Z TRANSFORM

SYNTHESIS OF A Z TRANSFORMED VIA SYSTEM

OBJECTIVE S

The objective of this work is to synthesize a system from its z-transform. This role must meet certain characteristics that must be demonstrated.

Specific objectives:

- Using the z Transform, design a real function whose amplitude is decreasing;
- For this function draw the pole -zero diagram, the pole position, and the Fourier transform.

THEORETICAL FOUNDATION

Z TRANSFORM

The z-transform of a signal x(n) is defined as:

$$X(z) = \sum_{n=-\infty}^{\infty} x(n) z^{-n}$$

The z parameter is a complex variable, represented in polar form by $z = re^{j\Omega}$, where $r$is the magnitude and Ωis the phase of z.

Taking an example of a function and its transform

$$\alpha^n u(n) \leftrightarrow \frac{1}{1-\alpha z^{-1}} \;\; com \; |z| > |\alpha|$$

In this function we can assign the value to the pole and thereby define the behavior of *x(n)* .

Knowing the expression of X(z) it is possible to obtain the frequency response replacing z by $e^{j\Omega}$in the expression.

RESULTS

To obtain the results, the software used was matlab , the developed code is available at https://github.com/felipale/transform-z.git .

α (alpha) was defined . To meet the criteria mentioned in the objective of this work, that is, for the function to be decreasing *α* must be less than 1.

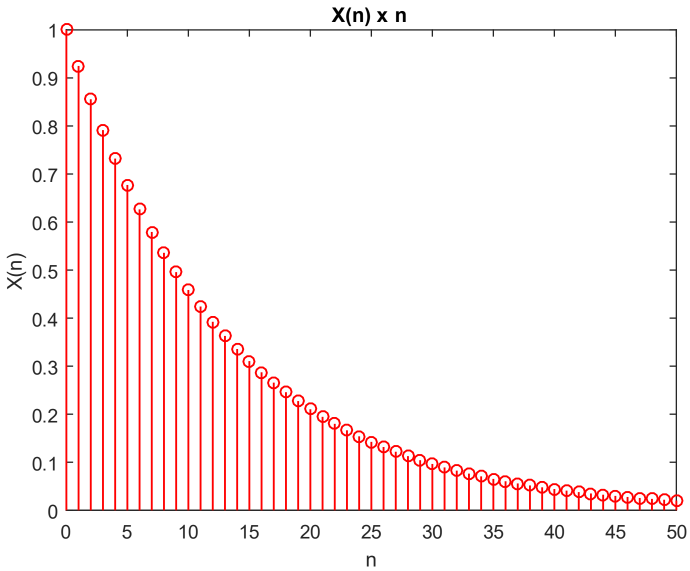
if αbelongs to the reals , for $0 < \alpha < 1$, the amplitude of the signal $x(n) = \alpha^n u(n)$, decreases as n increases. Therefore, the chosen *α value* was **0.925** .

$$\alpha^n u(n) \leftrightarrow \frac{1}{1 - \alpha z^{-1}} \quad com\ |z| > |\alpha|$$

The sign of the z-transform for this case can be simplified by multiplying it by $\frac{z}{z}$. Then we get $x(z) = \frac{z}{z-\alpha}$and $x(z) = \frac{z}{z-0{,}925}$.

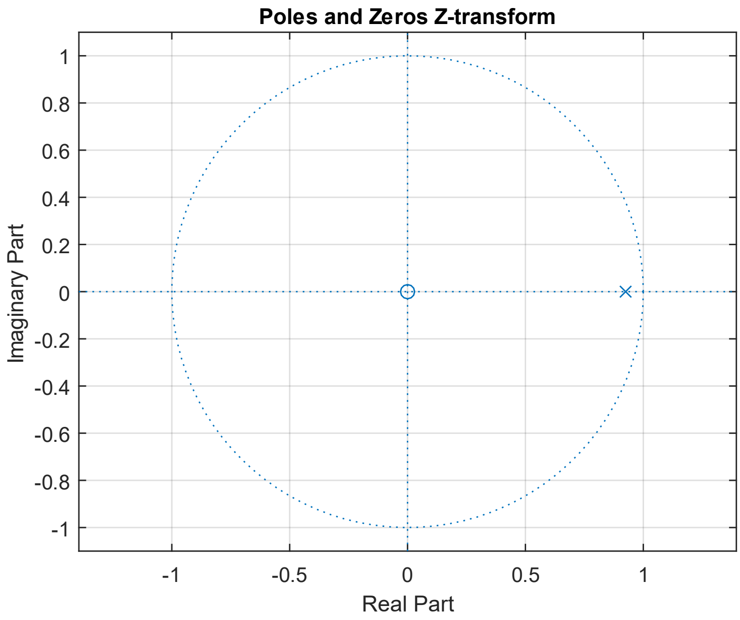
Therefore the sign $x(n) = \alpha^n u(n) = 0{,}925^n u(n)$. The signal x(n) obtained from the z-transform is shown in Figure 1.

Figure 1 – Discrete signal obtained from the z transform



From this, the diagram of poles and zeros was plotted, which can be seen in Figure 2. It can be noted that there is a zero at **z = 0** and a pole at **z = 0.925** .

Figure 2 – Diagram of poles and zeros.



Finally, the z transform was obtained, which is obtained by substituting z for $e^{j\Omega}$. Soon,

$$x(\Omega) = \frac{e^{j\Omega}}{e^{j\Omega} - \alpha} = \frac{e^{j\Omega}}{e^{j\Omega} - 0{,}925}$$

Figure 3 shows the magnitude and phase diagram of the Fourier transform.

Figure 3 – Magnitude and phase Fourier transform.