# MAP Optimization with NFs

**Maximum a posteriori (MAP):**

$$\mathbf{x}_{MAP} = \underset{\mathbf{x}}{\text{argmax}} \;\; p(\mathbf{x}\,|\,\mathbf{y})$$

$$\underset{\mathbf{x}}{\text{argmax}} \;\; p(\mathbf{y}\,|\,\mathbf{x})\; p(\mathbf{x}) = \underset{\mathbf{x}}{\text{argmax}} \;\; log\; p(\mathbf{y}\,|\,\mathbf{x}) + log\; p(\mathbf{x})$$

If noise is Gaussian then the data likelihood is given by $\ell_2$-norm data misfit

$$\underset{\mathbf{x}}{\text{argmin}} \;\; \frac{1}{2}\|A\mathbf{x} - \mathbf{d}\|_2^2 + \log R(\mathbf{x})$$

**How do you choose regularization?**



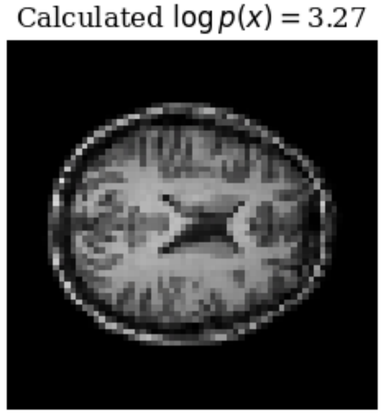
Asim, Muhammad, et al. "Invertible generative models for inverse problems: mitigating representation error and dataset bias." International Conference on Machine Learning. PMLR, 2020.

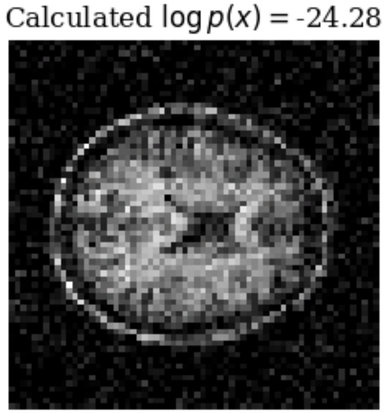
# Normalizing Flow as a trained prior

Allow for exact likelihood evaluation

$$p_x(x = \text{  }) = 0.99$$

$$p_x(x = \text{  }) = 0.01$$




Train network and use as prior in bayesian formulation:

$$\underset{\mathbf{x}}{\text{argmax}} \; p(\mathbf{x}|\mathbf{y}) = \underset{\mathbf{x}}{\text{argmax}} \; \log \; p(\mathbf{y}|\mathbf{x}) + \log \; p(\mathbf{x})$$