Tatsis, Konstantinos E., Vasilis K. Dertimanis, and Eleni N. Chatzi. "Sequential Bayesian Inference for Uncertain Nonlinear Dynamic Systems: A Tutorial." arXiv preprint arXiv:2201.08180 (2022).

Sequential Bayesian Inference

Calculate *posterior* $p\left(\mathbf{x}_k \mid \mathbf{y}_{1:k}\right)$ for the *state,* $\mathbf{x}_k$, recursively, via the *predictive* distribution:

$$p\left(\mathbf{x}_k \mid \mathbf{y}_{1:k-1}\right) = \int p\left(\mathbf{x}_k \mid \mathbf{x}_{k-1}\right) p\left(\mathbf{x}_{k-1} \mid \mathbf{y}_{1:k-1}\right) \mathrm{d}\mathbf{x}_{k-1}$$

$$= \mathbb{E}_{\mathbf{x}_k \sim p\left(\mathbf{x}_{k-1} \mid \mathbf{y}_{1:k-1}\right)} \left[p\left(\mathbf{x}_k \mid \mathbf{x}_{k-1}\right) \right]$$

followed by the *correction* step involving Bayes formula:

$$p(\mathbf{x}_k \mid \mathbf{y}_{1:k}) = \frac{\overbrace{p(\mathbf{y}_k \mid \mathbf{x}_k)}^{\text{likelihood}} \overbrace{p(\mathbf{x}_k \mid \mathbf{y}_{1:k-1})}^{\text{"prior"}}}{\int p(\mathbf{y}_k \mid \mathbf{x}_k) p(\mathbf{x}_k \mid \mathbf{y}_{1:k-1}) \mathrm{d}\mathbf{x}_k}$$

- **Marginalization over state $\mathbf{x}_{k-1}$ in Chapman-Kolmogorov integral, and**
- **Integral for evidence are both computationally unfeasible!**

Complexities

dynamical model for CO_2 plumes

CO_2 monitoring w/ Data Assimilation is also complicated by

- complexity & nonlinearities of the *dynamics* & *measurement* models
- lack of knowledge on the reservoir properties (permeability)

Nonlinear multi-modal *dynamical* problem w/ control:

$$\mathbf{x}_k = \mathscr{M}_{k-1}(\mathbf{x}_{k-1}, \mathbf{K}; \mathbf{q}_{k-1}) \quad \text{with} \quad K \sim p(K)$$

$$\mathbf{y}_k = \mathscr{H}_k(\mathbf{x}_k) + \epsilon_k$$

- ***permeability*** **is treaded as a** ***random variable*** **that can be** ***sampled***
- ***injection*** **rates,** $\mathbf{q}_{k-1}$**, need to be** ***controlled*** **to avoid** ***fracturing*** **the** ***seal***
- **well placement,** $\mathscr{H}_k$**, needs to be** ***optimized*** **to** ***reduce*** **uncertainty & costs**