Timothee Guerin
260447866

Comp 330 – Assignment 6

Question 1

As there is only one letter this come to check if we can make two string of the same size. So we have multiple case. First case for all domino the size of the top is larger than the size of the bottom so there is no solution, similarly if all the bottoms are larger than the tops there is no solution. Last case for each domino we can assign a number to it which is the size of top minus the size of bottom. Now we for each subset

we just have to sums up those values and if the result is 0 then it's a solution.

Question 2

We can reduce the post correspondence problem to this one. Let $d_1, \ldots, d_n$ be a set of dominos. Now for $i = 1, \ldots, n$ we have the top to be w_i and the bottom v_i

$$W \rightarrow\ w_1W\ d_1 \mid w_2W\ d_2 \mid \ldots \mid w_nW\ d_n \mid w_1\ d_1 \mid w_2\ d_2 \mid \ldots \mid w_n\ d_n \mid$$

and

$$V \rightarrow\ v_1Vd_1 \mid v_2X\ d_2 \mid \ldots \mid v_nX\ d_n \mid v_1\ d_1 \mid v_2\ d_2 \mid \ldots \mid v_n\ d_n \mid$$

We see that the PCS has a match exactly when the intersection of the languages generated by the resulting grammars above is nonempty. But we know that PCS is undecidable when the matching set is empty so this is undecidable too.

Question 3

We are going to check radius by radius. So the square of 2by2 is radius 0. So we start at radius 0(Position doesn't matter) but to check this radius we need 4 step so the submarine would have moved 4 times. So we are going to increase the radius by 0,1,2,1,2,3,1,2,3,4,… and as the submarine have a finite speed we will catch him sooner or later. Now as it took us n step to check a radius(4 in the first one) we need to multiply the increase by this n. This insure we will catch the submarine at a finite number of steps.

Question 4

1. We can make a NFA that either stay at the same state(so skip the letter) or go to the DFA for the language L. So the only words that work are the words that finish with a word in L. Then this language is regular.
2. Let $N = \{a^n b^n | n \geq 0\}$, and $L = N\#\Sigma^* \cup \Sigma^*\#L(G)$. Let suppose $L(G) = \Sigma^*$ then we can rewrite L to be $\Sigma^*\#\Sigma^*$ which is regular by part 1 as Σ^* is regular. Now let's suppose $L(G)$ is not Σ^* , let's take $w \notin L(G)$ then $L \cap \Sigma^*\#w = N\#w$ which is not regular then L is not regular. So L is regular if and only if $L(G) = \Sigma^*$. Now Let's take a context free grammar G' such that $L(G') = N\#\Sigma^* \cup \Sigma^*\#L(G)$. But to prove $L(G')$ is regular we also have to prove $L(G)$ is Σ^* but its undecidable so prove $L(G')$ is regular is undecidable.

Question 5

- $R \subset L$

This is undecidable. Let's for example take the regular language $R = \Sigma^*$ then for $R \subset L$ to hold we must have $L = \Sigma^*$ but its undecidable if a context free language can produce Σ^*

- $L \subset R$

This is decidable. We can take the complement of R, if $\overline{R}$ is regular then $\overline{R}$ then $\overline{R} \cap L = \emptyset$. So checking the intersection between a regular language and a context free lamague is decidable. If $\overline{R}$ is not regular then return false.