CS220: Lecture Notes

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1 Boolean Algebra

- Boolean algebra defines operations and rules for working with the set $\{0, 1\}$.

1.1 Boolean Operations and Functions

Complement Denoted by a bar:

$$\overline{0} = 1 \qquad \overline{1} = 0$$

Boolean sum Denoted as + / OR:

$$1 + 1 = 1 \qquad 1 + 0 = 1$$
$$0 + 1 = 1 \qquad 0 + 0 = 0$$

Boolean product Denoted as $\cdot$ / AND:

$$1 \cdot 1 = 1 \qquad 1 \cdot 0 = 0$$
$$0 \cdot 1 = 0 \qquad 0 \cdot 0 = 0$$

Definition 1.1 (Boolean variable)**.** Variable x is a **Boolean variable** only if $x \in \{0, 1\}$.

1.2 Identities

1.3 Defintition of a Boolean Algebra

- All the properties of Boolean functions and expression apply to other mathematical structures such as propositions and sets and the operations defined on them.
- If we can show that a particular structure is a Boolean algebra, then we know that all results established about Boolean algebras apply to this structure.

- For this purpose, we need an abstract definition of a Boolean algebra.

Definition 1.2 (Boolean Algebra)**.** A Boolean algebra is a set B with two binary operators $\wedge$ and $\vee$, elements 0 and 1, and a unary operation $-$ such that the following properties hold fo all x, y, and z in B:

- $x \vee 0 = x$ and $x \wedge 1 = x$ (identity laws).

2 Relations

- If we want to describe a relationship between elements of two sets A and B, we can use ordered pairs with an element taken from A and an element taken from B.
- Since this is a relation between two sets, it is called a *binary relation.*

Definition 2.1 (Binary Relation)**.** Let A and B be sets. A **binary relation** from A to B is a subset of $A \times B$.

- In other words, for a binary relation R we have $R \subseteq A \times B$. We use the notation aRb to denote that $(a, b) \in R$ and a$\underline{\text{R}}$b to denote that $(a, b) \notin R$.
- When (a, b) belongs to R, a is said to be related to b by R.
- **Example:** Let P be a set of people, C be a set of cars, and D be the relation describing which person drives which car(s).
  - $P = \{$Carl,Suzanne,Peter,Carla$\}$
  - $C = \{$Mercedes,BMW,tricycle$\}$
  - $D = \{$(Carl,Mercedes),(Suzanne,Mercedes),(Suzanne,BMW),(Peter,tricycle)$\}$

  This means that Carl drives a Mercedes, Suzanne drives a Mercedes and a BMW, Peter drivevs a tricycle, and Carla does not drive of these vehicles.