MAP Optimization with NFs

Maximum a posteriori (MAP):

$$\mathbf{x}_{MAP} = \underset{\mathbf{x}}{\mathrm{argmax}} \ p(\mathbf{x} \,|\, \mathbf{y})$$

$$\underset{\mathbf{x}}{\mathrm{argmax}} \ p(\mathbf{y} \,|\, \mathbf{x}) \, p(\mathbf{x}) = \underset{\mathbf{x}}{\mathrm{argmax}} \ log \, p(\mathbf{y} \,|\, \mathbf{x}) + log \, p(\mathbf{x})$$

If noise is Gaussian then the data likelihood is given by ℓ_2-norm data misfit

$$\underset{\mathbf{x}}{\mathrm{argmin}} \ \frac{1}{2}\|A\mathbf{x} - \mathbf{d}\|_2^2 + \log R(\mathbf{x})$$ **How do you choose regularization?**
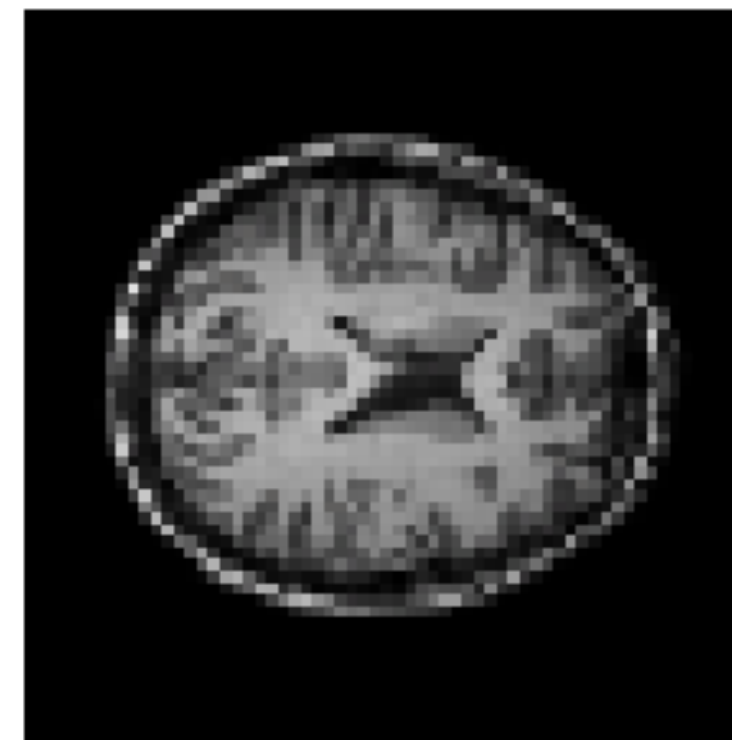
 Asim, Muhammad, et al. "Invertible generative models for inverse problems: mitigating representation error and dataset bias." International Conference on Machine Learning. PMLR, 2020.

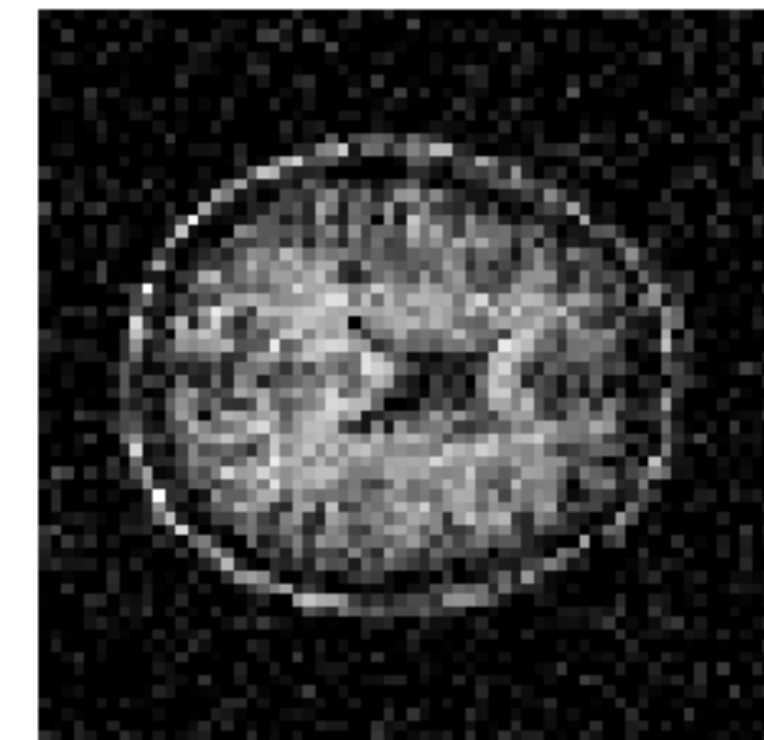Allow for exact likelihood evaluation

$$p_x(x = \text{[image]}) = 0.99$$

$$p_x(x = \text{[image]}) = 0.01$$

Calculated $\log p(x) = 3.27$ Calculated $\log p(x) = -24.28$

Train network and use as prior in bayesian formulation:

$$\underset{\mathbf{x}}{\mathrm{argmax}} \ p(\mathbf{x} \,|\, \mathbf{y}) = \underset{\mathbf{x}}{\mathrm{argmax}} \ log \ p(\mathbf{y} \,|\, \mathbf{x}) + log \ p(\mathbf{x})$$