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Advanced Parallel Computing for Scientific Applications
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Solution 8

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Question 1: Fox's Algorithm

a) It is important to see that the formula

$$C_{ij} = A_{i0}B_{0j} + A_{i1}B_{1j} + \ldots + A_{i,q-1}B_{q-1,j}$$

works no matter what the size q of the submatrices is:

i) For $q = 1$: $c_{23} = a_{20}b_{03} + a_{20}b_{03} + a_{20}b_{03} + a_{20}b_{03}$

ii) For $q = 2$: $C_{11} = A_{10}B_{01} + A_{11}B_{11}$

b) From A, all the submatrices from the same row i, and from B, all submatrices from the same column j, have to be sent to a process $\mathcal{P}_{ij}$ to calculate C_{ij}. In this particular case:

- $\mathcal{P}_{01}$ sends B_{01} to $\mathcal{P}_{11}$.
- $\mathcal{P}_{10}$ sends A_{10} to $\mathcal{P}_{11}$.

c) Iteration $k = 0$:

$$\begin{array}{lll} C_{00} = A_{00} \cdot B_{00} & C_{01} = A_{00} \cdot B_{01} & C_{02} = A_{00} \cdot B_{02} \\ C_{10} = A_{11} \cdot B_{10} & C_{11} = A_{11} \cdot B_{11} & C_{12} = A_{11} \cdot B_{12} \\ C_{20} = A_{22} \cdot B_{20} & C_{21} = A_{22} \cdot B_{21} & C_{22} = A_{22} \cdot B_{22} \end{array} \tag{1}$$

Iteration $k = 1$:

$$\begin{array}{lll} C_{00} += A_{01} \cdot B_{10} & C_{01} += A_{01} \cdot B_{11} & C_{02} += A_{01} \cdot B_{12} \\ C_{10} += A_{12} \cdot B_{20} & C_{11} += A_{12} \cdot B_{21} & C_{12} += A_{12} \cdot B_{22} \\ C_{20} += A_{20} \cdot B_{00} & C_{21} += A_{20} \cdot B_{01} & C_{22} += A_{20} \cdot B_{02} \end{array} \tag{2}$$

Iteration $k = 2$:

$$\begin{array}{lll} C_{00} += A_{02} \cdot B_{20} & C_{01} += A_{02} \cdot B_{21} & C_{02} += A_{02} \cdot B_{22} \\ C_{10} += A_{10} \cdot B_{00} & C_{11} += A_{10} \cdot B_{01} & C_{12} += A_{10} \cdot B_{02} \\ C_{20} += A_{21} \cdot B_{10} & C_{21} += A_{21} \cdot B_{11} & C_{22} += A_{21} \cdot B_{12} \end{array} \tag{3}$$

Observation 1: The submatrix A_{ij} is always shared along the rows. The process that 'owns' this submatrix is highlighted in red. At $k = 0$ these processes are on the diagonal. For each iteration the process that 'owns' the submatrix is next to the right of the previous one (with periodic boundaries).
Observation 2: At $k = 0$ the submatrices B_{ij} are already at the right position. For each iteration, the submatrices B_{ij} are shifted upwards (with periodic boundaries).