Interview Skills

Graph Algorithms

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Fordham University CS Society

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What is a Graph?

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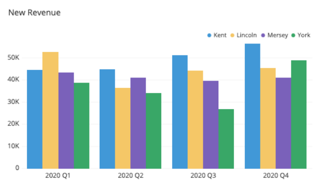

Figure 1: Not This Kind of Graph!

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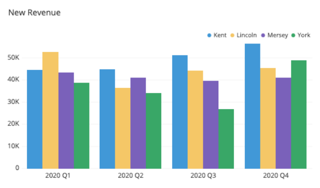

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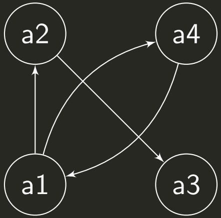

Figure 2: This Kind of Graph!

Characteristics of a Graph

- Set of Nodes S
- Set of Edges $E \subset S^{2*}$

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  - Network Connections†
  - Intersections and Streets

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  - Islands & Bridges
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  - Intersections and Streets
- Position of nodes is for communication only
- Edges may have a "weight" assigned to them, which may represent distance in some cases

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Adjacency Matrix

- Very useful for mathematical proofs

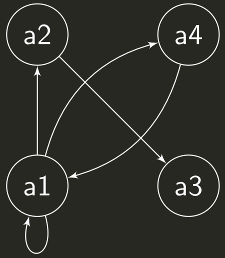

Figure 3: Visual Representation of a Graph

$$\begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{pmatrix}$$

Figure 4: The Same Graph as 3, in an Adjacency Matrix

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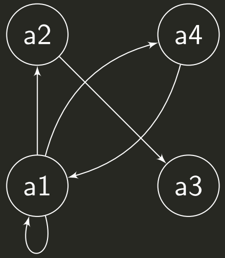

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- Memory usage causes issues when used in programs.

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- Graphs are most often represented as:
  - Node Lists
  - Edge Lists

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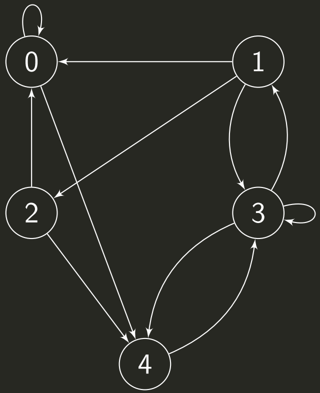
Node List




```cpp
vector<vector<int>> graph = {
      {0, 4},
      {2, 3, 0},
      {0, 4},
      {3, 1, 4},
      {3}
};
```

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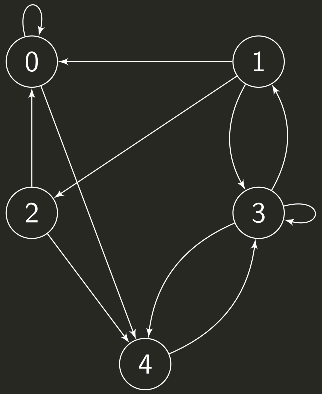
Edge List




```cpp
vector<vector<int>> graph = {
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  - adjacent(x,y): bool
  - neighbors(x): list of edges
  - get_vertex_value(x): value type
  - get_edge_value(...): value type[‡]

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  - get_vertex_value(x): value type
  - get_edge_value(...): value type[‡]
- Modify Operations[§]
  - set_vertex_value(x, v): void
  - set_edge_value(..., v): void
  - add / remove edges / vertices

[‡] Argument is either edge key or the two vertices it connects, depending on representation.
[§] We won't be modifying already constructed graphs in this workshop.

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  - It it possible to end up back where you started without re-using edges?
  - Most of the time problems that involve acyclic graphs are phrased as tree problems, so if you aren't told otherwise, assume you have to deal with cycles in your code.

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- Values on Edges / Nodes
  - Sometimes all that really matters is the connections, sometimes you have values attached.
  - For e.g. calculating the travel time between two cities on a graph representing a map.
  - Representation in code will get more complex, but luckily you'll usually be given starter code.

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Most Basic Graph Algorithm: Depth First Search

- Simple and straightforward (although not efficient) way to identify any path between two nodes.
- Path is not guaranteed to be optimal by any metric.

Problem 1: Flower Garden

You have N gardens, labeled 1 to N - 1. In each garden, you want to plant one of 4 types of flowers (labeled 0 to 3). You are supplied with an array paths such that paths[i] = [x,y] means that there is a bidirectional path between nodes x and y. You may assume that no garden has more than 3 paths connecting to it.

Your task is to determine which type of flower to plant in each garden, such that no two connected gardens have the same flower. Your code should return an array of size N, where each element represents the flower planted in each garden.

Solution 1 (Ineficient but Easy to Understand)

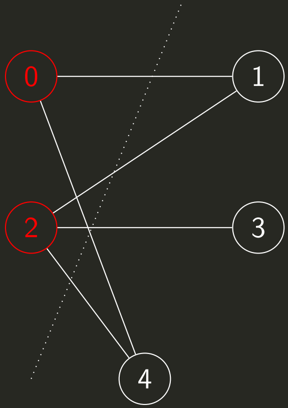
```cpp
vector<int> gardenNoAdj(int N, vector<vector<int>>& paths) {
    vector<int> result(N, -1);
    for (int i = 1; i <= N; ++i) {
        int color = 1;
        for (; color <= 4; ++color) {
            bool conflict = false;
            for (auto& edge : paths) {
                if (edge[0] == i) {
                    if (result[edge[1] - 1] == color) {
                        // Conflict found for this color
                        conflict = true;
                        break;
                    }
                }
                if (edge[1] == i) {
                    if (result[edge[0] - 1] == color) {
                        // Conflict found for this color
                        conflict = true;
                        break;
                    }
                }
            }
            if (!conflict) {
                break;
            }
        }
        result[i - 1] = color;
    }
    return result;
}
```

Problem 2: Bipartite

Given an undirected graph, determine whether it is bipartite. A bipartite graph is simply one that can be split into two sets of nodes such that no nodes in the same set are connected to each other.

In more graphical terms, this means you can draw a line that crosses every edge.

Your function should take input as a list of nodes and return a boolean. ¶



¶ `bool is_bipartite(const vector<vector<int>>& graph)`

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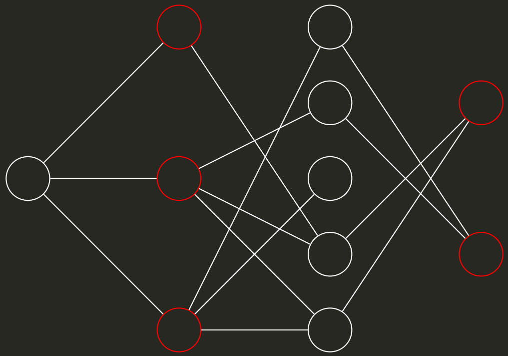
Solution Problem 2

There about a million ways you could code this, so I'll just show the theory behind the solution.



You create a list that stores the "color" of each node: red, black, or uncolored.