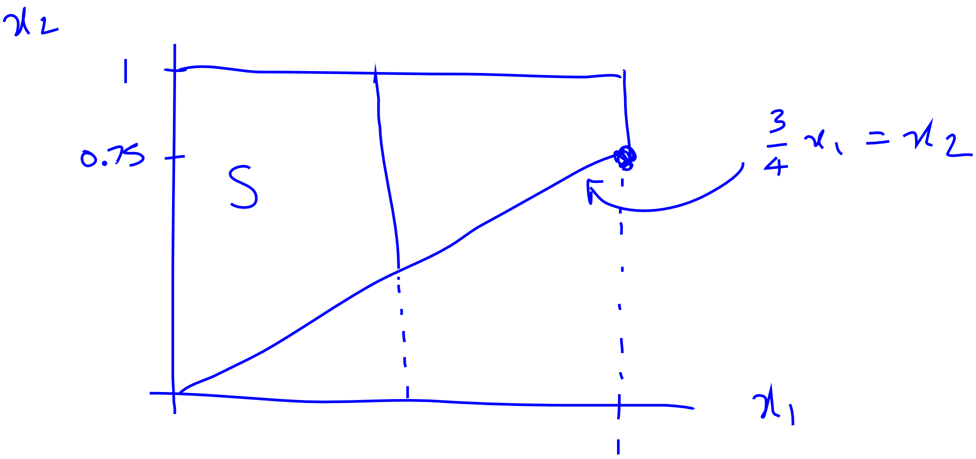
Exercise 6.25

First draw a diagram of the rejection region S as a subset of the unit square. Since $S = \{3x_1 < 4x_2\} = \{\frac{3}{4}x_1 < x_2\}$ we have:



The joint density of X_1 and X_2 is (by independence):

$$f_{X_1X_2}(x_1,x_2) = \theta x_1^{\theta-1}\,\theta x_2^{\theta-1} = \theta^2(x_1x_2)^{\theta-1}$$

Hence

$$E_\theta\varphi^* = P(3X_1 < 4X_2) = \iint_S f_{X_1X_2}(x_1,x_2)\,dx_1\,dx_2$$

$$= \int_0^1\int_{\frac{3}{4}x_1}^1 \theta^2(x_1x_2)^{\theta-1}\,dx_2\,dx_1$$

$$= \theta^2\int_0^1 x_1^{\theta-1}\left[\frac{x^\theta}{\theta}\right]_{\frac{3}{4}x_1}^1 dx_1$$

$$= \theta^2\int_0^1 x_1^{\theta-1}\cdot\left[\frac{1}{\theta} - \frac{1}{\theta}\left(\frac{3}{4}x_1\right)^\theta\right]dx_1$$

$$= \theta\left[\int_0^1 x_1^{\theta-1}\,dx - \left(\frac{3}{4}\right)^\theta\int_0^1 x_1^{2\theta-1}\,dx_1\right]$$

$$= \theta\left[\frac{x_1^\theta}{\theta}\Big|_0^1 - \left(\frac{3}{4}\right)^\theta \frac{x_1^{2\theta}}{2\theta}\Big|_0^1\right]$$

$$= 1 - \frac{1}{2}\left(\frac{3}{4}\right)^\theta$$

Size: $E_{\theta=1}\varphi^* = 1 - \frac{1}{2}\left(\frac{3}{4}\right)^1 = \frac{5}{8}$

This is not a good test since the size is too high!

Exercise 6.27

Take any $\sigma_1 > \sigma_0$ fixed and test $H_0 : \sigma = \sigma_0$ vs $H_1 : \sigma = \sigma_1$.

Then the NP lemma leads to the test

$$\varphi^*(X) = \begin{cases} 1 & \text{if } \dfrac{L(X,\sigma_1)}{L(X,\sigma_0)} > C \\ 0 & \text{if } \dfrac{L(X,\sigma_1)}{L(X,\sigma_0)} < C \end{cases}$$

Now
$$\frac{L(X,\sigma_1)}{L(X,\sigma_0)} = \frac{(2\pi\sigma_1^2)^{-n/2} \exp\left(-\frac{1}{2\sigma_1^2}\sum_{i=1}^n X_i^2\right)}{(2\pi\sigma_0^2)^{-n/2} \exp\left(-\frac{1}{2\sigma_0^2}\sum_{i=1}^n X_i^2\right)}$$

$$= \underbrace{\left(\frac{\sigma_0}{\sigma_1}\right)^n}_{>0} \exp\left(\frac{1}{2}\sum_{i=1}^n X_i^2 \underbrace{\left(\frac{1}{\sigma_0^2} - \frac{1}{\sigma_1^2}\right)}_{>0}\right) \geq k$$

Since $\sigma_1^2 > \sigma_0^2 \Rightarrow \frac{1}{\sigma_1^2} < \frac{1}{\sigma_0^2} \Rightarrow 0 < \frac{1}{\sigma_0^2} - \frac{1}{\sigma_1^2}$

Hence $\frac{L(X,\sigma_1)}{L(X,\sigma_0)}$ is monotonically increasing in the statistic $\sum_{i=1}^n X_i^2$

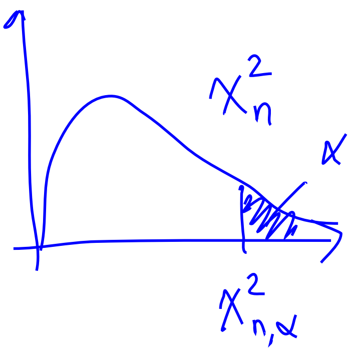
and hence our rejection region reduces to $\left\{\sum_{i=1}^n X_i^2 \geq C\right\}$.

To determine C we "exhaust the α-level":

$$\alpha = E_{\sigma_0}\varphi^* = P_{\sigma_0}\left(\sum_{i=1}^n X_i^2 \geq C\right)$$

$$= P_{\sigma_0}\left(\frac{\sum_{i=1}^n X_i^2}{\sigma_0^2} \geq \frac{C}{\sigma_0^2}\right)$$

$$= P\left(\chi_n^2 \geq \frac{C}{\sigma_0^2}\right)$$



$$\Rightarrow \frac{C}{\sigma_0^2} = \chi_{n,\alpha}^2 \Rightarrow C = \sigma_0^2 \chi_{n,\alpha}^2$$

Therefore, for testing $H_0: \sigma = \sigma_0$ vs $\bar{H}_1: \sigma = \sigma_1$ we have

$$\varphi^* = \begin{cases} 1 & \text{if } \sum_{i=1}^n X_i^2 \geq \sigma_0^2 \chi^2_{n,\alpha} \\ 0 & \text{if } \sum_{i=1}^n X_i^2 < \sigma_0^2 \chi^2_{n,\alpha} \end{cases}.$$

Instead of applying the monotonicity argument of $\frac{L(X,\sigma_1)}{L(X,\sigma_1)}$ in $T = \sum_{i=1}^n X_i$ we could rearrange the rejection region:

$$\left\{ \frac{L(X,\sigma_1)}{L(X,\sigma_0)} \geq k \right\}$$

as follows (requires some algebra):

$$\left\{ \sum_{i=1}^n X_i^2 \geq \frac{2\log\left(k\left(\frac{\sigma_0}{\sigma_1}\right)^n\right)}{\left(\frac{1}{\sigma_0^2} - \frac{1}{\sigma_1^2}\right)} = k' \right\}.$$

Since $\frac{1}{\sigma_0^2} - \frac{1}{\sigma_1^2} > 0$.

Also, this same test φ^* turns out to be UMP α-test of $H_0: \sigma = \sigma_0$ vs the original $H_1: \sigma > \sigma_0$. Indeed, although it was constructed as the most powerful test of a simple hypothesis against a simple alternative $\bar{H}_1: \sigma = \sigma_1$, the structure of the test does <u>NOT</u> involve the specific alternative value σ_1. Hence, it's power is maximal for any $\sigma_1 > \sigma_0$!

Exercise 6.28

Since $N(\theta,1)$ is a one-parameter exponential family with $d(x) = x$ and $c(\theta) = \theta$ which is monotonically increasing, we have the MLR property in the statistic $T(X) = \sum_{i=1}^{n} X_i$.

Direct application of Blackwell and Girshick Theorem gives the structure of the UMP α-test:

$$\varphi^*(X) = \begin{cases} 1 & \text{if} \quad T(X) = \sum_{i=1}^{n} X_i > K \\ 0 & \text{if} \quad T(X) = \sum_{i=1}^{n} X_i < K \end{cases}.$$

To find K we exhaust the level:

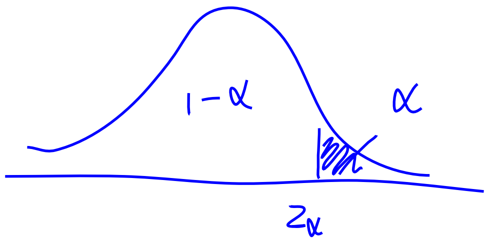
$$E_{\theta_0}\varphi^* = \alpha = 1 \cdot P\left(\sum_{i=1}^{n} X_i > K \mid \theta = \theta_0\right)$$

$$= P\left(\bar{X} > \frac{K}{n} \mid \theta = \theta_0\right) \qquad \text{now } \bar{X} \sim N(\theta_0, \tfrac{1}{n}) \text{ under } H_0$$

$$= P\left(\frac{\bar{X} - \theta_0}{1/\sqrt{n}} > \frac{\frac{K}{n} - \theta_0}{1/\sqrt{n}} \mid \theta = \theta_0\right)$$

$$= P\left(Z > \sqrt{n}\left(\frac{K}{n} - \theta_0\right)\right)$$

where $Z \sim N(0,1)$.



$P(Z > z_\alpha) = \alpha$

Therefore $\sqrt{n}\left(\frac{K}{n} - \theta_0\right) = z_\alpha$ must hold and

$$K = n\left(\frac{z_\alpha}{\sqrt{n}} + \theta_0\right) = \sqrt{n}\, z_\alpha + n\theta_0$$

Hence, the UMP α-test is

$$\varphi^*(X) = \begin{cases} 1 & \text{if} \quad \sum_{i=1}^{n} X_i > n\theta_0 + \sqrt{n}\, z_\alpha \\ 0 & \text{if} \quad \sum_{i=1}^{n} X_i < n\theta_0 + \sqrt{n}\, z_\alpha \end{cases}$$

This is the same test as

$$\varphi^*(X) = \begin{cases} 1 & \text{if} \quad \bar{X} > \theta_0 + \frac{1}{\sqrt{n}} z_\alpha \\ 0 & \text{if} \quad \bar{X} < \theta_0 + \frac{1}{\sqrt{n}} z_\alpha \end{cases}$$

as in Example 6.48

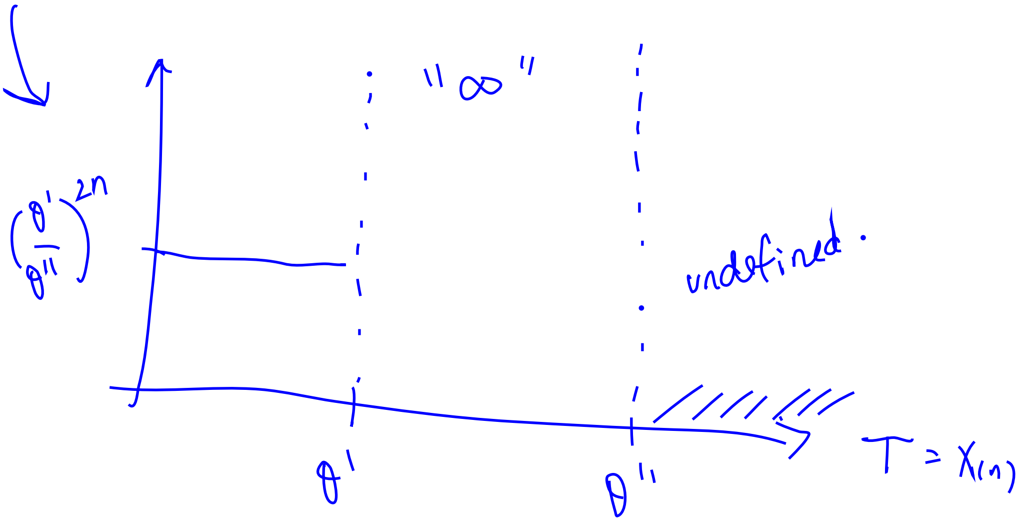
Exercise 6.29

Claim: the family $L(X,\theta)$ is a MLR family in the statistic $T = X_{(n)}$.

$$L(X,\theta) = \prod_{i=1}^{n} \frac{2x_i}{\theta^2} I_{(x_i,\infty)}(\theta) = \frac{2^n}{\theta^{2n}} \prod_{i=1}^{n} X_i \, I_{(X_{(n)},\infty)}(\theta)$$

Now take two values $0 < \theta' < \theta''$ and then

$$\frac{L(X,\theta'')}{L(X,\theta')} = \left(\frac{\theta'}{\theta''}\right)^{2n} \frac{I_{(X_{(n)},\infty)}(\theta'')}{I_{(X_{(n)},\infty)}(\theta')} \quad \text{"function of } T\text{"}$$



Hence we have the MLR property in $T = X_{(n)}$. Then the BG theorem provides the following UMP α-test for testing H_0 vs H_1 given by

$$\varphi^* = \begin{cases} 1 & \text{if } X_{(n)} > k \\ 0 & \text{if } X_{(n)} < k \end{cases}$$

To find k we must exhaust the α-level:

$$\alpha = E_{\theta_0}\varphi^* = P_{\theta_0}(X_{(n)} > k) = 1 - P_{\theta_0}(X_{(n)} < k)$$

$$= 1 - [P_{\theta_0}(X_1 < k)]^n$$

$$F(x) = \int_0^x \frac{2t}{\theta^2} dt = \frac{t^2}{\theta^2}\Big|_0^x = \frac{x^2}{\theta^2}$$

$$= 1 - \left(\frac{k^2}{\theta_0^2}\right)^n = 1 - \left(\frac{k}{\theta_0}\right)^{2n} = \alpha$$

Hence $k = \theta_0(1-\alpha)^{1/2n}$ and φ^* is completely determined:

$$\varphi^* = \begin{cases} 1 & \text{if } X_{(n)} > k = \theta_0(1-\alpha)^{1/2n} \\ 0 & \text{if } X_{(n)} < k = \theta_0(1-\alpha)^{1/2n} \end{cases}$$

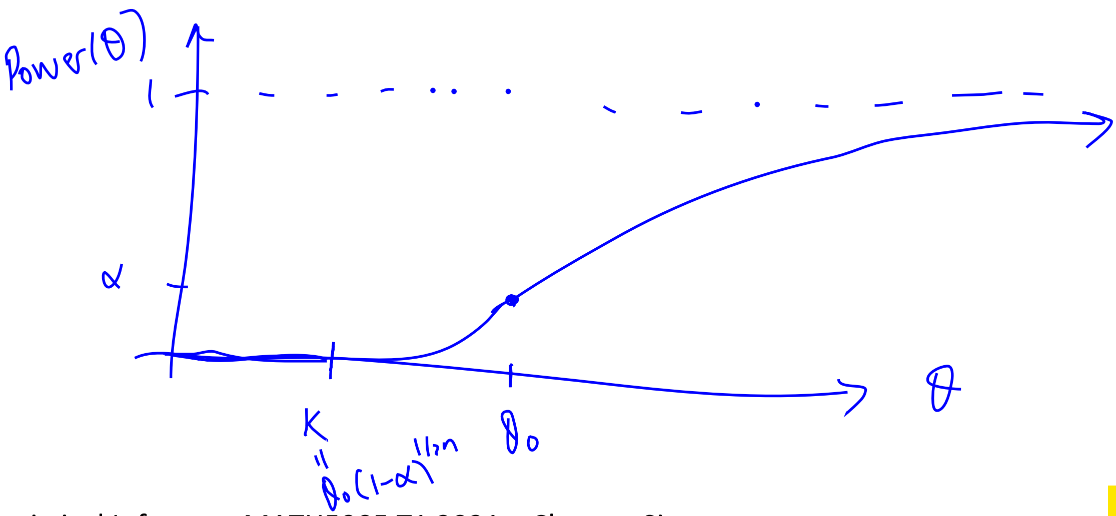
Now the power function is:

$$\begin{aligned} Power(\theta) = E_\theta \varphi^* &= P(X_{(n)} > \theta_0(1-\alpha)^{1/2n} \mid \theta = \theta) \\ &= 1 - P(X_{(n)} < \theta_0(1-\alpha)^{1/2n} \mid \theta = \theta) \\ &= 1 - [P_\theta(X_1 < \theta_0(1-\alpha)^{1/2n})]^n \\ &= 1 - \left(\frac{\theta_0(1-\alpha)^{1/2n}}{\theta}\right)^{2n} \\ &= 1 - \left(\frac{\theta_0}{\theta}\right)^{2n}(1-\alpha), \quad \theta > k = \theta_0(1-\alpha)^{1/2n} \end{aligned}$$

$$Power(k) = 1 - \left(\frac{\cancel{\theta_0}}{\cancel{\theta_0}(1-\alpha)^{1/2n}}\right)^{2n}(1-\alpha) = 1 - \frac{1-\alpha}{1-\alpha} = 1 - 1 = 0$$

$$E_{\theta_0}\varphi^* = Power(\theta_0) = 1 - \left(\frac{\cancel{\theta_0}}{\cancel{\theta_0}}\right)^{2n}(1-\alpha) = 1 - 1 + \alpha = \alpha$$

$$\lim_{\theta \to \infty} Power(\theta) = 1 - (1-\alpha)\lim_{\theta \to \infty}\left(\frac{\theta_0}{\theta}\right)^{2n} = 1 - (1-\alpha).0 = 1$$



Exercise 6.30

First the UMP α-test exist because of the Blackwell and Girshick Theorem since

$$f(x,\theta) = \theta^x (1-\theta)^{1-x} = (1-\theta)\left(\frac{\theta}{1-\theta}\right)^x = (1-\theta)\exp\left(x\log\frac{\theta}{1-\theta}\right)$$

is one-parameter exponential family with $c(\theta) = \log\frac{\theta}{1-\theta}$ monotonically increasing in $\theta \in (0,1)$ and $d(x) = X$. Hence we have the MLR property in the statistic $T(X) = \sum_{i=1}^n X_i$.

The B-G Theorem tells us that the structure of the UMP α-test is

$$\varphi^*(X) = \begin{cases} 1 & \text{if } \sum_{i=1}^n X_i > K \\ \gamma & \text{if } \sum_{i=1}^n X_i = K \\ 0 & \text{if } \sum_{i=1}^n X_i < K \end{cases}$$

To determine K and γ we have to find the smallest natural number k for which $P_\theta(T > K) \le \alpha$. Since

$$T = \sum_{i=1}^{25} X_i \sim \text{Binomial}(25, 0.15)$$

under the borderline $\theta_0 = 0.15$ value, we can find these probabilities

$$P_{\theta_0}(T = t) = \binom{25}{t} 0.15^t\, 0.85^{25-t} \qquad t = 0,1,2,\dots,25$$

and tabulate the entire CDF (or ask the computer to do it for us) to extract the distribution of T:

X	....	7	8	9	....
$P(T \le X)$		0.974532	0.99207	0.99786	
		X	✓	✓	

We want $P(T > x) < 0.01$ and x should be the smallest with this property, which implies that $K = 8$. Then to exhaust the level:

$$\alpha = E_{\theta_0}\varphi^* = 1 \cdot P(T > 8) + \gamma \underbrace{P(T = 8)}_{P(T \le 8) - P(T \le 7)}$$

$$0.01 = (1 - 0.99207) + \gamma(0.99207 - 0.974532)$$

$$0.01 = 0.00793 + 0.017538\gamma$$

which implies that

$$\gamma = \frac{0.01 - 0.00793}{0.017538} = 0.118$$

and the UMP α-test $(\alpha = 0.01)$ is completely determined:

$$\varphi^*(x) = \begin{cases} 1 & \text{if} \quad \sum_{i=1}^{25} X_i > 8 \\ 0.118 & \text{if} \quad \sum_{i=1}^{25} X_i = 8 \\ 0 & \text{if} \quad \sum_{i=1}^{25} X_i < 8 \end{cases}$$

Exercise 6.31

Since $N(0,\sigma^2)$ belongs to the one-parameter exponential family, it has the MLR property in the statistic $T(X)=\sum_{i=1}^n X_i^2$. Hence, the BA theorem gives the following structure for the UMP α-test:

$$\varphi^*(X)=\begin{cases}1 & \text{if } \quad T(X)=\sum_{i=1}^n X_i^2 > C\\ 0 & \text{if } \quad T(X)=\sum_{i=1}^n X_i^2 < C\end{cases}.$$

when testing $H_0:\sigma\le\sigma_0$ versus $H_1:\sigma>\sigma_0$. To determine C we must exhaust the level:

$$\alpha = E_{\sigma_0}\varphi^* = P\left(\sum_{i=1}^n X_i^2 > C \mid \sigma=\sigma_0\right)$$

$$= P\left(\sum_{i=1}^n \frac{X_i^2}{\sigma_0^2} > \frac{C}{\sigma_0^2} \mid \sigma=\sigma_0\right)$$

$$= P\left(\chi^2_n > \frac{C}{\sigma_0^2}\right) \qquad \text{since } \sum_{i=1}^n \frac{X_i^2}{\sigma_0^2}\sim\chi^2_n \text{ undr } H_0$$

Therefore $\frac{C}{\sigma_0^2}=\chi^2_{n,\alpha}$ where $\chi^2_{n,\alpha}$ is the upper $\alpha\times 100\%$ percentage point of the chi-squared distribution with n degrees of freedom. Then $C=\sigma_0^2\chi^2_{n,\alpha}$ and the UMP α-test becomes

$$\varphi^*(X)=\begin{cases}1 & \text{if } \quad \sum_{i=1}^n X_i^2 \geq \sigma_0^2\chi^2_{n,\alpha}\\ 0 & \text{if } \quad \sum_{i=1}^n X_i^2 < \sigma_0^2\chi^2_{n,\alpha}\end{cases}$$

The power function is:

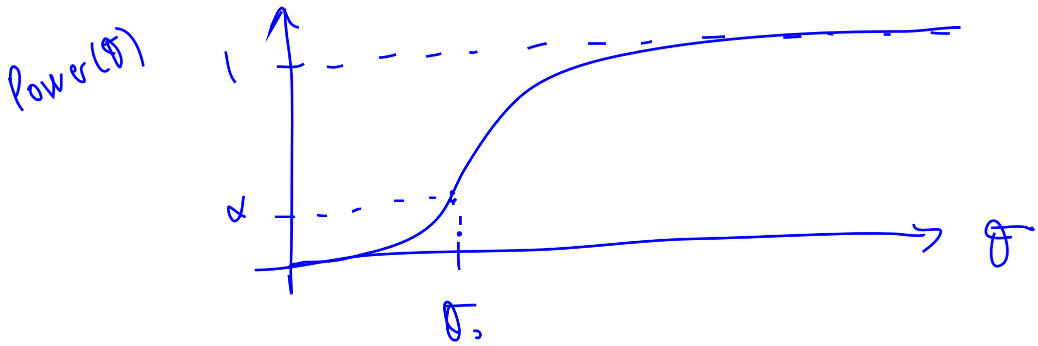
$$\text{Power}(t) = E_{\sigma=t}(\varphi^*) = P\left(\sum_{i=1}^n X_i^2 \geq \sigma_0^2\chi^2_{n,\alpha} \mid \sigma=t\right)$$

$$= P\left(\sum_{i=1}^n \frac{X_i^2}{t^2} \geq \left(\frac{\sigma_0}{t}\right)^2\chi^2_{n,\alpha} \mid \sigma=t\right)$$

$$= P\left(\chi^2_n \geq \left(\frac{\sigma_0}{t}\right)^2\chi^2_{n,\alpha}\right)$$

The power function is an increasing function on $t \in (0,\infty)$ and satisfies:

$$\text{Power}(0) = P(\chi^2_n > \infty) = 0$$
$$\text{Power}(\sigma_0) = P(\chi^2_n > \chi^2_{n,\alpha}) = \alpha$$
$$\lim_{\sigma \to \infty} \text{Power}(\sigma) = P(\chi^2_n > 0) = 1$$



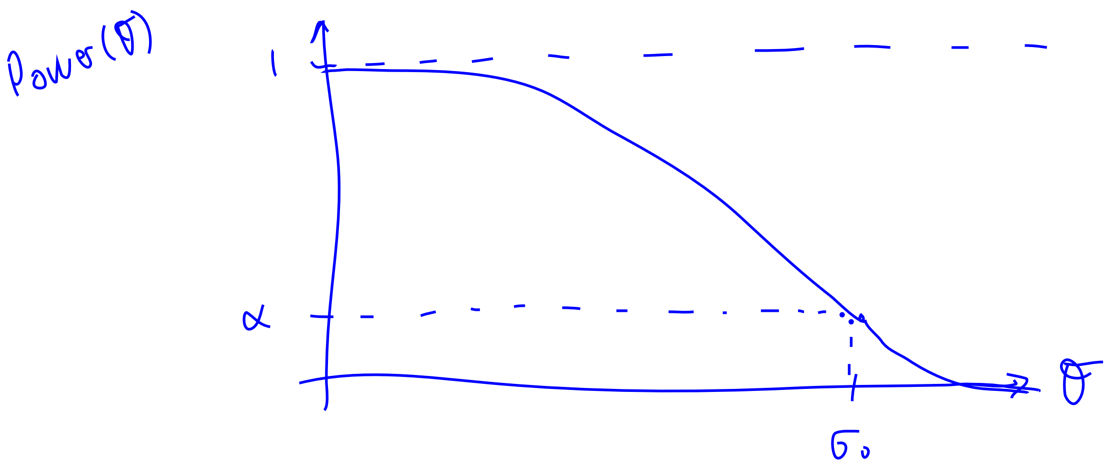
Now suppose that we were testing $H_0: \sigma \geq \sigma_0$ vs $H_1: \sigma < \sigma_0$ then the α-UMP test is given by (show this!):

$$\varphi^*(X) = \begin{cases} 1 & \text{if} \quad \sum_{i=1}^n X_i^2 \leq \sigma_0^2 \chi^2_{n,1-\alpha} \\ 0 & \text{if} \quad \sum_{i=1}^n X_i^2 > \sigma_0^2 \chi^2_{n,1-\alpha} \end{cases}$$

with power function $\text{Power}(t) = P\left(\chi^2_n \leq \left(\frac{\sigma_0}{t}\right)^2 \chi^2_{n,1-\alpha}\right)$.

The graph is now "reversed" since the hypothetical and alternative region have been changed:

$$\text{Power}(0) = P(\chi^2_n \leq 0) = 0$$
$$\text{Power}(\sigma_0) = P(\chi^2_n \leq \chi^2_{n,1-\alpha}) = \alpha$$
$$\lim_{\sigma \to \infty} \text{Power}(\sigma) = P(\chi^2_n \leq \infty) = 1$$



Exercise 6.32

Since f is one-parameter exponential we have $T(X) = \sum_{i=1}^{n} X_i$ has the MLR property. Therefore by the BG Thm the UMP α-test is

$$\varphi^* = \begin{cases} 1 & \text{if } \sum_{i=1}^{n} X_i \geq k \\ 0 & \text{if } \sum_{i=1}^{n} X_i < k \end{cases}$$

Asymptotically for large values of n, we can apply the CLT and use the fact that $EX_i = \theta$ and $Var X_i = \theta^2$ then to exhaust the α-level:

$\bar{X} \overset{a}{\sim} N(\theta, \frac{\theta_0^2}{n})$ under H_0

$$\alpha = E_{\theta_0}\varphi^* = P(\sum X_i \geq k \mid \theta = \theta_0) =$$

$$= P(\bar{X} \geq \frac{k}{n} \mid \theta = \theta_0)$$

$$= P\left(\frac{\bar{X} - \theta_0}{\theta_0/\sqrt{n}} \geq \frac{(\frac{k}{n}) - \theta_0}{\theta_0/\sqrt{n}} \mid \theta = \theta_0\right)$$

$$= P\left(Z \geq \frac{\sqrt{n}(\frac{k}{n} - \theta_0)}{\theta_0}\right)$$

$P(Z > z_\alpha) = \alpha$

which $\frac{\sqrt{n}(\frac{k}{n} - \theta_0)}{\theta_0} = z_\alpha \implies k \approx n\theta_0 + \sqrt{n}\,\theta_0 z_\alpha$

Exact k: First we apply the density transformation formula

$$f_Y(y) = f_X(w^{-1}(y)) \left|\frac{dw^{-1}(y)}{dy}\right|$$

Since $X \sim f(x,\theta) = \frac{1}{\theta} e^{-x/\theta}$ $x > 0$, $\theta > 0$. Let $Y = \frac{X}{\theta}$ then

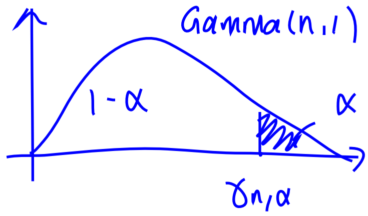
$X = \theta Y$ and $\frac{dX}{dY} = \theta$ and

$$f_Y(y) = f_X(y\theta)\,|\theta| = \frac{1}{\theta} e^{-y\theta/\theta} \cdot \theta = e^{-y} \qquad y > 0$$

and $Y \sim Exp(1)$.

Then using properties of the gamma distribution

$$\sum_{i=1}^{n} Y_i = \sum_{i=1}^{n} \frac{X_i}{\theta} \sim \text{Gamma}(n,1)$$



with density $f_Y(y) = \dfrac{e^{-y} y^{n-1}}{\Gamma(n)}$ $y>0$ and upper-α percentage point $\gamma_{n,\alpha}$ i.e

$$P(\text{Gamma}(n,1) > \gamma_{n,\alpha}) = \alpha .$$

Then to find K we must exhaust the level:

$$\alpha = E_{\theta_0}\varphi^* = P\left(\sum_{i=1}^{n} X_i > k \mid \theta = \theta_0\right)$$

$$= P\left(\sum_{i=1}^{n} \frac{X_i}{\theta_0} \geq \frac{k}{\theta_0} \mid \theta = \theta_0\right) \qquad \text{now } \sum_{i=1}^{n} \frac{X_i}{\theta_0} \sim \text{Gamma}(n,1)$$

$$= P\left(\text{Gamma}(n,1) \geq \frac{K}{\theta_0}\right)$$

Hence the threshold must satisfy $\dfrac{K}{\theta_0} = \gamma_{n\alpha}$ or $K = \theta_0 \gamma_{n,\alpha}$.

This is an exact result:

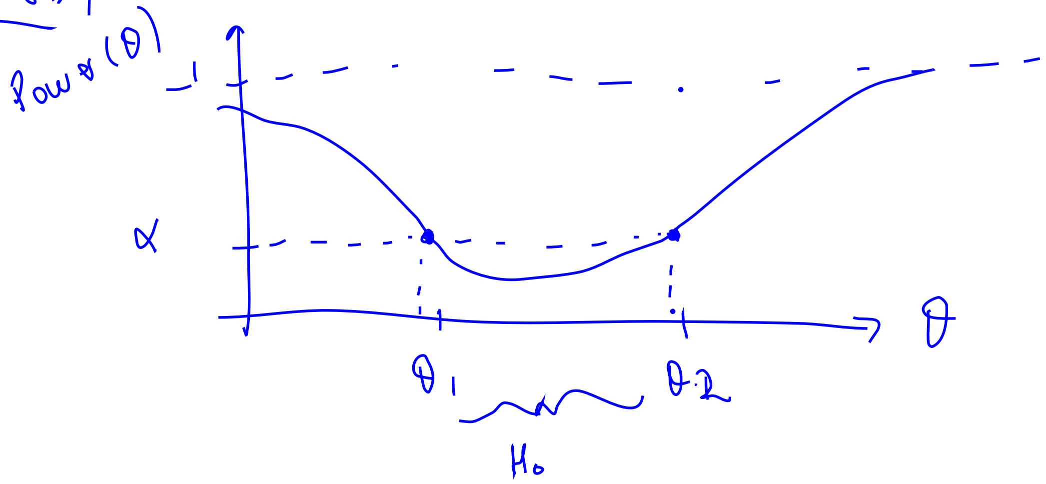
$$\varphi^*(X) = \begin{cases} 1 & \text{if } \sum_{i=1}^{n} X_i \geq \theta_0 \gamma_{n,\alpha} \\ 0 & \text{if } \sum_{i=1}^{n} X_i < \theta_0 \gamma_{n,\alpha} \end{cases} .$$

and the approximate result based on CLT is:

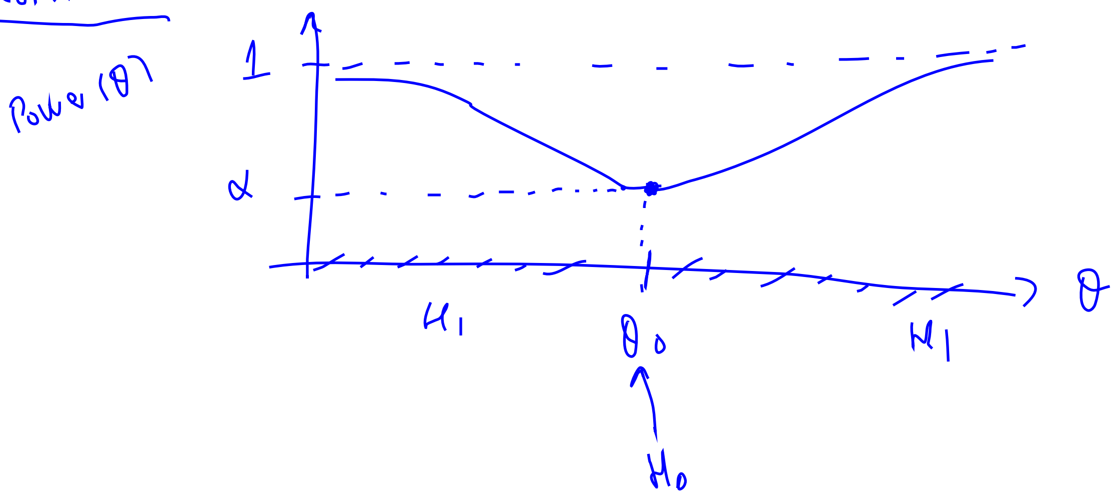
$$\bar{\varphi}(X) = \begin{cases} 1 & \text{if } \sum_{i=1}^{n} X_i \geq n\theta_0 + \theta_0 \sqrt{n}\, z_\alpha \\ 0 & \text{if } \sum_{i=1}^{n} X_i < n\theta_0 + \theta_0 \sqrt{n}\, z_\alpha \end{cases}$$

in order to have an α-sized test asymptotically.

Theorem 6.24



Theorem 6.25



Theorem 6.25 can be interpreted as the limiting case (when θ_1 and θ_2 collapse to a single point θ_0).

Example 6.52

Write $C_1 = 2\theta_0 - C_2$ then

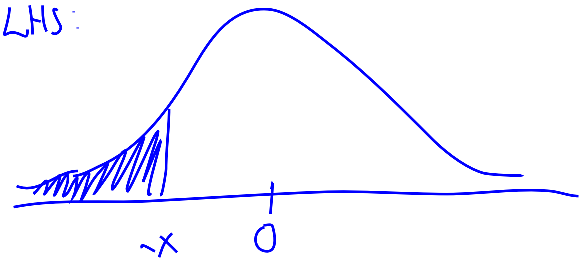
$$\Phi\left(\frac{\sqrt{n}(C_1-\theta_0)}{\sigma}\right) + 1 - \Phi\left(\frac{\sqrt{n}(C_2-\theta_0)}{\sigma}\right) = \alpha$$

$$\Phi\left(\frac{\sqrt{n}(2\theta_0-C_2-\theta_0)}{\sigma}\right) + 1 - \Phi\left(\frac{\sqrt{n}(C_2-\theta_0)}{\sigma}\right) = \alpha \quad \text{by substituting } C_1$$

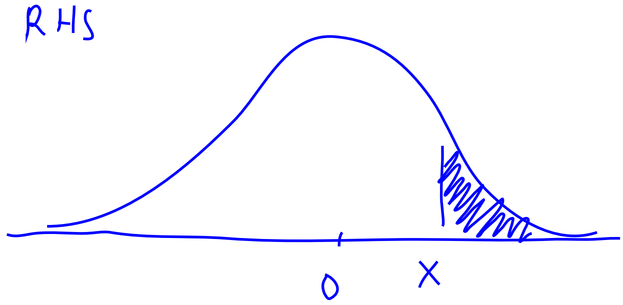
$$\Phi\left(-\frac{\sqrt{n}(C_2-\theta_0)}{\sigma}\right) + 1 - \Phi\left(\frac{\sqrt{n}(C_2-\theta_0)}{\sigma}\right) = \alpha$$

Now a well-known property of the CDF Φ is $\Phi(-x) = 1 - \Phi(x)$

LHS:

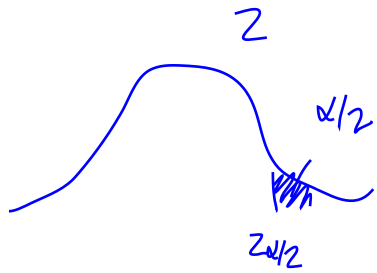


RHS



$$\Rightarrow 1 - \Phi\left(\frac{\sqrt{n}(C_2-\theta_0)}{\sigma}\right) + 1 - \Phi\left(\frac{\sqrt{n}(C_2-\theta_0)}{\sigma}\right) = \alpha$$

$$\Rightarrow 2\left[1 - \Phi\left(\frac{\sqrt{n}(C_2-\theta_0)}{\sigma}\right)\right] = \alpha$$



$$\Rightarrow P\left(Z > \frac{\sqrt{n}(C_2-\theta_0)}{\sigma}\right) = \alpha/2$$

$$\Rightarrow \frac{\sqrt{n}(C_2-\theta_0)}{\sigma} = Z_{\alpha/2}$$

$$\Rightarrow C_2 - \theta_0 = \frac{\sigma}{\sqrt{n}} Z_{\alpha/2}$$

$$\Rightarrow C_2 = \theta_0 + \frac{\sigma}{\sqrt{n}} Z_{\alpha/2} \quad //$$

$$C_1 = 2\theta_0 - C_2 = \theta_0 - \frac{\sigma}{\sqrt{n}} Z_{\alpha/2}$$