -instcombine -barrier -elim-avail-extern -basiccg -rpo-functionattrs -globalopt -globaldc
e -basiccg -globals-aa -float2int -domtree -loops -loop-simplify -lcssa-verification -lcssa -basicaa -aa -scalar-evolution -loop-rotate
-loop-accesses -lazy-branch-prob -lazy-block-freq -opt-remark-emitter -loop-distribute -branch-prob -block-freq -scalar-evolution -basic
aa -aa -loop-accesses -demanded-bits -lazy-branch-prob -lazy-block-freq -opt-remark-emitter -loop-vectorize -loop-simplify -scalar-evolu
tion -aa -loop-accesses -loop-load-elim -basicaa -aa -lazy-branch-prob -lazy-block-freq -opt-remark-emitter -instcombine -simplifycfg -d
omtree -loops -scalar-evolution -basicaa -aa -demanded-bits -lazy-branch-prob -lazy-block-freq -opt-remark-emitter -slp-vectorizer -opt-
remark-emitter -instcombine -loop-simplify -lcssa-verification -lcssa -scalar-evolution -loop-unroll -lazy-branch-prob -lazy-block-freq
-opt-remark-emitter -instcombine -loop-simplify -lcssa-verification -lcssa -scalar-evolution -licm -alignment-from-assumptions -strip-de
ad-prototypes -globaldce -constmerge -domtree -loops -branch-prob -block-freq -loop-simplify -lcssa-verification -lcssa -basicaa -aa -sc
alar-evolution -branch-prob -block-freq -loop-sink -lazy-branch-prob -lazy-block-freq -opt-remark-emitter -instsimplify -div-rem-pairs -
simplifycfg -verify
```

-O3

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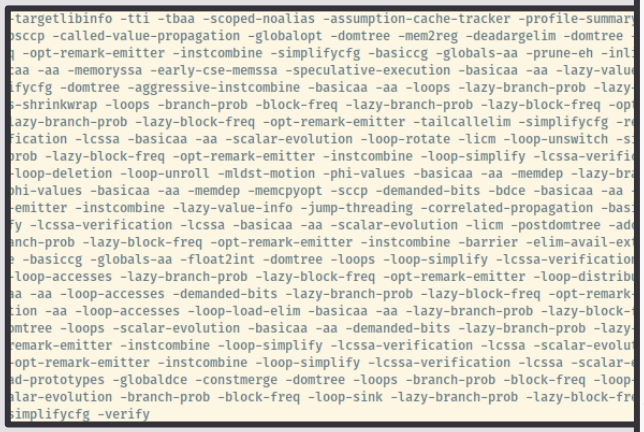
C++



-O3

Compiler Passes

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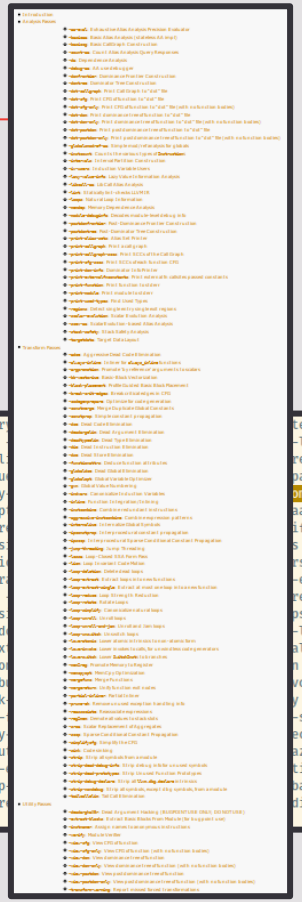
How do we optimize programs today?

General Purpose Compilers

C++



Compiler Passes

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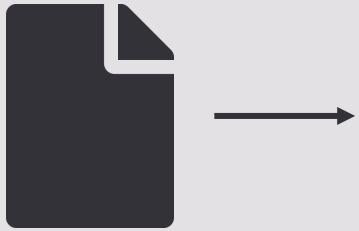
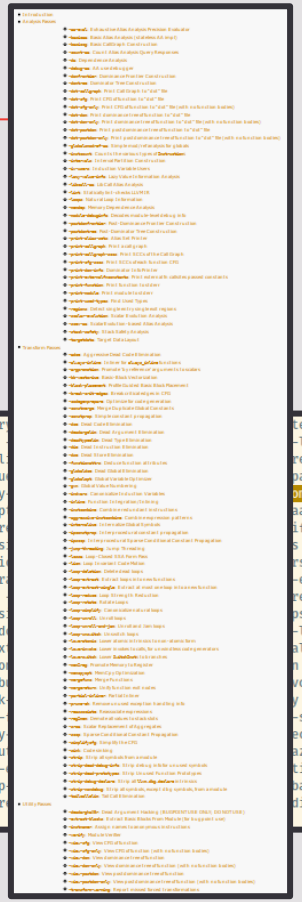
General Purpose Compilers



C++

-O3



Compiler Passes

Controlling optimisations is hard, because:

We have no clue what's going on inside the compiler!

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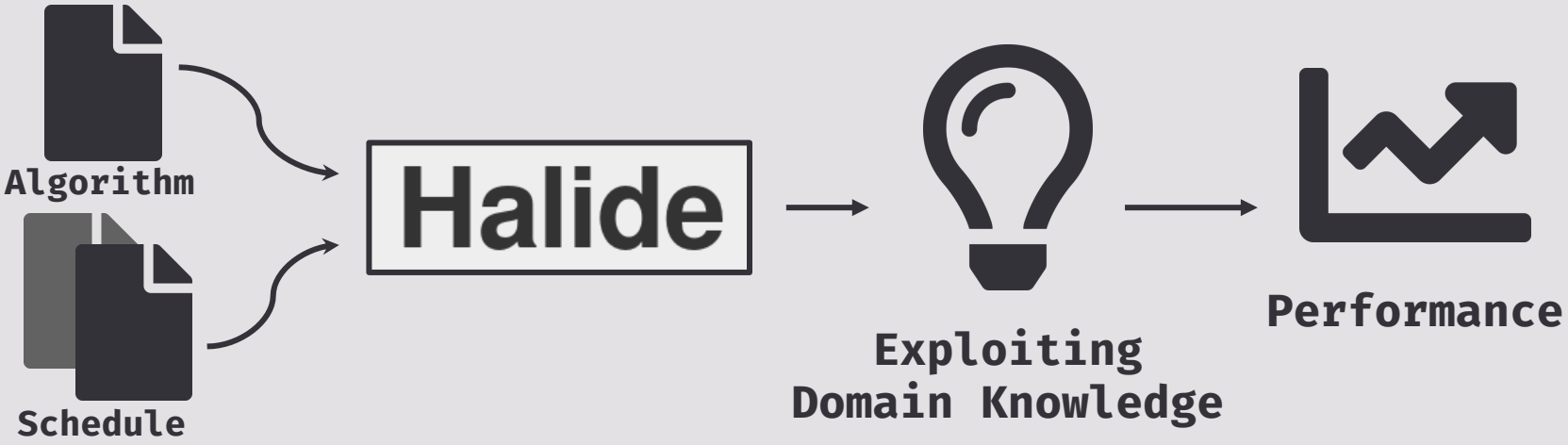
How do we optimize programs today?

Domain-Specific Compilers



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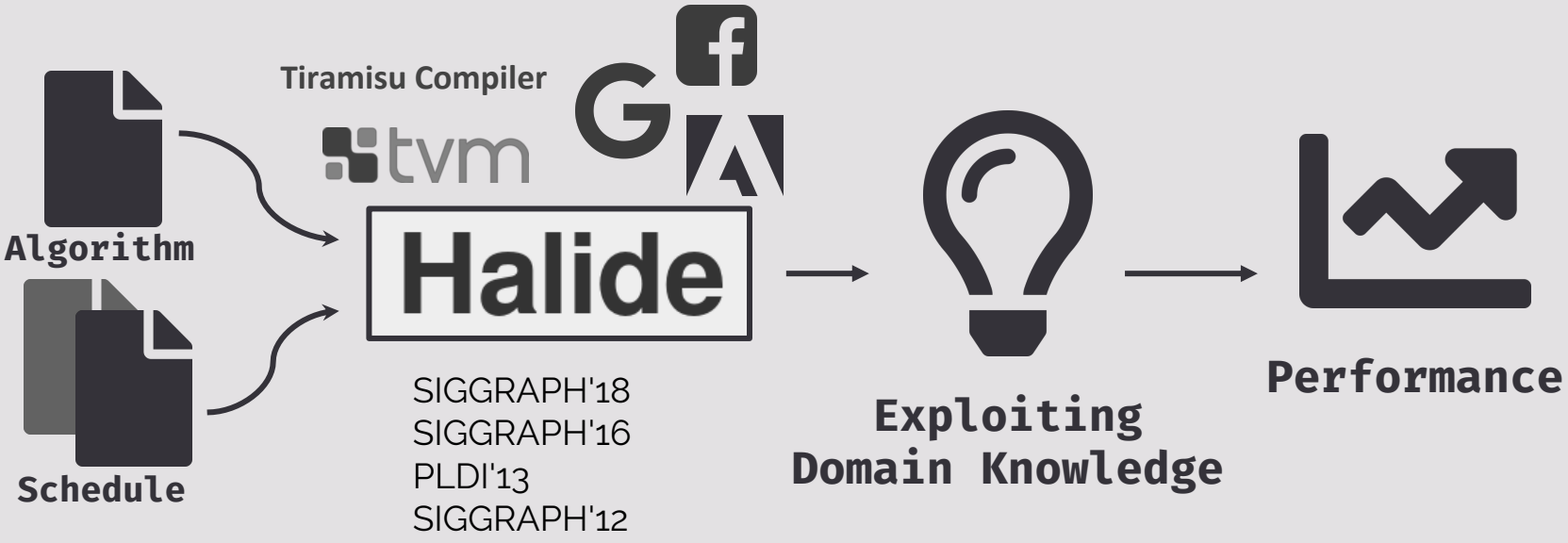
How do we optimize programs today?

Domain-Specific Compilers



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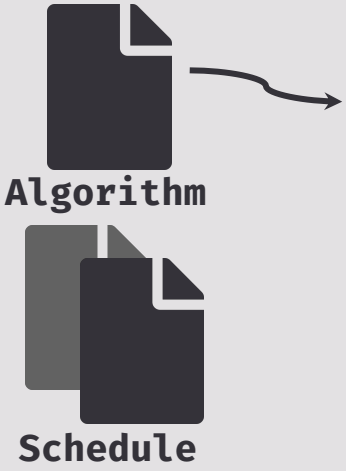
Domain-Specific Compilers



Matrix Multiplication

```
Func prod(“prod");
RDom r(0, size);
prod(x, y) +=
    A(x, r) * B(r, y);
out(x, y) = prod(x, y);
```

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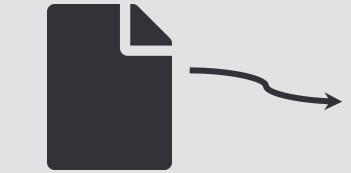
How do we optimize programs today?

Domain-Specific Compilers

Matrix Multiplication



Algorithm

```
Func prod("prod");
RDom r(0, size);
prod(x, y) +=
    A(x, r) * B(r, y);
out(x, y) = prod(x, y);
```

Schedule

```
const int warp_size = 32;
const int vec_size = 2;
const int x_tile = 3;
const int y_tile = 4;
const int y_unroll = 8;
const int r_unroll = 1;
Var xi, yi, xio, xii, yii, xo, yo, x_pair, xiio, ty;
RVar rxo, rxi;
out.bound(x, 0, size)
   .bound(y, 0, size)
   .tile(x, y, xi, yi, x_tile * vec_size * warp_size, y_tile *
y_unroll)
   .split(yi, ty, yi, y_unroll)
   .vectorize(xi, vec_size)
   .split(xi, xio, xii, warp_size)
   .reorder(xio, yi, xii, ty, x, y)
   .unroll(xio)
   .unroll(yi)
   .gpu_blocks(x, y)
   .gpu_threads(ty)
   .gpu_lanes(xii);
prod.store_in(MemoryType!::Register)
                        .compute_at(out, x)
                        .split(x, xo, xi, warp_size * vec_size,
TailStrategy!::RoundUp)
                        .split(y, ty, y, y_unroll)
                        .gpu_threads(ty)
                        .unroll(xi, vec_size)
                        .gpu_lanes(xi)
                        .unroll(xo)
                        .unroll(y)
                        .update()
                        .split(x, xo, xi, warp_size * vec_size,
TailStrategy!::RoundUp)
                        .split(y, ty, y, y_unroll)
                        .gpu_threads(ty)
                        .unroll(xi, vec_size)
                        .gpu_lanes(xi)
                        .split(r.x, rxo, rxi, warp_size)
                        .unroll(rxi, r_unroll)
                        .reorder(xi, xo, y, rxi, ty, rxo)
                        .unroll(xo)
                        .unroll(y);
Var Bx = B.in().args()[0], By = B.in().args()[1];
Var Ax = A.in().args()[0], Ay = A.in().args()[1];
B.in()
                        .compute_at(prod, ty)
                        .split(Bx, xo, xi, warp_size)
                        .gpu_lanes(xi)
                        .unroll(xo).unroll(By);
A.in()
                        .compute_at(prod, rxo)
                        .vectorize(Ax, vec_size)
                        .split(Ax, xo, xi, warp_size)
                        .gpu_lanes(xi)
                        .unroll(xo).split(Ay, yo, yi, y_tile)
                        .gpu_threads(yi).unroll(yo);
A.in().in().compute_at(prod, rxi)
                        .vectorize(Ax, vec_size)
                        .split(Ax, xo, xi, warp_size)
                        .gpu_lanes(xi)
                        .unroll(xo).unroll(Ay);
set_alignment_and_bounds(A, size);
set_alignment_and_bounds(B, size);
set_alignment_and_bounds(out, size);
```

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How do we optimize programs today?

Domain-Specific Compilers

Matrix Multiplication

Algorithm

```
Func prod("prod");
RDom r(0, size);
prod(x, y) +=
    A(x, r) * B(r, y);
out(x, y) = prod(x, y);
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Schedule

```
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RVar rxo, rxi;
out.bound(x, 0, size)
   .bound(y, 0, size)
   .tile(x, y, xi, yi, x_tile * vec_size * warp_size, y_tile *
y_unroll)
   .split(yi, ty, yi, y_unroll)
   .vectorize(xi, vec_size)
   .split(xi, xio, xii, warp_size)
   .reorder(xio, yi, xii, ty, x, y)
   .unroll(xio)
   .unroll(yi)
   .gpu_blocks(x, y)
   .gpu_threads(ty)
   .gpu_lanes(xii);
prod.store_in(MemoryType!::Register)
                        .compute_at(out, x)
                        .split(x, xo, xi, warp_size * vec_size,
TailStrategy!::RoundUp)
                        .split(y, ty, y, y_unroll)
                        .gpu_threads(ty)
                        .unroll(xi, vec_size)
                        .gpu_lanes(xi)
                        .unroll(xo)
                        .unroll(y)
                        .update()
                        .split(x, xo, xi, warp_size * vec_size,
TailStrategy!::RoundUp)
                        .split(y, ty, y, y_unroll)
                        .gpu_threads(ty)
                        .unroll(xi, vec_size)
                        .gpu_lanes(xi)
                        .split(r.x, rxo, rxi, warp_size)
                        .unroll(rxi, r_unroll)
                        .reorder(xi, xo, y, rxi, ty, rxo)
                        .unroll(xo)
                        .unroll(y);
Var Bx = B.in().args()[0], By = B.in().args()[1];
Var Ax = A.in().args()[0], Ay = A.in().args()[1];
B.in()
                        .compute_at(prod, ty)
                        .split(Bx, xo, xi, warp_size)
                        .gpu_lanes(xi)
                        .unroll(xo).unroll(By);
A.in()
                        .compute_at(prod, rxo)
                        .vectorize(Ax, vec_size)
                        .split(Ax, xo, xi, warp_size)
                        .gpu_lanes(xi)
                        .unroll(xo).split(Ay, yo, yi, y_tile)
                        .gpu_threads(yi).unroll(yo);
A.in().in().compute_at(prod, rxi)
                        .vectorize(Ax, vec_size)
                        .split(Ax, xo, xi, warp_size)
                        .gpu_lanes(xi)
                        .unroll(xo).unroll(Ay);
set_alignment_and_bounds(A, size);
set_alignment_and_bounds(B, size);
set_alignment_and_bounds(out, size);
```

Schedules are much harder to write than algorithms

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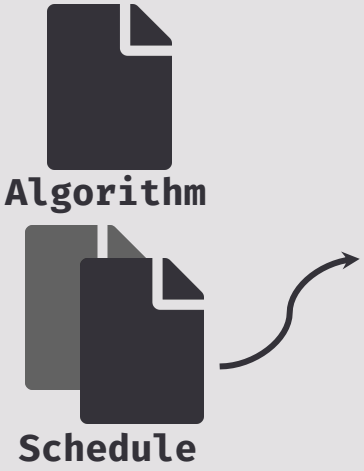
How do we optimize programs today?

Domain-Specific Compilers




```
...
out.bound(x, 0, size)
.bound(y, 0, size)
.tile(x, y, xi, yi, x_tile * vec_size * warp_size, y_tile * y_unroll)
.split(yi, ty, yi, y_unroll)
.vectorize(xi, vec_size)
.split(xi, xio, xii, warp_size)
.reorder(xio, yi, xii, ty, x, y)
.unroll(xio)
.unroll(yi)
.gpu_blocks(x, y)
.gpu_threads(ty)
.gpu_lanes(xii);
...
```

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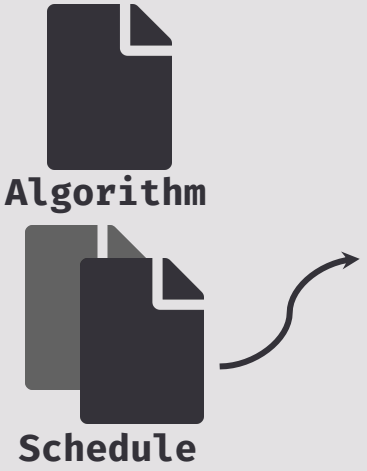
How do we optimize programs today?

Domain-Specific Compilers

fixed set of optimizations ⇒ lack of extensibility




```
...
out.bound(x, 0, size)
.bound(y, 0, size)
.tile(x, y, xi, yi, x_tile * vec_size * warp_size, y_tile * y_unroll)
.split(yi, ty, yi, y_unroll)
.vectorize(xi, vec_size)
.split(xi, xio, xii, warp_size)
.reorder(xio, yi, xii, ty, x, y)
.unroll(xio)
.unroll(yi)
.gpu_blocks(x, y)
.gpu_threads(ty)
.gpu_lanes(xii);
...
```

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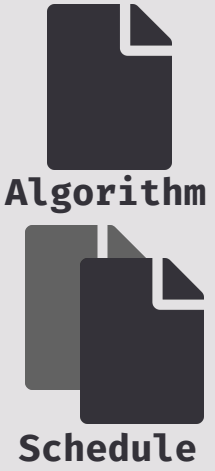
Domain-Specific Compilers

fixed set of optimizations ⇒ lack of extensibility




```
...
out.bound(x, 0, size)
.bound(y, 0, size)
.tile(x, y, xi, yi, x_tile * vec_size * warp_size, y_tile * y_unroll)
.split(yi, ty, yi, y_unroll)
.vectorize(xi, vec_size)
.split(xi, xio, xii, warp_size)
.reorder(xio, yi, xii, ty, x, y)
.unroll(xio)
.unroll(yi)
.gpu_blocks(x, y)
.gpu_threads(ty)
.gpu_lanes(xii);
...
```

what happens if the order of these is swapped ?

⇒ unclear semantics

⇒ unclear how to automatically generate schedules

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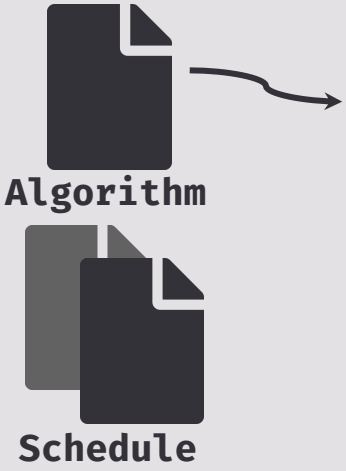
How do we optimize programs today?

Not always a clear separation of algorithm and schedule

Domain-Specific Compilers




```
Func prod("prod");
prod(j, i) = A_(j, i) * x_(j);

RDom k(0, sum_size_vecs, "k");
Func accum_vecs("accum_vecs");
accum_vecs(j, i) += prod(k * vec_size + j, i);

Func accum_vecs_transpose("accum_vecs_transpose");
accum_vecs_transpose(i, j) = accum_vecs(j, i);

RDom lanes(0, vec_size);
Func sum_lanes("sum_lanes");
sum_lanes(i) += accum_vecs_transpose(i, lanes);

RDom tail(sum_size_vecs * vec_size, sum_size - sum_size_vecs * vec_size);
Func sum_tail("sum_tail");
sum_tail(i) = sum_lanes(i);
sum_tail(i) += prod(tail, i);

Func Ax("Ax");
Ax(i) = sum_tail(i);
result(i) = b_ * y_(i) + a_ * Ax(i);
```

Matrix Vector Multiplication

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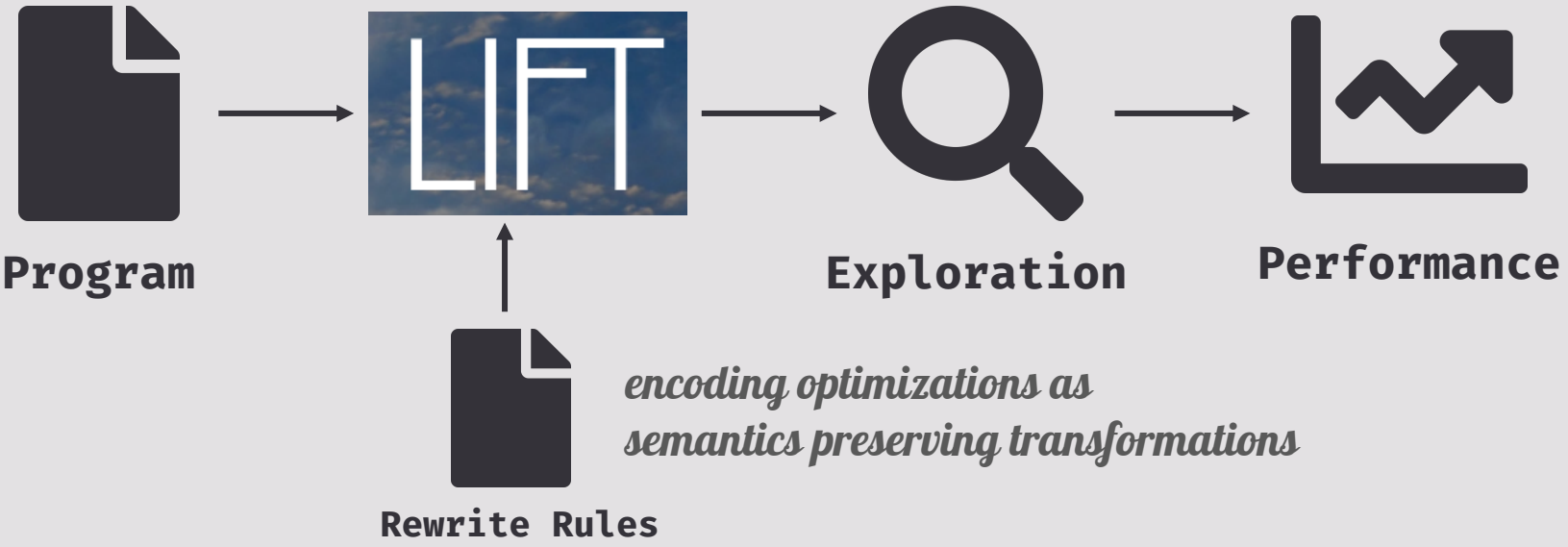
How do we optimize programs today?

Functional Domain-Specific Compilers



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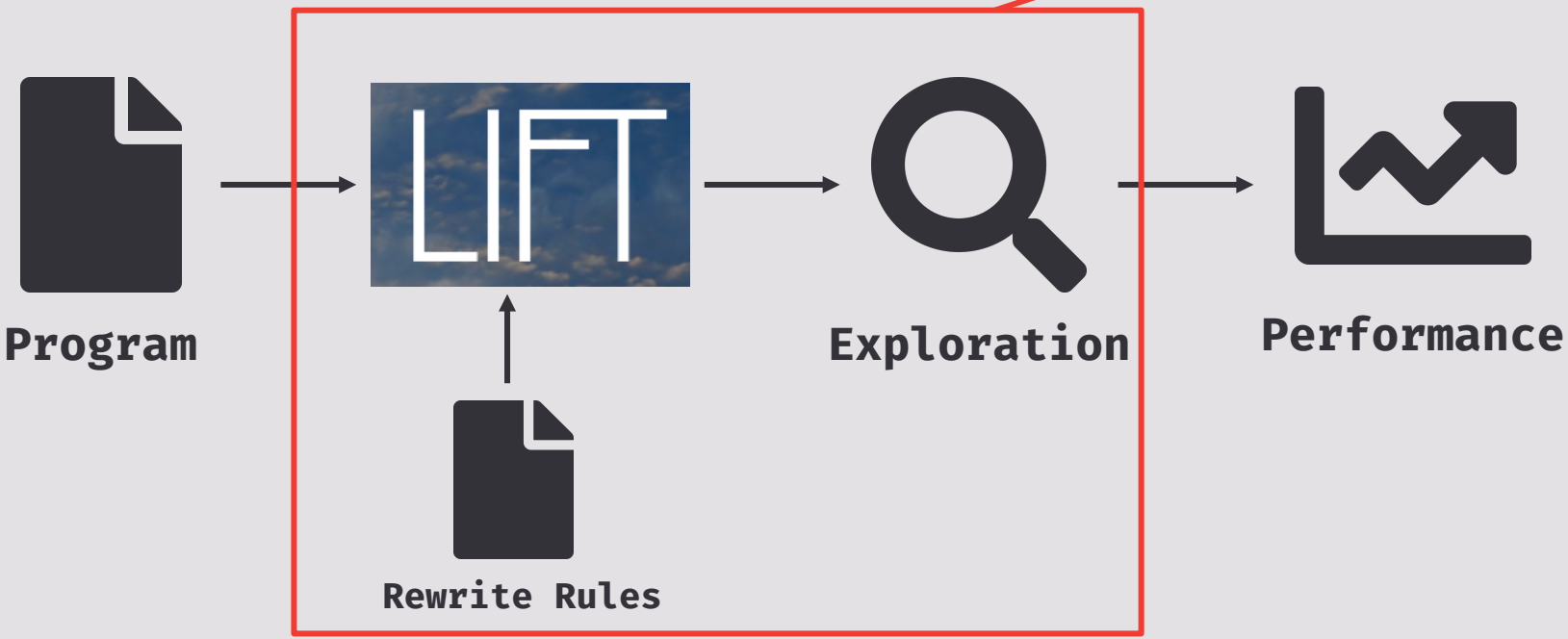
How do we optimize programs today?

Functional Domain-Specific Compilers

How does this actually work?



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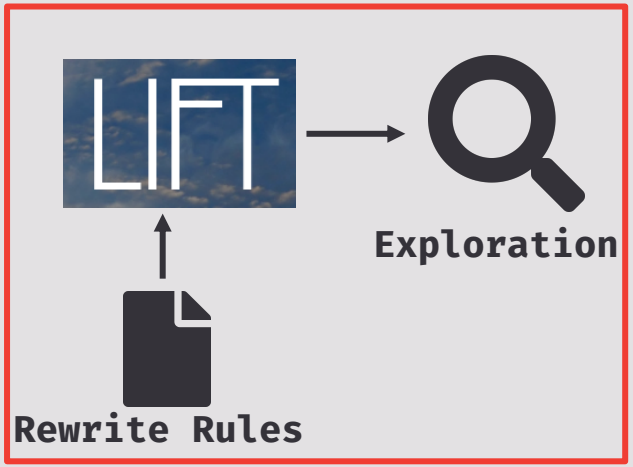
How do we optimize programs today?

Functional Domain-Specific Compilers

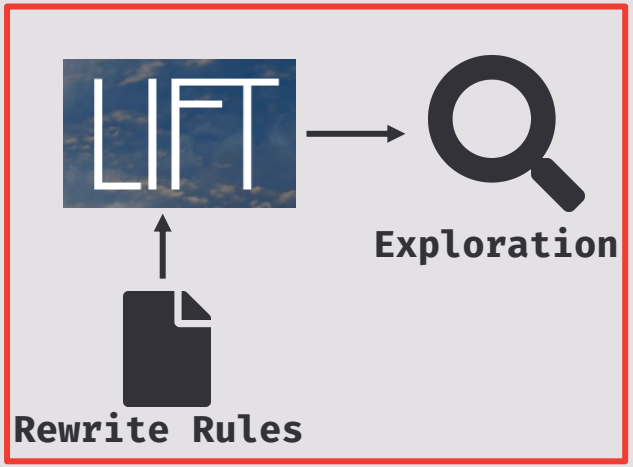


CGO'18
CGO'17
CASES'16
GPGPU'16
ICFP'15

MOTIVATION

How do we optimize programs today?

Functional Domain-Specific Compilers



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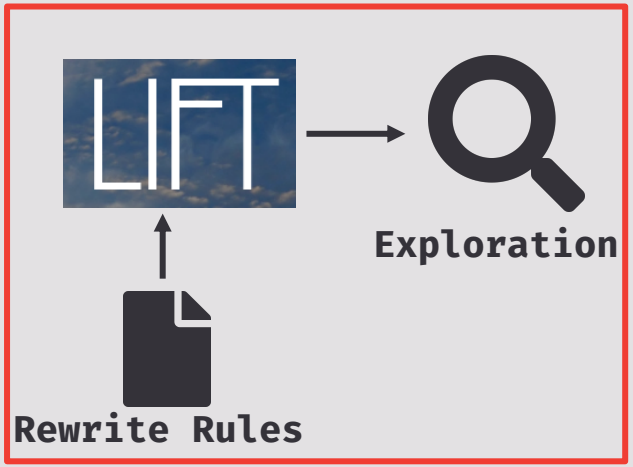

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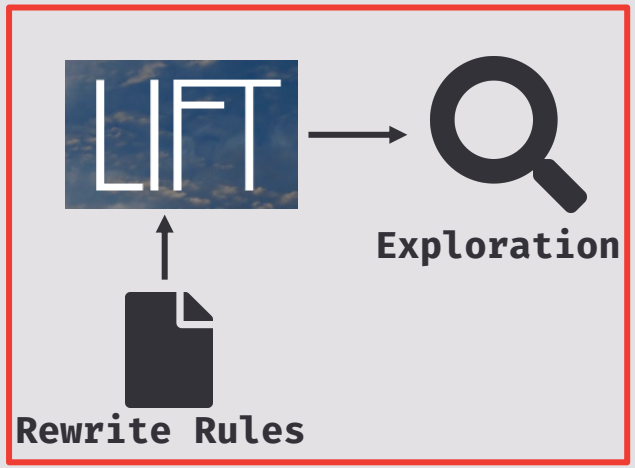


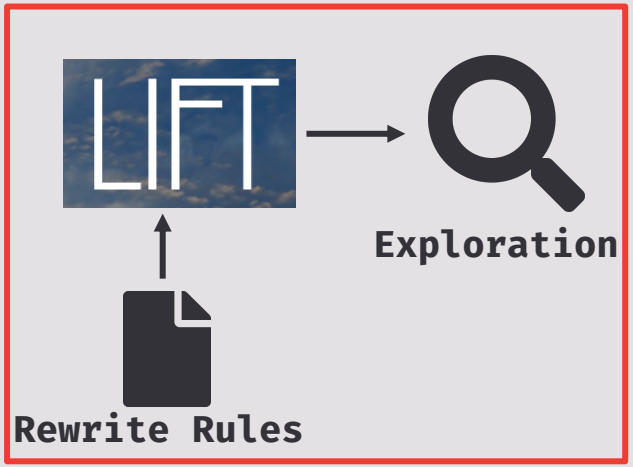
~~CGO'18~~
~~CGO'17~~
CASES'16
GPGPU'16
ICFP'15

> …the resulting space is extremely large, even potentially unbounded,which opens up a ***new research challenge***.
> …
> We present here a first, ***simple and heuristic-based*** pruning strategy to tackle the space complexity problem. ***Future research*** will investigate more advanced techniques to fully automate the pruning process.
> …

MOTIVATION

How do we optimize programs today?

Functional Domain-Specific Compilers



~~CGO'18~~
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ICFP'15

"…guide the automatic rewrite process by grouping rewrite rules together into ***macro rules***

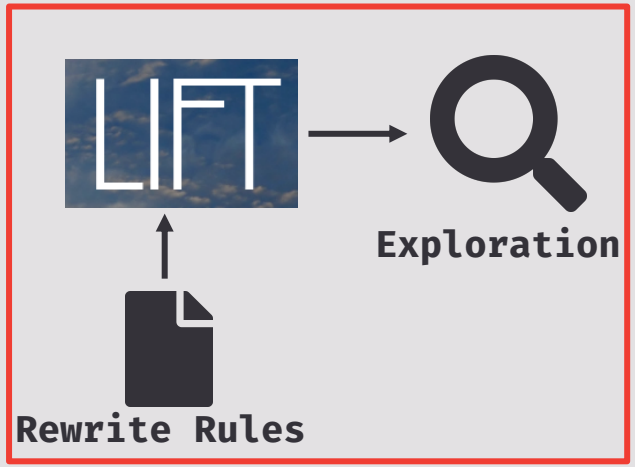
…

These macro rules are more flexible than the simple rules. They ***try to apply different sequences*** of rewrites to achieve their optimization goal."

MOTIVATION

How do we optimize programs today?

Functional Domain-Specific Compilers



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GPGPU'16
ICFP'15

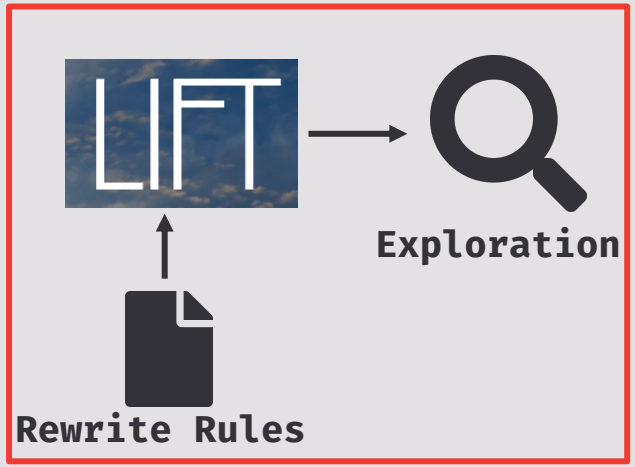
> …guide the automatic rewrite process by grouping rewrite rules together into ***macro rules***
>
> …
>
> These macro rules are more flexible than the simple rules. They ***try to apply different sequences*** of rewrites to achieve their optimization goal.

unclear semantics of macro rules
unclear how to define macro rules

MOTIVATION

How do we optimize programs today?

Functional Domain-Specific Compilers



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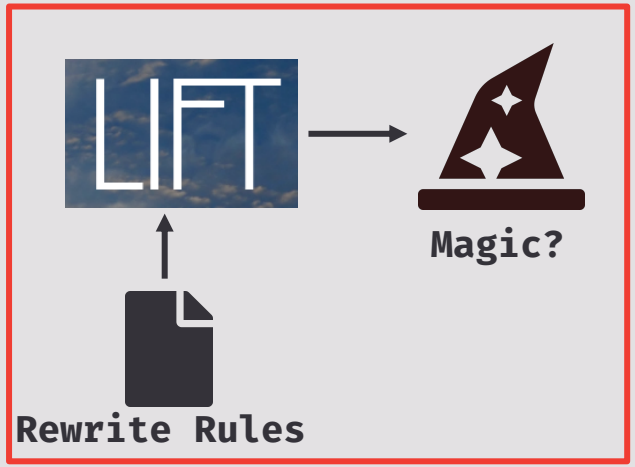
~~GPGPU'16~~
ICFP'15

> …we have developed a ***simple automatic search*** strategy
> …
> Our current search strategy is ***rather basic*** and just designed to prove that it is possible to find good implementations automatically.
> …
> We ***envision replacing this*** exploration strategy in the future

MOTIVATION

How do we optimize programs today?

Functional Domain-Specific Compilers



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MOTIVATION

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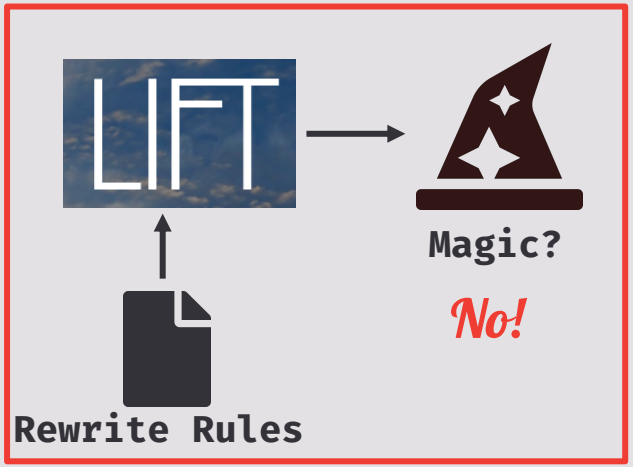

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The future research is here...

STRATEGIES

Optimizing Programs like it's ~~1998~~ 2019*

* Elevate's design is inspired by Eelco Visser and others work,
e.g. the 1998 ICFP paper *"Building program optimizers with rewriting strategies"*

ELEVATE

lessons from the past

- A ***Strategy*** is a function: ***Program → Program***
- A ***Transformation*** is the simplest strategy:

```
map(f) → join ∘ map(map(f)) ∘ split(n)
```

- ***isDefined*** tests if a strategy is defined for a given program

```
isDefined : Strategy → Program → Bool
```

- ***apply*** applies a strategy at a particular location in the program *(and might fail)*

```
apply : Strategy → Location → Strategy
```

- A ***Location*** and ***Traversal*** are ADTs:

```
data Location = Position(Traversal, Int) | FindFirst(Traversal, Program → Bool)
data Traversal = BFS | DFS
```

COMPOSING SRATEGIES

defining simple building blocks

id: Strategy
id = λp . p

seq: Strategy → Strategy -> Strategy
seq = λf . λs . λp . s (f p)

leftChoice: Strategy → Strategy → Strategy
leftChoice = λf . λs . λp . `try` (f p) `catch` (s p)

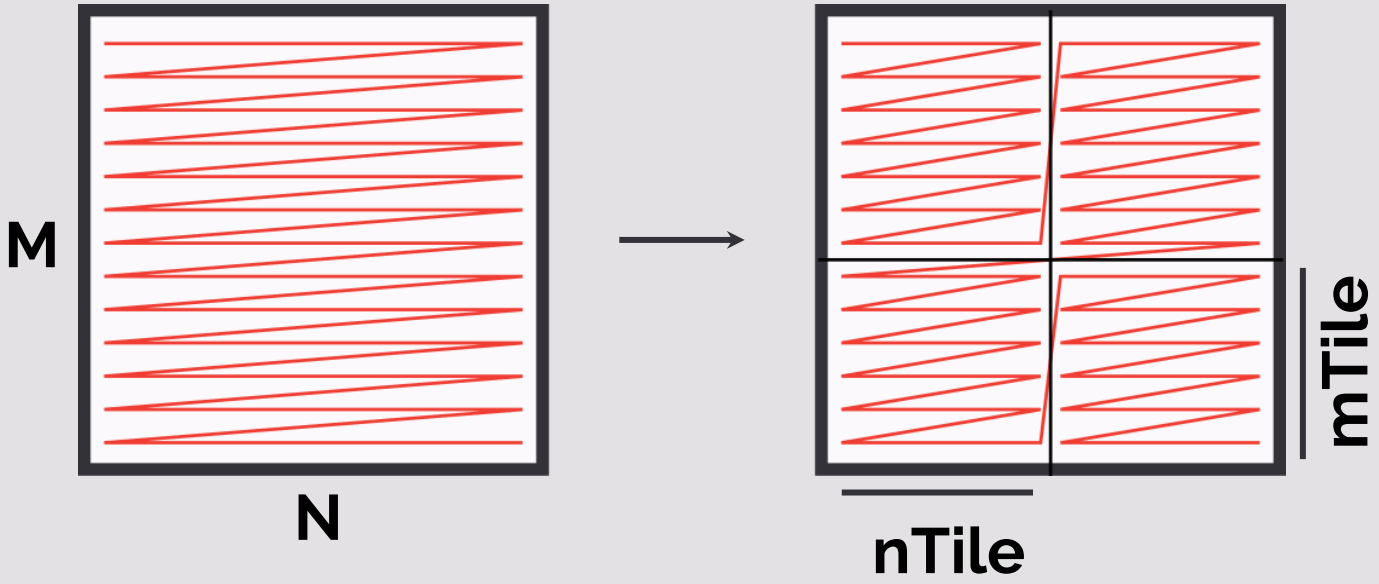
try: Strategy → Strategy
try = λs . *leftChoice* s id

repeat: Strategy → Strategy
repeat = λs . *try* (s ; (*repeat* s))

normalize: Strategy → Strategy
normalize: λs . *repeat*(*apply* s *FindFirst*(BFS, isDefined s))

DATA LAYOUT STRATEGY

what is tiling and why is it important for performance



Benefits of tiling:

- Exposes more ***parallelism***
- Enables to exploit ***locality***

→ **performance improvements of orders of magnitudes**

DATA LAYOUT STRATEGY

let's define Halide's `.tiling` *in Elevate*

```
tiling: Int → Strategy
tiling = λn . λp .
  ((tileEveryDimension n) ;        // step 1
    rewriteNormalForm ;            // step 2
      rearrangeDimensions) p       // step 3
```

DATA LAYOUT STRATEGY

let's define Halide's `.tiling` *in Elevate*

Short form for *seq*

```
tiling: Int → Strategy
tiling = λn . λp .
  ((tileEveryDimension n) ;      // step 1
    rewriteNormalForm ;          // step 2
     rearrangeDimensions) p      // step 3
```

We have *decomposed* the tiling Strategy into three *conceptual steps* that we define also as Strategies

DATA LAYOUT STRATEGY

let's define Halide's `.tiling` *in Elevate*

Elevate:

```
tiling: Int → Strategy
tiling =  λn . λp .
  ((tileEveryDimension n) ;
    rewriteNormalForm ;
     rearrangeDimensions) p
```

Lift:

$$**f$$

OpenCL:

```
for(int i = 0; i < M; i++) {
 for(int j = 0; j < N; j++) {
  out[i][j] = f(in[i][j]);
}}
```

Lift abbreviations: * = map | **S** = split | **J** = join | **T** = transpose

DATA LAYOUT STRATEGY

let's define Halide's `.tiling` *in Elevate*

Elevate:

step 1

```
tileEveryDimension: Int → Strategy
tileEveryDimension = λn . λp .
fold (λ (p, l). try (apply (splitJoin n) l p)
        p
        (findAll (isDefined (splitJoin n)) p))
```

(*splitJoin* size) = ***f* → **J** ∘ ****f* ∘ **S**(size)
fold: (List L) → (P → L → P) → P → P
findAll: (Program → Bool) → Program → (List Location)

Lift:

```
**f
```

OpenCL:

```
for(int i = 0; i < M; i++) {
 for(int j = 0; j < N; j++) {
  out[i][j] = f(in[i][j]);
}}
```

Lift abbreviations: * = map | **S** = split | **J** = join | **T** = transpose

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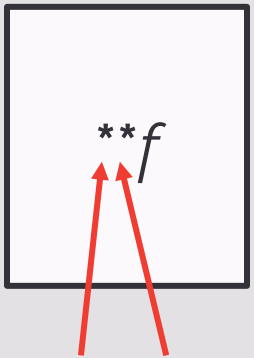

2 locations where splitJoin rule is defined

OpenCL:

```
for(int i = 0; i < M; i++) {
 for(int j = 0; j < N; j++) {
  out[i][j] = f(in[i][j]);
}}
```

Lift abbreviations: * = map | **S** = split | **J** = join | **T** = transpose

DATA LAYOUT STRATEGY

let's define Halide's `.tiling` *in Elevate*

Elevate:

step 1

tileEveryDimension: Int → Strategy
tileEveryDimension = λn . λp .
fold (λ (p, l). **try** (**apply** (splitJoin n) l p)
p
(**findAll** (**isDefined** (*splitJoin* n)) p))

(*splitJoin* size) = ***f* → **J** ∘ ****f* ∘ **S**(size)
fold: (List L) → (P → L → P) → P → P
findAll: (Program → Bool) → Program → (List Location)

Lift:

J ∘
****(**J** ∘ ****f* ∘ **S**) ∘
S

OpenCL:

```
for(int i = 0; i < M; i++) {
 for(int ii = 0; ii < s; ii++) {
  for(int j = 0; j < N; j++) {
   for(int jj = 0; jj < s; jj++) {
    int i_ = i * iTile + ii;
    int j_ = j * jTile + jj;
    out[i_][j_] = f(in[i_][j_]);
}}}}
```

Lift abbreviations: * = map | **S** = split(s) | **J** = join | **T** = transpose

DATA LAYOUT STRATEGY

let's define Halide's `.tiling` *in Elevate*

Elevate:

step 1

tileEveryDimension: Int → Strategy
tileEveryDimension = λn . λp .
fold (λ (p, l). **try** (**apply** (splitJoin n) l p)
p
(**findAll** (**isDefined** (*splitJoin* n)) p))

(*splitJoin* size) = ***f* → **J** ∘ ****f* ∘ **S**(size)
fold: (List L) → (P → L → P) → P → P
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Lift:

J ∘
****(J** ∘ ****f* ∘ **S)** ∘
S

OpenCL:

```
for(int i = 0; i < M; i++) {
 for(int ii = 0; ii < s; ii++) {
  for(int j = 0; j < N; j++) {
   for(int jj = 0; jj < s; jj++) {
    int i_ = i * iTile + ii;
    int j_ = j * jTile + jj;
    out[i_][j_] = f(in[i_][j_]);
}}}}
```

tiling the M & N dimension

Lift abbreviations: * = map | **S** = split(s) | **J** = join | **T** = transpose

DATA LAYOUT STRATEGY

let's define Halide's `.tiling` *in Elevate*

Elevate:

step 2

rewriteNormalForm: Strategy
rewriteNormalForm = λp .
(**normalize** *mapFission*) p

mapFission = *(*f* ∘ *g*) → **f* ∘ **g*

Lift:

J ∘
(J** ∘ **f* ∘ **S**) ∘
S

OpenCL:

```
for(int i = 0; i < M; i++) {
 for(int ii = 0; ii < s; ii++) {
  for(int j = 0; j < N; j++) {
   for(int jj = 0; jj < s; jj++) {
    int i_ = i * iTile + ii;
    int j_ = j * jTile + jj;
    out[i_][j_] = f(in[i_][j_]);
}}}}
```

Lift abbreviations: * = map | **S** = split(s) | **J** = join | **T** = transpose

DATA LAYOUT STRATEGY

let's define Halide's `.tiling` *in Elevate*

Elevate:

step 2

rewriteNormalForm: Strategy
rewriteNormalForm = λp . **repeat**
(**apply** *mapFission*
findFirst (*isDefined mapFission*))
p

mapFission = *(*f* ∘ *g*) → **f* ∘ **g*

Lift:

J ∘
(J** ∘ **f* ∘ **S**) ∘
S

OpenCL:

```
for(int i = 0; i < M; i++) {
 for(int ii = 0; ii < s; ii++) {
  for(int j = 0; j < N; j++) {
   for(int jj = 0; jj < s; jj++) {
    int i_ = i * iTile + ii;
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Lift abbreviations: * = map | **S** = split(s) | **J** = join | **T** = transpose

DATA LAYOUT STRATEGY

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Elevate:

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p

mapFission = *(*f* ∘ *g*) → **f* ∘ **g*

Lift:

J ∘ ****J** ∘
*****f* ∘
****S** ∘ **S**

OpenCL:

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 for(int ii = 0; ii < s; ii++) {
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   for(int jj = 0; jj < s; jj++) {
    int i_ = i * iTile + ii;
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    out[i_][j_] = f(in[i_][j_]);
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```

Lift abbreviations: * = map | **S** = split(s) | **J** = join | **T** = transpose

DATA LAYOUT STRATEGY

let's define Halide's `.tiling` *in Elevate*

Elevate:

step 3

```
rearrangeDimensions: Int → Strategy
rearrangeDimensions =  λd . λp .
 d match
  case <2 : p
  case 2 : (shuffleDimension d) p
  case _ : (rearrangeDimension (d-1) ;
              (shuffleDimension d)) p
```

Lift:

$$J \circ {}^{**}J \circ {}^{****}f \circ {}^{**}S \circ S$$

OpenCL:

```
for(int i = 0; i < M; i++) {
 for(int ii = 0; ii < s; ii++) {
  for(int j = 0; j < N; j++) {
   for(int jj = 0; jj < s; jj++) {
    int i_ = i * iTile + ii;
    int j_ = j * jTile + jj;
    out[i_][j_] = f(in[i_][j_]);
}}}}
```

Lift abbreviations: * = map | **S** = split(s) | **J** = join | **T** = transpose

DATA LAYOUT STRATEGY

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  case _ : (rearrangeDimension (d-1) ;
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```

Lift:

J ∘ **J ∘
***T ∘ ****f ∘ *T ∘**
****S ∘ S**

OpenCL:

```
for(int i = 0; i < M; i++) {
 for(int ii = 0; ii < s; ii++) {
  for(int j = 0; j < N; j++) {
   for(int jj = 0; jj < s; jj++) {
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J ∘ **J ∘
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****S ∘ S**

OpenCL:

```
for(int i = 0; i < M; i++) {
 for(int j = 0; j < N; j++) {
  for(int ii = 0; ii < s; ii++) {
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    int i_ = i * iTile + ii;
    int j_ = j * jTile + jj;
    out[i_][j_] = f(in[i_][j_]);
}}}}
```

swapped loop-order

Lift abbreviations: * = map | **S** = split(s) | **J** = join | **T** = transpose

DATA LAYOUT STRATEGY

Why is defining tiling as a strategy a good idea?

ELEVATE

```
tiling: Int → Strategy
tiling = λs . λp .
  ((tileEveryDimension s) ;        // step 1
    rewriteNormalForm ;            // step 2
      rearrangeDimensions) p       // step 3
```

vs

Halide

```
...
out.bound(x, 0, size)
.bound(y, 0, size)
.tile(x, y, xi, yi, x_tile * vec_size * …)
.split(y, ty, yi, y_unroll)
.vectorize(xi, vec_size)
.split(xi, xio, xii, warp_size)
...
```

- Strategy not built-in in the compiler
- Works for arbitrary dimensions

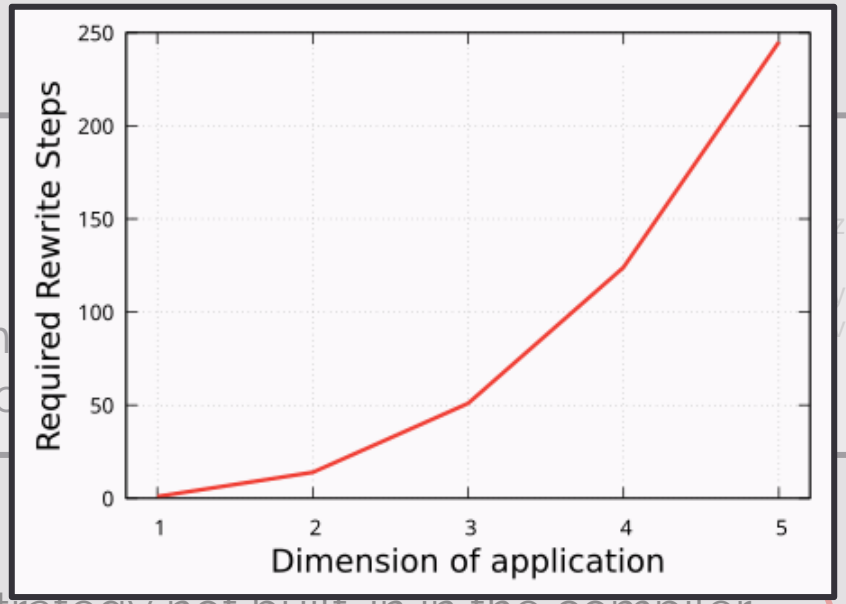
2D specific

DATA LAYOUT STRATEGY

Why is defining tiling as a strategy a good idea?

ELEVATE

```
tiling: Int → Strategy
tiling = λs . λp .
  ((tileEveryDimension
    rewriteNormalForm
      rearrangeDimensio
```



Halide

```
size)
ze)
yi, x_tile * vec_size * …)
yi, y_unroll)
vec_size)
xii, warp_size)
```

- Strategy not built-in in the compiler

Strategies are crucial for tiling in higher dimensions

DATA LAYOUT STRATEGY

Why is defining tiling as a strategy a good idea?

ELEVATE

```
overlapTiling: Int → Strategy
overlapTiling = λs . λp .
  ((tileEveryDimension s) ;        // step 1
    rewriteNormalForm ;            // step 2
     rearrangeDimensions) ;        // step 3
      (try overlapping) p          // step 4
```

vs

Halide

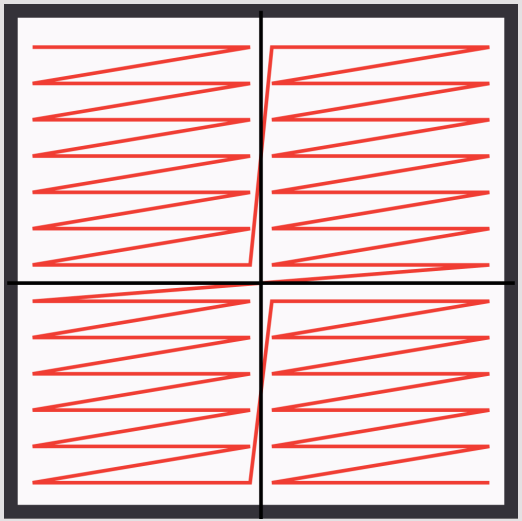
```
...
out.bound(x, 0, size)
.bound(y, 0, size)
.tile(x, y, xi, yi, x_tile * vec_size * …)
.split(yi, ty, yi, y_unroll)
.vectorize(xi, vec_size)
.split(xi, xio, xii, warp_size)
...
```

?

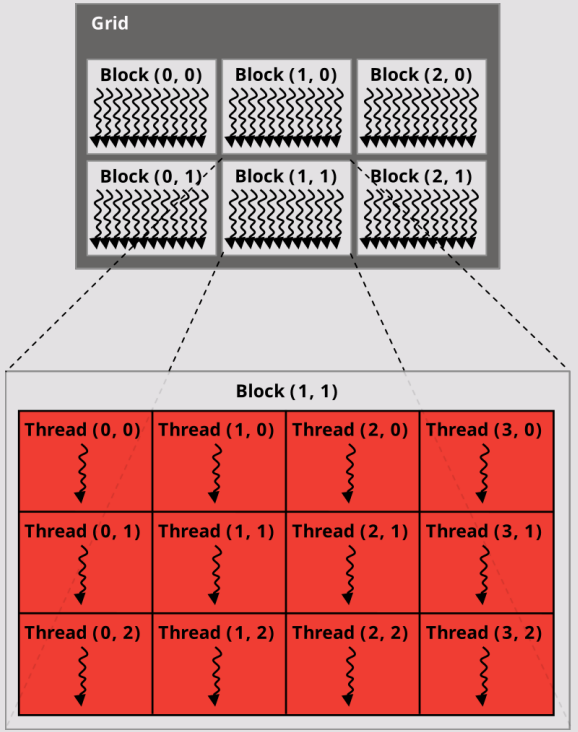
- Strategy not built-in in the compiler
- Works for arbitrary dimensions
- Easy to extend and reuse existing strategies

PARALLELISM STRATEGY

How to best exploit parallelism in the hardware



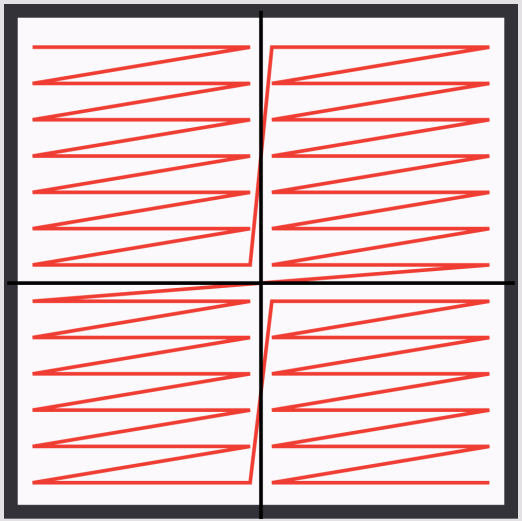
Application



GPU Parallelism Model

PARALLELISM STRATEGY

How to best exploit parallelism in the hardware



Application

Futhark @ PPoPP'19

Incremental Flattening for Nested Data Parallelism

Troels Henriksen
University of Copenhagen
athas@sigkill.dk

Frederik Thorøe
University of Copenhagen
wkj532@alumni.ku.dk

Martin Elsman
University of Copenhagen
mael@di.ku.dk

Cosmin Oancea
University of Copenhagen
cosmin.oancea@diku.dk

Abstract

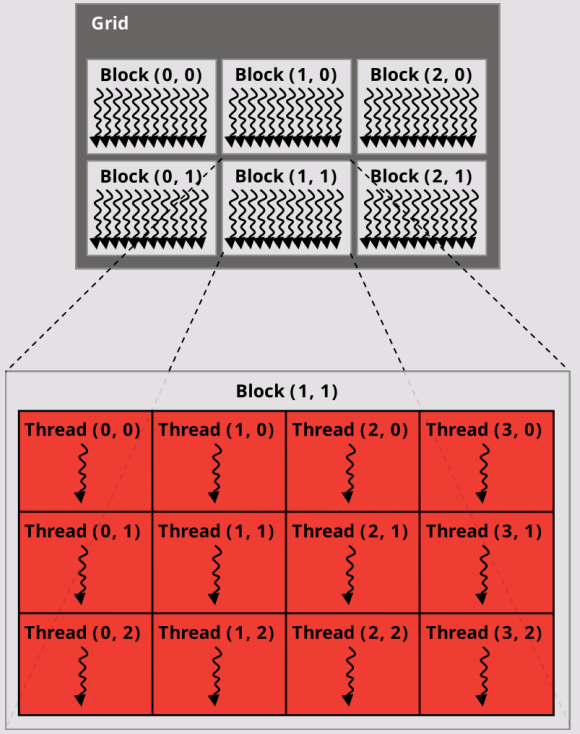
Compilation techniques for nested-parallel applications that can adapt to hardware and dataset characteristics are vital for unlocking the power of modern hardware. This paper proposes such a technique, which builds on flattening and is applied in the context of a functional data-parallel language. Our solution uses the degree of utilised parallelism as the driver for generating a multitude of code versions, which together cover all possible mappings of the application's regular nested parallelism to the levels of parallelism supported by the hardware. The versions are combined into a single program by guarding them with predicates, whose threshold values are automatically tuned to hardware and dataset characteristics. Our unsupervised method—of statically clustering datasets to code versions—is different from autotuning work that typically searches for the combination of code transformations producing a single version, best suited for a specific dataset or on average for all datasets.

We demonstrate—by fully integrating our technique in the repertoire of a compiler for the Futhark programming language—significant performance gains on two GPUs for three real-world applications, from the financial domain, and for six Rodinia benchmarks.

CCS Concepts • **Computing methodologies** → **Parallel programming languages**; • **Software and its engineering** → **Source code generation**; *Software performance*;

Keywords functional language, parallel, compilers, GPGPU.

Goal:
Minimize parallelism while still fully utilize hardware



GPU Parallelism Model

PARALLELISM STRATEGY

let's define Halide's .tiling

Elevate:

```
workgroupMapping: Strategy
workgroupMapping = λp . (
  (applyOutermost (mapWrg 1)) ;
  (applyOutermost (mapWrg 0)) ;
  (applyOutermost (mapLcl 1)) ;
  (applyOutermost (mapLcl 0)) ;
  (normalize mapSeq)) p
```

applyOutermost: Strategy → Strategy
applyOutermost = λs . λp .
apply *s* (FindFirst DFS (*isDefined s*)) p

Lift:

J ∘ **J ∘ *T ∘
****f ∘
*T ∘ **S ∘ S

OpenCL:

```
for(int i = 0; i < M; i++) {
 for(int j = 0; j < N; j++) {
  for(int ii = 0; ii < s; ii++) {
   for(int jj = 0; jj < s; jj++) {
          // f
}}}}
```

Lift abbreviations: * = map | **S** = split(s) | **J** = join | **T** = transpose

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*********f* **∘**
***T ∘ **S ∘ S**

(mapWrg i) = *map f* → $mapWrg_i\ f$
(mapLcl i) = *map f* → $mapLcl_i\ f$

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f must write to memory

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J ∘ **J ∘ *T ∘
map(map(
map(map *f*))) ∘
***T ∘ **S ∘ S**

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Lift:

$J \circ {}^{**}J \circ {}^{*}T \circ mapWrg_1(map(map(map\ f))) \circ {}^{*}T \circ {}^{**}S \circ S$

(mapWrg i) = *map f* → $mapWrg_i\ f$
(mapLcl i) = *map f* → $mapLcl_i\ f$

OpenCL:

```
int i = get_group_id(1);
 for(int j = 0; j < N; j++) {
  for(int ii = 0; ii < s; ii++) {
   for(int jj = 0; jj < s; jj++) {
          // f
}}}}
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(mapWrg i) = *map f* → $mapWrg_i\ f$
(mapLcl i) = *map f* → $mapLcl_i\ f$

OpenCL:

```
int i = get_group_id(1);
 int j = get_group_id(0);
  int ii = get_local_id(1);
   int jj = get_local_id(0);
          // f
}}}}
```

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```

Lift:

map(map f))

OpenCL:

```
for(int i = 0; i < M; i++) {
  for(int j = 0; j < N; j++) {
          out[i][j] = f(in[i][j]);
}}
```

What if we applied this strategy to our non-tiled Lit program?

Lift abbreviations: * = map | **S** = split(s) | **J** = join | **T** = transpose

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!

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  for(int j = 0; j < N; j++) {
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What if we applied this strategy to our non-tiled Lit program?

Lift abbreviations: * = map | **S** = split(s) | **J** = join | **T** = transpose

PARALLELISM STRATEGY

let's define Halide's .tiling

Elevate:

```
globalMapping: Strategy
globalMapping = λp . (
  (applyOutermost (mapGlb 1)) ;
  (applyOutermost (mapGlb 0)) ;
  (normalize mapSeq)) p
```

Lift:

$mapGlb_1($
$mapGlb_0\ f))$

OpenCL:

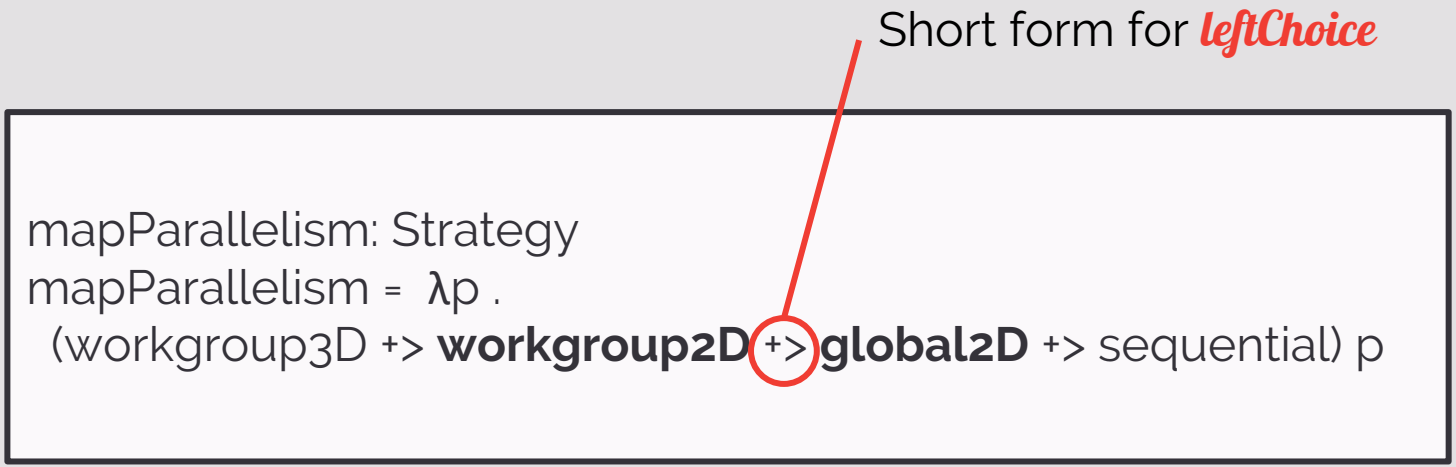
```
int i = get_global_id(1);
 int j = get_global_id(0);
   out[i][j] = f(in[i][j]);
}}
```

What if we applied this strategy to our non-tiled Lit program?

Lift abbreviations: * = map | **S** = split(s) | **J** = join | **T** = transpose

PARALLELISM STRATEGY

exploiting intentional failing of strategies



Trying to ***exploit all available parallelism*** and ***gradually fall back*** to strategies which make use of less parallelism

Achieves the same goal as Futhark's incremental flattening

ELEVATE

Specifying Compiler Optimizations:

- ***Principled:***
  One principled way to understand, write and apply compiler optimizations as Strategies
- ***Extensible:***
  not be fix and built-in.
  let programmers define abstractions and inject domain & expert knowledge

One Language - Many Optimizations:

Controlling different categories of optimizations with the same language

Algorithmic	*Hardware-specific*
Data-Layout	Memory
Computation	Parallelism