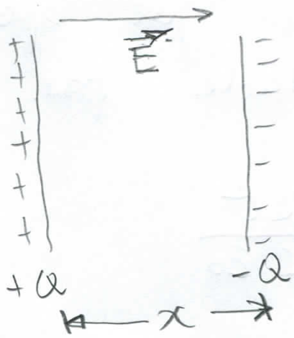
Soluciones

③ 24.29.

ⓐ



La energía almacenada por un condensador es.

$$U = \frac{Q^2}{2C}$$

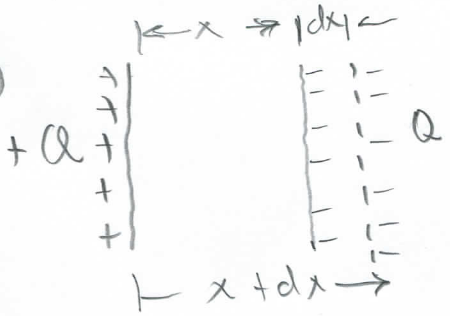
y la capacitancia para un condensador de placas paralelas de acuerdo a la geometría es. $C = \frac{\varepsilon_0 A}{d} = \frac{\varepsilon_0 A}{x}$.

luego de acuerdo con lo anterior la energía es.

$$U = \frac{Q^2 x}{2\varepsilon_0 A}.$$

cuyas variables son conocidas de acuerdo a los datos del problema.

ⓑ



luego. $U(x) = \frac{x Q^2}{2\varepsilon_0 A} \Rightarrow \frac{dU(x)}{dx} = \frac{Q^2}{2\varepsilon_0 A}$.

$$\Rightarrow dU = \frac{Q^2}{2\varepsilon_0 A} dx$$

ⓒ $dU = dW = F dx \Rightarrow dU = \frac{Q^2}{2\varepsilon_0 A} dx = F dx$

entonces $\boxed{F = \frac{Q^2}{2\varepsilon_0 A}}$.

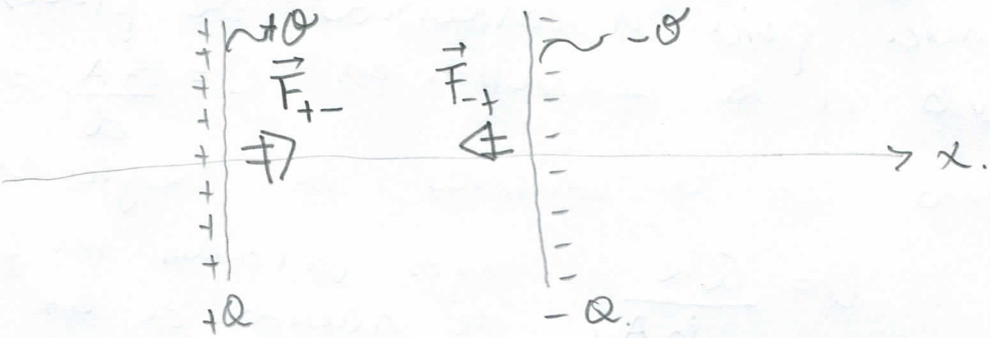
ⓓ El campo debido a ambas placas para un condensador está dado por $E = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$. Para una sola placa la ley de Gauss establece que $E = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2\varepsilon_0 A}$ luego.

$F = \frac{Q^2}{2\varepsilon_0 A}$ puesto que $\boxed{F = QE}$.

↳ Fuerza que ejerce una placa sobre la otra.

⑥ De acuerdo con el problema ③ en la parte ⓓ se tiene que la fuerza que ejerce una placa sobre la otra está dada por.

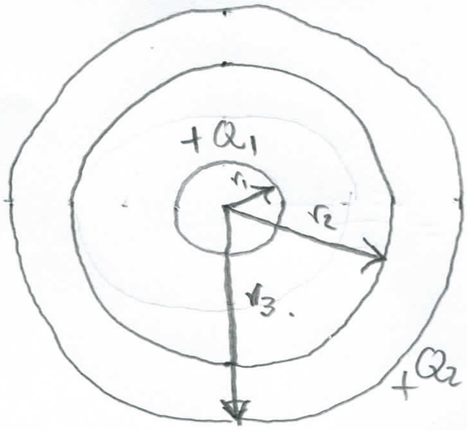
$$F = \frac{Q^2}{2\varepsilon_0 A} = \frac{Q\sigma}{2\varepsilon_0}$$



$$\vec{F}_{+-} = \frac{Q^2}{2\varepsilon_0 A}\hat{\imath} = +\frac{Q\sigma_-}{2\varepsilon_0}\hat{\imath}.$$

$$\vec{F}_{-+} = \frac{-Q^2}{2\varepsilon_0 A}\hat{\imath} = -\frac{Q\sigma_+}{2\varepsilon_0}\hat{\imath}.$$

(9).



$r_1 = 10$ cms

$r_2 = 30$ cms

$r_3 = 40$ cms.

$Q_1 = 1 \times 10^{-5}$ C.

$Q_2 = 8 \times 10^{-5}$ C.

Dentro de la bola metálica de radio r_1 el campo es nulo, luego $E(r<r_1) = 0 = -\frac{dV}{dr} \Rightarrow V = cte!$

En el exterior de la bola metálica se tiene que

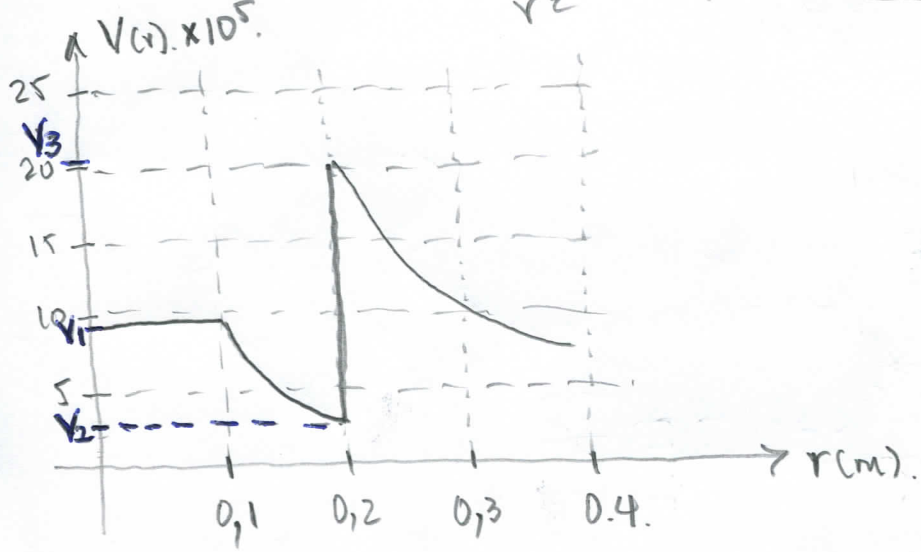
$$E(r_1<r<r_2) = \frac{KQ_1}{r^2} \Rightarrow V = \frac{KQ_1}{r}.$$

Entre el interior y el exterior del caparazón se tiene.

$$E(r_2<r<r_3) = 0 = -\frac{dV}{dr} \Rightarrow V = cte!$$

En el exterior de la la coraza se tiene que

$$E(r>r_3) = \frac{K(Q_1+Q_2)}{r^2} \Rightarrow V = \frac{K(Q_1+Q_2)}{r}, \text{ luego.}$$



$$V_1 = \frac{KQ_1}{r_1} = 9 \times 10^5 \text{ v.}$$

$$V_2 = \frac{K(Q_1+Q_2)}{r_3} = 20.3 \times 10^5 \text{ V.}$$

$$V_3 = \frac{KQ_1}{r_2} = 3 \times 10^5 \text{ v.}$$

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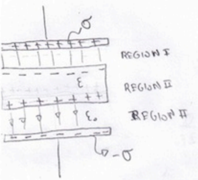

The electric field in the region I and III is

$$E = 2\left(\frac{-\sigma}{2\varepsilon_0}\right) = \frac{-\sigma}{\varepsilon_0}$$

Now the field inside the metal slab is 0 therefore

$$\vec{E}(z) = \hat{z}\begin{cases} -\sigma/\varepsilon_0 & \text{if } 0<z<\frac{1}{2}(d-a) \text{ or } \left(\frac{d+a}{2}\right)<z<d \\ 0 & \text{otherwise}\end{cases}$$

potential is $V = -\int E\,dz$

$$V(z) = \begin{cases} z\,\sigma/\varepsilon_0 + c & \text{if } <z<\frac{1}{2}(d-a) \text{ or } \frac{(d+a)}{2}<z<d \\ \text{CONST} \end{cases}$$

$$V\left(\tfrac{1}{2}(d-a)\right) = \frac{\sigma}{\varepsilon_0}\frac{(d-a)}{2} + C_1 = \text{const} = \frac{\sigma}{\varepsilon_0}\frac{(d+a)}{2} + C_2 = V\left(\tfrac{1}{2}(d+a)\right)$$

Hence $C_1 - C_2 = \frac{\sigma}{\varepsilon_0}\left(\frac{d+a}{2}\right) - \left(\frac{d-a}{2}\right)\frac{\sigma}{\varepsilon_0} = \frac{\sigma}{\varepsilon_0}a$

a result

$$= V(0) - V(d) = (C_1) - \left(d\frac{\sigma}{\varepsilon_0} + C_2\right) = (C_1 - C_2) - d\frac{\sigma}{\varepsilon_0} = \boxed{\frac{\sigma}{\varepsilon_0}(a-d) = V_{12}}$$

capacitance is

$$= \left|\frac{Q}{V_{12}}\right| = \left|\frac{\sigma A}{\sigma/\varepsilon_0\,(a-d)}\right| \Rightarrow \boxed{C = \varepsilon_0\left(\frac{A}{d-a}\right)} \quad \text{if } a=0 \Rightarrow C = C_0 = \varepsilon_0\frac{A}{d}$$

$$C = \frac{\varepsilon_0 A}{d}\left(\frac{d}{d-a}\right) \Rightarrow \boxed{C = C_0\left(\frac{d}{d-a}\right)}$$

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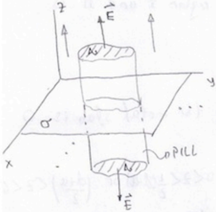
finite sheet



Using the Gauss law

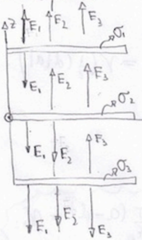
$$\oint_{\text{PILL}} \vec{E}\cdot d\vec{S} = \frac{Q_{int}}{\varepsilon_0}$$

$$2EA_0 = \frac{\sigma A_0}{\varepsilon_0} \Rightarrow E = \frac{\sigma}{2\varepsilon_0}$$

$$\boxed{\vec{E}(z) = \operatorname{sgn}(z)\frac{\sigma}{2\varepsilon_0}\hat{z}}$$

Electric field of a single sheet placed at $z=0$

urrent problem is the following



$$E = \frac{1}{2\varepsilon_0}(\sigma_1+\sigma_2+\sigma_3)$$

$$E = \frac{1}{2\varepsilon_0}(\sigma_2+\sigma_3-\sigma_1)$$

$$E = \frac{1}{2\varepsilon_0}(\sigma_3-\sigma_1-\sigma_2)$$

$$E = \frac{-1}{2\varepsilon_0}(\sigma_1+\sigma_2+\sigma_3)$$

The field in the region $z\in[-h_2,h_1]$ is given by

$$\vec{E} = \frac{\hat{z}}{2\varepsilon_0}\cdot\begin{cases}\sigma_2+\sigma_3-\sigma_1 & \text{if } z>0\\ \sigma_3-\sigma_1-\sigma_2 & \text{if } z<0\end{cases}$$

Now $\vec{E} = -\frac{\partial V}{\partial z}\hat{z}$ hence

$$V(z) = -\int E\,dz$$

ce

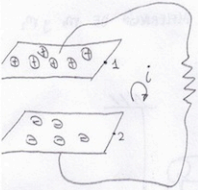
$$\boxed{V(z) = -\frac{z}{2\varepsilon_0}\cdot\begin{cases}\sigma_2+\sigma_3-\sigma_1 & \text{if } 0<z<h_1\\ \sigma_3-\sigma_1-\sigma_2 & \text{if } h_2<z<0\end{cases} + \text{Constant}}$$

e $V(h_1) = -\frac{h_1}{2\varepsilon_0}(\sigma_2+\sigma_3-\sigma_1)$ and $V(-h_2) = +\frac{h_2}{2\varepsilon_0}(\sigma_3-\sigma_1-\sigma_2)$. THE Potential ference between the upper and lower sheet is

$$V(h_1)-V(-h_2) = V_{12} = -\frac{h_1}{2\varepsilon_0}(\sigma_2+\sigma_3-\sigma_1) - \frac{h_2}{2\varepsilon_0}(\sigma_3-\sigma_1-\sigma_2) = \frac{(\sigma_1-\sigma_3)(h_1+h_2)}{2\varepsilon_0} + \frac{\sigma_2}{2\varepsilon_0}(h_2-h_1)$$

EM 24.27 WELL, the solution is too simple $W=\frac{1}{2}C\mathcal{V}_0^2$ with $C=450\,\mu F$ and $\mathcal{V}_0=295V$. However, there is more physics in this problem. Let us see

The conservation of energy implies



$$\mathcal{V}_{12}=\frac{q}{C}+Ri=\mathcal{V}_0$$

The current is $i=\frac{dq}{dt}$ then

$$\frac{dq}{dt}+\frac{1}{RC}q=\frac{\mathcal{V}_0}{R}$$

$q(t)=Q_A e^{\alpha t}+Q_0$, then $\frac{dq}{dt}=\alpha Q_A e^{\alpha t}$ Hence (1) takes the form

with $Q_A, \alpha \in \mathbb{R}$

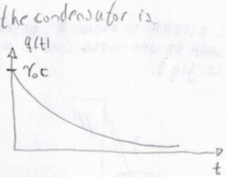
$$Q_A e^{\alpha t}+\frac{1}{RC}(Q_A e^{\alpha t}+Q_0)=\frac{\mathcal{V}_0}{R}$$

$$\left(\alpha+\frac{1}{RC}\right)Q_A e^{\alpha t}+\frac{Q_0}{RC}=\frac{\mathcal{V}_0}{R}$$ if we choose $\frac{Q_0}{RC}=\frac{\mathcal{V}_0}{R} \Rightarrow \alpha+\frac{1}{RC}=0$

Hence $\boxed{Q_0=\mathcal{V}_0 C}$ AND $\boxed{\alpha=-1/RC}$

a result, the charge in one of the plates of the condensator is

$$\boxed{q=\mathcal{V}_0 C e^{-t/RC}}$$ this charge is decreasing exponencially with time



potential is dissipated by de resistence (R) of wire. This is

$$\mathcal{V}_{dis}=R\frac{di}{dt}=R\left(-\frac{\mathcal{V}_0 C}{RC}e^{-t/RC}\right) \Rightarrow \boxed{\mathcal{V}_{dis}=-\mathcal{V}_0 e^{-t/RC}}$$

$$=dq\,\mathcal{V}_{dis}=\underbrace{\left[\mathcal{V}_0\cancel{C}e^{-t/RC}\left(-\frac{dt}{R\cancel{C}}\right)\right]}_{dq}\underbrace{\left(-\mathcal{V}_0 e^{-t/RC}\right)}_{\mathcal{V}_{dis}}=\frac{\mathcal{V}_0^2}{R}e^{-2t/RC}dt$$

$$W=\frac{\mathcal{V}_0^2}{\cancel{R}}\left(-\frac{\cancel{R}C}{2}\right)e^{-t/RC}\Big|_0^t \Rightarrow \boxed{W(t)=\frac{1}{2}C\mathcal{V}_0^2\left[1-e^{-t/RC}\right]}$$ ENERGY DISSIPATED IN THE WIRE