CSE423 - Computer Graphics

Final Practice Sheet [Spring 2025]

N.B. *This is merely a reference to the problems, inclusive and exclusive to the questions that will be set in the exam. This practice sheet does not include all kinds of questions that may come in the examination. The sole purpose of this practice sheet is to facilitate a clear and thorough understanding of the concepts.*

1. Derive a 4x4 **simple purpose** perspective projection matrix using the appropriate figure, showing P, P′, COP, PP. Also, this matrix can be converted into a simple perspective projection matrix. Possible cases are -
    a. Origin is at COP and
    b. Origin on the projection plane.
2. Derive a 4x4 **general-purpose** perspective projection matrix using the appropriate figure, showing PP′, COP, PP.
3. For a point P (10, 20, -40), calculate the projected point P' using Orthographic Projection.
    a. For the projection plane of XY.
    b. For the projection plane of YZ.
    c. For the projection plane of ZX.
    d. For the projection plane of y = -13
4. For a point P (10, 20, -40), calculate the projected point P' using Cavalier Projection, orientation angle is 30° for the projection plane of XY.
5. For a point P (10, 20, -40), calculate the projected point P' using Cabinet Projection, where the orientation angle is 30°
    a. For the projection plane of XY.
    b. For the projection plane of x = 7.
    c. For the projection plane of ZX.
6. For COP at origin, calculate the projected point P' for a given point P(50, 60, -300), if the plane is 200 units in the Z axis away from the COP.
7. For PP at origin, calculate the projected point P' for a given point P(30, 20, 100), if the plane is 200 units in the Z axis away from the COP.
8. Let a 3D point (423, -423, 423) be projected on a projection plane.

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Given that the center of the projection plane is (0, 0, 400) and the coordinate of the COP is (4, 2, 3). Determine the coordinate of that 3D point on the projection plane using a general-purpose perspective projection matrix. How far is the Projection Plane (PP) from the Center of Projection (COP) in terms of units?

9. For COP at (100, 90, 0), calculate the projected point P' for a point P(30, 50, -250) where the Projection Plane (PP) is 200 units away from COP.
10. A 3D vertex P(40, 30, 70) is projected on the projection plane. Suppose near clipping plane distance, z_p (check whether it is 40). From the below general purpose perspective projection matrix;

$$\begin{bmatrix} 1 & 0 & -0.3333 & 13.3333 \\ 0 & 1 & 0.6667 & -26.6667 \\ 0 & 0 & -1.3333 & 93.3333 \\ 0 & 0 & -0.0333 & 2.3333 \end{bmatrix}$$

    a. Find out the Center of Projection (COP).
    b. How far is the Projection Plane (PP) from the Center of Projection (COP) in terms of units?
    c. Now, determine the projected coordinate P' on the projection plane.
    d. Can you identify the equations for this 4x4 projection matrix?
11. Let (50, 70, 100) be the coordinates of a light source of intensity 0.95 units. The light is illuminating a quad consisting of $P_0(10, 10, 5)$, $P_1(-10, 10, 5)$, $P_2(-10, -10, 6)$ and $P_3(10, -10, 6)$ vertices. Determine the intensity of the reflected light at the center of the quad using the diffuse reflection model. Given that the diffuse absorption coefficient of the quad surface is 0.8 units.
12. Let (30, 10, 500) be the coordinates of a light source of intensity 0.5 units. The light is illuminating a sphere whose center is at C(10, -15, 6). Determine the total intensity of the reflected light from a point P(20, 10, 120) on the sphere using the diffuse reflection model. Given that the diffuse absorption coefficient of the surface is 0.8 units.
13. A light source with an intensity of 15 and a radius of influence of 80 is located at (4,2,3) from which you are called to calculate the illumination of a point on the y = 4 plane. The camera is set at (2, 1, 5) and the light is reflected from points (3, 4, 4) of the plane. The ambient, diffuse, and specular coefficient is given at 0.4, 0.2, and 0.3. The shininess factor of the surface is 3. If the ambient light intensity is at 2, calculate the total reflected light intensity using Phong's Lighting Model.
14. Suppose there are two light sources in the scene. One light source is

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located at (4, 2, 3) with an intensity of 15 and a radius of influence of 20 and another one is located at (10, 40, 50) with an intensity of 5 and a radius of influence of 80. You are called to calculate the illumination of a point on the yz plane. The camera is set at (2, 1, 5) and the light is reflected from points (0, 3, 2) of the plane. The ambient, diffuse, and specular coefficients of the surface are given at 0.23, 0.7, and 0.5. The shininess factor of the surface is 5. If the ambient light intensity is at 6, calculate the total reflected light intensity using Phong's Lighting Model.

15. Write an algorithm for converting the RGB color values into HLS/HSL color values.
16. Write an algorithm for converting the RGB color values into HSV/HSB color values.
17. For a grayscale color with a green component value of 0.74, determine the corresponding values of the red and blue components.
18. For an achromatic color with a green component value of 0.74, determine the corresponding values of the red and blue components.
19. For a monochrome color with a green component value of 0.74, determine the corresponding values of the red and blue components.
20. Convert the RGB colors into both HLS/HSL and HSV color values.
    a. (0.25, 0.3, 1.0)
    b. (0.01, 1.0, 0.09)
    c. (0.8, 0.8, 0.35)
    d. (0,0, 0.4, 0.4)
    e. (1.0, 1.0, 0.5)
    f. (0.7, 0.71, 0.7)
    g. (0.5, 0.5, 0.5)
    h. (1.0, 1.0, 1.0)
    i. (0.39, 0.398, 0.2)
21. Convert the HSV values into RGB color values: H = 236^0, S = 0.3, V = 60%
22. Convert the HSV values into RGB color values: H = 135^0, S = 0.9, V = 80%
23. Convert the HSV values into CMY color values: H = 336^0, S = 0.7, V = 40%
24. To answer some of the following questions, you will need four variables A, B, C, and D, sequentially the first, second, third, and fourth pair of digits from the left in your student ID. For example, if your ID is 15101208, then A = 15, B = 10, C = 12, and D = 8.

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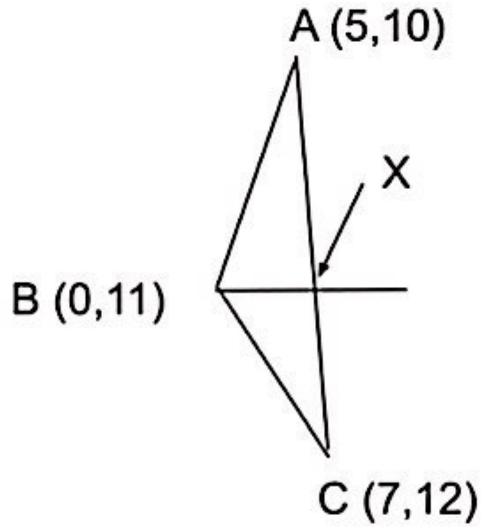

a. A color is given in CMY form with the values (0.A, 0.C, 0.D). Convert the color into an equivalent HSV model. Show the calculation in detail.

b. The color at vertex A is the value B of your student ID, and at vertex C is D of your student ID. Now, calculate the color at point X using Gouraud shading. Can a specular light on point X be captured using the above model? Why or why not?

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Final
Practice Sheet
Solve by Azmani Sultana

Projection

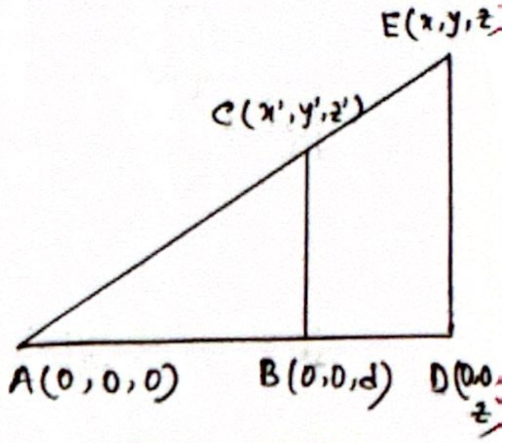
① a) COP is at origin (0, 0, 0).

$x' \cdot w = (1)x$

$y' \cdot w = (1)y$

$z' \cdot w = (1)z$

$w = \left(\frac{1}{d}\right)z$



For simple purpose

$$\begin{bmatrix} x' \cdot w \\ y' \cdot w \\ z' \cdot w \\ w \end{bmatrix} = \begin{bmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1/d & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}.$$

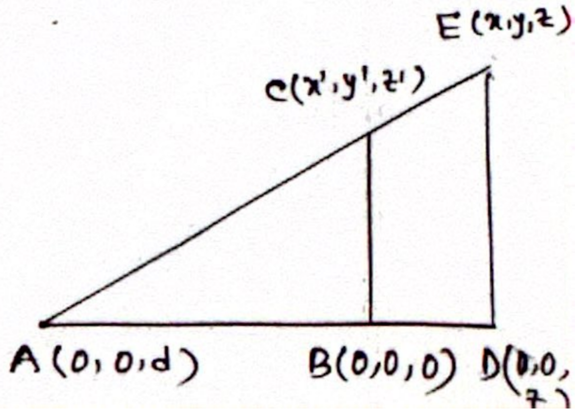
b) Origin on the projection plane.

$x' \cdot w = (1)x$

$y' \cdot w = (1)y$

$z' \cdot w = 0$

$w = \left(-\frac{1}{d}\right)z + 1$



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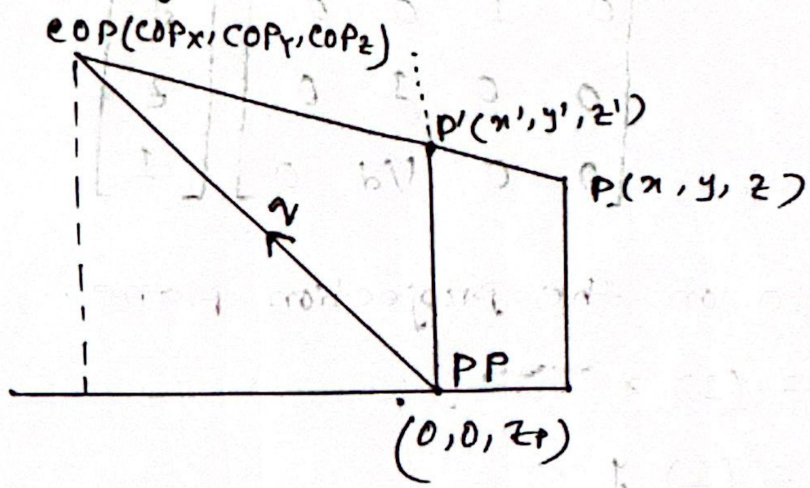
$$\begin{bmatrix} x' \cdot w \\ y' \cdot w \\ z' \cdot w \\ w \end{bmatrix} = \begin{bmatrix} a_1 & b_1 & c_1 & d_1 \\ a_2 & b_2 & c_2 & d_2 \\ a_3 & b_3 & c_3 & d_3 \\ a_4 & b_4 & c_4 & d_4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1/d & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

(2) Derive a 4X4 general-purpose perspective projection matrix using the appropriate figure, showing PP', COP, PP.



$$\begin{bmatrix} x'.w \\ y'.w \\ z'.w \\ w \end{bmatrix} = \begin{bmatrix} 1 & 0 & -q_x/q_z & z_p \cdot \frac{q_x}{q_z} \\ 0 & 1 & -q_y/q_z & z_p \cdot \frac{q_y}{q_z} \\ 0 & 0 & -z_p/q_z & z_p + \frac{z_p^2}{q_z} \\ 0 & 0 & -1/q_z & 1 + \frac{z_p}{q_z} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

③ For a point P(10, 20, -40), calculate the projected point P' using Orthographic Projection.

a. For the projection plane of XY

$$\begin{bmatrix} 1 & 0 & \lambda\cos\beta & 0 \\ 0 & 1 & \lambda\sin\beta & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x'.w \\ y'.w \\ z'.w \\ w \end{bmatrix}$$

For orthographic projection, $\lambda = 0$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 20 \\ -40 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 20 \\ 0 \\ 1 \end{bmatrix}$$

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b. For the projection plane of YZ

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 20 \\ -40 \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 20 \\ -40 \\ 1 \end{bmatrix}$$

c. For the projection plane of ZX.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 20 \\ -40 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 0 \\ -40 \\ 1 \end{bmatrix}$$

d. For the projection plane of Y=-13

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -13 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 20 \\ -40 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ -13 \\ -40 \\ 1 \end{bmatrix}$$

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(4) For a point P (10, 20, -40), calculate the projected point P' using Cavalier Projection. Orientation angle is 30° for the projection plane of XY. (slide)

For cavalier, $\alpha = 45°$. • $\beta = 30°$ (given)

$$\begin{bmatrix} 1 & 0 & \frac{\cos\beta}{\tan\alpha} & 0 \\ 0 & 1 & \frac{\sin\beta}{\tan\alpha} & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x' \cdot w \\ y' \cdot w \\ z' \cdot w \\ w \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0.866 & 0 \\ 0 & 1 & 0.5 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 20 \\ -40 \\ 1 \end{bmatrix} = \begin{bmatrix} -24.64 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

(5) For a point P(10, 20, -40), calculate the projected point P' using Cabinet Projection, where the orientation angle is 30°.

a) For the projection plane of XY

$\beta = 30°$

$\lambda = 0.5$

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$$\begin{bmatrix} 1 & 0 & \lambda\cos\beta & 0 \\ 0 & 1 & \lambda\sin\beta & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} x'\cdot w \\ y'\cdot w \\ z'\cdot w \\ w \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0.433 & 0 \\ 0 & 1 & 0.25 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 20 \\ -40 \\ 1 \end{bmatrix} = \begin{bmatrix} -7.32 \\ 10 \\ 0 \\ 1 \end{bmatrix}$$

b. for projection plane of $x=7$.

$$\begin{bmatrix} 0 & 0 & 0 & 7 \\ \lambda\cos\beta & 1 & 0 & 0 \\ \lambda\sin\beta & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 20 \\ -40 \\ 1 \end{bmatrix} = \begin{bmatrix} 7 \\ 24.33 \\ -37.5 \\ 1 \end{bmatrix}$$

c. For projection plane of ZX.

$$\begin{bmatrix} 1 & 0.5\sin 30^\circ & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0.5\cos 30^\circ & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 10 \\ 20 \\ -40 \\ 1 \end{bmatrix} = \begin{bmatrix} 15 \\ 0 \\ -31.33 \\ 1 \end{bmatrix}$$

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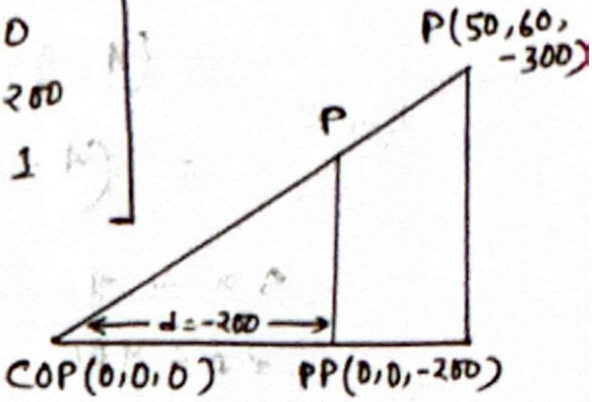
6 For COP at origin, calculate the projected point P' for a given point P(50, 60, -300), if the plane is 200 units in the z axis away from the COP.

(slide)

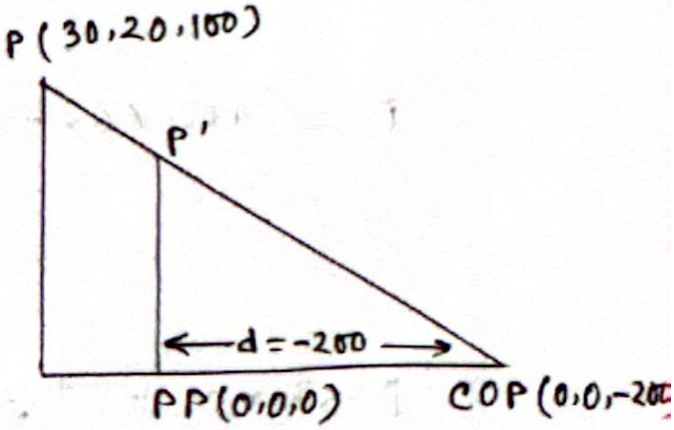
$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1/200 & 0 \end{bmatrix} \begin{bmatrix} 50 \\ 60 \\ -300 \\ 1 \end{bmatrix} = \begin{bmatrix} 50 \\ 60 \\ -300 \\ 3/2 \end{bmatrix} = \begin{bmatrix} 33.33 \\ 40 \\ -200 \\ 1 \end{bmatrix}$$



$P' = (33.33, 40, -200)$

7 For PP at origin, calculate the projected point P' for a given point P(30, 20, 100), if the plane is 200 units in the z axis away from the COP.

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 1/200 & 1 \end{bmatrix} \begin{bmatrix} 30 \\ 20 \\ 100 \\ 1 \end{bmatrix} = \begin{bmatrix} 30 \\ 20 \\ 0 \\ 1.5 \end{bmatrix}$$



$$= \begin{bmatrix} 20 \\ 13.33 \\ 0 \\ 1 \end{bmatrix}$$

$P' = (20, 13.33, 0)$

⑧ P(123, −123, 123)

COP (1, 2, 3)

$q = COP - (0, 0, z_p)$

$= (1, 2, 3) - (0, 0, 400)$

$= (1, 2, -397)$

$q_x = 1, \quad q_y = 2, \quad q_z = -397$

$z_p = 400$

$$\begin{bmatrix} 1 & 0 & -1/-397 & 400 \cdot \frac{1}{-397} \\ 0 & 1 & -2/-397 & 400 \cdot \frac{2}{-397} \\ 0 & 0 & -400/-397 & 400 + \frac{400^2}{-397} \\ 0 & 0 & -1/-397 & 1 + \frac{400}{-397} \end{bmatrix} \begin{bmatrix} 123 \\ -123 \\ 123 \\ 1 \end{bmatrix} = \begin{bmatrix} 123.23 \\ -122.8 \\ 123.17 \\ 1.0579 \end{bmatrix}$$

$$P' = \begin{bmatrix} 100.06 \\ -399.66 \\ 400.01 \\ 1 \end{bmatrix}$$

$\therefore P' = (100.06, -399.66, 400.01)$

Distance $= |3 - 400| = 397$ units.

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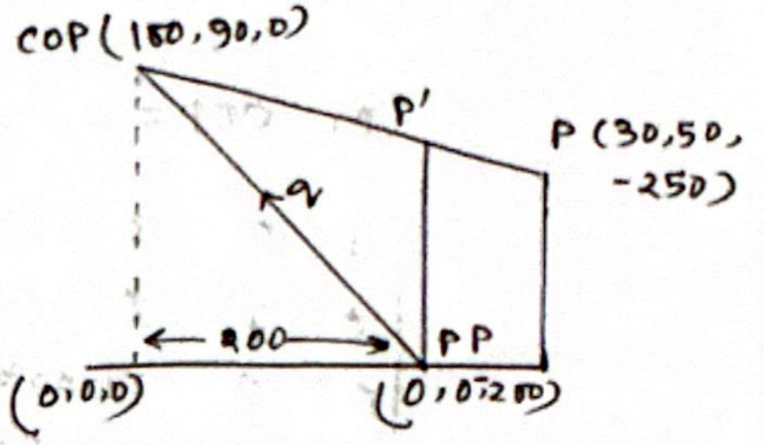
(19) For COP at (100, 90, 0), calculate the projected point P' for a point P (30, 50, −250) where the projection plane (PP) is 200 units away from COP. (slide)

$z_p = -200$



$q = COP - (0, 0, z_p)$

$= (100, 90, 0) - (0, 0, -200)$

$= (100, 900, 200)$

$q_x = 100$

$q_y = 90$

$q_z = 200$

$$\begin{bmatrix} 1 & 0 & -q_x/q_z & z_p \cdot \frac{q_x}{q_z} \\ 0 & 1 & -q_y/q_z & z_p \cdot \frac{q_y}{q_z} \\ 0 & 0 & -z_p/q_z & z_p + \frac{z_p^2}{q_z} \\ 0 & 0 & -1/q_z & 1 + \frac{z_p}{q_z} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -1/2 & -100 \\ 0 & 1 & -9/20 & -90 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1/200 & 0 \end{bmatrix} \begin{bmatrix} 30 \\ 50 \\ -250 \\ 1 \end{bmatrix} = \begin{bmatrix} 55 \\ 72.5 \\ -250 \\ 1.25 \end{bmatrix} = \begin{bmatrix} 44 \\ 85 \\ -200 \\ 1 \end{bmatrix}$$

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Note: To project a point, $P' = M \cdot P$

To find the COP, $M \cdot COP = 0$

(10) a)

$M \cdot COP = 0$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -0.3333 & 13.3333 \\ 0 & 1 & 0.6667 & -26.6667 \\ 0 & 0 & -1.3333 & 93.3333 \\ 0 & 0 & -0.0333 & 2.3333 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

$1x + 0 \cdot y - 0.3333z + 13.3333 = 0$

$\Rightarrow x - 0.3333z + 13.3333 = 0$

$0 \cdot x + 1 \cdot y + 0.6667z - 26.6667 = 0$

$\Rightarrow y + 0.6667z - 26.6667 = 0$

$-1.3333z + 93.3333 = 0$

$\Rightarrow z = \dfrac{93.3333}{1.3333} = 70$

$-0.0333z + 2.3333 = 0$

$\Rightarrow z = \dfrac{2.3333}{0.0333} = 70$

$\therefore x = 0.3333 \times 70 - 13.3333 = 10$

$\therefore y = 26.6667 - 0.6667 \times 70 = -20$

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$\therefore$ COP $= (x, y, z) = (10, -20, 70)$

b) $z_p = 40$

Distance $= |70 - 40| = 30$ units.

c)

$$\begin{bmatrix} 1 & 0 & -0.3333 & 13.3333 \\ 0 & 1 & 0.6667 & -26.6667 \\ 0 & 0 & -1.3333 & 93.3333 \\ 0 & 0 & -0.0333 & 2.3333 \end{bmatrix} \begin{bmatrix} 40 \\ 30 \\ 70 \\ 1 \end{bmatrix} = \begin{bmatrix} 30.0023 \\ 50.0023 \\ 2.3 \times 10^{-3} \\ 2.3 \times 10^{-3} \end{bmatrix}$$

$$= \begin{bmatrix} 13044.48 \\ 21740.13 \\ 1 \\ 1 \end{bmatrix}$$

d) $x' = x - 0.3333z + 13.3333$

$y' = y + 0.6667z - 26.6667$

$z' = -1.3333 + 93.3333$

$w' = -0.0333z + 2.3333$

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Final

Lighting

(11) $L=(50,70,100)$

$I_p = 0.95$

$P_0\ (10, 10, 5)$

$P_1\ (-10, 10, 5)$

$P_2\ (-10, -10, 6)$

$P_3\ (10, -10, 6)$

$$P_c = \left(\frac{10-10-10+10}{4}, \frac{10+10-10-10}{4}, \frac{5+5+6+6}{4}\right)$$

$$= (0, 0, 5.5)$$

$$u = P_1 - P_0 = (-20, 0, 0)$$

$$v = P_3 - P_0 = (0, -20, 1)$$

$$N = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -20 & 0 & 0 \\ 0 & -20 & 1 \end{vmatrix} = 0\hat{i} + 20\hat{j} + 400\hat{k}$$

$$|N| = \sqrt{0^2 + 20^2 + 400^2} = 400.5$$

$$\hat{N} = \left(0, \frac{20}{400.5}, \frac{400}{400.5}\right)$$

$$= (0, 0.04994, 0.99875)$$

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$L - P_c = (50-0, 70-0, 100-5.5) = (50, 70, 94.5)$

$|L - P_c| = \sqrt{50^2 + 70^2 + 94.5^2} = 127.8$

$\hat{L} = \left(\frac{50}{127.8}, \frac{70}{127.8}, \frac{94.5}{127.8}\right)$

$= (0.3912, 0.5477, 0.7396)$

$\hat{L} \cdot \hat{N} = (0.3912 \times 0 + 0.5477 \times 0.04994 + 0.7396 \times 0.99875)$

$= 0.76601$

$D = I_p k_d \max(\hat{L} \cdot \hat{n}, 0)$

$= 0.95 \times 0.8 \times 0.76601$

$= 0.581$

(12) $L = (30, 10, 500)$

$I_p = 0.5$

$c = (10, -15, 6)$

$P(20, 10, 120)$

$k_d = 0.8$

$N = P - C = (20-10, 10-(-15), 120-6)$

$= (10, 25, 114)$

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$$|N| = \sqrt{10^2 + 25^2 + 114^2} = 117.17$$

$$\hat{N} = \left(\frac{10}{117.17}, \frac{25}{117.17}, \frac{114}{117.17}\right)$$

$$= (0.08534, 0.21335, 0.97295)$$

$$L - P = (30 - 20, 10 - 10, 500 - 120)$$

$$= (10, 0, 380)$$

$$|L - P| = \sqrt{10^2 + 0^2 + 380^2} = 380.41$$

$$\hat{L} = \left(\frac{10}{380.41}, 0, \frac{380}{380.41}\right)$$

$$= (0.02628, 0, 0.99892)$$

$$\hat{N} \cdot \hat{L} = (0.08534 \times 0.02628 + 0.21335 \times 0$$
$$+ 0.97295 \times 0.99892)$$

$$= 0.97422$$

$$D = I_p k_d \hat{N} \cdot \hat{L}$$

$$= 0.5 \times 0.8 \times 0.97422$$

$$= 0.389688$$

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(13) $I_p = 15$

$r = 80$

$L = (4, 2, 3)$

$V = (2, 1, 5)$

$P = (3, 4, 4)$

$n = 3$

$N = (0, 1, 0)$

$I_a = 2$

$K_a = 0.4$

$K_d = 0.2$

$K_s = 0.3$

$$L - P = (4 - 3, 2 - 4, 3 - 4) = (1, -2, -1)$$

$$|L - P| = (\sqrt{1^2 + (-2)^2 + (-1)^2}) = \sqrt{6} = d$$

$$f_{att} = 1 - \left(\frac{d}{r}\right)^2$$
$$= 0.99$$

$$\hat{L} = \left(\frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\right)$$

$$\hat{L} \cdot \hat{N} = \left(\frac{1}{\sqrt{6}} \cdot \frac{-2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\right)(0, 1, 0)$$

$$= -0.816 < 0$$

$$\therefore \hat{L} \cdot \hat{N} = 0$$

$$R = 2(\hat{L} \cdot \hat{N})N - \hat{L}$$

$$= 2\left(-\frac{2}{\sqrt{6}}\right)(0, 1, 0) - \left(\frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}}\right)$$

$$= \left(-\frac{1}{\sqrt{6}}, -\frac{2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$$

$$V - P = (2 - 3, 1 - 4, 5 - 4) = (-1, -3, 1)$$

$$|V - P| = \sqrt{(-1)^2 + (-3)^2 + 1^2} = \sqrt{11}$$

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$$\hat{V} = \left(\frac{-1}{\sqrt{11}}, \frac{-3}{\sqrt{11}}, \frac{1}{\sqrt{11}}\right)$$

$$\hat{R}\cdot\hat{V} = \left(-\frac{1}{\sqrt{6}}, \frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)\left(\frac{-1}{\sqrt{11}}, \frac{-3}{\sqrt{11}}, \frac{1}{\sqrt{11}}\right)$$

$$= 0.985$$

$$I = I_a k_a + I_p (0 + k_s (\hat{R}\cdot\hat{V})^n)$$

$$= 2\times 0.4 + 15^{\times 0.99}(0 + 0.3 \times (0.985)^3)$$

$$\cancel{= 5.0975}$$

$$= 5.058$$

(11) $L_1 = (4, 2, 3)$

$I_{p_1} = 15$

$r_1 = 20$

$L_2 = (10, 40, 50)$

$I_{p_2} = 5$

$r_2 = 80$

$N = (1, 0, 0)$

$V = (2, 1, 5)$

$P = (0, 3, 2)$

$n = 5$

$I_a = 6$

$k_a = 0.23$

$k_d = 0.7$

$k_s = 0.5$

$$f_{att1} = 1 - \left(\frac{d_1}{r_1}\right)^2$$

$$= 1 - \left(\frac{4.243}{20}\right)^2$$

$$= 0.95$$

$$f_{att2} = 1 - \left(\frac{d_2}{r_2}\right)^2$$

$$= 1 - \left(\frac{61.42}{80}\right)^2$$

$$= 0.41$$

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$L_1 - P = (4-0,\ 2-3,\ 3-2) = (4, -1, 1)$

$|L_1 - P| = \sqrt{4^2 + (-1)^2 + 1^2} = 4.243 = d_1$

$\hat{L}_1 = \left(\frac{4}{4.243}, \frac{-1}{4.243}, \frac{1}{4.243}\right)$

$= (0.943, -0.236, 0.236)$

$\ddagger L_2 - P \ddagger = (10-0,\ 40-3,\ 50-2) = (10, 37, 48)$

$|L_2 - P| = 61.42 = d_2$

$\hat{L}_2 = \left(\frac{10}{61.42}, \frac{37}{61.42}, \frac{48}{61.42}\right) = (0.163, 0.602, 0.781)$

$V - P = (2-0,\ 1-3,\ 5-2) = (2, -2, 3)$

$|V - P| = \sqrt{17} = 4.123$

$\hat{V} = \left(\frac{2}{4.123}, \frac{-2}{4.123}, \frac{3}{4.123}\right) = (0.485, -0.485, 0.727)$

$\hat{N} \cdot \hat{L}_1 = (1,0,0)(0.943, -0.236, 0.236)$

$= 0.943$

$\hat{N} \cdot \hat{L}_2 = (1,0,0)(0.163, 0.602, 0.781)$

$= 0.163$

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$$R_1 = 2(\hat{N}\cdot\hat{L}_1)N - \hat{L}_1$$

$$= 2(0.943)(1,0,0) - (0.943, -0.236, 0.236)$$

$$= (0.943, 0.236, -0.236)$$

$$R_2 = 2(\hat{N}\cdot\hat{L}_2)N - \hat{L}_2$$

$$= 2(0.163)(1,0,0) - (0.163, 0.602, 0.781)$$

$$= (0.163, -0.602, -0.781)$$

$$\hat{R}_1\cdot\hat{V} = (0.943)(0.485) + (0.236)(0.485) + (-0.236)(0.727)$$

$$= 0.457 - 0.114 - 0.171 = 0.172$$

$$\hat{R}_2\cdot\hat{V} = (0.163)(0.485) + (-0.602)(-0.485) + (-0.781)(0.727)$$

$$= -0.197 < 0$$

$$\therefore \hat{R}_2\cdot\hat{V} = 0$$

so viewer is not in the direction of the perfect reflection.

$D_1 = I_{p_1} k_d \times \hat{N} \cdot \hat{L}_1$

$= 15 \times 0.7 \times 0.943$

$= 9.9015$

$S_1 = I_{p_1} k_s (\hat{R}_1 \cdot \hat{V})^n$

$= 15 \times 0.5 \times (0.172)^5$

$= 1.129 \times 10^{-3}$

$D_2 = I_{p_2} k_d \times \hat{N} \cdot \hat{L}_2$

$= 5 \times 0.7 \times 0.163$

$= 0.5705$

$\times \cancel{S_2 = I_{p_2} k_s (\hat{R}_2 \cdot \hat{V})^n}$

$\cancel{= 5 \times 0.5 (0.197)^5}$

$S_2 = 0$

$I_a k_a = 6 \times 0.23 = 1.38$

$I = 1.38 + \overset{0.95}{(9.9015 + 1.129 \times 10^{-3})} + \overset{0.41}{(0.5705 + 0)}$

$\cancel{= 11.853}$

$= 11.02$

(15) Write an algorithm for converting the RGB color values into HLS/HSL color values.

Converting RGB into HLS

1. Normalise
2. Compute cmax, cmin, difference
3. $L = (cmax + cmin)/2$
4. Hue calculation :

   if diff = 0, then h = 0
   if cmax = r, H = (g-b)/diff
   if cmax = g, H = 2 + (b-r)/diff
   if cmax = b, H = 4 + (r-g)/diff
   H = H * 60
   if H < 0 → H = H + 360

5. Saturation computation :

   if diff = 0, then S = 0
   S = diff / (cmax + cmin) if L <= 0.5
   S = diff / [2 - (cmax + c min)] if L > 0.5

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Azmari Sultana

(16) Write an algorithm for converting the RGB color values into HSV / HSB color values.

Converting RGB to HSV

1. Divide r, g, b by 255 (if the scale is 0 – 255, otherwise skip)

2. compute cmax, cmin, difference

3. Hue calculation:

if cmax = 0, h = 0
cmax = r, h = (g − b) / diff
cmax = g, h = 2 + (b − r) / diff
cmax = b, h = 4 + (r − g) / diff
h = h * 60
if h < 0 → h = h + 360

4. Saturation computation:

cmax = 0, then s = 0
cmax ≠ 0, s = (diff / cmax) * 100

5. Value computation:

v = cmax * 100

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(17) For a grayscale color with a green component value of 0.74, determine the corresponding values of the red and blue component.

- For a grayscale color, all three RGB values (Red, green, blue) must be equal. because grayscale has no hue or saturation, only brightness.

if green = 0.74

then red = blue = 0.74

(18) For an achromatic color with a green component value of 0.74, determine the corresponding values of the red and blue components.

- Achromatic colors have equal red, green and blue components.

Green = 0.74

red = blue = green = 0.74

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(19) For a monochrome color with a green component value of 0.74, determine the corresponding values of the red and blue components.

- Unlike achromatic or grayscale colors, the RGB components are also necessarily equal. for monochrome color

Green = 0.74

Red = blue = 0.74 ~~(for pure green)~~

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(20) a) (0.25 , 0.3, 1.0)

RGB to HLS

$c_{max} = \cancel{0.3}\ 1.0$

$c_{min} = 0.25$

$\text{difference} = 1 - 0.25 = 0.75$

$L = (c_{max} + c_{min}) / 2$

$= (1 + 0.25)/2 = 0.625$

$c_{max} = b$

$H = 60(4 + (r - g)/ \text{diff})$

$= 60(4 + (0.25 - 0.3)/0.75)$

$= 236°$

$L > 0.5$; $S = \text{diff} / [2 - (c_{max} + c_{min})]$

$= 0.75/[2 - (1 + 0.25)]$

$= 1$

$HLS = (236°, \cancel{ko}\ 0.625, 1)$

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RGB to HSV :

RGB = (0.25, 0.3, 1.0)

cmax = 1.0

cmin = 0.25

difference = 0.75

cmax = b

$h = 60(4 + (r - g)/diff)$

$= 236°$

$S = \frac{diff}{cmax} = \frac{0.75}{1} = 0.75$

$V = cmax = 1$

HSV = (236°, 0.75, 1)

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(21) Convert the HSV values into RGB color values:

$H = 236°$, $S = 0.3$; $V = 60\%$.

$$C = V \times S = 0.6 \times 0.3 = 0.18$$

$$x = C \times (1 - |(H/60 \% 2) - 1|)$$

$$= 0.18\left(1 - \left|\left(\frac{236}{60} \% 2\right) - 1\right|\right)$$

$$= 0.18(1 - |1.93 - 1|)$$

$$= 0.0126$$

$$m = V - C = 0.6 - 0.18 = 0.42$$

$180 \leq H < 240$; $R = 0$, $G = X$; $B = C$

$$(R+m, G+m, B+m) = (0 + 0.42, 0.0126 + 0.42, 0.18 + 0.42)$$

$$= (0.42, 0.4326, 0.6)$$

$$R, G, B = (0.42, 0.43, 0.6)$$

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22. convert the HSV values into RGB color values : $H = 135°, S = 0.9, V = 80\%$.

$$C = V \times S$$
$$= 0.8 \times 0.9 = 0.72$$

$$X = C \times (1 - |(H/60 \ \% \ 2) - 1|)$$
$$= 0.72 \left(1 - \left|\left(\frac{135}{60} \ \% \ 2\right) - 1\right|\right)$$
$$= 0.18$$

$$m = V - C = 0.8 - 0.72 = 0.08$$

$120 \leq H < 180$; $R = 0, G = C, B = X$

$$(R+m, G+m, B+m) = (0+0.08, 0.72+0.08, 0.18+0.08)$$
$$= (0.08, 0.8, 0.26)$$

$$(R, G, B) = (0.08, 0.8, 0.26)$$

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23. convert the HSV values into CMY color values: $H = 336°$, $S = 0.7$, $V = 40\%$.

$$C = V \times S = 0.4 \times 0.7 = 0.28$$

$$X = C \times \left(1 - \left|\left(\frac{H}{60} \% 2\right) - 1\right|\right)$$

$$= 0.28\left(1 - \left|\left(\frac{336}{60} \% 2\right) - 1\right|\right)$$

$$= 0.112$$

$$m = V - C = 0.4 - 0.28 = 0.12$$

$330 \le H < 360$; $R = C$, $G = 0$, $B = X$

$$R+m, G+m, B+m = (0.28 + 0.12,\ 0 + 0.12,\ + 0.112 + 0.12)$$

$$= (0.4,\ 0.12,\ 0.232)$$

$$(R, G, B) = (0.4,\ 0.12,\ 0.23)$$

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(21) Id = 22201949

A = 22

B = 20

C = 19

D = 49

(a). CMY = (0.A, 0.C, 0.D)

= (0.22, 0.19, 0.49)

R = 1 - C = 1 - 0.22 = 0.78

G = 1 - M = 1 - 0.19 = 0.81

B = 1 - Y = 1 - 0.49 = 0.51

(R, G, B) = (0.78, 0.81, 0.51)

C_{max} = 0.81

C_{min} = 0.51

difference = 0.81 - 0.51 = 0.3

C_{max} = g

$\therefore$ h = 60(2 + (b - r)/diff)

= 60(2 + (0.51 - 0.78)/0.3)

= 66°

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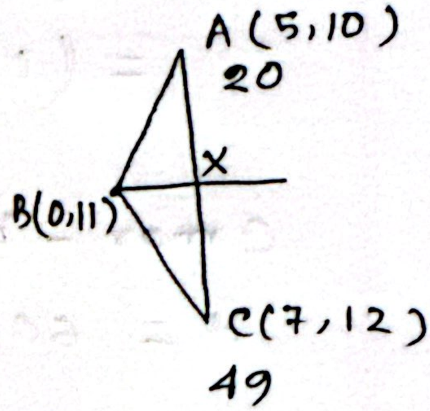
Azmani

$s = diff / c_{max} = 0.3 / 0.81 = 0.37$

$v = c_{max} = 0.81$

$HSV = (66°, 0.37, 0.81)$

(b)



Question is incomplete

$$x = C + (A - C)\frac{11 - 12}{10 - 12}$$

$$= 49 + (20 - 49)\frac{11 - 12}{10 - 12}$$

$$= 34.5$$

A specular light on point X cannot be captured using Gouraud shading model. Because in this model, lighting is only computed at vertices and colors are interpolated across the surface from those vertex colors. So, it can't accurately capture the color at the middle point of a polygon.

Scanned with CamScanner

25. Determine **C(1) and G(1)** continuity of the following functions at the given points:

a. At **t = 2π**,

$$(x(t), y(t)) = \begin{cases} (t, \sin t) & \text{for } t \leq 2\pi \\ (t, 1 - \cos t) & \text{for } t > 2\pi \end{cases}$$

b. At **t = π/4**,

$$(x(t), y(t)) = \begin{cases} (t, \sin t) & \text{if } t \leq \frac{\pi}{4} \\ (t, 1 - \cos t) & \text{if } t > \frac{\pi}{4} \end{cases}$$

c. At **t = 1**,

$$(x(t), y(t)) = \begin{cases} (6t, t^3) & \text{if } t \leq 1 \\ (t^4 + 5, t^2) & \text{if } t > 1 \end{cases}$$

d. At **t = 1**,

$$(x(t), y(t)) = \begin{cases} (t, t^2) & \text{for } t \leq 1 \\ (t, t^2 + (t-1)^3) & \text{for } t > 1 \end{cases}$$

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26. Find the explicit **representation** of a **quadratic** curve going through the following 3 points using the **Lagrange** Polynomial:

$P0 = (0, 0)$, $P1 = (1, 2)$, $P2 = (2, 0)$

27. Derive the **Basis Matrix** for the **cubic Bézier** curve.

28. Given four control points $P0 = (1,1)$, $P1 = (2,3)$, $P2 = (4,3)$, and $P3 = (5,1)$, find the point on the **cubic Bézier** curve at **$t = 0.5$**.

29. Given the four control points in 3D:

$P0 = (0,0,0)$, $P1 = (3,6,0)$, $P2 = (6,6,6)$, $P3 = (9,0,6)$

Find the point on the **cubic Bézier** curve at **$t = 0.5$**.

30. Given the four control points in 2D:

$P0 = (0,0)$, $P1 = (2,2)$, $P2 = (4,-2)$, $P3 = (6,0)$

Find the point on the **cubic Bézier** curve at **$t = 0.75$**.

31. Given the first three control points of a cubic **Bézier** curve:

$P0 = (2, 1)$, $P1 = (3, 4)$, $P2 = (5, 6)$

and the point on the curve at $t = 0.5$:

$f(0.5) = (4, 5)$

Find the **fourth control point**, $P3 = (x3, y3)$.

32. You are going to draw 3 cubic **Bézier** curves joined together to form a single smooth composite curve. You have already decided upon the control points for the first and last Bézier curves:

- First Bézier curve (Curve A):
  $A0 = (0, 0)$, $A1 = (1, 2)$, $A2 = (2, 2)$, $A3 = (3, 0)$
- Third Bézier curve (Curve C):
  $C0 = (6, 0)$, $C1 = (7, -2)$, $C2 = (8, -2)$, $C3 = (9, 0)$

You want to insert a **Bézier curve (Curve B)** between them such that the entire 3-curve segment is **C(1) continuous**.

Find the **4 control points-** B0, B1, B2, B3 of the **middle Bézier** curve (Curve B).

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Practice sheet (Curve)

(25) Determine $C(1)$ and $G(1)$ continuity of the following functions at the given points:

(a). At $t = 2\pi$,

$$(x(t), y(t)) = \begin{cases} (t, \sin t) & \text{for } t \le 2\pi \\ (t, 1-\cos t) & \text{for } t > 2\pi \end{cases}$$

To check for C^1 continuity, we need to check C^0.

At $t = 2\pi$;

$(t, \sin t) \rightarrow (2\pi, \sin 2\pi) = (2\pi, 0)$

$(t, 1-\cos t) \rightarrow (2\pi, 1-\cos 2\pi) = (2\pi, 1-1)$
$= (2\pi, 0)$

So, at $t = 2\pi$, it's C^0 and G^0 continuous.

For C^1 continuity:

$$\frac{d}{dt}(x(t), y(t)) = \begin{cases} (1, \cos t) \\ (1, \sin t) \end{cases}$$

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At $t = 2\pi$,

$(1, \cos t) = (1, 1)$

$(1, \sin t) = (1, 0)$

not equal, so it's not C^1 continuous.

To check for G^1 continuity: Take unit vectors of the 2 velocity vectors and check if equal.

$\vec{v_1} = (1, 1)$

$|\vec{v_1}| = \sqrt{1^2 + 1^2} = \sqrt{2}$

$\hat{v}_1 = \dfrac{1i + 1j}{\sqrt{2}} = \left(\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right)$

$\vec{v_2} = (1, 0)$

$|\vec{v_2}| = \sqrt{1^2 + 0} = 1$

$\hat{v}_2 = \dfrac{1i + 0j}{1} = (1, 0)$

$\left(\dfrac{1}{\sqrt{2}}, \dfrac{1}{\sqrt{2}}\right) \neq (1, 0)$

So, it's not G^1 continuous.

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(b) At $t = \pi/4$,

$$(x(t), y(t)) = \begin{cases} (t, \sin t) & \text{if } t \le \frac{\pi}{4} \\ (t, 1-\cos t) & \text{if } t > \frac{\pi}{4} \end{cases}$$

For C^0 continuous:

At $t = \frac{\pi}{4}$

$$(t, \sin t) = \left(\pi/4, \frac{1}{\sqrt{2}}\right)$$

$$(t, 1-\cos t) = \left(\frac{\pi}{4}, 1-\frac{1}{\sqrt{2}}\right) = \left(\frac{\pi}{4}, \frac{2-\sqrt{2}}{2}\right)$$

$$\left(\pi/4, \frac{1}{\sqrt{2}}\right) \ne \left(\frac{\pi}{4}, \frac{2-\sqrt{2}}{2}\right)$$

So it's not C^0 continuous. If not C^0 then it cannot be C^1 continuous and G^0 continuous. So, it also cannot be G^1 continuous.

(c) At $t = 1$,

$$(x(t), y(t)) = \begin{cases} (6t, t^3) & \text{if } t \le 1 \\ (t^4+5, t^2) & \text{if } t > 1 \end{cases}$$

For C^0 continuous: at $t = 1$

$$(6t, t^3) = (6, 1)$$

$$(t^4+5, t^2) = (6, 1)$$

equal, so it's C^0 and G^0 continuous.

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For C^1 continuous:

$$\frac{d}{dt}((x(t)),(y(t))) = \begin{cases} (6, 3t^2) \\ (4t^3, 2t) \end{cases}$$

At $t=1$,

$(6, 3t^2) = (6, 3)$

$(4t^3, 2t) = (4, 2)$

$(6, 3) \neq (4, 2)$

So it's not C^1 continuous.

For G^1 continuous:

$\vec{V_1} = (6, 3)$

$$\hat{V_1} = \frac{\vec{V_1}}{|\vec{V_1}|} = \frac{(6,3)}{\sqrt{6^2+3^2}} = \left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$$

$\vec{V_2} = (4, 2)$

$$\hat{V_2} = \frac{\vec{V_2}}{|\vec{V_2}|} = \frac{(4,2)}{\sqrt{4^2+2^2}} = \left(\frac{2}{\sqrt{5}}, \frac{1}{\sqrt{5}}\right)$$

$\hat{V_1} = \hat{V_2}$

So, it's G^1 continuous.

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(d) At $t = 1$,

$$(x(t), y(t)) = \begin{cases} (t, t^2) & \text{for } t \le 1 \\ (t, t^2 + (t-1)^3) & \text{for } t > 1 \end{cases}$$

For C^0 continuous:

At $t = 1$:

$(t, t^2) = (1, 1)$

$(t, t^2 + (t-1)^3) = (1, 1)$

So, it's C^0 and G^0 continuous.

For C^1 continuous:

$$\frac{d}{dt}(x(t), y(t)) = \begin{cases} (1, 2t) \\ 1, 2t + 3(t-1)^2 \end{cases}$$

at $t = 1$:

$(1, 2t) = (1, 2)$

$(1, 2t + 3(t-1)^2) = (1, 2)$

Equal, so it's C^1 continuous. If C^1 continuous then it's automatically G^1 continuous.

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(26) Find the explicit representation of a quadritic curve going through the following 3 points using the lagrange polynomial:

$P_0 = (0,0)$, $P_1 = (1,2)$, $P_2 = (2,0)$.

degree of P(x)

$$\sum_{k=0}^{n} y_k l_k(x) = f(x) \qquad \Big|\ n = 3-1 = 2$$

$$P_2(x) = l_0(x)\overset{y_0}{f(x_0)} + l_1(x)\overset{y_1}{f(x_1)} + l_2(x)\overset{y_2}{f(x_2)}$$

$$l_0(x) = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)} = \frac{(x-1)(x-2)}{(0-1)(0-2)} = \frac{(x-1)(x-2)}{2}$$

$$l_1(x) = \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)} = \frac{(x-0)(x-2)}{(1-0)(1-2)} = -(x)(x-2)$$

$$l_2(x) = \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)} = \frac{(x-0)(x-1)}{(2-0)(2-1)} = \frac{x(x-1)}{2}$$

$$P_2(x) = \frac{(x-1)(x-2)}{2} \times 0 \; -2x(x-2) + \frac{x(x-1)}{2} \times 0$$

$$= -2x^2 + 4x$$

$$\therefore P_2(x) = 4x - 2x^2$$

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(27) Derive the Basis Matrix for the cubic Bézier curve.

$$Q(t) = (1 - 3t + 3t^2 - t^3) P1 + (3t - 6t^2 + 3t^3) P2 + (3t^2 - 3t^3) P3 + t^3 P4$$

$$Q(t) = \begin{bmatrix} (1 - 3t + 3t^2 - t^3) & (3t - 6t^2 + 3t^3) & (3t^2 - 3t^3) & t^3 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix}$$

$$Q(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix}$$

$$Q(t) = T \cdot M_B \cdot G_B$$

here M_B is the basis matrix of Bézier curve.

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(28) Given four control points $P_0 = (1,1)$, $P_1 = (2,3)$, $P_2 = (4,3)$, $P_3 = (5,1)$. Find the point on the cubic Bézier curve at $t = 0.5$.

$$Q(t) = (1-t)^3 P_0 + 3t(1-t)^2 P_1 + 3t^2(1-t) P_2 + t^3 P_3$$

$$Q(0.5) = (1-0.5)^3 (1,1) + 3(0.5)(1-0.5)^2 (2,3) + 3(0.5)^2(1-0.5)(4,3) + 0.5^3 (5,1)$$

$$= (0.125, 0.125) + (0.75, 1.125) + (1.5, 1.125) + (0.625, 0.125)$$

$$= (3.00, 2.5)$$

(29) Given four control points in 3D:

$P_0 = (0,0,0)$, $P_1 = (3,6,0)$, $P_2 = (6,6,6)$, $P_3 = (9,0,6)$

Find the point on the cubic Bezier curve at $t = 0.5$

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$$Q(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

$$Q(0.5) = \begin{bmatrix} 0.125 & 0.25 & 0.5 & 1 \end{bmatrix} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

$$= \begin{bmatrix} 0.125 & 0.375 & 0.375 & 0.125 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

$$= 0.125 P_0 + 0.375 P_1 + 0.375 P_2 + 0.125 P_3$$

$$= 0.125(0,0,0) + 0.375(3,6,0) + 0.375(6,6,6) + 0.125(9,0,6)$$

$$= (0 + 1.125 + 2.25 + 1.125,\ 0 + 2.25 + 2.25 + 0,\ 0 + 0 + 2.25 + 0.75)$$

$$= (4.5,\ 4.5,\ 3)$$

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30) $P_0 = (0,0)$, $P_1 = (2,2)$, $P_2 = (4,-2)$, $P_3 = (6,0)$

Find the point on the cubic Bézier curve at $t = 0.75$

$$Q(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

$$Q(0.75) = \begin{bmatrix} 0.42 & 0.56 & 0.75 & 1 \end{bmatrix} \begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 3 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{bmatrix}$$

$= 0.01 P_0 + 0.01 P_1 + 0.01 P_2 + 0.01 P_3$

$= 0.01(0,0) + 0.01(2,2) + 0.01(4,-2) + 0.01(6,0)$

$= (0,0) + (0.02, 0.02) + (0.04, -0.02) + (0.06, 0)$

$= (0.12, 0)$

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(31) Given the first three control points of a cubic Bézier curve:

$P_0 = (2,1)$, $P_1 = (3,4)$, $P_2 = (5,6)$

and the point on the curve at $t = 0.5$:

$f(0.5) = (4,5)$. Find the fourth control point, $P_3 = (x_3, y_3)$

$$f(t) = (1-t)^3 P_0 + 3t(1-t)^2 P_1 + 3t^2(1-t) P_2 + t^3 P_3$$

$$\Rightarrow (4,5) = (1-0.5)^3 P_0 + 3(0.5)(1-0.5)^2 P_1 + 3(0.5)^2(1-0.5) P_2 + 0.5^3 P_3$$

$$\Rightarrow (4,5) = 0.125\,(2,1) + 0.375\,(3,4) + 0.375\,(5,6) + 0.125\,(x_3, y_3)$$

$$\Rightarrow (4,5) = (0.25, 0.125) + (1.125, 1.5) + (1.875, 2.25) + 0.125\,(x_3, y_3)$$

$$\Rightarrow (4,5) - (3.25, 3.875) = 0.125\,(x_3, y_3)$$

$$\Rightarrow (x_3, y_3) = (6, 9)$$

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(32)

For the spline to be C^1 continuous, we have to first ensure C^0 continuity.

Hence, all 3 curve segments need to be connected. So, whenever curve A will end, B needs to start from there.

$\therefore$ End point of A, A_3 = starting point of B, B_0

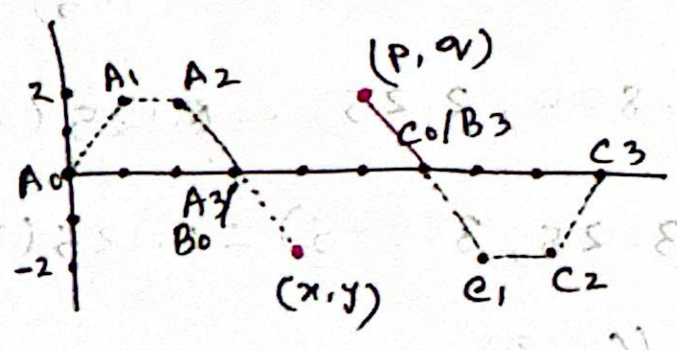
$\therefore A_3 = \boxed{B_0 = (3,0)}$

Similarly, whenever curve B will end, C needs to start from there.

End point of B, B_3 = starting point of C, C_0

$\therefore C_0 = \boxed{B_3 = (6,0)}$

For B1 and B2:



B1 needs to be pointed (x, y) place to maintain C^1 continuity, the target vector at A3 must be equal on both sides.

So A3 is the mid point of the line segment from A2 to B1.

$$\therefore (3, 0) = \left(\frac{x+2}{2}, \frac{y+2}{2}\right)$$

$$\Rightarrow \frac{x+2}{2} = 3 \qquad \text{again } \frac{y+2}{2} = 0$$

$$\therefore x = 4 \qquad \therefore y = -2$$

$$\boxed{B1 = (4, -2)}$$

Similarly, B2 must be in the pointed (p, q) place, so that the magnitude and direction of target at C0 is equal on both sides.

So, C0 is the midpoint of the line segment from B2 to C1.

$$\therefore (6, 0) = \left(\frac{p+7}{2}, \frac{q-2}{2}\right)$$

$$\frac{p+7}{2} = 6 \qquad \frac{q-2}{2} = 0$$

$$\therefore p = \cancel{6}5 \qquad \therefore q = 2$$

$$\boxed{B2 = (5, 2)}$$

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Alternative:

for C^1 continuous:

$$P5 = 2P4 - P3$$

P3 = A2 (3rd control point of A curve)

P4 = A3 (4th " " of A ")

P5 = B1 (2nd " " of B ")

$\therefore$ $B1 = 2(3,0) - (2,2)$

$= (6,0) - (2,2) = (4,-2)$

Similarly,

P3 = B2 (3rd control point of B curve)

P4 = B3 (4th " " of B ")

P5 = C1 (2nd " " of C curve)

$\therefore$ $C1 = 2(6,0) - (x,y)$

$\Rightarrow (7,-2) = (12,0) - (x,y)$

$\therefore$ $(x,y) = (12,0) - (7,-2)$

$B2 = (5,2)$

$B0 = (3,0)$

$B3 = (6,0)$

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