Brief Article

The Author

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1 Problem statement

$\Omega \subset \mathbb{R}^2$ or $\mathbb{R}^3$ – the domain, i.e. the shape we operate on

$H_j \subset \Omega,\ j = 1, ..., m$ – control handles. A handle can be single point, region, skeleton bone or a cage vertex

The user defines an affine transformation $T_j \in \mathbb{R}^{2x3}$ or $\mathbb{R}^{3x4}$ for each handle H_j
Every point $p \in \Omega$ is deformed according to:

$$p' = \sum_{j=1}^{m} w_j(p)\, T_j p = \sum_{j=1}^{m} w_j(p)\ (T_j p) = \left(\sum_{j=1}^{m} w_j(p)\, T_j \right) \cdot p$$

1.1 A subsection

More text.

2 Setting the Weights

$$W \leftarrow \underset{w_j,\ j=1,\ldots,m}{\arg\min} \sum_{j=1}^{m} \frac{1}{2} \int_{\Omega} \|\Delta w_j\|^2 dV$$

$$W \in \mathbb{R}^{|V| \times m}$$

subject to:

1) $w_j|_{H_k} = \delta_{jk}$
2) $w_j|_F$ is linear $\forall F \in \mathcal{F}_C$
3) $\sum_{j=1}^{m} w_j(p) = 1 \quad \forall p \in \Omega$
4) $0 \leq w_j(p) \leq 1,\ j = 1, ..., m \quad \forall p \in \Omega$

$$\sum_{j=1}^{m} \frac{1}{2} \int_{\Pi} \rho \|\nabla w_j\|^2 dV$$

$$\sum_{j=1}^{m} \frac{1}{2} \int_{\Omega} \|\Delta w_j\|^2 dV + \sum_{j=1}^{m} \frac{1}{2} \int_{\Pi} \rho \|\nabla w_j\|^2 dV$$

3 Properties

$$(\sum_{j=1}^{m} w_j(p) = 1 \quad \text{for every } p \in \Omega)$$

$$\sum_{j=1}^{m} w_j(p) = 1 \quad \forall p \in \Omega \text{ along with } p' = \underbrace{\left(\sum_{j=1}^{m} w_j(p)\, T_j \right)}_{T_p} \cdot p$$

$$T_p \in \mathrm{ConvHull}\left(\{T_j\}\right)$$

4 Implementation

$$\sum_{j=1}^{m} \frac{1}{2} \int_{\Omega} \|\Delta w_j\|^2 dV \;\approx\; \sum_{j=1}^{m} \frac{1}{2} (M^{-1} L w_j)^T M (M^{-1} L w_j) \;=\; \frac{1}{2} \sum_{j=1}^{m} w_j^T (L M^{-1} L) w_j$$

$$\sum_{j=1}^{m} \frac{1}{2} \int_{\Pi} \rho \|\nabla w_j\|^2 dV \approx \sum_{j=1}^{m} \frac{1}{2} w_j^T (G^T R \bar{M} G) w_j$$

$$\underset{w_j,\ j=1,\ldots,m}{\arg\min}\ \frac{1}{2} \sum_{j=1}^{m} w_j^T (L M^{-1} L) w_j + \sum_{j=1}^{m} \frac{1}{2} w_j^T (G^T R \bar{M} G) w_j$$

subject to:

1) $w_j|_{H_k} = \delta_{jk}$
2) $w_j|_F$ is linear $\forall F \in \mathcal{F}_C$
3) $\displaystyle\sum_{j=1}^{m} w_j(p) = 1 \quad \forall p \in \Omega$
4) $0 \le w_j(p) \le 1,\ j = 1, \ldots, m \quad \forall p \in \Omega$