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Comp 330 - Assignment 4

Question 1

The given language generate any string with a equals number of a and b. The grammar can produce either ϵ, $aSb\ or\ bSa$ so it's always in pair so we always have an equals number of a and b's. Now it can also be a combination of two of those string but as both have them same number of a and b's then its still working.

We can prove using induction the inverse.

Base case: the empty string with no a's

Let's assume that any string s in L with at most n a's is generated by the grammar. The goal is to show any string with $n + 1$ a's in L is also generated by the grammar. We now have two case:

- W starts and ends with the same letter (a or b) $w = aw_0a$ so w_0 contains to more bs than as and we can split w in two part(x and y) such that both part contains exactly one more b than a. So we have $w = axya$ so $ax\ and\ ya$ now have the same number of as and bs So it can be generated by the grammar: $S \to SS$
- W starts and ends with different letters, $w = aw_0b$(orbw_0a) So as w_0 as less than n a's then we can use $S \to aSb$ to generate this string.

So by induction it's true for all $n \geq 0$

Question 2

a. Let's make the string a+++a using two different tree

$$S \to V + V$$

$$\to a + + + I$$

$$\to a + + + a$$

$$S \to V + V$$

$$S \to I + + + a$$

$$S \to a + + + a$$

So we have the same string a+++a but they don't mean the same a++ + a and a + ++a

b. Grammar 1:

$$S \to I\ +\ V | V + V | I + I$$
$$V \to < pre > | < post >$$
$$I \to a | b | c | \ldots$$

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$$< pre > \to + + I$$

Grammar 2

$$S \to \ V + I | V + V | I + I$$
$$V \to < pre > | < post >$$
$$I \to a | b | c | \ldots$$
$$< post > \to I + +$$

So we remove the ambiguity by putting the two different generation tree that were ambiguous in those to grammar.

Question 3

$$S \to ASB | AB | CSD | CD | TT | e$$

$$S \to ASB | AB | CSD | CD | TT$$

$$A \to ($$

$$B \to)$$

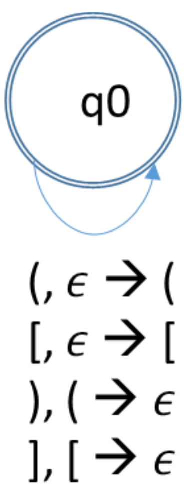
$$C \to [$$

$$D \to]$$

ASB and CSD for 2., A

Question 4

The stack can contains two element (and [so we can keep track of both () and []



Question 5

$$S \to T | AU | SC$$

$$A \to aA | \epsilon$$

$$B \to bB | \epsilon$$

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$$C \to Cc|\epsilon$$

$$T \to aTc|B$$

$$U \to bTc|\epsilon$$

We have two cases, the first check we have no less c's than a's and the th second that we have no less b's than c's. So T will do a word with the same amount of as and cs and any amount of b's in the middle (B)

U will generate a word with the same amount of c and bs. So AU will add any amount of A's at the beginning

Let's prove this grammar can generate any word in L. So let $w = a^n b^m c^p$ now let $q = \min(m, n)$

So we can rewrite w: $w = a^n b^m c^q c^{p-q} = w_1 w_2, s.t\ w_1 = a^n b^m c^q\ and\ w_2 = c^{p-q}$. w_2 is generated by C so if w_1 is also generated by the grammar then w is too($S \to SC$). Suppose we have $n \leq m$ then $w_1 = a^n b^m c^n$ So we can use the rule T to generate n as and cs then the rule B tp have m bs. Simirlarty if $m \geq n$ then we can use the rule$S \to AU$ with U generating n b and c and A m as. So we can generate w_1 and w_2 using the grammar and thus w can also be generated using the grammar $S \to SC$

Question 6

Let find a counter example by taking the previous language where m,n,p are all strictly positive. MIN (L) is in fact the special case where $p = \min(m, n)$. So we have $MIN(L) = \{a^n b^m c^{\min(m,n)}, m, n \geq 0\}$

Let take $s = a^p b^p c^p$

Now let $s = uvxyz$, with u, v, x, y, z substring of s and $|vxy| \leq p$ and $|vy| \geq 1$. So we can easely see that with those restriction vxy can only be composed of 2 letters. So we have the following cases:

- vxy is only compose of a or b's then uv^0xy^0z then the amount of a's (or b's) is smaller than the amount of c and as c need to be the minimum it's not working
- vxy is only compose of c then uv^0xy^0z then the amount of c is smaller than the amount of a or b
- vxy is compose of a's and b's then uv^2xy^2z must have more a's and b's than c's
- vxy is compose of b's and c's then uv^2xy^2z then we have more c's than a's that cannot be possible as the number of c's needs to be the min(n,m)

So all cases fail to be in $Min(L)$ so by the pumping lemma $Min(L)$ is not context free