Introduction to Data Science

Group Assignment 1

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Exercise 1: Prove Lemma 1.14

Given a probability triple $(\Omega, \mathcal{F}, \mathbb{P})$ then for $A \in \mathcal{F}$ with $\mathbb{P}(A) \neq 0$,

$$P(\cdot|A) : \mathcal{F} \to [0,1]$$

is a probability measure as in Definition 1.10 over $(\Omega, \mathcal{F})$.

We need to show that

$$P(\cdot|A) : \mathcal{F} \to [0,1] \tag{1}$$

satisfies the following conditions:

1. $\mathbb{P}(A|\Omega) = 1$, and

State what the conditions are called: axiom 1 “something ha

2. for $B, C \in \mathcal{F}$: $B \cap C = \varnothing \Rightarrow \mathbb{P}(A \cup B) = \mathbb{P}(B|A) + P(C|A)$.

For the first condition, we have:

$$\mathbb{P}(A|\Omega) = \frac{\mathbb{P}(\Omega \cap A)}{\mathbb{P}(A)} = \frac{\mathbb{P}(A)}{\mathbb{P}(A)} = 1\,. \tag{2}$$

For the second condition, if $B, C \in \mathcal{F}$ and are mutually exclusive, i.e. $B \cap C = \varnothing$, then, using the fact that (P) is a probability, we have

$$\begin{aligned}
\mathbb{P}(B \cup C) &= \frac{\mathbb{P}((B \cup C) \cap A)}{\mathbb{P}(A)} \\
&= \frac{\mathbb{P}((B \cap A) \cup (C \cap A))}{\mathbb{P}(A)} \\
&= \frac{\mathbb{P}((B \cap A)) + \mathbb{P}((C \cap A))}{\mathbb{P}(A)} \\
&= \mathbb{P}(B|A) + \mathbb{P}(C|A)\,.
\end{aligned} \tag{3}$$

Explain why the two sides are equal (c

Extra sets of parenthesis in this line

Explain why they are equal (c

Therefore, it is a probability measure.

Explain why they are equal (c

Exercise 1: Prove Lemma 2.8

Given a probability triple $(\Omega, \mathcal{F}, \mathbb{P})$ and an event $A \in \mathcal{F}$, the following properties hold:

1. $\mathbb{1}_A = 1 - \mathbb{1}_{A^c}$, *(complementation behaves like the probability)*
2. $\mathbb{1}_{A\cap B} = \mathbb{1}_A \mathbb{1}_B$ *(intersection becomes product)*
3. $\mathbb{1}_{A\cup B} = \mathbb{1}_A + \mathbb{1}_B - \mathbb{1}_A \mathbb{1}_B$ *(union becomes addition–intersection)*

For property 1 we have:

1. If $a \in A$ then $\mathbb{1}_A = 1$ and $\mathbb{1}_{A^c} = 0$, therefore, it follows that $\mathbb{1}_A = 1 = 1 - 0 = 1 - \mathbb{1}_{A^c}$.
2. If $a \in A^c$ then $\mathbb{1}_A = 0$ and $\mathbb{1}_{A^c} = 1$, therefore, it follows that $\mathbb{1}_{A^c} = 1 = 1 - 0 = 1 - \mathbb{1}_A \Leftrightarrow \mathbb{1}_A = 1 - \mathbb{1}_{A^c}$.

For property 2, if $a \in A \cap B$ we have that $\mathbb{1}_{A\cap B} = 1$. Therefore,

$$\begin{aligned} \mathbb{1}_{A\cap B} = 1 &\Leftrightarrow a \in A \cap B \\ &\Leftrightarrow a \in A \text{ and } a \cap B \\ &\Leftrightarrow \mathbb{1}_A \mathbb{1}_B \,. \end{aligned} \tag{4}$$

For property 3, if $a \in A \cup B$ we have that $a \in A$ or $a \in B$, which means $\{\mathbb{1}_A = 1, \mathbb{1}_B = 0\}$ or $\{\mathbb{1}_A = 0, \mathbb{1}_B = 1\}$, but not both at the same time. Also, we have that $A \cup B = A \cup (A^c \cap B)$ and $B = (A \cap B) \cup (A^c \cap B)$. Therefore,

$$\mathbb{1}_{A\cup B} = \mathbb{1}_{A\cup(A^c\cap B)} = \mathbb{1}_A + \mathbb{1}_{A^c\cap B} = \mathbb{1}_A + \mathbb{1}_B - \mathbb{1}_A \mathbb{1}_B \,, \tag{5}$$

where

$$\mathbb{1}_B = \mathbb{1}_{(A\cap B)\cup(A^c\cap B)} = \mathbb{1}_{(A\cap B)} + \mathbb{1}_{(A^c\cap B)} \Leftrightarrow \mathbb{1}_{(A^c\cap B)} = \mathbb{1}_B - \mathbb{1}_{(A\cap B)} \tag{6}$$

Missing:Proof property 4 of Theorem 2.18Solution Exercise 2.59Proof “tower pro