

Sila nabijene šipke na naboj

Vektorski zapis diferencijala sile iznosi:

$$d\vec{F}_q = k \frac{q \cdot dQ}{r_{12}^2} \hat{r}_{21} \quad (1)$$

$$= k \frac{q \cdot \lambda dL}{r_{12}^2} \hat{r}_{21} \quad (2)$$

$$= k \frac{q \cdot \lambda dL}{x^2 + L^2} \hat{r}_{21} \quad (3)$$

Diferencijal po vektorskim komponentama iznosi:

$$dF_x = dF \frac{x}{\sqrt{x^2 + L^2}} \quad (4)$$

$$dF_y = dF \frac{L}{\sqrt{x^2 + L^2}} \quad (5)$$

Iznos komponente u odnosu na $|d\vec{F}_q|$ (3) proizlazi iz geometrije zadatka:

$$dF_x = dF \frac{x}{\sqrt{x^2 + L^2}} \quad (6)$$

$$= k \frac{q\lambda dL}{x^2 + L^2} \frac{x}{\sqrt{x^2 + L^2}} \quad (7)$$

$$= kq\lambda x \frac{dL}{\sqrt{(x^2 + L^2)^3}} \quad (8)$$

Ukupna sila F_x iznosi, gdje (12) proizlazi iz integrala (15):

$$F_x = \int dF_x \quad (9)$$

$$= \int_0^a kq\lambda x \frac{dL}{\sqrt{(x^2 + L^2)^3}} \quad (10)$$

$$= kq\lambda x \int_0^a \frac{dL}{\sqrt{(x^2 + L^2)^3}} \quad (11)$$

$$= kq\lambda x \left[\frac{1}{x^2} \frac{L}{\sqrt{L^2 + x^2}} \right]_0^a \quad (12)$$

$$= kq\lambda x \frac{1}{x^2} \left(\frac{a}{\sqrt{a^2 + x^2}} - 0 \right) \quad (13)$$

$$= kq \frac{\lambda a}{x\sqrt{a^2 + x^2}} = \frac{kQq}{x\sqrt{a^2 + x^2}} \quad (14)$$

Prikazani integral možemo riješiti tan substitucijom:

$$\int \frac{dL}{\sqrt{(x^2 + L^2)^3}} = \frac{1}{x^3} \int \frac{dL}{\left(1 + \frac{L^2}{x^2}\right)^{\frac{3}{2}}} \quad (15)$$

$$= \left| \frac{\tan \theta = \frac{L}{x}}{\frac{1}{\cos^2 \theta} d\theta = \frac{dL}{x}} \right| \quad (16)$$

$$= \frac{1}{x^3} \int \frac{x}{\cos^2 \theta} \frac{d\theta}{(1 + \tan^2 \theta)^{\frac{3}{2}}} \quad (17)$$

$$= \frac{x}{x^3} \int \frac{1}{\cos^2 \theta} \frac{d\theta}{\left(\frac{1}{\cos^2 \theta}\right)^{\frac{3}{2}}} \quad (18)$$

$$= \frac{1}{x^2} \int \cos \theta d\theta \quad (19)$$

$$= \frac{1}{x^2} \sin \theta + C \quad (20)$$

$$= \frac{1}{x^2} \sin \tan^{-1} \left(\frac{L}{x} \right) + C \quad (21)$$

$$= \frac{1}{x^2} \frac{L}{\sqrt{L^2 + x^2}} + C \quad (22)$$

Ukupna sila F_y iznosi, gdje (26) proizlazi iz integrala (28):

$$F_y = \int dF \frac{L}{\sqrt{x^2 + L^2}} \quad (23)$$

$$= \int_0^a k \frac{q\lambda dL}{x^2 + L^2} \frac{L}{\sqrt{x^2 + L^2}} \quad (24)$$

$$= kq\lambda \int_0^a \frac{LdL}{(x^2 + L^2)^{\frac{3}{2}}} \quad (25)$$

$$= kq\lambda \left[-\frac{1}{\sqrt{x^2 + L^2}} \right]_0^a \quad (26)$$

$$= \frac{kqQ}{a} \left(\frac{1}{x} - \frac{1}{\sqrt{x^2 + a^2}} \right) \quad (27)$$

Prikazani integral možemo riješiti u -substitucijom:

$$\int \frac{LdL}{(x^2 + L^2)^{\frac{3}{2}}} = \left| \frac{u = x^2 + L^2}{du = 2LdL} \right| \quad (28)$$

$$= \frac{1}{2} \int \frac{du}{u^{\frac{3}{2}}} \quad (29)$$

$$= -\frac{1}{2 \cdot \frac{1}{2}} u^{-\frac{1}{2}} + C \quad (30)$$

$$= -\frac{1}{\sqrt{x^2 + L^2}} + C \quad (31)$$