

Dynamic Time-Dependent Routing in Road Networks Through Sampling

Feb 2020
Motahhareh Nadimi



Abstract

Studying the earliest arrival in road networks with:

- **TIME DEPENDENT** functions
- **DYNAMIC UPDATES**
- It compute with **SMALL ERROR**
- Doesn't suffer from **MEMORY LACK** on large instances
- This algorithm is the only that is able to answer to queries below **50 ML**

Introduction

The road network as a weighted graph, directed graph :

Nodes: positions

Edges : road segments

Weight : travel time a car need to traverse the road segments .

A common assumption : travel times are time independent

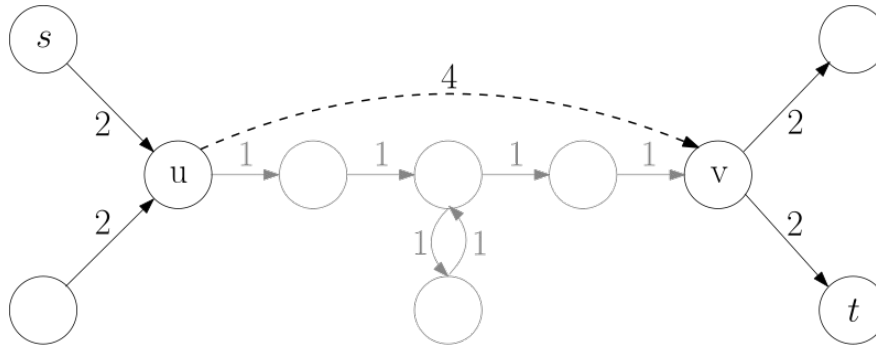
Result: problem will be a classical shortest path problem

Problem : road networks are huge and Dijkstra algorithm is slow!

Solution: contraction hierarchy (CH)

contraction Hierarchies

Contraction hierarchies (CHs) are a well-known algorithm for fast and exact route planning on road networks with constant travel costs



To find a path from s to t the algorithm can skip over the Grey vertices and use the dashed shortcut instead. This reduces the number of vertices the algorithm has to look at.

contraction Hierarchies:

- skip over unimportant vertices
- road networks are highly hierarchical. for example highway , are "more important" and higher up in the hierarchy than for example a junction leading into a dead end. the algorithm doesn't have to consider the full path at query time.

PROBLEM DEFINITION

The input of earliest problem:

NODES $> S$ AND T

Departure time $> s$

The task is consist of computing **ST- path with minimum arrival time.**

We want a routing system that accounts for **predicted and real time congestion**
(dynamic time dependent routing)

TD-S : Solve the earliest arrival problem with predicted congestion

TD-S+P : solve profile problem

TD-S+D: take real time congestion into account

Computing Errors

When taking predicted congestion into account, many proposed algorithms compute paths with an error

A shortest path with respect to the prediction is not necessarily shortest with respect to reality.

Let **d** denote the length of the computed path and **dopt** the length of a shortest path

Absolute error: $|d - dopt|$

Relative error : $|d - dopt| / dopt.$

A solution to the time-independent routing problem

Free flow Heuristic

assumption

No Congestion

The min value of e 's travel
time function
of f_e

Step

Find the shortest
time independent
path H

Compute **time** dependent
travel time along
H given departure time

The first step can be fasten using any time independent speed up: such as CH

Avg flow heuristic : the same but it use average travel time

Free flow Heuristic

Sampling means the variety of departure time in free flow

TD-S

Free flow never reroutes based on the current traffic situation

Steps

1

Compute sub graph H

2

Run time dependent extension of Dijkstra algorithm on sub graph

Compute the sub graph using a sampling approach

TD-S

| | |
|---|--|
| 1 | Define a constant number of K time intervals Better $k \leq 10$ |
| 2 | For every interval : obtain a time – independent graph ! |
| 3 | within each time intervals average the time dependent travel times. |

IN STEP 3, THEY USE A SAMPLING APPROACH,
AVG THE TIME DEPENDENT TRAVEL TIME WITHIN EACH INTERVAL
FOR EX: 7:10, 7:15 , 7:50

TD-S

4

For every interval : obtain a time – independent **graph** !

It s time independent
Because we calculate avg

5

For every graph we compute a shortest time independent **path**

6

The **union** of these paths is the sub graph H

They use Dijkstra here !

TD-S

**They compute for each time-interval
a time-independent CH**

The first step of TD+S executes k CH-queries.

Computing profile: TD-S+P

Steps

1

Compute
sub graph H

Just for example
For a sampling
rate 10 minute

2

Sample at regular intervals the travel time by
running the time-dependent extension of Dijkstra's
algorithm restricted to the sub graph H.

The sub graph computation step
is the same as for TD-S.

For a sampling rate of 10min,

- 1- algorithm first computes the sub graph H ,
- 2- then runs Dijkstra's algorithm with the departure times 0:00, 0:10, 0:20...23:50 restricted to H .

Denote by $a_1, a_2, a_3 \dots a_{144}$ the computed arrival times.

For example, the computed arrival time for departure time 0:07 is $0.3 a_1 + 0.7 a_2$

TD-S+P

TD-S+P is **faster** than iteratively running TD-S as the sub graph is only computed once.

Further, Dijkstra's algorithm iteratively runs on the same small sub graph H.

The first run loads H into the cache and thus all subsequent runs incur nearly no cache misses

TD-S+P

Dynamic TD-Routing Through Sampling

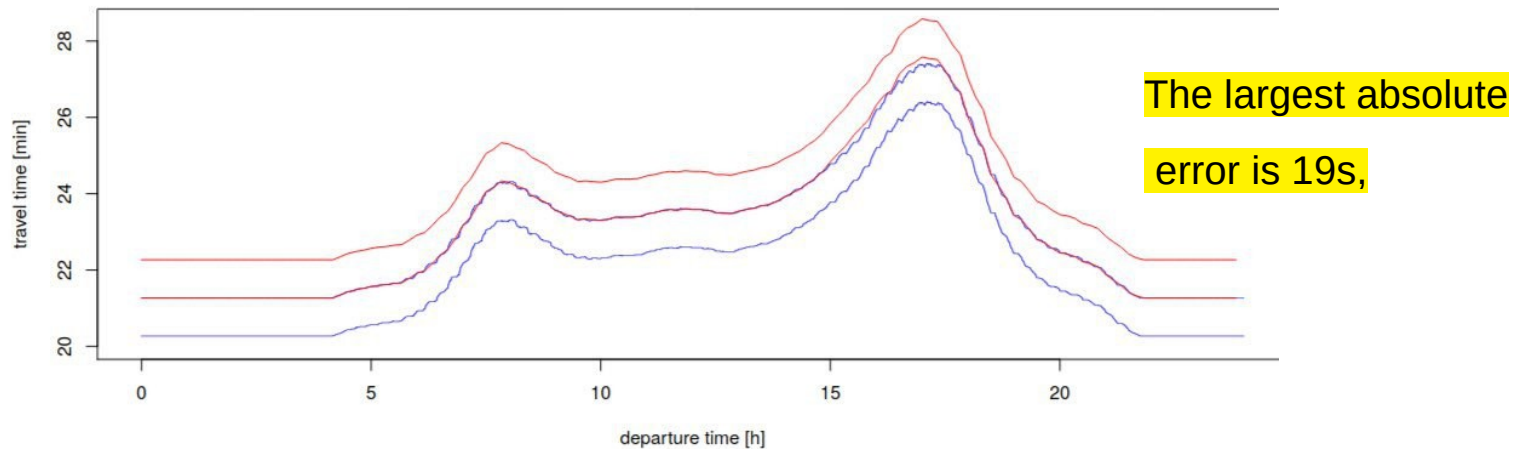


Figure 2

Example profile over 24h. The red curve (top) was computed with TD-S+P, while the blue one (bottom) is the exact solution. The middle overlapping curves are the actual profiles. To improve readability

TD-S+D

However, in many applications we must also take real time congestion into account.

1- Adapting TD-S by modifying the computation of the sub graph H yielding TD-S+D.

2-The second step is left unchanged

TD-S+D

The sub graph H is the union of k paths

TD-S+D adds a shortest st -path according to the current real time traffic as $k+1$ -th path to the union.

A efficient solution to the routing problem with real time congestion is needed. They use CCH algorithm .

TD-S+D

In the pre processing step, TD-S+D computes a CCH in addition to the k CHs of TD-S.

At regular time intervals, such as every 10s, TD-S+D updates the CCH edge weights to reflect the real time traffic situation

A TD-S+D query consists of running k CH queries and one CCH query.

The sub graph H is the union of the $k+1$ shortest paths. Finally, Dijkstra's algorithm is run to the sub graph H .

Simulating Traffic

Unfortunately, they do not have access to good measured real time traffic. they **simulate real time congestion** to study the performance of TD-S+D

Simulating Traffic

- For an earliest arrival time query from s to t with departure time τ , we **first compute the shortest time-dependent path P** with respect to the historic travel times
- On P we **generate three traffic congestion** by picking three random start edges.
- **From each of these edges we follow P for 4min**, yielding three sub paths

- 1- Travel time function f of e
- 2- by doubling the travel time at $f(\tau)$ and assume that it remains constant for some time
- 3- The congestion should be gone at $\tau + 1$ h.
- 4- FIFO-property: a modified function must have slope of -1 before $\tau + 1$ before joining the predicted travel time
- 5- ON THIS $2f(\tau) < f(\tau + 1)$, we do not generate a congestion.

Simulating Traffic

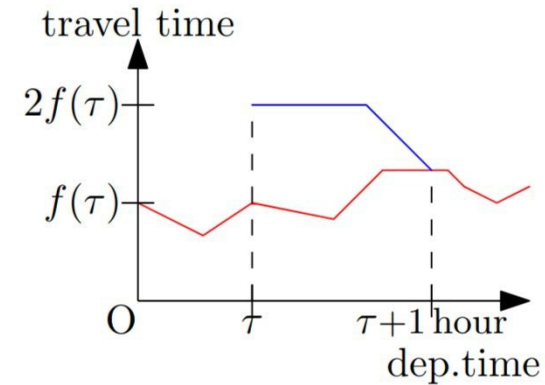


Table 2 Number of exact time-dependent queries and absolute and relative errors for Freeflow, TD-S+4, and TD-S+9. “Q99” refers to the 99%-quantile and “Q99.9” the 99.9%-quantile.

| Graph | Algo | Exact [%] | Relative Error [%] | | | | Absolute Error [s] | | | |
|-------|----------|-----------|--------------------|------|-------|------|--------------------|-----|-------|-------|
| | | | Avg | Q99 | Q99.9 | Max | Avg | Q99 | Q99.9 | Max |
| Lux | Freeflow | 80.0 | 0.244 | 5.1 | 11.5 | 28.1 | 5.0 | 106 | 235 | 356 |
| Lux | Avgflow | 81.0 | 0.123 | 2.5 | 6.4 | 19.4 | 2.5 | 49 | 143 | 329 |
| Lux | TD-S+4 | 97.7 | 0.008 | 0.2 | 1.5 | 4.9 | 0.2 | 4 | 30 | 141 |
| Lux | TD-S+9 | 99.6 | <0.001 | 0.0 | 0.1 | 1.7 | <0.1 | 0 | 3 | 27 |
| Ger | Freeflow | 67.9 | 0.085 | 1.5 | 3.1 | 12.4 | 11.1 | 200 | 417 | 825 |
| Ger | Avgflow | 69.2 | 0.044 | 0.8 | 1.9 | 10.3 | 5.9 | 113 | 284 | 587 |
| Ger | TD-S+4 | 94.6 | 0.005 | 0.1 | 1.0 | 3.0 | 0.8 | 17 | 159 | 474 |
| Ger | TD-S+9 | 98.2 | 0.001 | <0.1 | 0.4 | 3.0 | 0.3 | 1 | 76 | 374 |
| OGer | Freeflow | 60.7 | 0.140 | 2.0 | 4.7 | 12.4 | 15.9 | 219 | 465 | 1 104 |
| OGer | Avgflow | 68.8 | 0.050 | 0.9 | 2.2 | 6.5 | 5.7 | 96 | 227 | 619 |
| OGer | TD-S+4 | 96.4 | 0.002 | 0.1 | 0.4 | 2.0 | 0.3 | 6 | 47 | 333 |
| OGer | TD-S+9 | 98.5 | 0.001 | <0.1 | 0.2 | 2.0 | 0.1 | 1 | 24 | 276 |
| CEur | Freeflow | 54.9 | 0.089 | 1.4 | 2.7 | 10.8 | 26.4 | 428 | 833 | 1 477 |
| CEur | Avgflow | 55.8 | 0.048 | 0.8 | 1.7 | 6.6 | 14.2 | 235 | 507 | 1 069 |
| CEur | TD-S+4 | 91.1 | 0.006 | 0.2 | 0.7 | 3.8 | 1.8 | 47 | 226 | 547 |
| CEur | TD-S+9 | 96.8 | 0.001 | <0.1 | 0.3 | 1.2 | 0.5 | 6 | 109 | 397 |

The number of exactly solved queries decreases with instance size as the paths lengths grow with size

as most queries are answered exactly.

TD-S+4 has larger errors as it uses fewer time windows.

the query running times and the memory footprint of TD-S+4 are lower.

Analysis

TD-S with two selections of time windows:

TD-S+4 uses the windows 0:00–5:00, 6:00–9:00, 11:00–14:00, and 16:00–19:0

TD-S+9 uses the windows 0:00–4:00, 5:50–6:10, 6:50–7:10, 7:50– 8:10, 10:00–12:00, 12:00–14:00, 16:00–17:00, 17:00–18:00

the query running times
and the memory footprint
of TD-S+4 are lower.

TCH is lacking from
memory .they couldn't
run on 128 gig

TD-S+4 only needs
about 2.4 times the
memory required by the
input. TD-S+9 needs 4.1
times the memory.

TD-S+4 has an
about a factor 10
lower pre
processing time
than TCH

■ **Table 3** Average preprocessing and running times and memory consumption of various algorithms.

| Graph | TD-Dijkstra | Freeflow | Avgflow | TD-S+4 | TD-S+9 | TCH |
|--|-------------|----------|---------|--------|--------|----------|
| Average Query Running Time [ms] | | | | | | |
| Lux | 4 | 0.02 | 0.02 | 0.11 | 0.26 | 0.18 |
| Ger | 1 116 | 0.19 | 0.20 | 0.99 | 3.28 | 1.81 |
| OGer | 813 | 0.12 | 0.14 | 0.97 | 2.09 | 1.12 |
| CEur | 4 440 | 0.42 | 0.29 | 3.83 | 6.85 | OOM |
| Max. Query Memory [MiB] | | | | | | |
| Lux | 13 | 17 | 17 | 29 | 47 | 328 |
| Ger | 1 550 | 2 132 | 2 130 | 3 630 | 6 127 | 42 857 |
| OGer | 461 | 855 | 854 | 1 880 | 3 589 | 8 153 |
| CEur | 4 980 | 7 058 | 7 053 | 12 411 | 21 336 | >131 072 |
| Total Preprocessing Running Time [min] | | | | | | |
| Lux | — | <0.1 | <0.1 | <0.1 | 0.1 | 0.6 |
| Ger | — | 1.6 | 2.2 | 7.6 | 14.7 | 86.2 |
| OGer | — | 1.5 | 1.6 | 5.9 | 16.4 | 26.8 |
| CEur | — | 8.6 | 8.1 | 33.9 | 70.7 | 381.4 |

On the Luxembourg instance only 1.6% of the queries are solved optimally.

TD-S+D reduces these errors. Over all instances, the minimum number of optimally solved queries is 92.9%.

| Graph | Algo | Exact [%] | Relative Error [%] | | | | Absolute Error [s] | | | |
|-------|-----------|-----------|--------------------|------|-------|------|--------------------|-----|-------|------|
| | | | Avg | Q99 | Q99.9 | Max | Avg | Q99 | Q99.9 | Max |
| Lux | Predict.P | 1.6 | 17.228 | 56.1 | 75.0 | 93.8 | 323.0 | 739 | 826 | 997 |
| Lux | TD-S+D4 | 94.7 | 0.017 | 0.5 | 2.3 | 6.2 | 0.6 | 15 | 93 | 231 |
| Lux | TD-S+D9 | 95.0 | 0.016 | 0.5 | 2.2 | 6.2 | 0.5 | 14 | 89 | 231 |
| Ger | Predict.P | 55.1 | 1.2 | 17.9 | 36.9 | 79.3 | 78.5 | 552 | 741 | 1001 |
| Ger | TD-S+D4 | 90.9 | 0.032 | 1.0 | 2.6 | 7.0 | 3.5 | 116 | 233 | 474 |
| Ger | TD-S+D9 | 93.4 | 0.026 | 0.9 | 2.5 | 6.2 | 2.8 | 99 | 216 | 469 |
| OGer | Predict.P | 52.3 | 1.352 | 18.7 | 38.3 | 65.8 | 84.9 | 563 | 738 | 934 |
| OGer | TD-S+D4 | 91.5 | 0.031 | 1.0 | 2.6 | 5.4 | 3.2 | 108 | 224 | 462 |
| OGer | TD-S+D9 | 92.9 | 0.028 | 0.9 | 2.5 | 5.4 | 2.9 | 102 | 219 | 462 |
| CEur | Predict.P | 72.6 | 0.392 | 7.0 | 25.9 | 81.9 | 41.0 | 443 | 653 | 1870 |
| CEur | TD-S+D4 | 89.5 | 0.015 | 0.5 | 1.6 | 5.2 | 3.3 | 106 | 244 | 547 |
| CEur | TD-S+D9 | 94.0 | 0.011 | 0.3 | 1.4 | 5.2 | 1.9 | 69 | 205 | 397 |

Table 6 Number of exact dynamic, time-dependent queries and absolute and relative errors for the predicted path, TD-S+D4, and TD-S+D9.

Dynamic Time-Dependent Routing

In the dynamic scenario, we consider two types of congestion:

- (a) the predicted congestion.
- (b) the real time congestion.

The Predicted Path heuristic (Predict. P) as baseline, TD-S+D4, and TD- S+D9.

The Predicted Path heuristic computes a shortest path P with respect to only the predicted congestion NOT REAL TIME CONGESTION

P then is then evaluated with respect to both congestion types TD-S+D4, and TD-S+D9.

Predicted Path P

Free flow and Predicted Path are similar in spirit.

Free flow solves the time-dependent routing problem by ignoring predicted congestion.

Similarly, Predicted Path solves the dynamic time-dependent routing problem by ignoring real time congestion.

The Free flow heuristic produces surprisingly small errors

Conclusion

Introducing TD-S, a simple and efficient solution to the earliest arrival problem with predicted congestion on road graphs.

We extend it to TD-S+P which is the only algorithm to solve the profile variant in at most 50ms on all test instances.

Further, we demonstrated with TD-S+D that additional real time congestion can easily be incorporated into TD-S.