



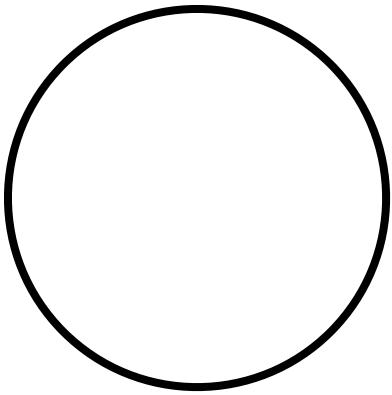
SLIM

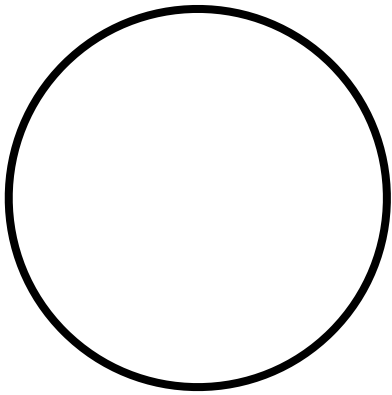




ML4Seismic

# *Learn* Sequential Bayesian Inference





**X**

***K***

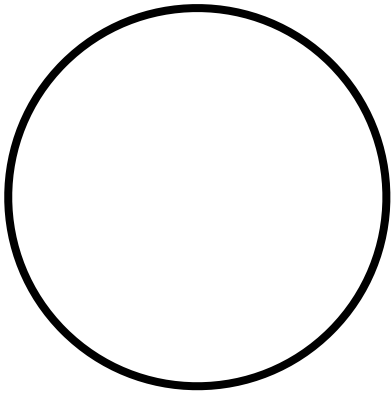
**—**

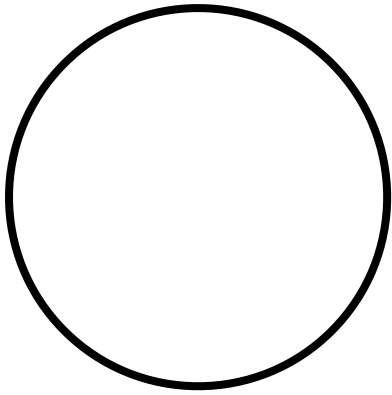
**1**

**X**

***K***







$y^0_{k-1}$

yo  
k

















sample from posterior  $\mathbf{x}_k \sim p(\mathbf{x}_k | y_k^o)$

**Note:** implicitly sampled from

$$p(\mathbf{x}_k | \mathbf{y}_k, \mathbf{y}_{1:k-1}) = \frac{p(\mathbf{y}_k | \mathbf{x}_k) p(\mathbf{x}_k | \mathbf{y}_{1:k-1})}{p(\mathbf{y}_k | \mathbf{y}_{1:k-1})}$$

$$p(\mathbf{x}_k | \mathbf{y}_{1:k-1}) = \mathbb{E}_{\mathbf{x}_{k-1} \sim p(\mathbf{x}_{k-1} | \mathbf{y}_{1:k-1})} \left[ p(\mathbf{x}_k | \mathbf{x}_{k-1}) \right]$$

Marginalizes over

- ▶ previous state  $\mathbf{x}_{k-1}$
- ▶ permeability  $\mathbf{K}$

**Sample from posterior  $\mathbf{x}_k \sim p(\mathbf{x}_k | y_k^0)$  via  $\mathbf{x}_k = f_{\hat{\phi}}^{-1}(\mathbf{z}; y_k^0)$**

**with  $\mathbf{z} \sim N(\mathbf{0}, \mathbf{I})$ .**

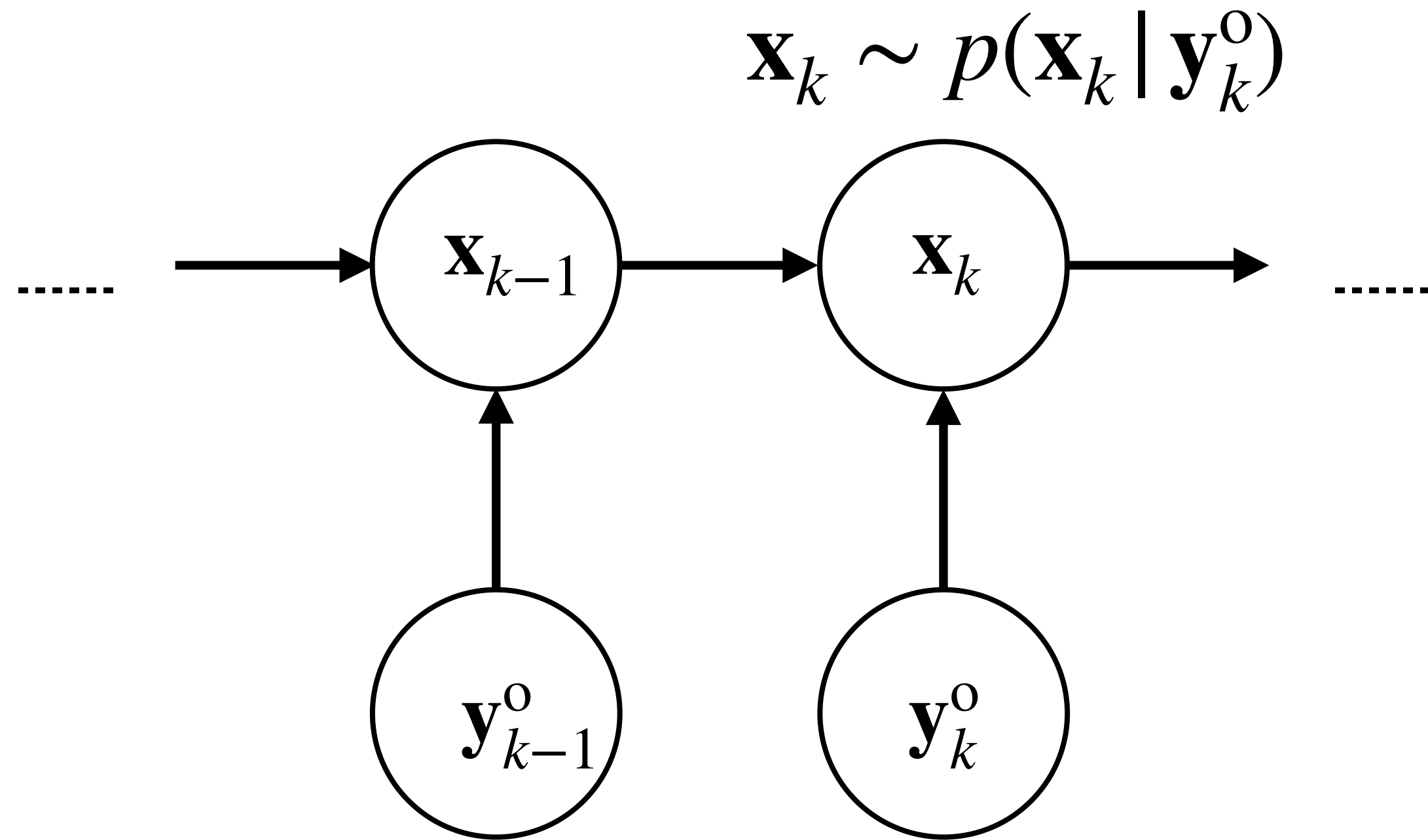
$$\mathbf{x}_k \sim p(\mathbf{x}_k | \mathbf{y}_k^o)$$

**HINT: Hierarchical Invertible Neural Transport for General and Sequential Bayesian inference,**  
**Detommaso, et. al., [arXiv:1905.10687](#)**



# Learned Sequential Bayesian Inference

sample from posterior  $\mathbf{x}_k \sim p(\mathbf{x}_k | \mathbf{y}_k^o)$



**Note:** implicitly sampled from

$$p(\mathbf{x}_k | \mathbf{y}_k, \mathbf{y}_{1:k-1}) = \frac{p(\mathbf{y}_k | \mathbf{x}_k) p(\mathbf{x}_k | \mathbf{y}_{1:k-1})}{p(\mathbf{y}_k | \mathbf{y}_{1:k-1})}$$

$$p(\mathbf{x}_k | \mathbf{y}_{1:k-1}) = \mathbb{E}_{\mathbf{x}_{k-1} \sim p(\mathbf{x}_{k-1} | \mathbf{y}_{1:k-1})} \left[ p(\mathbf{x}_k | \mathbf{x}_{k-1}) \right]$$

Marginalizes over

- ▶ previous state  $\mathbf{x}_{k-1}$
- ▶ permeability  $\mathbf{K}$

**Sample from posterior  $\mathbf{x}_k \sim p(\mathbf{x}_k | \mathbf{y}_k^o)$  via  $\mathbf{x}_k = f_{\hat{\phi}}^{-1}(\mathbf{z}; \mathbf{y}_k^o)$**

**with  $\mathbf{z} \sim N(\mathbf{0}, \mathbf{I})$ .**

# *Physical* simulations