Jansen, Jan Dirk. "Adjoint-based optimization of multi-phase flow through porous media–a review." *Computers & Fluids* 46.1 (2011): 40-51.

CO_2 dynamics
two-phase flow equations

mass balance equation:

$$\frac{\partial}{\partial t}(\phi S_i \rho_i) + \nabla \cdot (\rho_i \mathbf{v}_i) = \rho_i q_i, \quad i = 1,2$$

inject CO_2 to replace water

$$S_1 + S_2 = 1$$

Darcy's law:

$$\mathbf{v}_i = -\frac{K k_{ri}}{\tilde{\mu}_i}(\nabla P_i - g \rho_i \nabla Z), \quad i = 1,2$$

fluid pressure:

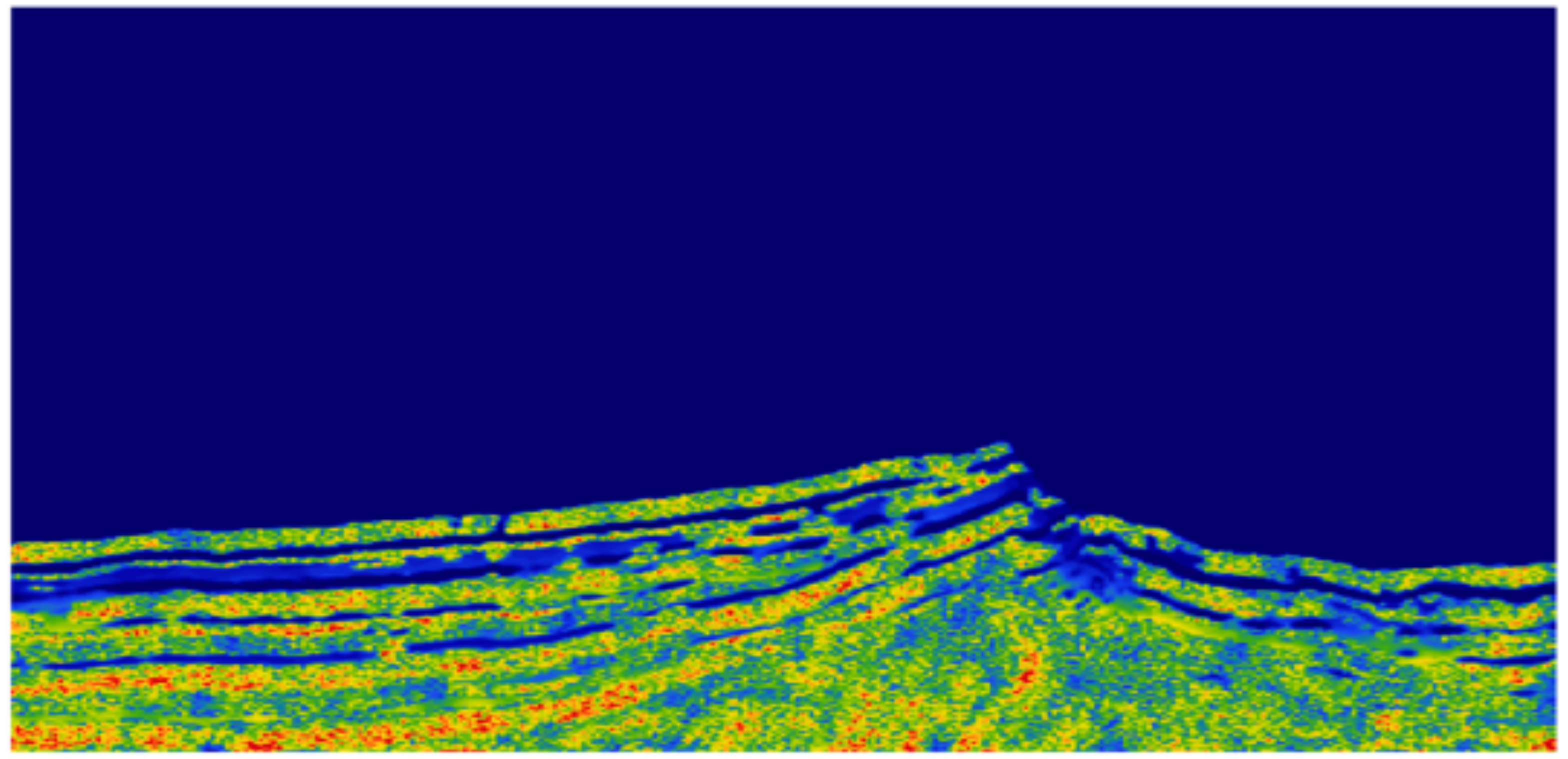
$$P_2 = P_1 - P_c(S_2)$$

Symbol	Meaning
K	permeability
ϕ	porosity
k_{ri}	relative permeability
S_i	fluid saturation
P_i	fluid pressure
P_c	capillary pressure
$\mathbf{v}_i$	Darcy's velocity
ρ_i	fluid density
$\tilde{\mu}_i$	fluid viscosity
q_i	injection/production rate
g	gravity constant
Z	vector of vertical direction

Training samples for the *state*

CO_2 saturation at $k = 1$

$K \sim p(K)$

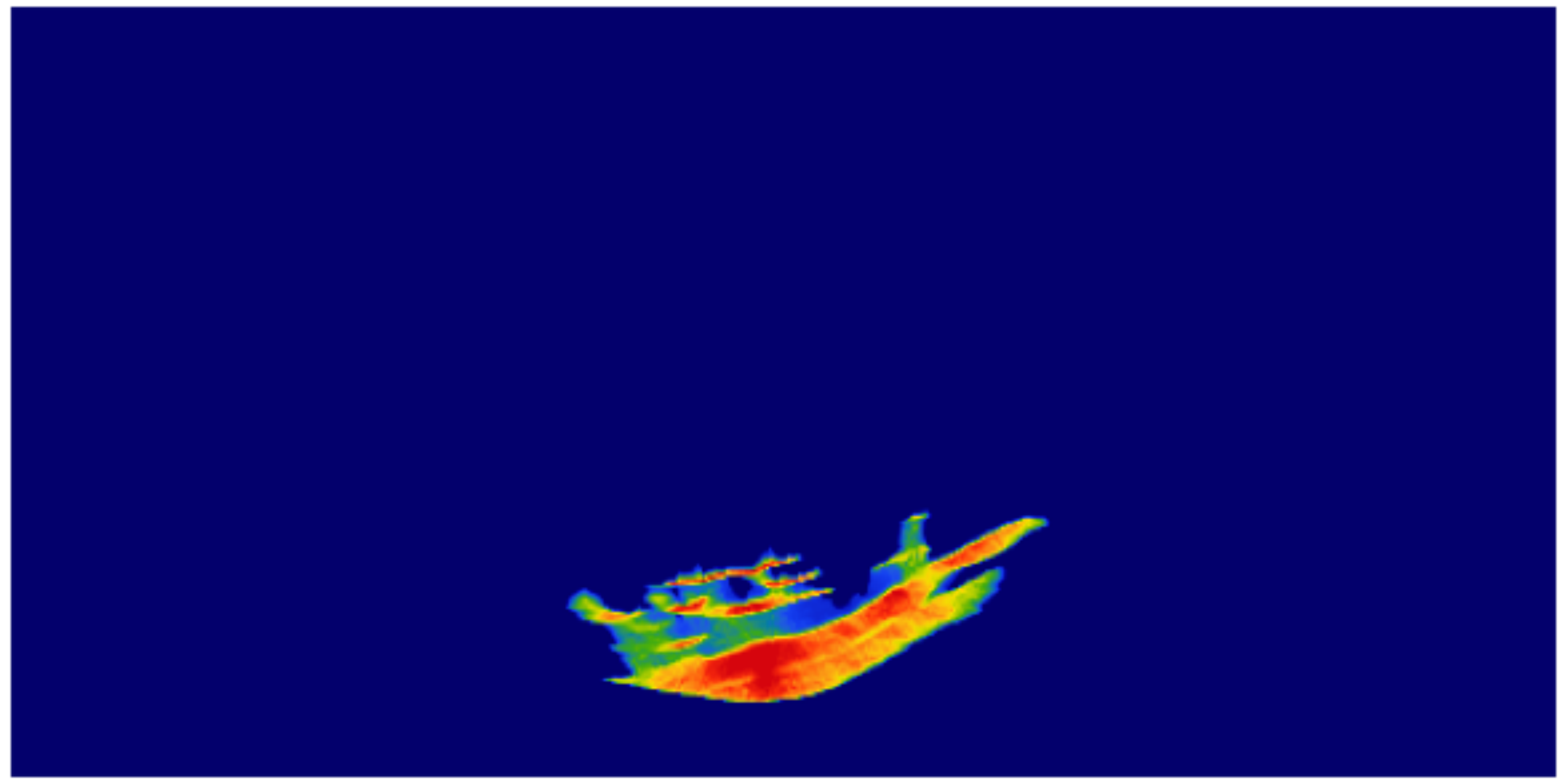


permeability

$\mathbf{x}_{k-1} \sim p(\mathbf{x}_{k-1})$

initial saturation

$\mathbf{x}_k \sim p(\mathbf{x}_k \mid \mathbf{x}_{k-1})$



transitioned saturation

$$\mathbf{x}_k = \mathscr{M}_{k-1}(\mathbf{x}_{k-1}, \mathbf{K}; \mathbf{q}_{k-1}) \quad \text{with} \quad \mathbf{x}_0 \sim p(\mathbf{x}_0), K \sim p(K)$$