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FACULTAT D'INFORMÀTICA DE BARCELONA

GEP — Deliverable 2

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# Analysis of the SVM-RFE algorithm for feature selection

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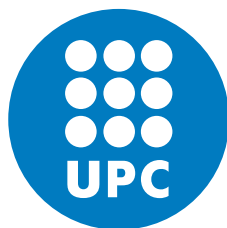
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## Chapter 1

# Introduction

This bachelor thesis of the Computer Engineering Degree, specialization in Computing, has been done in the Facultat d'Informàtica de Barcelona of the Universitat Politècnica de Catalunya and directed by Luis Antonio Belanche Muñoz, doctorate in Computer Science.

### 1.1 Context

In statistics, machine-learning, data-mining, and other related disciplines, it is often the case that there is redundant or irrelevant data in a dataset<sup>1</sup>. Indeed, before we can start working with the data, some form of data analysis and cleaning is required. Data cleaning may include removing duplicated rows or rows with missing values, removing observations that are clearly out-layers, removing irrelevant variables (e.g. name, surname, email address), etc.

With the new era of Big Data, datasets have increased in size, both in number of observations and in dimensions. Applying classical data-mining and machine-learning algorithms to this high-dimensional data rises multiple issues collectively known as “the curse of dimensionality”. One such issue is the elevated, usually intractable, cost and memory requirements derived from the non-linear (on number of dimensions) complexities of the algorithms. Another issue has to do with data in a high-dimensional space becoming sparse and negatively affecting the performance of algorithms designed to work in a low-dimensional space. And finally, a third issue is that with a high number of dimensions the algorithms tend to overfit, that is, they don't generalize enough and end up producing models that perform worse with real data than their predicted performance with the training data. (Li et al., 2017)

Simple manual data cleaning is not enough to achieve satisfactory amounts of dimensionality reduction. In this case we can use automatic techniques. We can classify such techniques in two categories, feature extraction, and feature selection.

#### 1.1.1 Feature Extraction

Feature extraction techniques transform the original high-dimensional space into a new low-dimensional space by extracting or deriving information from the original features. The premise is to compress the data in order to pack the same information at the expense of model explainability<sup>2</sup>. Continuing with our data compression analogy, virtually all feature extraction techniques perform lossy compression, that

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<sup>1</sup>A table with rows / records / observations and columns / variables / features / dimensions / predictors / attributes.

<sup>2</sup>The ability to explain why certain predictions are made. Also, interpretability.

is, some data is lost which makes the process irreversible. Notice that, if the process was reversible then feature extraction would not decrement explainability.

Some well known feature extraction algorithms include Principal Component Analysis (PCA) and auto-encoders, the first being a linear transformation over the feature-space and the second a neuronal network. PCA may be extended with a kernel method in order to make non-linear transformations. Similarly, we will also use kernels methods to extend SVM-RFE.

### 1.1.2 Feature Selection

In contrast, feature selection only selects a subset of the existing features, ideally the most relevant or useful. This may imply a greater loss of information compared to feature extraction, but it doesn't reduce explainability. Some problems require feature selection explicitly. In domains such as genetic analysis and text mining, feature selection is not necessarily used to build predictors. For example in micro-array analysis feature selection is used to identify genes (i.e. features) that discriminate between healthy and disease patients.

Feature selection methods may be classified by how they are constructed in three categories:

- **Filter:** A *feature ranking* criteria is used to sort the features in order of relevance, then select the  $k$ -most relevant.
- **Wrapper:** They use a learning machine (treated as a black box) to train and test the dataset with different subsets of variables. They rank the subsets based on the performance (score) of the model. A wide range of learning machines and search strategies can be used. Within the greedy strategies we find *forward selection* and *backward elimination*.
- **Embedded:** Like wrapper methods but more efficient. They use information from the trained model itself to make feature selection. Because they don't use the score, they can also skip testing the model.

In both wrapper and embedded methods greedy strategies can be used. **SVM-RFE** is a feature selection algorithm, of the embedded class, that uses Support Vector Machines (SVM) and a greedy strategy called Recursive Feature Elimination (RFE). It takes advantage of the fact that, for linear SVM, the variable with the smallest weight in each iteration is the one that needs to be eliminated. (Guyon and Elisseeff, 2003)

## 1.2 State of the art

The SVM-RFE algorithm was first proposed in a paper on the topic of cancer classification (Guyon et al., 2002).

*Stuff about how SVM-RFE has been used on cancer classification. Stuff about known and natural extensions of SVM-RFE. Maybe other interesting applications of SVM-RFE.*

*Justification, Identify the problem*

## 1.3 Objective

The main objective of this project is to research variants of the SVM-RFE algorithm and try to optimize it. Optimizations may be in the form of improved performance, a

reduction in time utilization or fewer memory required. We've classified the possible optimizations by the following categories:

- Optimizations that try to use the SVM more effectively. This includes exploring non-linear kernels and other values for the regularization parameter.
- Extensions to the RFE strategy. This may include reusing more information from previous iterations, sampling, etc.

### 1.3.1 Objective break down

To accomplish this objective, the project has been subdivided in two parts:

#### Theoretical Part

- Do research in SVM-RFE and in the methods that will be tackled in the project.
- For each method:
  - Design the algorithm and write its formalization in pseudocode.
  - Define the expected advantages or disadvantages of this method over the base SVM-RFE.
  - Compute the space and time complexities.

#### Practical Part

- Program the methods studied and the SVM-RFE extensions.
- For each method:
  - Analyze its behavior for artificial data sets.
  - Analyze its behavior for real-world data sets.
- Compare the results obtained with the ones computed in the theoretical part.
- Draw conclusions about all the results obtained in the project.

### 1.3.2 Stakeholders

This project is intended to be of use for many involved parties. The most directly involved group, is the tutor and the researcher. Luis Antonio Belanche Muñoz is the tutor of this project. Robert Planas Jimenez would be the researcher. Feature selection algorithms is one of the areas of research of the tutor, and he has wanted to explore extensions to the SVM-RFE algorithm. Thus, he will lead and guide the researcher for the correct development of the project. The researcher is responsible for planning, developing and documenting the project, as well as experimenting, analyzing and drawing conclusions.

The other group of interested parties would be stakeholders that do not interact with the project directly but still benefit from it. In the first place we have researchers on the fields of bioinformatics and data mining, that use machine learning methods (specifically, SVM-RFE) for analyzing micro-array analysis, text analysis, or other of its popular applications. Indirectly companies that make use of their findings will also benefit, and finally the general population may also benefit from better diagnostics and more effective drugs.

### 1.3.3 Potential obstacles and risks

Some of the obstacles and risks identified that could potentially prevent the correct execution of the project are:

- **Deadline of the project:** There is a deadline for the delivery of the project. This being a research project however, is considerably hard to estimate how much time tasks will take, or even decide whether a task has been finished or not.
- **Bugs on some libraries:** This is considered of low risk, but is still a possibility that errors on the software package used extend to code, making it work incorrectly.
- **Insufficient computational power:** Machine learning algorithms in general can be very resource intensive. It could be the case that our hardware can not handle some datasets.
- **Hardware related issues:** A hard drive failure could occur that would end in lost data, or a failure in a router could disconnect us from the internet.
- **Health related issues:** In addition to health issues that can occur at any time without prior notice, we're in the middle of a pandemic.

## 1.4 Methodology

### 1.4.1 Framework

The methodology that I will use for the project is a combination of Waterfall and Kanban methodologies. Waterfall will be used to define the general phases of the project, and Kanban for tracking the individual tasks. In Waterfall tasks can not start until the previous task has been completed, and thus following strict deadlines is important. Phases will be managed like this, one phase will not start until the previous phase ends. Each phase will be composed of multiple tasks, which will then be managed by a different methodology. That will be Kanban.

Kanban is much more flexible than Waterfall, its principal objective is to manage tasks in a general way, by assigning different status to them. Kanban stands out by its simplicity, and we will continue with that simplicity by managing the visual representation of the cards in a simple plain text file, with tabulation separated columns. Each card will be in a row, with the first column defining its name and the other its status. The status I've considered are:

- **To do:** The task has been defined, but I have not started to work on it yet.
- **In progress:** The task has started and some progress has been made.
- **On hold:** The development of the task has been paused for some unforeseen reason.
- **Completed:** The task is finished.

### 1.4.2 Validation

We will use a GitHub repository as a tool for version control, which will allow us to share code easily and recover from data loss. The repository will contain both the code for the different experiments, each in one subfolder, and code for the documentation. In order to verify the implemented code, it will be passed through various tests to see whether the code works as expected. In the practical part, hyperparameters will be selected using some model validation technique, such as the cross-validation. Each experiment will be done 3 times and the average of the results will be the final result. Lastly, face-to-face meetings will be scheduled once every two weeks with the tutor of the project. In these meetings it will be discussed the project status and the tasks to do during the following two weeks, before the next meeting. In case of problems in the project, extraordinary meetings can be arranged.

## Chapter 2

# Project Planning

This thesis is worth 15 ECTS credits, each of which with an estimate cost of 25 to 30 hours. Therefore, the total time allocated for this project, as indicated by the faculty, is of 375 to 450 hours. This time is to be distributed in 100 days, from 03/08/21 to 06/15/21, with an estimated work of 4 to 5 hours a day. The date of the oral defense is planned for the first week of July, this sets the deadline to be on the 06/18/21.

An extra 3 ECTS credits are to be used for project management, this is roughly 80 hours, which makes the total time estimate for the whole project (Theisis + Project Management) to be at best 450 hours and at worse 540 hours. In order to make a proper planning, we have defined the estimated cost to be at 500 hours.

## 2.1 Task definition

In this section it is presented all the tasks that will be carried out along the project. For each, a description, duration and a list of dependencies with other tasks are given.

Project management is a mandatory group of such tasks, albeit not very useful, considering that: One, this project is done by a single individual, with assistance from a project director; And two, this is a research project, which makes planning of specific tasks difficult, since it is the results from the research that drive the next steps to be done.

- **Context and scope:** We have to indicate the general objective(s) of the project, contextualize it and justify the reason for selecting this subject area.
- **Project planning:** This will help us not lose focus while we're working on the project.
- **Budget and sustainability:** For this specific project, this is irrelevant. The budget required is negligible and already defrayed; and the impact, beyond trivial matters, is likely zero and otherwise unknown.
- **Final project definition:** Review the work done in the project management tasks.
- **Meetings:** Online meetings are scheduled once every two weeks with the tutor of the project. Discussion of the status and next tasks to do will be done.

This project is research focused. Therefore, before starting the practical tasks research on the various topics needs to be done. This will involve collecting and analyzing previous studies that tried new methods and extensions to the SVM-RFE algorithm. We will also have to document ourselves in the SVM and feature selection



areas, as well as the algorithms and the statistical theory used in the studies. Some basic understanding of bioinformatics may also be required considering the use case of most if these studies.

- **Research** previous work on the literature on extensions of the algorithm and create a short **report** with the findings.
- Write the algorithm formalization in pseudocode.
- Define the expected advantages or disadvantages of this method over the base SVM-RFE.
- Compute the space and time complexities.

Once the initial research is done the algorithm must be codified and tested. This group is composed of the following tasks:

- **Program the base SVM-RFE algorithm.** For this we will use a library for the SVM. For the RFE part, since we need to be able to extend the algorithm, we will program it from scratch.
- **Program the extensions of the SVM-RFE algorithm** based on the research done and the pseudocode.
- **Test the new extensions with artificial data.** This requires creating models and testing their performance with artificial data sets.
- **Obtain data sets with real data.** Multiple papers refer to publically available data-sets, we could use those and compare our results.
- **Test the new extensions with real data.** This requires creating models and testing their performance with real data sets.
- **Analyze the results** obtained in the experiments and draw conclusions. A report will be made for reference during the final documentation.

Once finished, the final documenting phase will begin. Firstly, we will collect all the information obtained in the experimental and analysis part, which will be available in the form of the reports that we've done along the tasks. Afterwards, we can start writing the documentation of the project. This will include a fairly extensive review on concepts related to SVM, statistics, feature selection and RFE. Finally, we will have to prepare for the oral defense of the project.

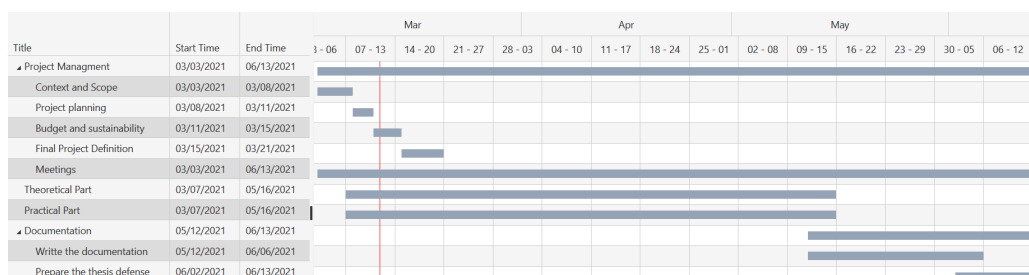


FIGURE 2.1: A summary of the tasks represented with a gantt chart. Notice that all the theoretical and practical tasks are done in parallel.

ID	Description	Hours	Dependencies
T1	Project Managment	80	
T1.1	Context and Scope	20	T2.1
T1.2	Project planning	10	
T1.3	Budget and sustainability	10	T1.2
T1.4	Final project definition	20	T1.1, T1.2, T1.3
T1.5	Meetings	20	
T2	Theoretical Part	160	
T2.1	Research	90	
T2.2	Formalize	20	T2.1
T2.3	Analyze	50	T2.2
T3	Practical Part	160	
T3.1	Program the base SVM-RFE algorithm	10	T2.1
T3.2	Program the extensions	50	T2.1, T3.1
T3.3	Test with artificial data	20	T3.2
T3.4	Test with real data	30	T3.2
T3.5	Analyze the results	50	T3.3, T3.4
T4	Documentation	100	
T4.1	Writte the documentation	80	T2, T3
T4.2	Prepare the thesis defense	20	T4.1

TABLE 2.1: Summary and time estimates of the tasks.

## 2.2 Resources

Our project needs resources to carry out its correct development. These resources have been divided in 4 different groups: human, hardware, software and material resources.

### 2.2.1 Human resources

There are three human resources that are directly involved in this project.

- **The researcher:** He is responsible for the development of the project, that is, he will have to plan, analyze, program, experiment and document the project.
- **The director/tutor:** He is responsible for leading and guiding the researcher for the correct development of the project.
- **The GEP tutor:** He is in charge of reviewing the project management tasks done in the initial stage of the project.

### 2.2.2 Hardware Resources

The most essential resource needed is a computer connected to the internet. In this project a personal computer will be used. Its specializations are 16 GB of RAM and a CPU *AMD Ryzen 7 4800HS*, with a base speed of 2.9 GHz and 8 cores. Hardware required for a connection to the internet (a router, an access point, etc) is also taken into account.

### 2.2.3 Software Resources

For project management tasks Google Calendar will be used. A number of other Google products such as Google Mail, Google Drive or Google Meet will also be used as tools required for communication with the director or storing of information.

The documentation will be written in  $\text{\LaTeX}$ , a document preparation system often used in academia. It has the advantage to integrate well with version control systems. A  $\text{\LaTeX}$  template named "Masters/Doctoral Thesis" will be used to facilitate the typesetting and styling of the document. This template was made by Vel and Johannes Böttcher and licensed under LPPL 1.3, based on previous work from Steve R. Gun and Sunil Patel and used with minor modifications. The original is available at <http://www.latextemplates.com/>. The document will be written with the Microsoft Visual Studio Code editor, using the LaTeX Workshop extension, and it will be compiled with the MiKTeX distribution installed on a Linux machine running virtualized within a WSL container on top of the actual operating system, a Microsoft Windows 10. For browsing the internet the latest available version of the Mozilla Firefox web browser will be used, and references to papers will be kept with the Zotero reference manager.

For the practical part, the programming language of choice will be Python3. This is currently one of the programming languages with better popularity in machine learning and related applications. Various libraries and software packages will be used for different tasks. For parsing datasets the pandas library will be used. For data visualization the matplotlib library will be used. Also, a general tool-set designed for machine learning, the library sklearn, will be used. Finally, code will be documented in-place during its development with the Jupyter Notebook software.

Other libraries and software packages could also be used if the need arises.

## 2.3 Risk Management

The potential risks and obstacles have already been introduced in section 1.3.2. In this section we will focus on a contingency plan to mitigate the risks.

- **Deadline of the project:** The flexibility of the Kanban methodology should help us modify our schedule and working hours if required. If it becomes apparent that the deadline will not be met, an extension of the deadline can be requested.
- **Bugs on some libraries:** Alternative libraries can be used. If no alternative is found, because most of the used libraries are open source, a *bugfix* could be implemented.
- **Insufficient computational power:** This can be mitigated by using a small sample of the data-set. This though, can induce a small performance reduction, and thus make the results not comparable with each other.
- **Hardware related issues:** To avoid data loss, all code and documentation will be routinely uploaded to the cloud in a GitHub repository and Google Drive account.

## Chapter 3

# Budget and Sustainability

In this section a budget estimation will be made. It will include personnel costs per task, generic costs and other costs. Moreover, some questions regarding the sustainability aspect of the project will be answered.

### 3.1 Budget

#### 3.1.1 Costs by role and activity

In this section we will add the personnel costs to the tasks defined in section 2.1. For each task a cost will be calculated based on the hourly pay wage per role and the time spend each. Four roles have been defined with their corresponding hourly wage, these are:

- The **project manager** (T1) who is responsible for leading the project direction, planning and correct development.
- The **researcher** (T2), who must perform research in the topic and related ideas, experiment, analyze the results and draw conclusions from them.
- The **programmer** (T3), who must set up the developer environment, code the algorithms in the specified programming language and test them for correctness.
- The **technical writer** (T4), who is responsible for writing this project documentation. This includes the reports for each task that requires it and the final thesis.

These roles have a clear mapping to the task groups defined in table 2.1. All roles will be played by the researcher (see section 2.2.1) except the project manager role, which will be played by the researcher, the director and the GEP tutor. For a detailed description of each task cost see table 3.1.

Notice that, although we aligned the project roles with the task groups to simplify our calculations, it is not required to do so. A more complicated scheme where multiple tasks are assigned to a given role is also possible. In such cases we would also have to think about whether the work distribution is even or not, and assign percentages if it isn't.

Role	Year (€)	Year +SS (€)	Hour +SS (€)	Task	Task Cost (€)
Project Manager	39 000	50 700	28.7	T1	2 296
Researcher	32 000	41 600	23.5	T2	3 760
Programmer	26 000	33 800	19.1	T3	3 065
Technical writer	22 000	28 600	16.2	T4	1 260
<b>Total</b>					<b>10 381</b>

TABLE 3.1: Anual estimated salary for the different project roles (*Salarios, ingresos, cohesión social 2017*). The amount of working hours in a year is defined to be 1764. +SS indicates “Social Security included”.

### 3.1.2 Generic costs

#### Amortization

In this section the amortization costs for the resources purchased in a single payment are calculated. Notice that since all the software used is free and open source, it doesn't contribute to the cost, thus it is not displayed here. In fact, the only resource that is valid for an amortization analysis is the computer specified in section 2.2.2.

The equation we use to compute the amortization cost for each resource is the following:

$$\text{Amortization (€)} = \text{Cost (€)} \times \frac{1}{4 \text{ years}} \times \frac{1}{100 \text{ days}} \times \frac{1}{5 \text{ hours}} \times \text{Hours Used} \quad (3.1)$$

If we apply the amortization equation to the computer, which we purchased for €999.95, and assuming 500 hours of usage, its estimated amortized cost is €249.98.

#### Electric cost

In this section we only calculate a rough estimate. Calculating accurately the cost of electricity involves many variables, and it's outside the scope of this thesis. The average cost of electricity in Spain, in terms of kWh, is €0.12. We only count the cost when the hardware is turned on. The following table (3.2) shows the individual and total cost per item.

Item	Power (W)	Time used (h)	Consumption (kWh)	Cost (€)
Computer	180	500	90	10.8
Router	10	500	5	0.6
<b>Total</b>				<b>11.4</b>

TABLE 3.2: Electric cost estimate.

#### Internet cost

The internet cost in my current location is €29.00 per month, this is roughly about €0.95 per day (variance is introduced because a month length is not constant). The

amount of working hours per day is assumed to be 5, thus the total cost the whole project is:

$$€0.95 / \text{day} \times 100 \text{ days} \times 5/24 \text{ hours} = €19.79$$

### Work space

This project will be developed at my parents home at Figueres, with a rent of €400. Since the amount of people living there is 2, the actual cost is €200.

### Total generic costs

Table 3.3 summarizes the total generic costs of this project.

Group	Cost (€)
Amortization	249.98
Electricity	11.4
Internet	19.79
Rent	200
<b>Total</b>	<b>481.17</b>

TABLE 3.3: Total generic cost estimate.

### 3.1.3 Other costs

#### Contingencies

Unexpected problems that were not foreseen may appear during the development of the project, which would take part of our budget. For this reason it is always a good idea to prepare a contingency budget calculated from the budgets we've calculated up to now. We will apply a 10% contingency margin, that is €108.62.

#### Incidental cost

Unexpected problems that were foreseen and can be mitigated are described in section 2.3. Some of these mitigations may incur some extra cost. To be able to handle that cost in this section we will calculate a budget based on the foreseeable problems, their chances of actually occurring, and the expected cost associated for solving them.

- **Deadline of the project:** If an extension of the deadline is requested, that would require extra hours expended by the researcher. Assuming the extra time required to be 50 hours that would give us a cost of €1,175.
- **Bugs on some libraries:** Implementing a *bugfix* we assume would cost around 20 hours, a task done by the programmer. The estimated cost is €382. The risk is small.
- **Insufficient computational power:** If using smaller datasets were not an option, we would abandon the project, since purchasing a new, more powerful, computer or renting a supercomputer is too expensive. The risk is very small.

- **Hardward related issues:** New hardware would be purchased, if possible only the faulty component would be replaced. This would have an estimated cost of about €100. The risk is small.

The following table summarized the expected cost due to foreseen incidents.

Risk	Expected (€)	Risk (%)	Cost (€)
Deadline of the project	1 175	30	352.5
Bugs on some libraries	382	10	38.2
Insufficient computational power		5	
Hardward related issues	100	10	10
<b>Total</b>			<b>400.7</b>

TABLE 3.4: Total incidental cost estimate.

### 3.1.4 Total cost

The total cost for the project is summarized in table 3.5. The cost has been computed as the sum of the justified costs explained in the other sections of the budget plan.

Section	Cost (€)
Costs by role and activity	10 381.00
Generic costs	481.17
Other costs	509.32
<b>Total</b>	<b>11 371.49</b>

TABLE 3.5: Total cost estimate.

## 3.2 Sustainability

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