ME2 Computing- Session 2: Gauss elimination

Let's start with a given set of five linear equations:

General form	Matrix form A·x = b	Numerical example	
$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 + a_{25}x_5 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 + a_{35}x_5 = b_3 \\ a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 + a_{45}x_5 = b_4 \\ a_{51}x_1 + a_{52}x_2 + a_{53}x_3 + a_{54}x_4 + a_{55}x_5 = b_5 \end{cases}$	$\begin{bmatrix} \mathbf{a_{11}} & \mathbf{a_{12}} & \mathbf{a_{13}} & \mathbf{a_{14}} & \mathbf{a_{15}} \\ \mathbf{a_{21}} & \mathbf{a_{22}} & \mathbf{a_{23}} & \mathbf{a_{24}} & \mathbf{a_{25}} \\ \mathbf{a_{31}} & \mathbf{a_{32}} & \mathbf{a_{33}} & \mathbf{a_{34}} & \mathbf{a_{35}} \\ \mathbf{a_{41}} & \mathbf{a_{42}} & \mathbf{a_{43}} & \mathbf{a_{44}} & \mathbf{a_{45}} \\ \mathbf{a_{51}} & \mathbf{a_{52}} & \mathbf{a_{53}} & \mathbf{a_{54}} & \mathbf{a_{55}} \end{bmatrix} \begin{bmatrix} \boldsymbol{x_1} \\ \boldsymbol{x_2} \\ \boldsymbol{x_3} \\ \boldsymbol{x_4} \\ \boldsymbol{x_5} \end{bmatrix} = \begin{bmatrix} \boldsymbol{b_1} \\ \boldsymbol{b_2} \\ \boldsymbol{b_3} \\ \boldsymbol{b_4} \\ \boldsymbol{b_5} \end{bmatrix}$	$\begin{bmatrix} \mathbf{2} & \mathbf{5} & \mathbf{-6} & \mathbf{7} & \mathbf{3} \\ \mathbf{4} & \mathbf{-6} & \mathbf{4} & \mathbf{2} & \mathbf{1} \\ \mathbf{3} & \mathbf{3} & \mathbf{4} & \mathbf{-4} & \mathbf{3} \\ \mathbf{1} & \mathbf{2} & \mathbf{-1} & \mathbf{3} & \mathbf{5} \\ \mathbf{2} & \mathbf{4} & \mathbf{3} & \mathbf{4} & \mathbf{-1} \end{bmatrix} \begin{bmatrix} \boldsymbol{x_1} \\ \boldsymbol{x_2} \\ \boldsymbol{x_3} \\ \boldsymbol{x_4} \\ \boldsymbol{x_5} \end{bmatrix} = \begin{bmatrix} \mathbf{2} \\ \mathbf{-3} \\ \mathbf{4} \\ \mathbf{8} \\ \mathbf{3} \end{bmatrix}$	**Eq. 1**

Set the zeros in the 1st column (eliminate the 2nd variable)

Eliminate on 2nd row: $row2 = row2 - a_{21}/a_{11} \cdot row1$

General form	Matrix form A·x = b	Numerical example	
$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 = b_1 \\ c_{22}x_2 + c_{23}x_3 + c_{24}x_4 + c_{25}x_5 = c_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 + a_{35}x_5 = b_3 \\ a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 + a_{45}x_5 = b_4 \\ a_{51}x_1 + a_{52}x_2 + a_{53}x_3 + a_{54}x_4 + a_{55}x_5 = b_5 \end{cases}$	$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ 0 & c_{22} & c_{23} & c_{24} & c_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} b_1 \\ c_2 \\ b_3 \\ b_4 \\ b_5 \end{bmatrix}$	$\begin{bmatrix} 2 & 5 & -6 & 7 & 3 \\ 0 & -16 & 16 & -12 & -5 \\ 3 & 3 & 4 & -4 & 3 \\ 1 & 2 & -1 & 3 & 5 \\ 2 & 4 & 3 & 4 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \\ 4 \\ 8 \\ 3 \end{bmatrix}$	**Eq. 2**

Eliminate on 3rd row: $row3 = row3 - a_{31}/a_{11} \cdot row1$

General form	Matrix form A·x = b	Numerical example	
$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 = b_1 \\ c_{22}x_2 + c_{23}x_3 + c_{24}x_4 + c_{25}x_5 = c_2 \\ c_{32}x_2 + c_{33}x_3 + c_{34}x_4 + c_{35}x_5 = c_3 \\ a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 + a_{45}x_5 = b_4 \\ a_{51}x_1 + a_{52}x_2 + a_{53}x_3 + a_{54}x_4 + a_{55}x_5 = b_5 \end{cases}$	$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ 0 & c_{22} & c_{23} & c_{24} & c_{25} \\ 0 & c_{32} & c_{33} & c_{34} & c_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} b_1 \\ c_2 \\ c_3 \\ b_4 \\ b_5 \end{bmatrix}$	$\begin{bmatrix} 2 & 5 & -6 & 7 & 3 \\ 0 & -16 & 16 & -12 & -5 \\ 0 & -4.5 & 13 & -14.5 & -1.5 \\ 1 & 2 & -1 & 3 & 5 \\ 2 & 4 & 3 & 4 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \\ 1 \\ 8 \\ 3 \end{bmatrix}$	**Eq. 3**

Eliminate on 4th row: $row4 = row4 - a_{41}/a_{11} \cdot row1$

General form	Matrix form A·x = b	Numerical example	
$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 = b_1 \\ c_{22}x_2 + c_{23}x_3 + c_{24}x_4 + c_{25}x_5 = c_2 \\ c_{32}x_2 + c_{33}x_3 + c_{34}x_4 + c_{35}x_5 = c_3 \\ c_{42}x_2 + c_{43}x_3 + c_{44}x_4 + c_{45}x_5 = c_4 \\ a_{51}x_1 + a_{52}x_2 + a_{53}x_3 + a_{54}x_4 + a_{55}x_5 = b_5 \end{cases}$	$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ 0 & c_{22} & c_{23} & c_{24} & c_{25} \\ 0 & c_{32} & c_{33} & c_{34} & c_{35} \\ 0 & c_{42} & c_{43} & c_{44} & c_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} b_1 \\ c_2 \\ c_3 \\ c_4 \\ b_5 \end{bmatrix}$	$\begin{bmatrix} 2 & 5 & -6 & 7 & 3 \\ 0 & -16 & 16 & -12 & -5 \\ 0 & -4.5 & 13 & -14.5 & -1.5 \\ 0 & -0.5 & 2 & -0.5 & 3.5 \\ 2 & 4 & 3 & 4 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \\ 1 \\ 7 \\ 3 \end{bmatrix}$	**Eq. 4**

Eliminate on 5th row: $row5 = row5 - a_{51}/a_{11} \cdot row1$

General form	Matrix form A·x = b	Numerical example	
$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 = b_1 \\ c_{22}x_2 + c_{23}x_3 + c_{24}x_4 + c_{25}x_5 = c_2 \\ c_{32}x_2 + c_{33}x_3 + c_{34}x_4 + c_{35}x_5 = c_3 \\ c_{42}x_2 + c_{43}x_3 + c_{44}x_4 + c_{45}x_5 = c_4 \\ c_{52}x_2 + c_{53}x_3 + c_{54}x_4 + c_{55}x_5 = c_5 \end{cases}$	$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ 0 & c_{22} & c_{23} & c_{24} & c_{25} \\ 0 & c_{32} & c_{33} & c_{34} & c_{35} \\ 0 & c_{42} & c_{43} & c_{44} & c_{45} \\ 0 & c_{52} & c_{53} & c_{54} & c_{55} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} b_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{bmatrix}$	$\begin{bmatrix} 2 & 5 & -6 & 7 & 3 \\ 0 & -16 & 16 & -12 & -5 \\ 0 & -4.5 & 13 & -14.5 & -1.5 \\ 0 & -0.5 & 2 & -0.5 & 3.5 \\ 0 & -1 & 9 & -3 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \\ 1 \\ 7 \\ 1 \end{bmatrix}$	**Eq. 5**

Set the zeros in the 2nd column (eliminate the 3rd variable)

Eliminate on 3rd row: $row3 = row3 - a_{32}/a_{22} \cdot row2$

General form	Matrix form A·x = b	Numerical example	
$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 = b_1 \\ c_{22}x_2 + c_{23}x_3 + c_{24}x_4 + c_{25}x_5 = c_2 \\ d_{33}x_3 + d_{34}x_4 + d_{35}x_5 = d_3 \\ c_{42}x_2 + c_{43}x_3 + c_{44}x_4 + c_{45}x_5 = c_4 \\ c_{52}x_2 + c_{53}x_3 + c_{54}x_4 + c_{55}x_5 = c_5 \end{cases}$	$\begin{bmatrix} \mathbf{a_{11}} & \mathbf{a_{12}} & \mathbf{a_{13}} & \mathbf{a_{14}} & \mathbf{a_{15}} \\ \mathbf{0} & \mathbf{c_{22}} & \mathbf{c_{23}} & \mathbf{c_{24}} & \mathbf{c_{25}} \\ \mathbf{0} & \mathbf{0} & \mathbf{d_{33}} & \mathbf{d_{34}} & \mathbf{d_{35}} \\ \mathbf{0} & \mathbf{c_{42}} & \mathbf{c_{43}} & \mathbf{c_{44}} & \mathbf{c_{45}} \\ \mathbf{0} & \mathbf{c_{52}} & \mathbf{c_{53}} & \mathbf{c_{54}} & \mathbf{c_{55}} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} b_1 \\ c_2 \\ d_3 \\ c_4 \\ c_5 \end{bmatrix}$	$\begin{bmatrix} 2 & 5 & -6 & 7 & 3 \\ 0 & -16 & 16 & -12 & -5 \\ 0 & 0 & 8.5 & -11.12 & -0.09 \\ 0 & -0.5 & 2 & -0.5 & 3.5 \\ 0 & -1 & 9 & -3 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \\ 2.96 \\ 7 \\ 1 \end{bmatrix}$	**Eq. 6**

Eliminate on 4th row: $row4 = row4 - a_{42}/a_{22} \cdot row2$

General form	Matrix form A·x = b	Numerical example	
$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 = b_1 \\ c_{22}x_2 + c_{23}x_3 + c_{24}x_4 + c_{25}x_5 = c_2 \\ d_{33}x_3 + d_{34}x_4 + d_{35}x_5 = d_3 \\ d_{43}x_3 + d_{44}x_4 + d_{45}x_5 = d_4 \\ c_{52}x_2 + c_{53}x_3 + c_{54}x_4 + c_{55}x_5 = c_5 \end{cases}$	$\begin{bmatrix} \mathbf{a_{11}} & \mathbf{a_{12}} & \mathbf{a_{13}} & \mathbf{a_{14}} & \mathbf{a_{15}} \\ \mathbf{0} & \mathbf{c_{22}} & \mathbf{c_{23}} & \mathbf{c_{24}} & \mathbf{c_{25}} \\ \mathbf{0} & \mathbf{0} & \mathbf{d_{33}} & \mathbf{d_{34}} & \mathbf{d_{35}} \\ \mathbf{0} & \mathbf{0} & \mathbf{d_{43}} & \mathbf{d_{44}} & \mathbf{d_{45}} \\ \mathbf{0} & \mathbf{c_{52}} & \mathbf{c_{53}} & \mathbf{c_{54}} & \mathbf{c_{55}} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} b_1 \\ c_2 \\ d_3 \\ d_4 \\ c_5 \end{bmatrix}$	$\begin{bmatrix} 2 & 5 & -6 & 7 & 3 \\ 0 & -16 & 16 & -12 & -5 \\ 0 & 0 & 8.5 & -11.12 & -0.09 \\ 0 & 0 & 1.5 & -0.12 & 3.65 \\ 0 & -1 & 9 & -3 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \\ 2.96 \\ 7.21 \\ 1 \end{bmatrix}$	**Eq. 7**

Eliminate on 5th row: $row5 = row5 - a_{52}/a_{22} \cdot row2$

General form	Matrix form A·x = b	Numerical example	
$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 = b_1 \\ c_{22}x_2 + c_{23}x_3 + c_{24}x_4 + c_{25}x_5 = c_2 \\ d_{33}x_3 + d_{34}x_4 + d_{35}x_5 = d_3 \\ d_{43}x_3 + d_{44}x_4 + d_{45}x_5 = d_4 \\ d_{53}x_3 + d_{54}x_4 + d_{55}x_5 = d_5 \end{cases}$	$\begin{bmatrix} \mathbf{a_{11}} & \mathbf{a_{12}} & \mathbf{a_{13}} & \mathbf{a_{14}} & \mathbf{a_{15}} \\ \mathbf{0} & \mathbf{c_{22}} & \mathbf{c_{23}} & \mathbf{c_{24}} & \mathbf{c_{25}} \\ \mathbf{0} & \mathbf{0} & \mathbf{d_{33}} & \mathbf{d_{34}} & \mathbf{d_{35}} \\ \mathbf{0} & \mathbf{0} & \mathbf{d_{43}} & \mathbf{d_{44}} & \mathbf{d_{45}} \\ \mathbf{0} & \mathbf{0} & \mathbf{d_{53}} & \mathbf{d_{54}} & \mathbf{d_{55}} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} b_1 \\ c_2 \\ d_3 \\ d_4 \\ d_5 \end{bmatrix}$	$\begin{bmatrix} 2 & 5 & -6 & 7 & 3 \\ 0 & -16 & 16 & -12 & -5 \\ 0 & 0 & 8.5 & -11.12 & -0.09 \\ 0 & 0 & 1.5 & -0.12 & 3.65 \\ 0 & 0 & 8 & -2.25 & -3.68 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \\ 2.96 \\ 7.21 \\ 1.43 \end{bmatrix}$	**Eq. 8**

Set the zeros in the 3rd column (eliminate the 4th variable)

Eliminate on 4th row: $row4 = row4 - a_{43}/a_{33} \cdot row3$

General form	Matrix form A·x = b	Numerical example	
$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 = b_1 \\ c_{22}x_2 + c_{23}x_3 + c_{24}x_4 + c_{25}x_5 = c_2 \\ d_{33}x_3 + d_{34}x_4 + d_{35}x_5 = d_3 \\ e_{44}x_4 + e_{45}x_5 = e_4 \\ d_{53}x_3 + d_{54}x_4 + d_{55}x_5 = d_5 \end{cases}$	$\begin{bmatrix} \mathbf{a_{11}} & \mathbf{a_{12}} & \mathbf{a_{13}} & \mathbf{a_{14}} & \mathbf{a_{15}} \\ \mathbf{0} & \mathbf{c_{22}} & \mathbf{c_{23}} & \mathbf{c_{24}} & \mathbf{c_{25}} \\ \mathbf{0} & \mathbf{0} & \mathbf{d_{33}} & \mathbf{d_{34}} & \mathbf{d_{35}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{e_{44}} & \mathbf{e_{45}} \\ \mathbf{0} & \mathbf{0} & \mathbf{d_{53}} & \mathbf{d_{54}} & \mathbf{d_{55}} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} b_1 \\ c_2 \\ d_3 \\ e_4 \\ d_5 \end{bmatrix}$	$\begin{bmatrix} 2 & 5 & -6 & 7 & 3 \\ 0 & -16 & 16 & -12 & -5 \\ 0 & 0 & 8.5 & -11.12 & -0.09 \\ 0 & 0 & 0 & 1.83 & 3.67 \\ 0 & 0 & 8 & -2.25 & -3.68 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \\ 2.96 \\ 6.69 \\ 1.43 \end{bmatrix}$	**Eq. 9**

Eliminate on 5th row: $row5 = row5 - a_{53}/a_{33} \cdot row3$

General form	Matrix form A·x = b	Numerical example	
$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 = b_1 \\ c_{22}x_2 + c_{23}x_3 + c_{24}x_4 + c_{25}x_5 = c_2 \\ d_{33}x_3 + d_{34}x_4 + d_{35}x_5 = d_3 \\ e_{44}x_4 + e_{45}x_5 = e_4 \\ e_{54}x_4 + e_{55}x_5 = e_5 \end{cases}$	$\begin{bmatrix} \mathbf{a_{11}} & \mathbf{a_{12}} & \mathbf{a_{13}} & \mathbf{a_{14}} & \mathbf{a_{15}} \\ \mathbf{0} & \mathbf{c_{22}} & \mathbf{c_{23}} & \mathbf{c_{24}} & \mathbf{c_{25}} \\ \mathbf{0} & \mathbf{0} & \mathbf{d_{33}} & \mathbf{d_{34}} & \mathbf{d_{35}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{e_{44}} & \mathbf{e_{45}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{e_{54}} & \mathbf{e_{55}} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} b_1 \\ c_2 \\ d_3 \\ e_4 \\ e_5 \end{bmatrix}$	$\begin{bmatrix} 2 & 5 & -6 & 7 & 3 \\ 0 & -16 & 16 & -12 & -5 \\ 0 & 0 & 8.5 & -11.12 & -0.09 \\ 0 & 0 & 0 & 1.83 & 3.67 \\ 0 & 0 & 0 & 8.22 & -3.59 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \\ 2.96 \\ 6.69 \\ -1.35 \end{bmatrix}$	**Eq. 10**

Set the zeros in the 4th column (eliminate the 5th variable)

Eliminate on 5th row: $row5 = row5 - a_{54}/a_{44} \cdot row4$

General form	Matrix form A·x = b	Numerical example	
$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 = b_1 \\ c_{22}x_2 + c_{23}x_3 + c_{24}x_4 + c_{25}x_5 = c_2 \\ d_{33}x_3 + d_{34}x_4 + d_{35}x_5 = d_3 \\ e_{44}x_4 + e_{45}x_5 = e_4 \\ f_{55}x_5 = f_5 \end{cases}$	$\begin{bmatrix} \mathbf{a_{11}} & \mathbf{a_{12}} & \mathbf{a_{13}} & \mathbf{a_{14}} & \mathbf{a_{15}} \\ \mathbf{0} & \mathbf{c_{22}} & \mathbf{c_{23}} & \mathbf{c_{24}} & \mathbf{c_{25}} \\ \mathbf{0} & \mathbf{0} & \mathbf{d_{33}} & \mathbf{d_{34}} & \mathbf{d_{35}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{e_{44}} & \mathbf{e_{45}} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{f_{55}} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} b_1 \\ c_2 \\ d_3 \\ e_4 \\ f_5 \end{bmatrix}$	$\begin{bmatrix} 2 & 5 & -6 & 7 & 3 \\ 0 & -16 & 16 & -12 & -5 \\ 0 & 0 & 8.5 & -11.12 & -0.09 \\ 0 & 0 & 0 & 1.83 & 3.67 \\ 0 & 0 & 0 & 0 & -20.02 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \\ 2.96 \\ 6.69 \\ -31.2 \end{bmatrix}$	**Eq. 11**

In summary, using the colour code adopted, you can work out the necessary loops needed to implement the method and which indices you need to play with in the arrays A and b.

Set the zeros in the 1st column

Eliminate on 2nd row ($row2 = row2 - a_{21}/a_{11} \cdot row1$)

Eliminate on 3rd row ($row3 = row3 - a_{31}/a_{11} \cdot row1$)

Eliminate on 4th row ($row4 = row4 - a_{41}/a_{11} \cdot row1$)

Eliminate on 5th row ($row5 = row5 - a_{51}/a_{11} \cdot row1$)

Set the zeros in the 2nd column

Eliminate on 3rd row ($row3 = row3 - a_{32}/a_{22} \cdot row2$)

Eliminate on 4th row ($row4 = row4 - a_{42}/a_{22} \cdot row2$)

Eliminate on 5th row ($row5 = row5 - a_{52}/a_{22} \cdot row2$)

Set the zeros in the 3rd column

Eliminate on 4th row ($row4 = row4 - a_{43}/a_{33} \cdot row3$)

Eliminate on 5th row ($row5 = row5 - a_{53}/a_{33} \cdot row3$)

Set the zeros in the 4th column

Eliminate on 5th row ($row5 = row5 - a_{54}/a_{44} \cdot row4$)

Once the matrix has been reduced to a triangular matrix, the set of solutions can be obtained, in sequence, as:

$x_5 = f_5/f_{55}$

$x_4 = e_4/e_{44} - e_{45}/e_{44} \cdot x_5$

$x_3 = d_3/d_{33} - d_{34}/d_{33} \cdot x_4 - d_{35}/d_{33} \cdot x_5$

etc.