

CS220: Lecture Notes

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1 Boolean Algebra

- Boolean algebra defines operations and rules for working with the set $\{0, 1\}$.

1.1 Boolean Operations and Functions

Complement Denoted by a bar:

$$\bar{0} = 1$$

$$\bar{1} = 0$$

Boolean sum Denoted as $+$ / OR:

$$1 + 1 = 1$$

$$1 + 0 = 1$$

$$0 + 1 = 1$$

$$0 + 0 = 0$$

Boolean product Denoted as \cdot / AND:

$$1 \cdot 1 = 1$$

$$1 \cdot 0 = 0$$

$$0 \cdot 1 = 0$$

$$0 \cdot 0 = 0$$

Definition 1.1 (Boolean variable). Variable x is a **Boolean variable** only if $x \in \{0, 1\}$.

1.2 Identities

1.3 Definition of a Boolean Algebra

- All the properties of Boolean functions and expression apply to other mathematical structures such as propositions and sets and the operations defined on them.
- If we can show that a particular structure is a Boolean algebra, then we know that all results established about Boolean algebras apply to this structure.

- For this purpose, we need an abstract definition of a Boolean algebra.

Definition 1.2 (Boolean Algebra). A Boolean algebra is a set B with two binary operators \wedge and \vee , elements 0 and 1 , and a unary operation $-$ such that the following properties hold for all x, y , and z in B :

- $x \vee 0 = x$ and $x \wedge 1 = x$ (identity laws).