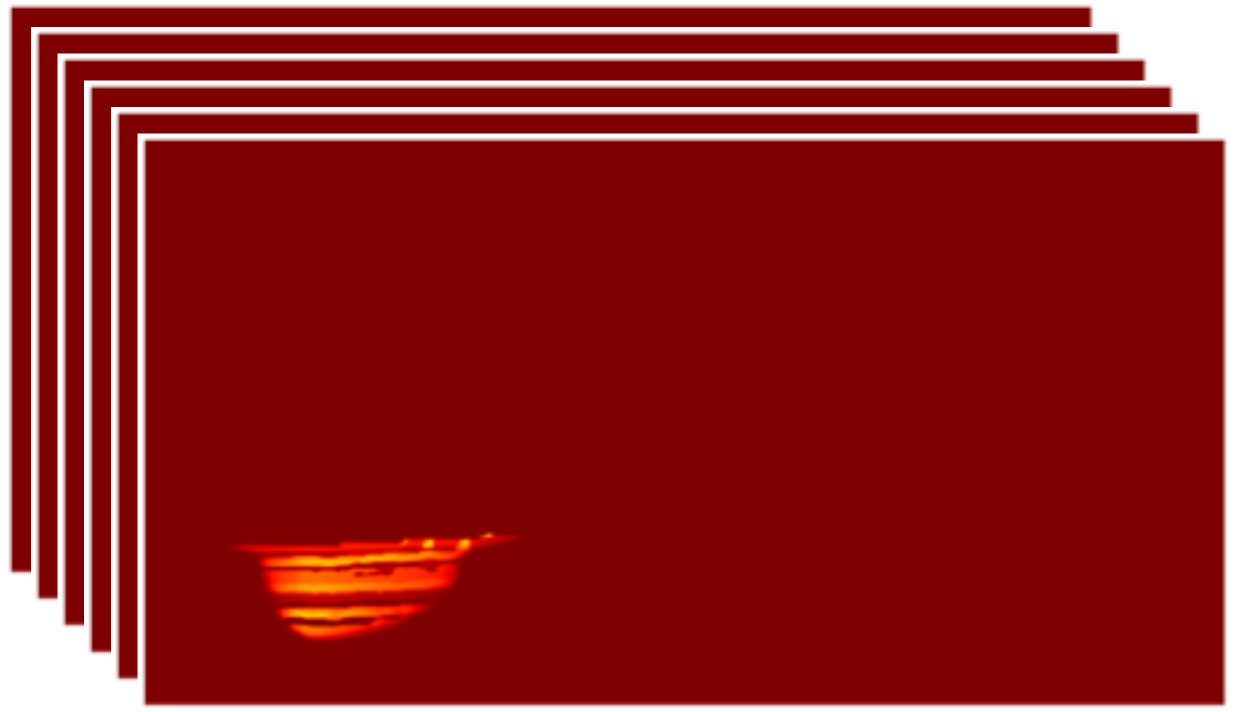
Rock physics

patchy saturation model at $k = 1$

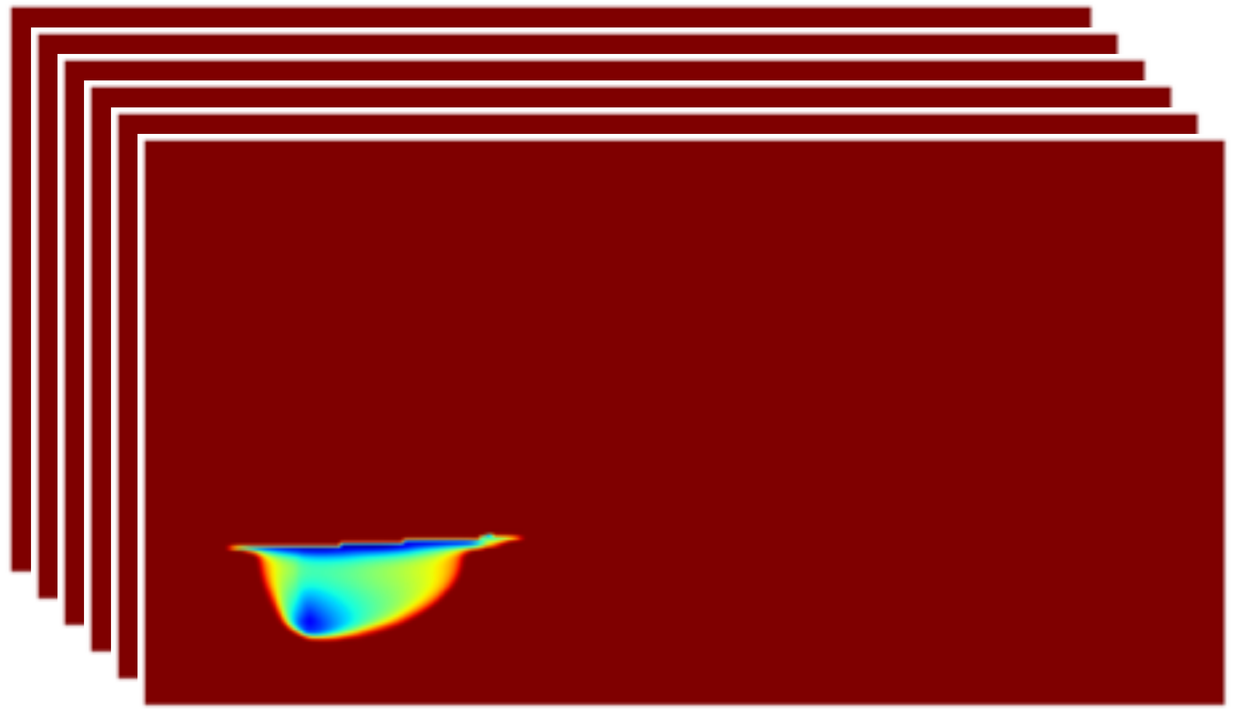
SLIM ML4Seismic

$\{\mathbf{x}_k^{(m)}\}_{m=1}^{M}$



saturations

$\{\mathbf{v}_k^{(m)} = \mathscr{R}(\bar{\mathbf{v}}_0, \mathbf{x}_k^{(m)})\}_{m=1}^{M}$



impedance change

Symbol	**Meaning**
B_{r_1}/B_{r_2}	bulk modulus of rock fully saturated with fluid 1/2
B_{f_1}/B_{f_2}	fluid bulk modulus
ρ_{f_1}/ρ_{f_2}	fluid density
μ_r	rock shear modulus
v_p/v_s	rock P/S-wave velocity
B_o	bulk modulus of rock grains
ρ_r	rock density
ϕ	rock porosity
S	CO_2 saturation

CO_2 concentration $\uparrow \rightarrow v_p$ & $\rho \downarrow$

v_p decrease by 0-300 m/s

localized time-lapse changes

1.68% change in acoustic impedance

$$
\begin{aligned}
B_{r1} &= \rho_r(v_p^2 - \tfrac{4}{3}v_s^2) \\
\mu_r &= \rho_r v_s^2 \\
\frac{B_{r2}}{B_o - B_{r2}} &= \frac{B_{r1}}{B_o - B_{r1}} - \frac{B_{f1}}{\phi(B_o - B_{f1})} + \frac{B_{f2}}{\phi(B_o - B_{f2})} \\
\hat{B}_r &= [(1-S)(B_{r1} + \tfrac{4}{3}\mu_r)^{-1} + S(B_{r2} + \tfrac{4}{3}\mu_r)^{-1}]^{-1} - \tfrac{4}{3}\mu_r \\
\hat{\rho}_r &= \rho_r + \phi S(\rho_{f2} - \rho_{f1}) \\
\hat{v}_p &= \sqrt{\frac{\hat{B}_r + \frac{4}{3}\mu_r}{\hat{\rho}_r}}
\end{aligned}
$$

Per Avseth, et al. Quantitative seismic interpretation: Applying rock physics tools to reduce interpretation risk. Cambridge university press, 2010.

Wave-based time-lapse imaging

wavespeeds & *fiducial* wavespeed at $k = 1$

Update wavespeeds & simulate M seismic datasets (start from baseline $\bar{\mathbf{v}}_0$)

$$\mathbf{v}_k^{(m)} = \mathscr{R}(\bar{\mathbf{v}}_0, \mathbf{x}_k^{(m)}), \quad m = 1 \cdots M \qquad \mathbf{y}_k^{(m)} = \mathscr{F}(\mathbf{v}_k^{(m)}), \quad m = 1 \cdots M$$

Update *reference* wavespeed & simulate *reference* dataset

$$\bar{\mathbf{v}}_k = \mathscr{R}(\bar{\mathbf{v}}_0, \bar{\mathbf{x}}_k), \quad \bar{\mathbf{x}}_k = \frac{1}{M}\sum_{m=1}^{M} \mathbf{x}_k^{(m)} \qquad \bar{\mathbf{y}}_k = \mathscr{F}(\bar{\mathbf{v}}_k)$$

Compute *fiducial* point & summarize M datasets

$$\widetilde{\mathbf{v}}_k = \mathrm{smooth}(\bar{\mathbf{v}}_k) \qquad \bar{\mathbf{y}}_k^{(m)} = \mathbf{J}^\top[\widetilde{\mathbf{v}}_k]\left(\bar{\mathbf{y}}_k - \mathbf{y}_k^{(m)}\right), \quad m = 1 \cdots M$$