Sequential Bayesian inference

Problem Statement

Bayesian filtering problem

Consider CO_2 monitoring as a Bayesian filtering problem:

$$\mathbf{x}_k \sim p(\mathbf{x}_k \mid \mathbf{x}_{k-1}) \qquad \mathbf{y}_k \sim p(\mathbf{y}_k \mid \mathbf{x}_k), \quad k = 0,1,2,\ldots$$

where

- $\mathbf{x}_k \in \mathbb{R}^n$ is the *state* (CO_2 saturation/pressure) vector at time $t = k\Delta t$
- $\mathbf{y}_k \in \mathbb{R}^m$ is the *observation* vector
- $p(\mathbf{x}_k \mid \mathbf{x}_{k-1}) = p(\mathbf{x}_k \mid \mathbf{x}_{1:k-1}, \mathbf{y}_{1:k-1})$ is the Markovian *transition* probability
- $p(\mathbf{y}_k \mid \mathbf{x}_k) = p(\mathbf{y}_k \mid \mathbf{x}_{1:k}, \mathbf{y}_{1:k-1})$ is the *likelihood* of the *measurement* model