# ME2 Computing- Session 2: Gauss elimination

# Let's start with a given set of five linear equations:

General form	Matrix form A·x = b	Numerical example	
$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 = b_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 + a_{25}x_5 = b_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 + a_{35}x_5 = b_3 \\ a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 + a_{45}x_5 = b_4 \\ a_{51}x_1 + a_{52}x_2 + a_{53}x_3 + a_{54}x_4 + a_{55}x_5 = b_5 \end{cases}$	$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \end{bmatrix}$	$\begin{bmatrix} 2 & 5 & -6 & 7 & 3 \\ 4 & -6 & 4 & 2 & 1 \\ 3 & 3 & 4 & -4 & 3 \\ 1 & 2 & -1 & 3 & 5 \\ 2 & 4 & 3 & 4 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 4 \\ 8 \\ 3 \end{bmatrix}$	Eq. 1

# Set the zeros in the 1<sup>st</sup> column (eliminate the 2<sup>nd</sup> variable)

#### Eliminate on 2<sup>nd</sup> row: $row2 = row2 - a_{21}/a_{11} \cdot row1$

General form	Matrix form A·x = b Numerical example	
$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 = b_1 \\ c_{22}x_2 + c_{23}x_3 + c_{24}x_4 + c_{25}x_5 = c_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 + a_{35}x_5 = b_3 \\ a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 + a_{45}x_5 = b_4 \\ a_{51}x_1 + a_{52}x_2 + a_{53}x_3 + a_{54}x_4 + a_{55}x_5 = b_5 \end{cases}$	$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ 0 & c_{22} & c_{23} & c_{24} & c_{25} \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} b_1 \\ c_2 \\ b_3 \\ b_4 \\ b_5 \end{bmatrix}$ $\begin{bmatrix} 2 & 5 & -6 & 7 & 3 \\ 0 & -16 & 16 & -12 & -5 \\ 3 & 3 & 4 & -4 & 3 \\ 1 & 2 & -1 & 3 & 5 \\ 2 & 4 & 3 & 4 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \\ 4 \\ 8 \\ 3 \end{bmatrix}$	Eq. 2

# Eliminate on 3<sup>rd</sup> row: $row3 = row3 - a_{31}/a_{11} \cdot row1$

General form	Matrix form A·x = b	Numerical example						
$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 = b_1 \\ c_{22}x_2 + c_{23}x_3 + c_{24}x_4 + c_{25}x_5 = c_2 \\ c_{32}x_2 + c_{33}x_3 + c_{34}x_4 + c_{35}x_5 = c_3 \\ a_{41}x_1 + a_{42}x_2 + a_{43}x_3 + a_{44}x_4 + a_{45}x_5 = b_4 \\ a_{51}x_1 + a_{52}x_2 + a_{53}x_3 + a_{54}x_4 + a_{55}x_5 = b_5 \end{cases}$	$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ 0 & c_{22} & c_{23} & c_{24} & c_{25} \\ 0 & c_{32} & c_{33} & c_{34} & c_{35} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} b_1 \\ c_2 \\ c_3 \\ b_4 \\ b_5 \end{bmatrix}$	$\begin{bmatrix} 2 & 5 & -6 & 7 & 3 \\ 0 & -16 & 16 & -12 & -5 \\ 0 & -4.5 & 13 & -14.5 & -1.5 \\ 1 & 2 & -1 & 3 & 5 \\ 2 & 4 & 3 & 4 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \\ 1 \\ 8 \\ 3 \end{bmatrix}$	Eq. 3					

#### Eliminate on 4<sup>th</sup> row: $row4 = row4 - a_{41}/a_{11} \cdot row1$

General form	Matrix form A·x = b	Numerical example					
$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 = b_1 \\ c_{22}x_2 + c_{23}x_3 + c_{24}x_4 + c_{25}x_5 = c_2 \\ c_{32}x_2 + c_{33}x_3 + c_{34}x_4 + c_{35}x_5 = c_3 \\ c_{42}x_2 + c_{43}x_3 + c_{44}x_4 + c_{45}x_5 = c_4 \\ a_{51}x_1 + a_{52}x_2 + a_{53}x_3 + a_{54}x_4 + a_{55}x_5 = b_5 \end{cases}$	$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ 0 & c_{22} & c_{23} & c_{24} & c_{25} \\ 0 & c_{32} & c_{33} & c_{34} & c_{35} \\ 0 & c_{42} & c_{43} & c_{44} & c_{45} \\ a_{51} & a_{52} & a_{53} & a_{54} & a_{55} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} b_1 \\ c_2 \\ c_3 \\ c_4 \\ b_5 \end{bmatrix}$	$\begin{bmatrix} 2 & 5 & -6 & 7 & 3 \\ 0 & -16 & 16 & -12 & -5 \\ 0 & -4.5 & 13 & -14.5 & -1.5 \\ 0 & -0.5 & 2 & -0.5 & 3.5 \\ 2 & 4 & 3 & 4 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \\ 1 \\ 7 \\ 3 \end{bmatrix}$	Eq. 4				

# Eliminate on 5<sup>th</sup> row: $row5 = row5 - a_{51}/a_{11} \cdot row1$

General form	Matrix form A·x = b	Numerical example					
$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 = b_1 \\ c_{22}x_2 + c_{23}x_3 + c_{24}x_4 + c_{25}x_5 = c_2 \\ c_{32}x_2 + c_{33}x_3 + c_{34}x_4 + c_{35}x_5 = c_3 \\ c_{42}x_2 + c_{43}x_3 + c_{44}x_4 + c_{45}x_5 = c_4 \\ c_{57}x_2 + c_{53}x_3 + c_{54}x_4 + c_{55}x_5 = c_5 \end{cases}$	$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ 0 & c_{22} & c_{23} & c_{24} & c_{25} \\ 0 & c_{32} & c_{33} & c_{34} & c_{35} \\ 0 & c_{42} & c_{43} & c_{44} & c_{45} \\ 0 & c_{52} & c_{53} & c_{54} & c_{55} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} b_1 \\ c_2 \\ c_3 \\ c_4 \\ c_5 \end{bmatrix}$	$\begin{bmatrix} 2 & 5 & -6 & 7 & 3 \\ 0 & -16 & 16 & -12 & -5 \\ 0 & -4.5 & 13 & -14.5 & -1.5 \\ 0 & -0.5 & 2 & -0.5 & 3.5 \\ 0 & -1 & 9 & -3 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \\ 1 \\ 7 \\ 1 \end{bmatrix}$	Eq. 5				

# Set the zeros in the 2<sup>nd</sup> column (eliminate the 3<sup>rd</sup> variable)

#### Eliminate on 3<sup>rd</sup> row: $row3 = row3 - a_{32}/a_{22} \cdot row2$

General form	Matrix form A·x = b Numerical example	
$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 = b_1 \\ c_{22}x_2 + c_{23}x_3 + c_{24}x_4 + c_{25}x_5 = c_2 \end{cases}$	$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ 0 & c_{22} & c_{23} & c_{24} & c_{25} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \begin{bmatrix} b_1 \\ c_2 \end{bmatrix} \begin{bmatrix} 2 & 5 & -6 & 7 & 3 \\ 0 & -16 & 16 & -12 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \begin{bmatrix} 2 \\ -7 \end{bmatrix}$	
$d_{33}x_3 + d_{34}x_4 + d_{35}x_5 = d_3$	$\begin{vmatrix} 0 & 0 & d_{33} & d_{34} & d_{35} \end{vmatrix} \begin{vmatrix} x_3 \end{vmatrix} = \begin{vmatrix} d_3 \end{vmatrix} \begin{vmatrix} 0 & 0 & 8.5 & -11.12 & -0.09 \end{vmatrix} \begin{vmatrix} x_3 \end{vmatrix} = \begin{vmatrix} 2.96 \end{vmatrix}$	Eq. 6
$c_{42}x_2 + c_{43}x_3 + c_{44}x_4 + c_{45}x_5 = c_4$	$\begin{vmatrix} 0 & c_{42} & c_{43} & c_{44} & c_{45} \end{vmatrix} \begin{vmatrix} x_4 \\ c_4 \end{vmatrix} + \begin{vmatrix} 0 & -0.5 & 2 & -0.5 & 3.5 \end{vmatrix} \begin{vmatrix} x_4 \\ c_4 \end{vmatrix} + \begin{vmatrix} 7 \\ c_4 \end{vmatrix}$	
$c_{52}x_2 + c_{53}x_3 + c_{54}x_4 + c_{55}x_5 = c_5$	$\begin{bmatrix} 1 & 0 & c_{52} & c_{53} & c_{54} & c_{55} \end{bmatrix} \begin{bmatrix} 1 & c_{5} \end{bmatrix} \begin{bmatrix} 1 & 0 & -1 & 9 & -3 & -4 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}$	

### Eliminate on 4<sup>th</sup> row: $row4 = row4 - a_{42}/a_{22} \cdot row2$

General form	Matrix form A·x = b	Numerical example					
$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 = b_1 \\ c_{22}x_2 + c_{23}x_3 + c_{24}x_4 + c_{25}x_5 = c_2 \\ d_{33}x_3 + d_{34}x_4 + d_{35}x_5 = d_3 \\ d_{43}x_3 + d_{44}x_4 + d_{45}x_5 = d_4 \end{cases}$	$\begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ 0 & c_{22} & c_{23} & c_{24} & c_{25} \\ 0 & 0 & d_{33} & d_{34} & d_{35} \\ 0 & 0 & d_{43} & d_{44} & d_{45} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ d_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ c_2 \\ d_3 \\ d_4 \end{bmatrix}$	$\begin{bmatrix} 2 & 5 & -6 & 7 & 3 \\ 0 & -16 & 16 & -12 & -5 \\ 0 & 0 & 8.5 & -11.12 & -0.09 \\ 0 & 0 & 1.5 & -0.12 & 3.65 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \\ 2.96 \\ 7.21 \end{bmatrix}$	Eq. 7				
$c_{52}x_2 + c_{53}x_3 + c_{54}x_4 + c_{55}x_5 = c_5$		$\begin{bmatrix} \begin{bmatrix} 1 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 1 \end{bmatrix} & \begin{bmatrix} 1 & 1 \end{bmatrix} \end{bmatrix}$					

#### Eliminate on 5<sup>th</sup> row: $row5 = row5 - a_{52}/a_{22} \cdot row2$

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	General form			Matri	( form	A∙x = b	Numerical example					
	$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 = b_1$	<sub>[</sub> a <sub>11</sub>	a <sub>12</sub>	a <sub>13</sub>	a <sub>14</sub>	$a_{15}$ <sub> [</sub> $x_{1}$ <sub> </sub> [ $b_1$ ]	<b>Γ2</b>	5	-6	7	$3$ <sub> </sub> $x_1$ <sub> </sub> $2$ <sub> </sub>	
	$c_{22}x_2 + c_{23}x_3 + c_{24}x_4 + c_{25}x_5 = c_2$	0	$\mathbf{c_{22}}$	$c_{23}$	$c_{24}$	$c_{25}   x_2    c_2 $	0	-16	16	-12	$-5  x_2   -7 $	
	$d_{33}x_3 + d_{34}x_4 + d_{35}x_5 = d_3$	0	0	$d_{33}$	$d_{34}$	$\mathbf{d_{35}}    x_3   =   d_3  $	0	0	8.5	-11.12	$-0.09   x_3  =  2.96 $	Eq. 8
	$d_{43}x_3 + d_{44}x_4 + d_{45}x_5 = d_4$	0	0	$d_{43}$	$d_{44}$	$d_{45}  x_4   d_4 $	0	0	1.5	-0.12	$3.65   x_4   7.21  $	
	$d_{53}x_3 + d_{54}x_4 + d_{55}x_5 = d_5$	L O	0	$\mathbf{d_{53}}$	$d_{54}$	$d_{55}$ ][ $x_5$ ] [ $d_5$ ]	L0	0	8	-2.25	-3.68 $1.43$	

# Set the zeros in the 3<sup>rd</sup> column (eliminate the 4<sup>th</sup> variable)

#### Eliminate on 4<sup>th</sup> row: $row4 = row4 - a_{43}/a_{33} \cdot row3$

General form	33	Matri	x form	A·x = b	Numerical example						
$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 = b_1 \\ c_{22}x_2 + c_{23}x_3 + c_{24}x_4 + c_{25}x_5 = c_2 \\ d_{33}x_3 + d_{34}x_4 + d_{35}x_5 = d_3 \\ e_{44}x_4 + e_{45}x_5 = e_4 \end{cases}$	0 c	a <sub>12</sub> a <sub>13</sub> c <sub>22</sub> c <sub>23</sub> 0 d <sub>33</sub> 0	a <sub>14</sub> c <sub>24</sub> d <sub>34</sub> e <sub>44</sub>	$\begin{bmatrix} a_{15} \\ c_{25} \\ d_{35} \\ e_{45} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ e_4 \end{bmatrix} = \begin{bmatrix} b_1 \\ c_2 \\ d_3 \\ e_4 \end{bmatrix}$	$\begin{bmatrix} 2 & 5 & -6 & 7 & 3 \\ 0 & -16 & 16 & -12 & -5 \\ 0 & 0 & 8.5 & -11.12 & -0.09 \\ 0 & 0 & 0 & 1.83 & 3.67 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 2 \\ -7 \\ 2.96 \\ 6.69 \end{bmatrix}$	Eq. 9					
$d_{53}x_3 + d_{54}x_4 + d_{55}x_5 = d_5$	L 0	0 d <sub>53</sub>	$d_{54}$	$d_{55}$ $\lfloor x_5 \rfloor - \lfloor d_5 \rfloor$	$\begin{bmatrix} 1 & 0 & 8 & -2.25 & -3.68 \end{bmatrix} \begin{bmatrix} x_5 \end{bmatrix} \begin{bmatrix} 1.43 \end{bmatrix}$						

#### Eliminate on 5<sup>th</sup> row: $row5 = row5 - a_{53}/a_{33} \cdot row3$

General form			Matri	x form	A•x = b	Numerical example						
$\begin{cases} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 = b_1 \\ c_{22}x_2 + c_{23}x_3 + c_{24}x_4 + c_{25}x_5 = c_2 \\ d_{33}x_3 + d_{34}x_4 + d_{35}x_5 = d_3 \\ e_{44}x_4 + e_{45}x_5 = e_4 \\ e_{54}x_4 + e_{55}x_5 = e_5 \end{cases}$	[a <sub>11</sub> 0 0 0 0 0 0 0	a <sub>12</sub> c <sub>22</sub> 0 0	a <sub>13</sub> c <sub>23</sub> d <sub>33</sub> 0	a <sub>14</sub> c <sub>24</sub> d <sub>34</sub> e <sub>44</sub>	$\begin{bmatrix} a_{15} \\ c_{25} \\ d_{35} \\ e_{45} \\ e_{55} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ e_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} b_1 \\ c_2 \\ d_3 \\ e_4 \\ e_5 \end{bmatrix}$	$\begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$	5 -16 0 0	-6 16 8.5 0	7 -12 -11.12 1.83	$\begin{bmatrix} 3 \\ -5 \\ -0.09 \\ 3.67 \\ -3.59 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} =$	$\begin{bmatrix} 2 \\ -7 \\ 2.96 \\ 6.69 \\ -1.35 \end{bmatrix}$	Eq. 10

#### Set the zeros in the 4<sup>th</sup> column (eliminate the 5<sup>th</sup> variable)

#### Eliminate on 5<sup>th</sup> row: $row5 = row5 - a_{54}/a_{44} \cdot row4$

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General form			Matri	x form	A∙x = b	Numerical example							
$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 + a_{15}x_5 = b_1$	<sub>[</sub> a <sub>11</sub>	a <sub>12</sub>	a <sub>13</sub>	a <sub>14</sub>	$a_{15}$ <sub> [</sub> $x_{1}$   [ $b_1$ ]	<b>г</b> 2	5	-6	7	<sup>ر</sup> ار3	17	г <b>2</b> 1	
$c_{22}x_2 + c_{23}x_3 + c_{24}x_4 + c_{25}x_5 = c_2$	0	$c_{22}$	$c_{23}$	$c_{24}$	$c_{25}   x_2    c_2 $	0	-16	16	-12	-5   <i>x</i>	2	-7	
$d_{33}x_3 + d_{34}x_4 + d_{35}x_5 = d_3$	0	0	$\mathbf{d_{33}}$	$d_{34}$	$\mathbf{d_{35}}    x_3   =   d_3  $	0	0	8.5	-11.12	-0.09  x	3 =	2.96	Eq. 11
$e_{44}x_4 + e_{45}x_5 = e_4$	0	0	0	$e_{44}$	$\mathbf{e_{45}}    x_4      e_4   $	0	0	0	1.83	3.67    x	4	6.69	
$f_{55}x_5 = f_5$	L 0	0	0	0	$f_{55} \rfloor \lfloor x_5 \rfloor \lfloor f_5 \rfloor$	LO	0	0	0	رالـ <u>20</u> .02 –	<sub>5</sub> ]	L-31.2	

In summary, using the colour code adopted, you can work out the necessary loops needed to implement the method and which indices you need to play with in the arrays A and b.

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Set the zeros in the 1<sup>st</sup> column

Eliminate on 2<sup>nd</sup> row (row2 = row2 - a_{21}/a_{11} \cdot row1)

Eliminate on 3<sup>rd</sup> row (row3 = row3 - a_{31}/a_{11} \cdot row1)

Eliminate on 4<sup>th</sup> row (row4 = row4 - a_{41}/a_{11} \cdot row1)

Eliminate on 5<sup>th</sup> row (row5 = row5 - a_{51}/a_{11} \cdot row1)

Set the zeros in the 2<sup>nd</sup> column

Eliminate on 3<sup>rd</sup> row (row3 = row3 - a_{32}/a_{22} \cdot row2)

Eliminate on 4<sup>th</sup> row (row4 = row4 - a_{42}/a_{22} \cdot row2)

Eliminate on 5<sup>th</sup> row (row5 = row5 - a_{52}/a_{22} \cdot row2)

Set the zeros in the 3<sup>rd</sup> column

Eliminate on 4<sup>th</sup> row (row4 = row4 - a_{43}/a_{33} \cdot row3)

Eliminate on 5<sup>th</sup> row (row5 = row5 - a_{53}/a_{33} \cdot row3)

Set the zeros in the 4<sup>th</sup> column

Eliminate on 5<sup>th</sup> row (row5 = row5 - a_{54}/a_{44} \cdot row4)
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Once the matrix has been reduced to a triangular matrix, the set of solutions can be obtained, in sequence, as:

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\begin{array}{l} x_5 = f_5/f_{55} \\ x_4 = e_4/e_{44} - e_{45}/e_{44} \cdot x_5 \\ x_3 = d_3/d_{33} - d_{34}/d_{33} \cdot x_4 - d_{35}/d_{33} \cdot x_5 \\ \text{etc.} \end{array}
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