Pushing–Medium Model: Core Formula Sheet

1. **Static refractive index field from point masses**

$$n(\mathbf{r}) = 1 + \sum_i \frac{\mu_i}{|\mathbf{r} - \mathbf{r}_i|}, \quad \mu_i = \frac{2GM_i}{c^2} \text{ (or chosen scaling)}$$

2. **Optional flow field (frame–drag analogue)**

$$\mathbf{u}_g(\mathbf{r}) = \sum_i \boldsymbol{\Omega}_i \times (\mathbf{r} - \mathbf{r}_i)$$

3. **Time–dependent perturbation (wave injection)**

$$\delta n_{\text{wave}}(\mathbf{r}, t) = A \cos\big(\mathbf{k} \cdot \mathbf{r} - \omega t\big)$$

4. **Total refractive index field**

$$n_{\text{total}}(\mathbf{r}, t) = n(\mathbf{r}) + \delta n_{\text{wave}}(\mathbf{r}, t)$$

5. **Ray equation of motion (Fermat's principle)**

$$\frac{d\hat{\mathbf{k}}}{ds} = \nabla_\perp \ln n_{\text{total}}(\mathbf{r}, t)$$

where $\nabla_\perp$ is the gradient perpendicular to the ray direction.

6. **Ray advection by flow field**

$$\frac{d\mathbf{r}}{dt} = \frac{c\,\hat{\mathbf{k}}}{n_{\text{total}}} + \mathbf{u}_g(\mathbf{r}, t)$$

7. **Massive particle acceleration (optical–mechanical analogy)**

$$\mathbf{a}_{\text{med}}(\mathbf{r}, t) = -\,c^2\,\nabla \ln n_{\text{total}}(\mathbf{r}, t)$$

8. **Newtonian gravity term for matter**

$$\mathbf{a}_{\text{grav}}(\mathbf{r}) = -\sum_i \frac{GM_i\,(\mathbf{r} - \mathbf{r}_i)}{|\mathbf{r} - \mathbf{r}_i|^3}$$

9. **Combined matter equation of motion**

$$\frac{d^2\mathbf{r}}{dt^2} = \mathbf{a}_{\text{grav}} + \mathbf{a}_{\text{med}} + \text{(optional flow coupling)}$$

10. **Index gradient from continuous mass density**

$$n(\mathbf{r}) = 1 + \frac{2G}{c^2} \int \frac{\rho(\mathbf{r}')}{|\mathbf{r} - \mathbf{r}'|} \, d^3r'$$