# *Physical* simulations

# $CO_2$ dynamics

## two-phase flow equations

**mass balance equation:**

$$\frac{\partial}{\partial t}(\phi S_i \rho_i) + \nabla \cdot (\rho_i \mathbf{v}_i) = \rho_i q_i, \quad i = 1,2$$

inject $CO_2$ to replace water

$$S_1 + S_2 = 1$$

**Darcy's law:**

$$\mathbf{v}_i = -\frac{K k_{ri}}{\tilde{\mu}_i}(\nabla P_i - g\rho_i \nabla Z), \quad i = 1,2$$

**fluid pressure:**

$$P_2 = P_1 - P_c(S_2)$$

| Symbol | Meaning |
|---|---|
| $K$ | permeability |
| $\phi$ | porosity |
| $k_{ri}$ | relative permeability |
| $S_i$ | fluid saturation |
| $P_i$ | fluid pressure |
| $P_c$ | capillary pressure |
| $\mathbf{v}_i$ | Darcy's velocity |
| $\rho_i$ | fluid density |
| $\tilde{\mu}_i$ | fluid viscosity |
| $q_i$ | injection/production rate |
| $g$ | gravity constant |
| $Z$ | vector of vertical direction |