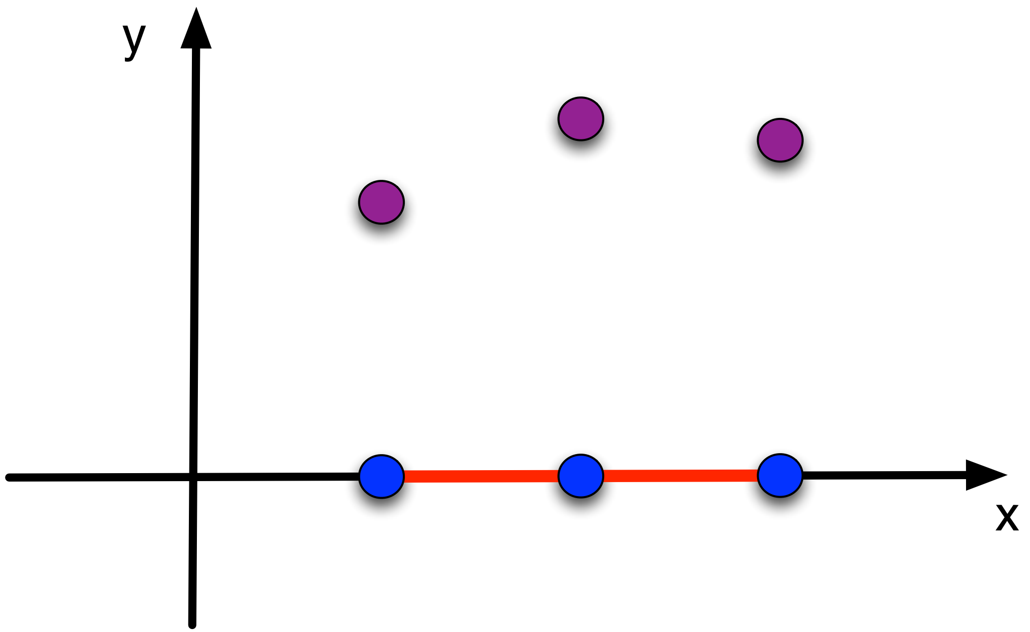
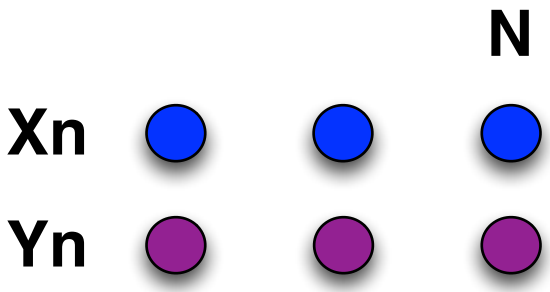
ME2 Computing

Numerical Interpolation

General concepts of interpolation



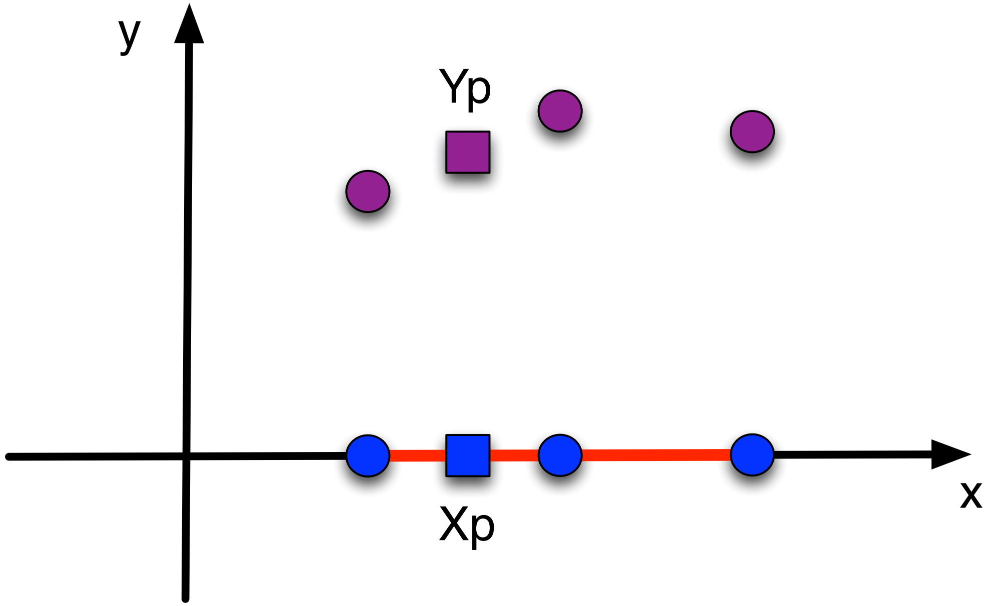
Nodal info:
two arrays of size N



Order of interpolation: n = N - 1

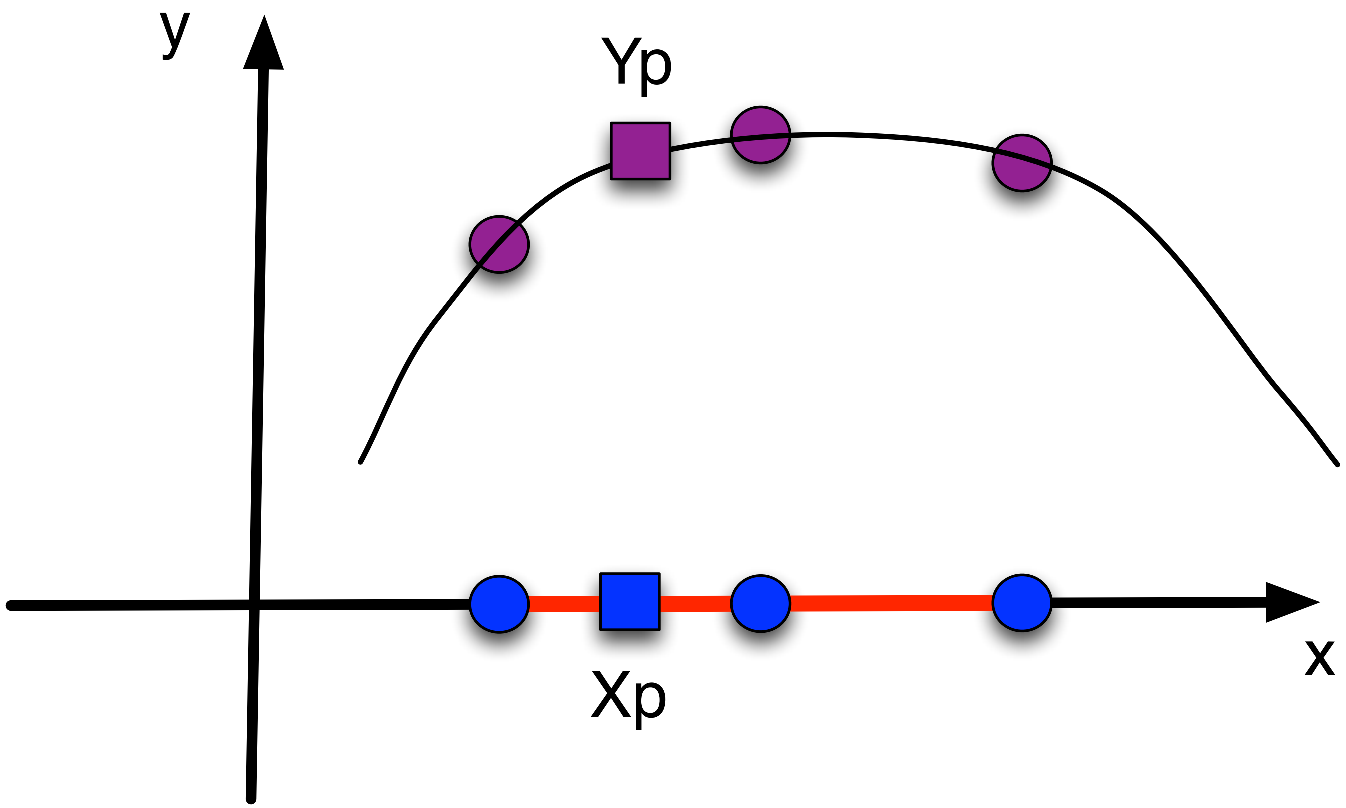
N = 2 linear
N = 3 quadratic
N = 4 cubic

General concepts of interpolation



We can assess the value Yp at any other point Xp, not being a node.

General concepts of polynomial interpolation



We approximate any other values with a polynomial of order N - 1

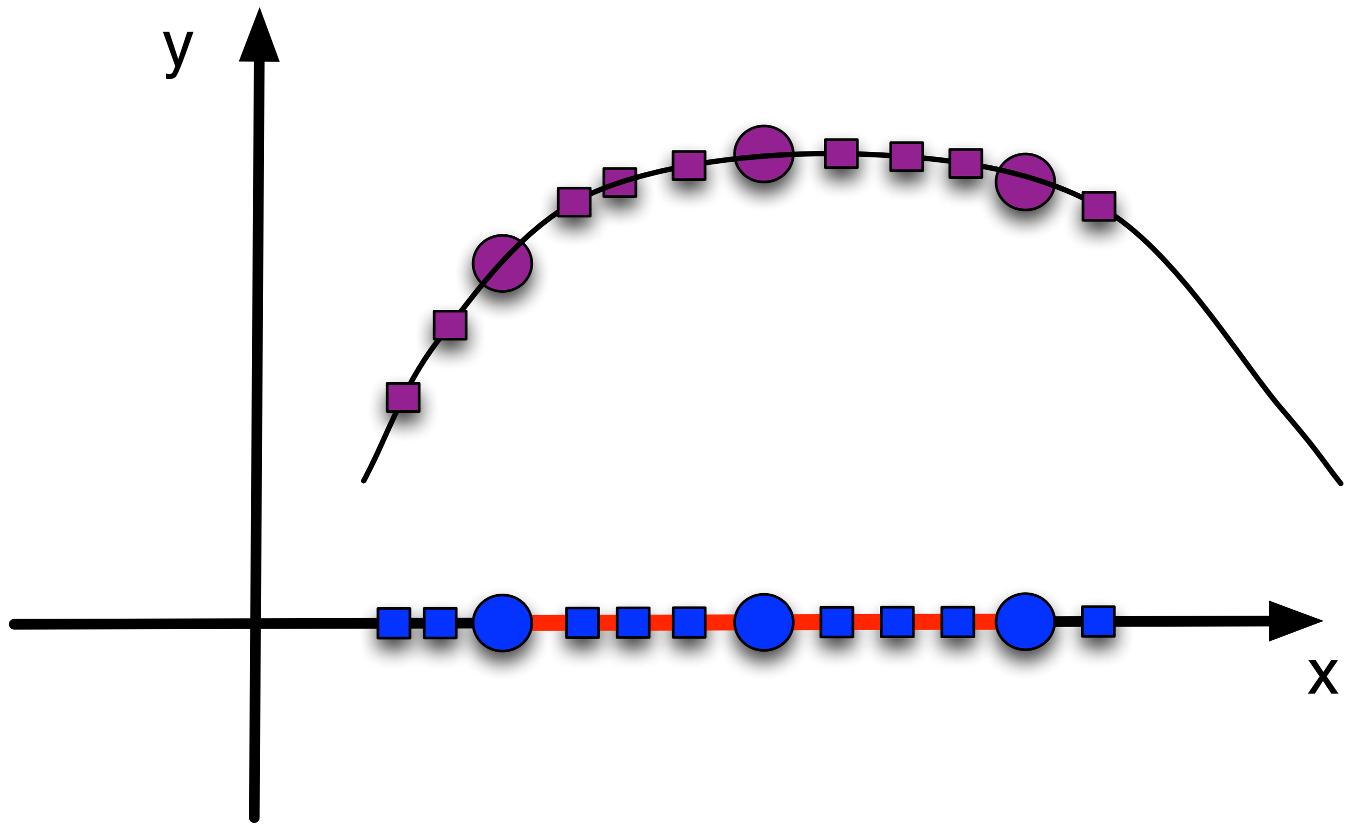
Lagrange interpolation

The interpolating polynomial are Lagrange polynomials

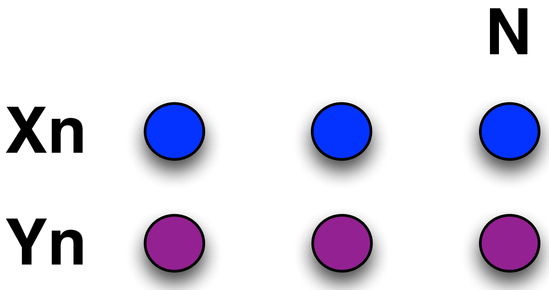
$$p_n(x_p) = \sum_{j=0}^{n} y_j L_j(x)$$

$$L_j(x) = \prod_{\substack{k=0 \\ k\neq j}}^{n} \frac{(x - x_k)}{(x_j - x_k)}$$

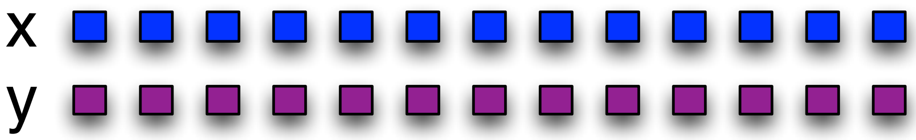
Interpolating at many points



Nodal info:
two arrays of size N



Interpolated points:
two arrays of some size



Newton’s Divided Difference

Node (x_j)	$f(x_j)$	1st Divided Difference	2nd Divided Difference	3rd Divided Difference
x_0	y_0			
		$y_{0,1} = \frac{y_1 - y_0}{x_1 - x_0}$		
x_1	y_1		$y_{0,1,2} = \frac{y_{1,2} - y_{0,1}}{x_2 - x_0}$	
		$y_{1,2} = \frac{y_2 - y_1}{x_2 - x_1}$		$y_{0,1,2,3} = \frac{y_{1,2,3} - y_{0,1,2}}{x_3 - x_0}$
x_2	y_2		$y_{1,2,3} = \frac{y_{2,3} - y_{1,2}}{x_3 - x_1}$	
		$y_{2,3} = \frac{y_3 - y_2}{x_3 - x_2}$		$y_{1,2,3,4} = \frac{y_{2,3,4} - y_{1,2,3}}{x_4 - x_1}$
x_3	y_3		$y_{2,3,4} = \frac{y_{3,4} - y_{2,3}}{x_4 - x_2}$	
		$y_{3,4} = \frac{y_4 - y_3}{x_4 - x_3}$		
x_4	y_4			

Newton's Divided Difference: recursive approach

$$f[x_0] \equiv f(x_0)$$

$$f[x_0, x_1, \dots, x_n] = \frac{f[x_0, \dots, x_{n-1}] - f[x_1, \dots, x_n]}{x_0 - x_n}$$

Newton's Divided Difference: iterative approach

N = 5, n = 4

		j = 0		j = 1		j = 2		j = 3
Node (x_j)	$f(x_j)$		1st Divided Difference		2nd Divided Difference		3rd Divided Difference	
x_0	y_0 f[0]							
		i = 0	$y_{0,1} = \frac{y_1 - y_0}{x_1 - x_0}$ f[0]					
x_1	y_1 f[1]			i = 0	$y_{0,1,2} = \frac{y_{1,2} - y_{0,1}}{x_2 - x_0}$ f[0]			
		i = 1	$y_{1,2} = \frac{y_2 - y_1}{x_2 - x_1}$ f[1]			i = 0	$y_{0,1,2,3} = \frac{y_{1,2,3} - y_{0,1,2}}{x_3 - x_0}$ f[0]	
x_2	y_2 f[2]			i = 1	$y_{1,2,3} = \frac{y_{2,3} - y_{1,2}}{x_3 - x_1}$ f[1]			i = 0 f[0]
		i = 2	$y_{2,3} = \frac{y_3 - y_2}{x_3 - x_2}$ f[2]			i = 1	$y_{1,2,3,4} = \frac{y_{2,3,4} - y_{1,2,3}}{x_4 - x_1}$ f[1]	
x_3	y_3 f[3]			i = 2	$y_{2,3,4} = \frac{y_{3,4} - y_{2,3}}{x_4 - x_2}$ f[2]			
		i = 3	$y_{3,4} = \frac{y_4 - y_3}{x_4 - x_3}$ f[3]					
x_4	y_4 f[4]							